ANJUMAN COLLEGE OF ENGINEERING & TECHNOLOGY
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DEPARTMENT OF CIVIL ENGINEERING

GEOTECHNICAL ENGINEERING – II

B.E. FIFTH SEMESTER

UNIT – III

LATERAL EARTH PRESSURE:

Earth pressure at rest, active & passive pressure, General & local states of plastic equilibrium in soil. Rankines and Coulomb‟s theories for earth pressure. Effects of surcharge, submergence. Rebhann‟s criteria for active earth pressure. Graphical construction by Poncelet and Culman for simple cases of wall-soil system for active pressure condition.

INTRODUCTION

This is required in designs of various earth retaining structures like: -

i) Retaining walls
ii) Sheeting & bracings in cuts / excavations
iii) Bulkheads
iv) Bridge abutments, tunnels, cofferdams etc.

Lateral earth pressure depends on:-

i) Type of soil.
ii) Type of wall movement:-
a) Translatory
b) Rotational
iii) Soil-structure interaction.

A retaining wall or retaining structure is used for maintaining the ground surface at different elevations on either side of it. The material retained or supported by the structure is called backfill which may have its top surface horizontal or inclined. The position of the backfill lying above a horizontal plane at the elevation of the top of a wall is called the surcharge, and its inclination to the horizontal is called surcharge angle β.

Lateral earth pressure can be grouped into 3 categories, depending upon the movement of the retaining wall with respect to the soil retained. The soil retained is also known as the backfill.

1) At-rest pressure: -

The lateral earth pressure is called at-rest pressure when the soil mass is not subjected to any lateral yielding or movement. This case occurs when the retaining wall is firmly fixed at its top and is not allowed to rotate or move laterally. Fig.1(a) shows the basement retaining walls which are restrained against the movement by the basement slab provided at their tops. Another example of the at-rest pressure is that of a bridge abutment wall which is restrained at its top by the bridge slab. The at-rest condition is also known as the elastic equilibrium, as no part of soil mass has failed and attained the plastic equilibrium.

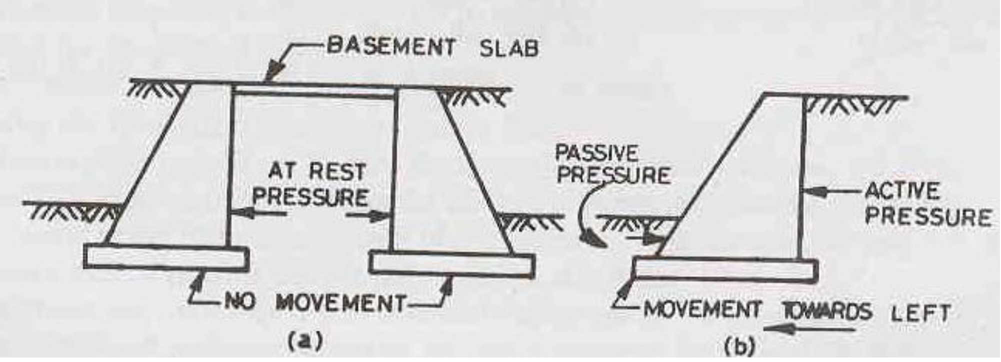


Fig.1

2) **Active earth pressure: -** A state of active pressure occurs when the soil mass yields in such a way that it tends to stretch horizontally. It is a state of plastic equilibrium as the entire soil mass is on the verge of failure. A retaining wall when moves away from the backfill, there is a stretching of the soil mass and the active state of earth pressure exists. In Fig.2, the active pressure develops on the rigid hand side when the wall moves towards left.

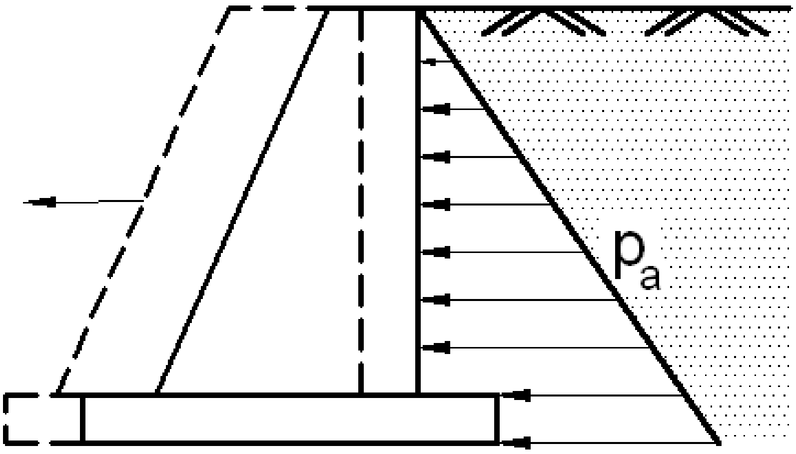
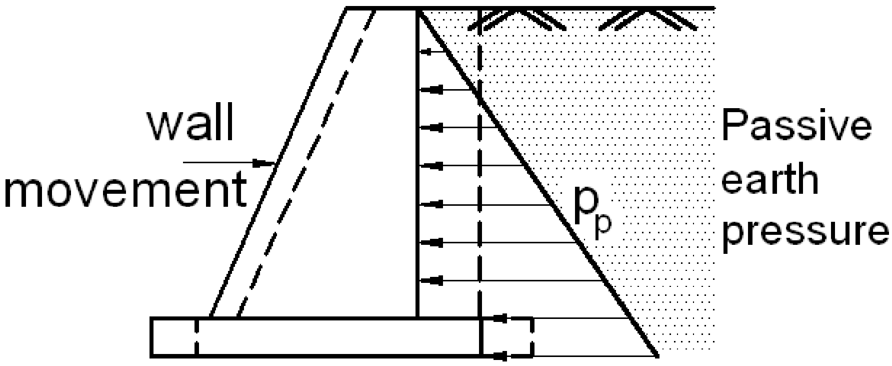


Fig.2

3) **Passive earth pressure: -** A state of active pressure occurs when the soil mass yields in such a way that it tends to stretch horizontally. It is another extreme of the limiting equilibrium condition. In Fig.3, the passive pressure develops on the left-side of the wall below the ground level, as the soil in this zone is compressed when the movement of the wall is towards left. Another example of the passive earth pressure is the pressure acting on an anchor block.



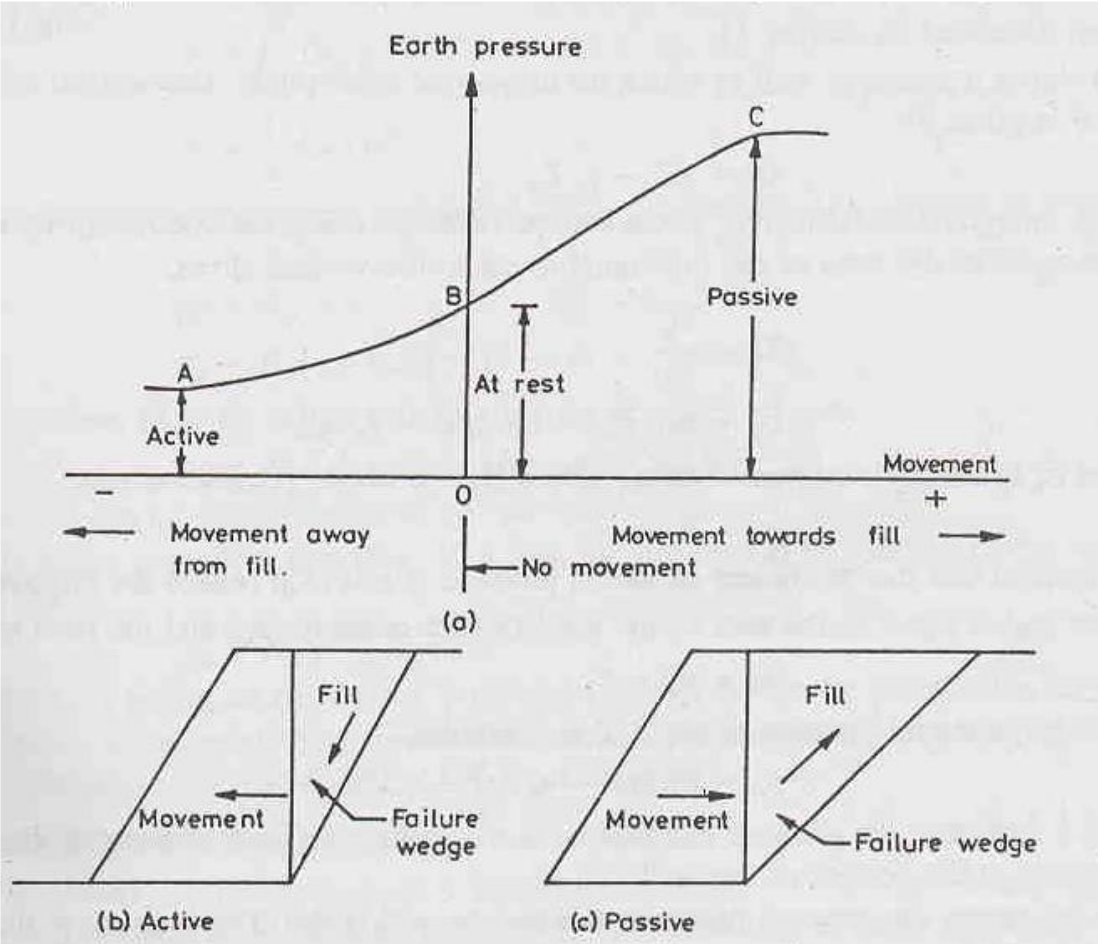


Fig.3

EARTH PRESSURE AT REST

The earth pressure at rest, exerted on the back of a rigid, unyielding retaining structure, can be calculated using theory of elasticity, assuming the soil to the semi-infinite, homogeneous, elastic and isotropic. Consider an element of soil at a depth 'z' being acted upon by vertical stress σ_v and horizontal stress σ_h. There will be no shear stress. The lateral strain ϵ_h in the horizontal direction is given by:

$$\epsilon_h = \frac{1}{E}\left[\sigma_h - \mu\left(\sigma_h - \sigma_v\right)\right]$$

The earth pressure at rest corresponding to the condition of zero lateral strain ($\epsilon_h = 0$). Hence

$$\sigma_h = \mu\left(\sigma_h + \sigma_v\right)$$

or

$$\frac{\sigma_h}{\sigma_v} = K_0' = \frac{\mu}{1-\mu}$$

where K_v is the coefficient of the earth pressure at rest.

Designating the lateral earth pressure (σ_h) at rest by p_0 and substituting $\sigma v = \gamma\, z$, we have,

$$p_0 = K_0 \gamma z$$

The pressure distribution diagram is thus triangular with zero intensity at z = 0 and an intensity of $K_0\gamma H$ at the base of the wall, where z = H. The total pressure P0 per unit length for the vertical height H is given by

$$P_0 = \int_0^H K_0 \gamma z . d_z = \frac{1}{2} K_0 \gamma H^2$$

The behavior of soil is not in accordance with the elastic theory and do not have a well-defined value of the Poisson's ratio.

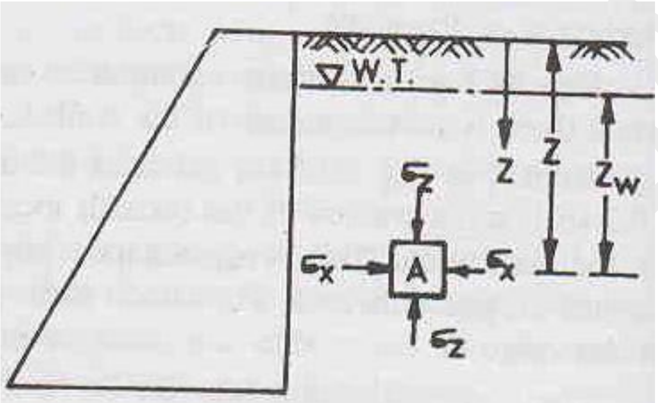
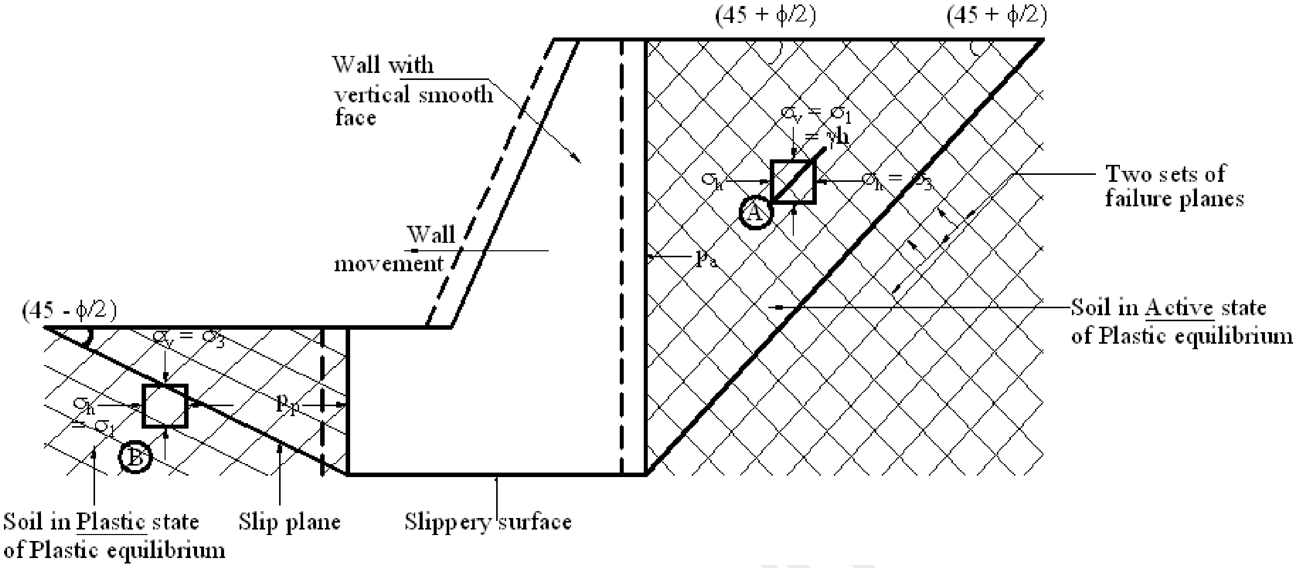


Fig.4

S.No.	Soil Type	K_0
1	Loose sand	0.4
2	Dense sand	0.6
3	Sand compacted in layers	0.8
4	Soft clay	0.6
5	Hard clay	0.5

PLASTIC EQUILIBRIUM CONDITIONS IN SOIL (ACTIVE & PASSIVE CASES): -



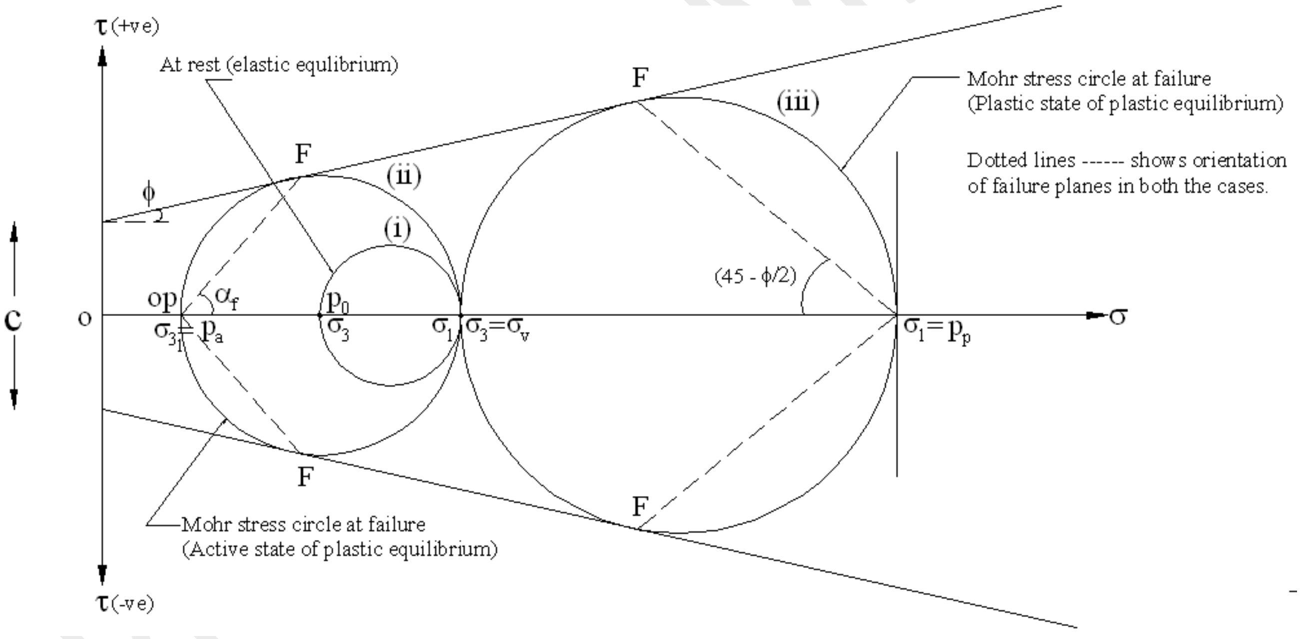


Fig.5 General State of Plastic Equilibrium condition (Active & Passive State).

Consider a gravity retaining wall (with vertical smooth faces) & c - ϕ soil fill.

1) When the wall is at rest (steady), soil is in equilibrium. Stress circle does not touch failure envelope.

$$\sigma_1 = \sigma_v \quad \& \quad \sigma_3 = \sigma_h = p_0$$

2) If wall is allowed to move laterally, soil in A - σ_3 reduces as wall moves away from backfill and at a perpendicular wall movement, σ_3 will attain such a value that the stress circle (i) will

touch the failure envelope, i.e., general state of plastic equilibrium will be developed at all points within the soil mass A. This is Active case of plastic equilibrium. In this

$$\sigma_1 = \sigma_v, \ \ \sigma_3 = \sigma_h = p_a$$

Two sets of failure planes developed in soil mass making (45+ϕ/2) angle with horizontal.

With wall movement σ_h in soil B will increase and with adequate movement of wall σ_h may become larger than σ_v. Thereafter σ_v will be σ_3 and $\sigma_h = \sigma_1$. At a particular value of σ_h (or σ_1), the stress circle again becomes tangential to strength envelope, failure is incipient and soil is said to be in Passive state of plastic equilibrium. In this, $\sigma_3 = \sigma_v$, $\sigma_1 = \sigma_h = p_p$. Two sets of failure planes developed in soil mass making (45 - ϕ/2) angle with horizontal.

3) Thus in Active case:-

$$p_a = \sigma_h = \sigma_3 = \sigma_1 . tan^2 \left(45 - \frac{\emptyset}{2}\right) - 2\, c. \tan\left(45 - \frac{\emptyset}{2}\right)$$

4) And for Passive case:-

$$p_p = \sigma_h = \sigma_1 = \sigma_3 . tan^2 \left(45 + \frac{\emptyset}{2}\right) + 2\, c. \tan\left(45 + \frac{\emptyset}{2}\right)$$

The above mentioned approach (i.e. stress condition at any point is the soil mass at failure) is adopted in Rankine's earth pressure theory.

ACTIVE EARTH PRESSURE: RANKINES THEORY

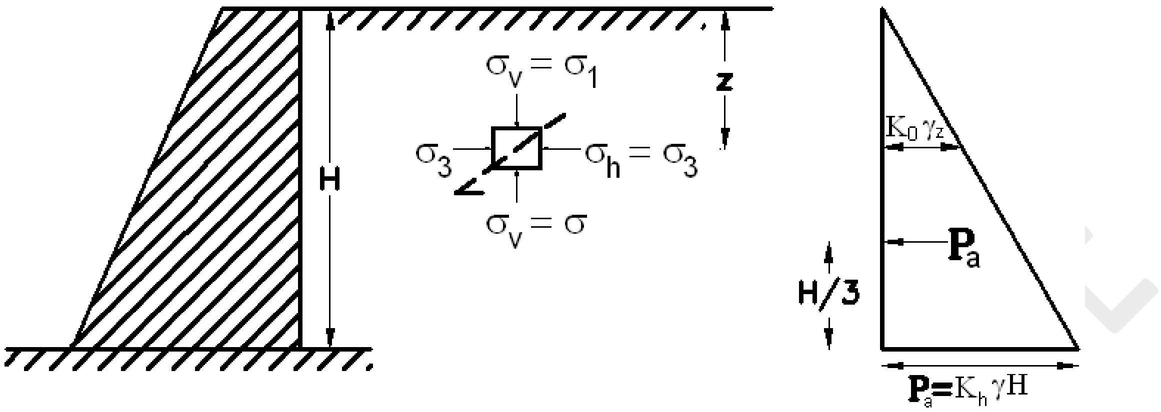
Rankine's theory of lateral earth pressure is applied to uniform cohesionless soil only. Following are the assumptions of the Rankine theory: -

1) The soil mass is semi-infinite, homogeneous, dry and cohesionless.
2) The ground surface is a plane which may be horizontal or inclined.
3) The back of the wall is vertical and smooth.
4) The wall yields about the base and thus satisfies the deformation condition for plastic deformation.

The following cases of cohesionless backfill will now be considered: -

1) Dry or moist backfill with no surcharge.
2) Submerged backfill.
3) Backfill with uniform surcharge.
4) Inclined back and surcharge.

1) **Dry or moist backfill with no surcharge: -**



(a) (b)

Consider an element at a depth 'z' below the ground surface. When the wall is at the point of moving outwards (i.e., away from the fill), the active state of plastic equilibrium is established. The horizontal pressure σ_h is then the minimum principal stress σ_3 and the vertical pressure σ_v is the major principal stressσ_1. From the stress relationship, we have,

$$\sigma_1 = \sigma_3 \tan^2\left(45^0 + \varnothing/_2\right)$$

$$\frac{\sigma_3}{\sigma_1} = \frac{\sigma_h}{\sigma_v} = \frac{1}{\tan^2\left(45^0 + \varnothing/_2\right)}$$

Now, σ_h = lateral earth pressure = p_a

σ_v = vertical pressure on the element = $\gamma.z$

So, $p_a = \gamma.z \cot^2\left(45^0 + \varnothing/_2\right) = K_a \gamma z$

K_a = co-efficient of active earth pressure. $= \cot^2\left(45^0 + \varnothing/_2\right) = \frac{1 - \sin\varnothing}{1 + \sin\varnothing}$

$P_a = K_a \gamma H$

Acting at H/3 above the base of the wall.

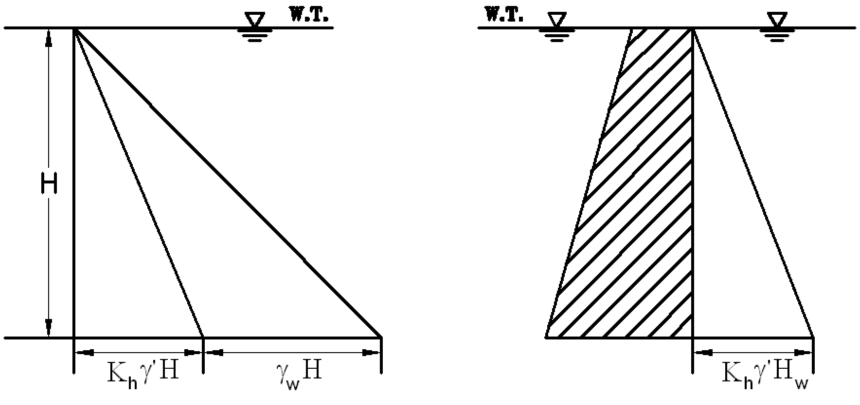
2) **Submerged backfill: -**

In this case, the sand fill behind the retaining wall is saturated with water. The lateral pressure is made up of two components:

i) Lateral pressure due to submerged weight γ' of the soil, and

ii) Lateral pressure due to water. Thus, at any depth 'z' below the surface,

$P_a = K_a\gamma' z + \gamma_w z$



(a) (b)

The pressure at the base of the retaining wall (z = H) is given by

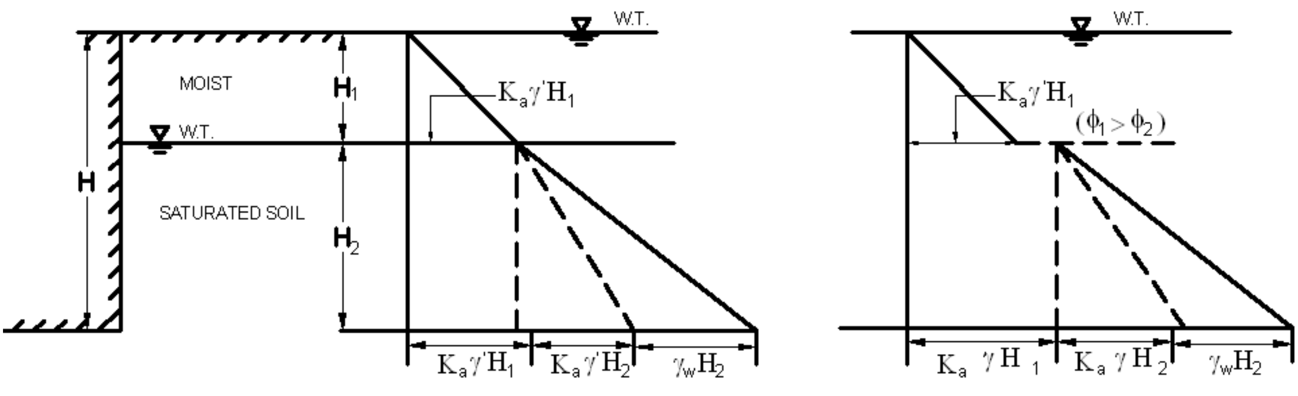
$$P_a = K_a\gamma' H + \gamma_w H$$

If the free water stands to both sides of the wall the water pressure need not be considered, and the net lateral pressure is given by,

$$P_a = K_a\gamma' H$$

If the backfill is partly submerged, the lateral pressure intensity at the base of the wall is given by,

$$P_a = K_a\gamma H_1 + K_a\gamma' H_2 + \gamma_w H_2$$



(a) (b)

The lateral intensity at the base of wall is given by;

$$P_a = K_{a2}\gamma H_1 + \gamma' K_a\gamma' H_2 + \gamma_w H_2$$

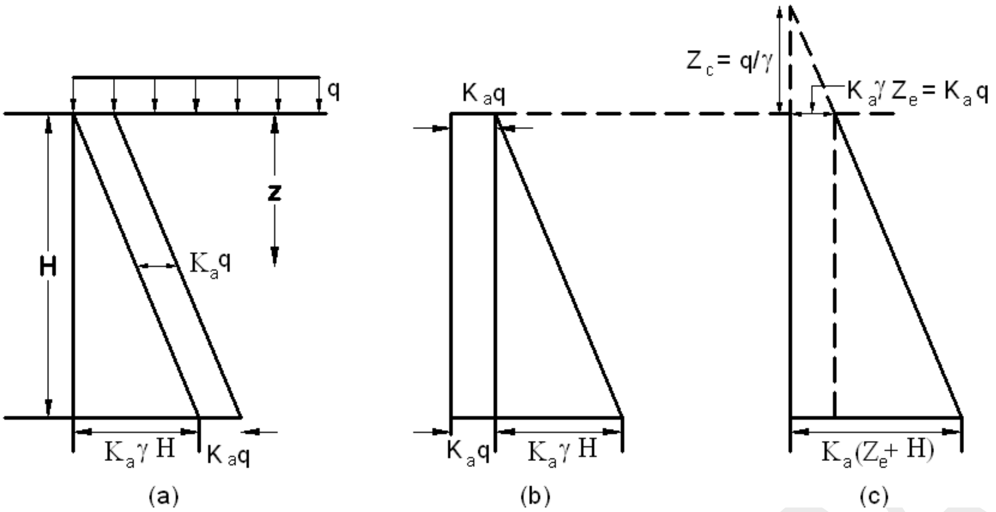
3) Backfill with uniform surcharge: -

If the backfill is horizontal and carries a surcharge of uniform intensity ‘q’ per unit area, the vertical pressure increment, at any depth ‘z’ will increase by ‘q’. The increase in the lateral pressure due to this will be $K_a q$. Hence the lateral pressure at any depth ‘z’ is given by,

$$P_a = K_a\gamma z + K_a q$$

At the base of the wall, the pressure intensity is

$$P_a = K_a\gamma z + K_a q$$



The height of fill z_e, equivalent to the uniform surcharge intensity is given by the relation,

$$K_a\gamma z_e = K_aq$$

4) Backfill with sloping surface: -

The sloping surface behind the wall is inclined at an angle β with the horizontal; β is called the surcharge angle. The total active earth pressure P_a for the wall of height H is given by;

$$P_a = \frac{1}{2} K_a \gamma H^2$$

The resultant acts H/3 above the base in direction parallel to the surface, as shown in fig.4

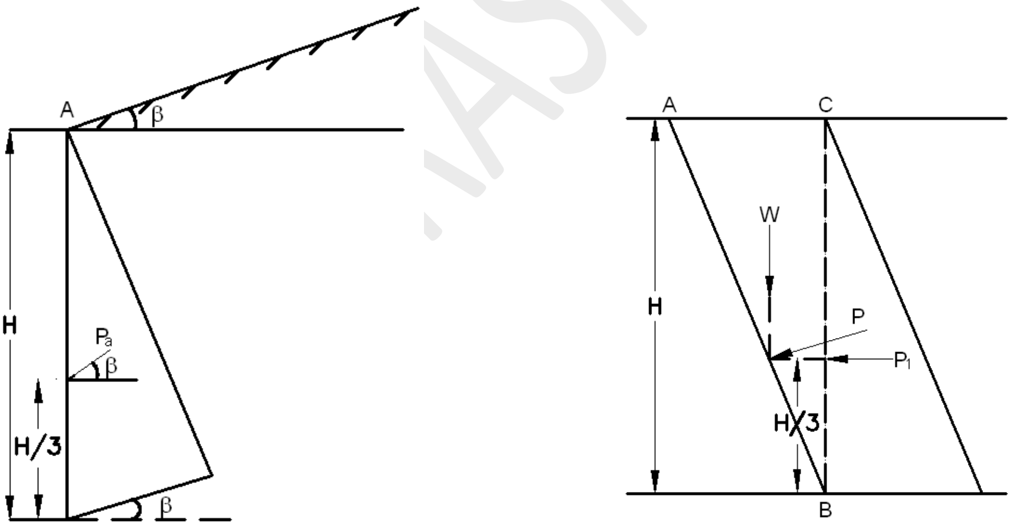


Fig.4

5) Inclined Back and surcharge: -

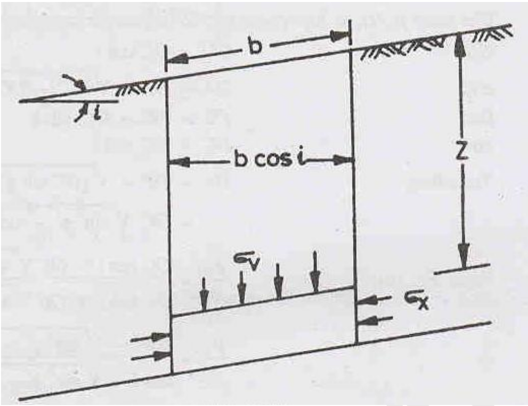
Fig.5 shows a retaining wall with an inclined back supporting a backfill with horizontal ground surface. The total active earth pressure P1 is first calculated on a vertical plane BC passing through the heel BC. The total pressure P is the resultant of the horizontal pressure P1 and the weight W of the wedge ABC:

where $P_1 = \frac{1}{2} K_a \gamma H^2$

$$P = \sqrt{P_1^2 + W^2}$$

RANKINE'S EARTH PRESSURE WHEN THE SURFACE IS INCLINED

Two stresses are called conjugate stresses when the direction of one stress is parallel to the plane on which the other stress acts. Rankine assumed that the vertical stress on an element of the soil within the inclined backfill and the lateral stress on the vertical plane of the element are conjugate stresses. In other words, he assumed that the lateral stress is parallel to the inclined backfill.



Let us consider an element of soil at depth 'Z' below the soil surface inclined at angle 'i' to horizontal. The angle 'i' is known as the angle of surcharge. The intensity of vertical stress (σ_v) on the element is given by

$$\sigma_v = \frac{\gamma\,(Z\,b\cos i)}{b}$$

$$or\ \sigma_v = \gamma\,Z\cos i$$

The other conjugate stress is the lateral stress (σ_x).

It may be mentioned that the vertical stress σ_v is not the principal stress, as a shear stress also exists on the inclined plane at the top of the element. Likewise, the lateral stress σ_x is also not a principal stress. A relationship between the lateral pressure and the vertical stress can be obtained for the active and passive cases as given below.

Active earth pressure: Fig.6 shows the Mohr circle corresponding to the active limiting conditions.

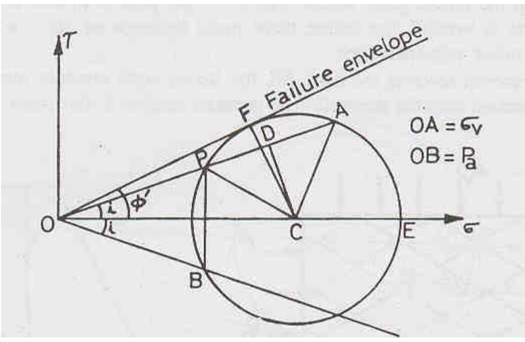


Fig.6

The vertical stress σ_v is represented by the line OA making an angle 'i' with the horizontal. At any depth, the value of σ_v is constant and equal to that above equation.

If the lateral expansion of the soil is sufficient to induce the state of active plastic equilibrium, the Mohr circle must pass through 'A' and it should be tangential to the failure envelope. The origin of planes'P' is obtained as the point of intersection of OA with the Mohr circle. The origin of planes is located by drawing from the point representing a stress (vertical stress, in this case) a line parallel to the plane on which it acts (plane inclined at 'I' in this case). A verticlaline through 'P' cuts the circle at 'B' below the σ - axis. The conjugate stress, which is the active pressure (p_a), is represented by OB. Numerically; the conjugate stress is also equal to OP.

From the fig.6, $$\frac{P_a}{\sigma_v} = \frac{OB}{OA} = \frac{OP}{OA} = \frac{OD - DP}{OD + DA}$$

The ratio P_a/σ_v is known as the conjugate stress ratio.

Now $$OD = OC\cos i$$

$$DA = DP = \sqrt{PC^2 - DC^2}$$

and $$PC = FC = OC\sin \text{Ø}'$$

But $$DC = OC\sin i$$

and $$DA = DP = \sqrt{(OC\sin \text{Ø}')^2 - (OC\sin i)^2}$$

Therefore,

$$= OC\sqrt{sin^2\text{Ø}' - sin^2 i}$$

$$\frac{P_a}{\sigma_v} = \frac{OC\cos i - OC\sqrt{sin^2\text{Ø}' - sin^2 i}}{OC\cos i + OC\sqrt{sin^2\text{Ø}' - sin^2 i}}$$

or $$\frac{P_a}{\sigma_v} = \frac{\cos i - \sqrt{sin^2\text{Ø}' - sin^2 i}}{\cos i + \sqrt{sin^2\text{Ø}' - sin^2 i}}$$

$$= \frac{\cos i - \sqrt{(1 - cos^2\text{Ø}') - (1 - cos^2 i)}}{\cos i + \sqrt{(1 - cos^2\text{Ø}') - (1 - cos^2 i)}}$$

$$\frac{P_a}{\sigma_v} = \frac{\cos i - \sqrt{(cos^2 i - cos^2\text{Ø}'}}{\cos i + \sqrt{(cos^2 i - cos^2\text{Ø}'}}$$

or $$P_a = \frac{\cos i - \sqrt{(cos^2 i - cos^2\text{Ø}'}}{\cos i + \sqrt{(cos^2 i - cos^2\text{Ø}'}}\ (\gamma Z\cos i)$$

or $$P_a = K_a\gamma Z$$

where K_a is the coefficient of active pressure, given by

$$K_a = \cos i \times \frac{\cos i - \sqrt{cos^2 i - cos^2\text{Ø}'}}{\cos i + \sqrt{cos^2 i - cos^2\text{Ø}'}}$$

It must be noted that p_a is parallel to the inclined surface.

For the special case, $$K_a = \frac{1 - \sin\text{Ø}'}{1 + \sin\text{Ø}'}$$ when i = 0

Passive earth pressure: This is similar to the one of the active case with one basic difference that the vertical stress is the smaller of the two conjugate stresses. In figure, OA represents the vertical stress (σ_v). The point 'P' shows the origin of planes, and OB represents the passive pressure. From the figure,

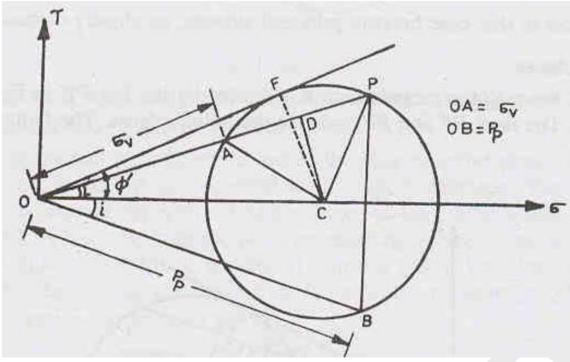


Fig.7

$$\frac{P_p}{\sigma_V} = \frac{OB}{OA} = \frac{OP}{OA} = \frac{OD + DP}{OD - DA}$$

Now $\quad OD = OC \cos i$

and $\quad Dp = DA = \sqrt{\sqrt{(AC)^2} - (DC)^2} = \sqrt{(FC)^2 - (DC)^2}$

$$= OC\sqrt{(OC \sin \emptyset')^2 - (OC \sin i)^2}$$

$$= \sqrt{(OC \sin \emptyset')^2 - (OC \sin i)^2}$$

$$= OC\sqrt{sin^2\emptyset' - sin^2 i}$$

$$\frac{P_p}{\sigma_v} = \frac{OC \cos i + OC\sqrt{sin^2 \emptyset' - sin^2 i}}{OC \cos i - OC\sqrt{sin^2 \emptyset' - sin^2 i}}$$

$$= \frac{\cos i + \sqrt{sin^2\emptyset' - sin^2 i}}{\cos i - \sqrt{sin^2\emptyset' - sin^2 i}}$$

$$= \frac{\cos i + \sqrt{cos^2 i - cos^2\emptyset'}}{\cos i - \sqrt{cos^2 i - cos^2\emptyset'}}$$

$$P_p = (\gamma\, Z \cos i)\, \frac{\cos i + \sqrt{cos^2 i - cos^2\emptyset'}}{\cos i - \sqrt{cos^2 i - cos^2\emptyset'}}$$

$$P_p = K_p\, \gamma\, Z$$

where K_p is given by $$K_p = \cos i \frac{\cos i + \sqrt{\cos^2 i - \cos^2 \varnothing'}}{\cos i - \sqrt{\cos^2 i - \cos^2 \varnothing'}}$$

For the special case, when $i = 0$

$$K_p = \frac{1 + \sin \varnothing'}{1 - \sin \varnothing'}$$

REHBANN'S CONSTRUCTION FOR ACTIVE PRESSURE

Rehbann (1871) gave a graphical method for the determination of the total active pressure according to Coulomb's theory. It is based on Poncelet's solution (1840), and is therefore, also known as Poncelet's method.

Construction: -

1) Draw a ground line and ϕ - line at an angle β and ϕ respectively with the horizontal to meet in point D.
2) Draw a semi – circle on BD as diameter.
3) Draw ψ – line (also called as pressure line) from ϕ – line at an angle ψ.
4) Through A, draw lie AG parallel to ψ – line.
5) Draw GJ perpendicular to BD, meeting the semicircle at J.
6) With B as centre, BJ as radius, draw an arc to cut BD in E.
7) Through E, draw EC parallel to the ψ – line. BC then represents slip plane.
8) With E as the centre and EC as radius draw an arc of circle CK upto BD.
9) Join Ck by straight line, from Δ KCE.
   $P_a = \gamma\ (area\ of\ \Delta KCE)$
10) Drop perpendicular on KE from C. F is foot of perpendicular.
   $P_a = \frac{1}{2} . \gamma . (CF \times KE)$

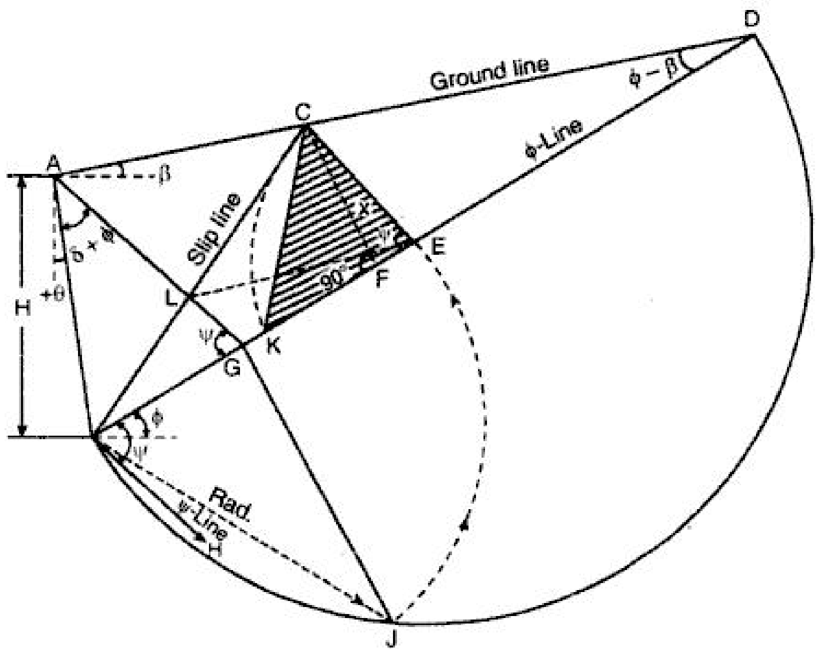


Fig.8 Rebhann's graphical method for

CULMANN'S CONSTUCTION FOR ACTIVE PRESSURE

Culmann (1866) developed a method which is more general than Rehbann's method. It can be used to determine Coulomb's earth pressure for ground surface of any configuration, for various types of surcharge loads and for layered back fills. Culmann's construction is, in fact the method of construction of the force triangle in a rotated oriented. The procedure consists of following steps:

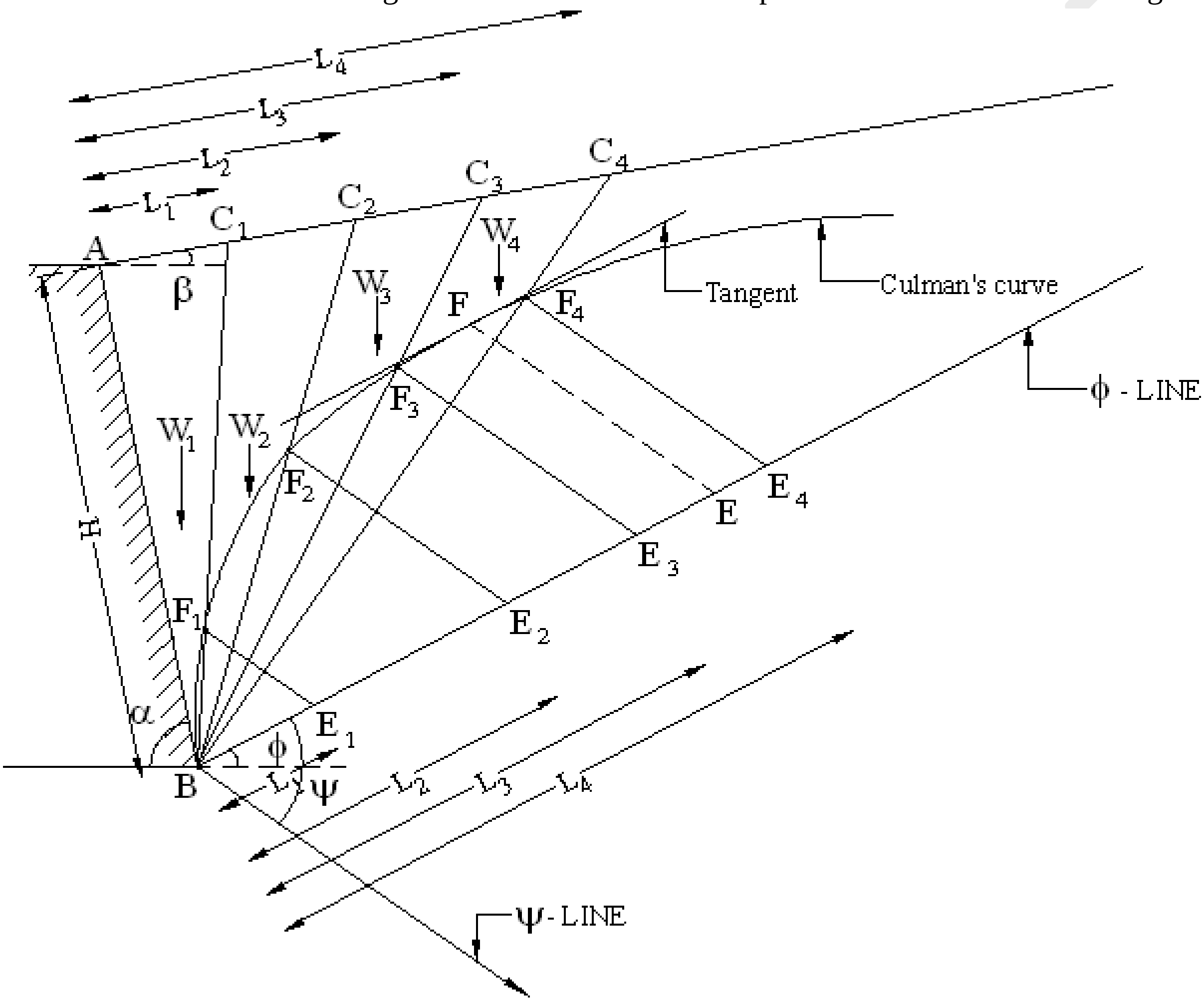


Fig.9 Culmann's graphical method for Active pressure.

1) Draw GL, ϕ – line and ψ – line.
2) Take a trial slip plane BC_1 and calculate the weight of the wedge ABC_1. Show it as BE_1 (to certain scale) on ϕ – line.
3) Through E1 draw E_1F_1 parallel to the ψ – line to cut the slip plane at F_1.
4) Similarly, second trial plane BC_2. Repeat as above.
5) Take number of slip planes BC_3, BC_4 etc., plot the weight of the corresponding wedges on the ψ – line and obtain points F_1, F_2, F_3, F_4 etc.

6) Draw a smooth curve through point B, F_1, F_2, F_3, F_4 etc. This curve is known as Culmann's line.
7) Draw a tangent to culmann's line , parallel to the ϕ – line.

8) Find F. Join BF and extend to C. BC is a critical slip plane.
9) Pa is represented by EF (to Scale).
10) For point of application, draw line parallel BC through CG of sliding wedge and get its intersection on wall AB.
11) Pa = ½ γ. H. (EF)

COULOMB'S WEDGE THEORY

Coulomb (1776) developed a method for the determination of the earth pressure in which he considered the equilibrium of the sliding wedge which is formed when the movement of the retaining wall takes place.

The following assumptions are made: -

1) The backfill is dry, cohesionless, homogeneous, isotropic and ideally plastic material.
2) The slip surface is a plane surface which passes through the heel of the wall.
3) The wall surface is rough. The resultant earth pressure on the wall is inclined at an angle δ is the angle of the friction between the wall and the backfill.
4) The sliding wedge itself acts as a rigid body.

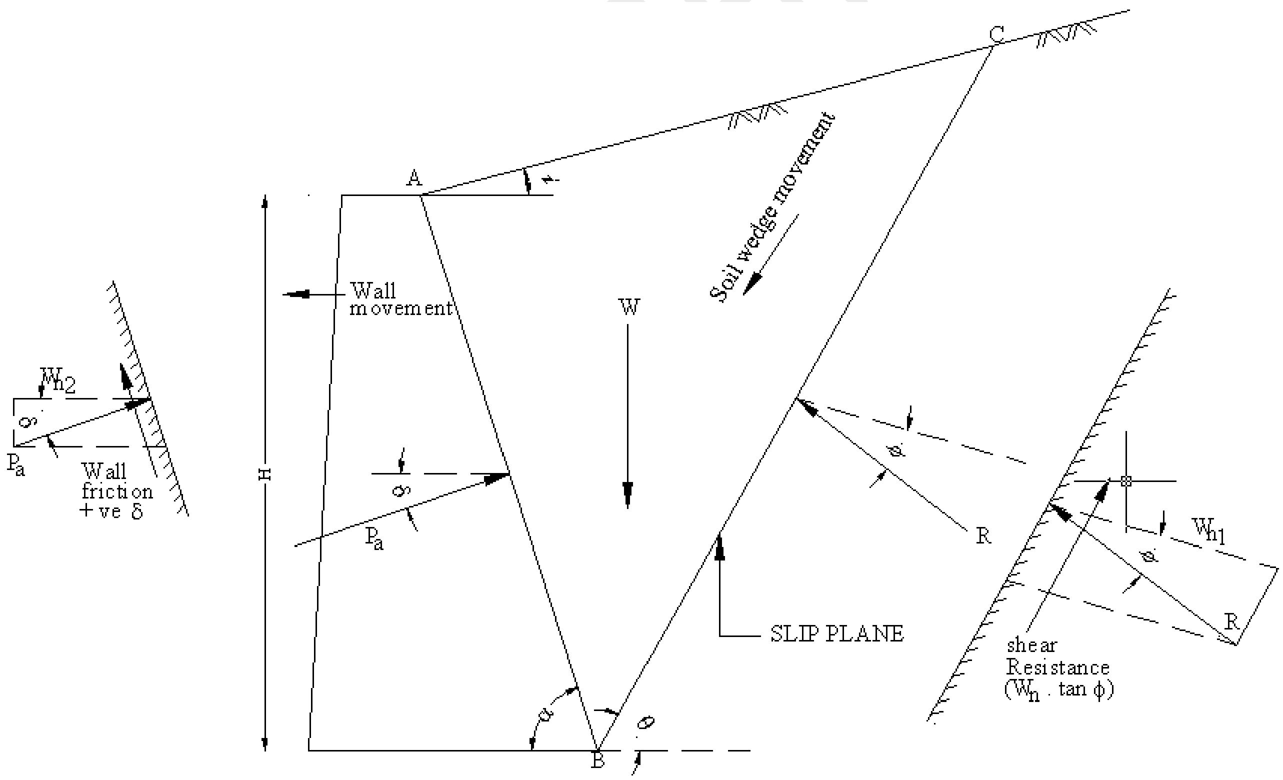
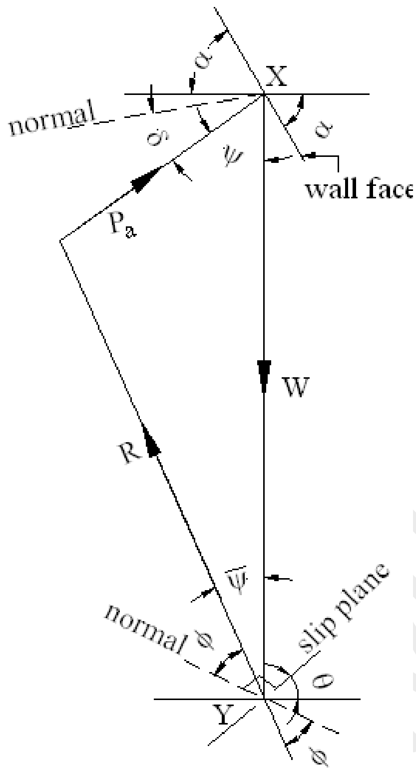


Fig.10



Forces acting on soil wedge ABC are: -

i) Weight of soil wedge W.
ii) Resultant reaction of soil on wedge, R.
iii) Resultant thrust of wall on wedge, Pa (= Earth force on wall).

These forces keep the wedge in static equilibrium.
Hence, the force Polygon (in this case force triangle) is as shown below: -

- ψ = angle made by P_a with vertical
  $90 = \delta + \psi + (90 - \alpha)$
  So, $\psi = \alpha - \delta$
  $\bar{\psi} = \theta - \Phi$
- $\bar{\psi} =$ *angle made by R with vertical*
  $90 = \emptyset + \bar{\psi} + (90 - \theta)$

Hence, for a force triangle XYZ, by sine rule,

$$\frac{P_a}{\sin(\theta - \emptyset)} = \frac{W}{\sin[180 - (\alpha - \delta) - (\theta - \emptyset)}$$

From the geometry of soil wedge,

$$W = \frac{1}{2} \cdot \frac{\gamma . H^2}{sin^2 \alpha} \cdot \sin(\alpha + \theta) . \frac{\sin(\alpha + i)}{\sin(\theta - i)} \qquad \text{(i)}$$

$$\therefore \ P_a = \frac{\gamma . H^2}{2 . sin^2 \alpha} \cdot \frac{\sin(\theta - \emptyset)}{\sin(180 - \alpha + \delta - \theta + \emptyset)} \cdot [\sin(\alpha + \theta) . \frac{\sin(\alpha + i)}{\sin(\theta - i)} \qquad \text{(ii)}$$

(it is seen that Pa is function of i, α, ϕ, δ & θ),

i, α, ϕ, δ = These are constant for a given wall – soil system.

θ = varies.

$\therefore \ \theta_c = critical\ angle\ of\ failure\ surface.$

From eqn (ii) , values of P_a depends on θ,

$P_a = 0$, when $\theta = \phi$. As θ increases beyond ϕ, Pa also increases and reaching a max. value, it again returns & $P_a = 0$, when $\theta = 180 - \alpha$).