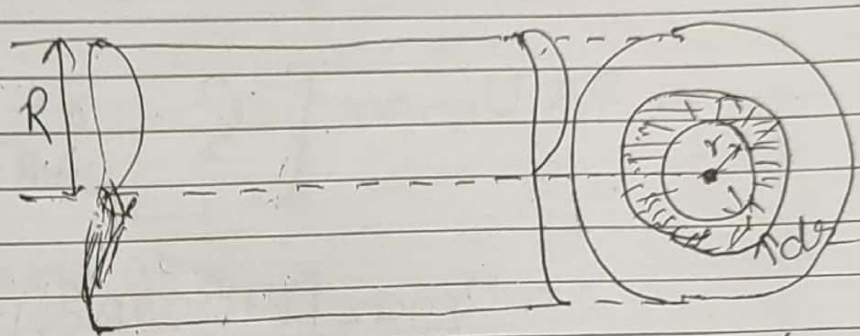
## Discharge ob the bluid blowing through ~~a pip~~ a circular pipe.




let us suppose that a bluid is blowing through a circular pipe ob radius 'R'. consider a bluid element ob thickness $dr$ at a radius $r$ as shown in the bigure.

Now, the discharge ob the elementary bluid is given by

$dQ$ = crosssection area ob element × velocity ob bluid passing through the

$$dQ = 2\pi r\,dr \times u_{max}\left(1-\frac{r^2}{R^2}\right)$$

$$Q = \int 2\pi r\,dr \times u_{max}\left(1-\frac{r^2}{R^2}\right)$$

$$= \int_0^R 2\pi U_{max}\, r\left(1-\frac{r^2}{R^2}\right) dr$$

$$= 2\pi U_{max}\int_0^R r\left(1-\frac{r^2}{R^2}\right) dr$$

$$= 2\pi U_{max} \int_0^R \left(r - \frac{r^3}{R^2}\right) dr$$

$$= 2\pi U_{max} \left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R$$

$$= 2\pi U_{max} \left[\left(\frac{R^2}{2} - \frac{R^4}{4R^2}\right) - (0-0)\right]$$

$$= 2\pi U_{max} \left[\frac{R^2}{2} - \frac{R^2}{4}\right]$$

$$= 2\pi U_{max} \left[\frac{2R^2 - R^2}{4}\right)$$

$$= \cancel{2}\pi U_{max} \times \frac{R^2}{\cancel{4}_2}$$

$$Q = \frac{\pi U_{max} \times R^2}{2} \quad \text{—} \; (v)$$

$$= \frac{\pi \times \frac{-1}{4\mu}\left(\frac{\partial P}{\partial x}\right) R^2 \times R^2}{2}$$

$$\boxed{Q = \frac{\pi}{8\mu}\left(\frac{-\partial P}{\partial x}\right) R^4}$$

# Average velocity ($\bar{v}$) : or mean velocity

$$\bar{v} = \frac{Q}{A}$$

$$= \frac{\cancel{\pi} \times u_{max} \times \cancel{R^2}}{2 \times \cancel{\pi} \times \cancel{R^2}}$$

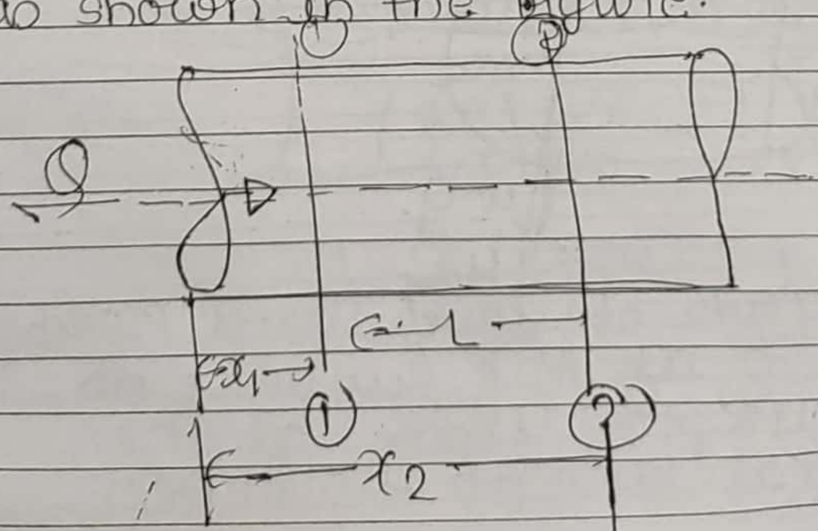
$$\boxed{\bar{v} = \frac{u_{max}}{2}} \; **$$

It is defined as the ratio of total discharge to the ~~tot~~ total area of flow and is given by.

$$\bar{v} = \frac{u_{max}}{2}$$

# pressure drop in a given length (L) of the pipe.

Let us suppose that a fluid is flowing through a circular pipe of radius 'R' considered two sections 1 – 1 and 2 - 2 as shown in the figure.



Let $P_1$ = Pressure intensity at 1-1
$P_2$ = pressure intensity at 2-2
$L$ = length of the pipe between 1-1 & 2-2

Now,

$$V = \frac{u_{max}}{2}$$

$$V = \frac{1}{2}\left[\frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right) \cdot R^2\right]$$

$$V = \frac{1}{8\mu}\left(-\frac{\partial P}{\partial x}\right) R^2$$

$$\Rightarrow V = \frac{1}{8\mu}\left(-\frac{\partial P}{\partial x}\right) \cdot \frac{D^2}{4}$$

$$V = \frac{1}{32\mu}\left[-\frac{\partial P}{\partial x}\right] D^2$$

$$\Rightarrow \partial P = -\frac{32\mu V}{D^2} \partial x$$

$$\Rightarrow \int_{P_1}^{P_2} \partial P = -\frac{32\mu V}{D^2} \int_{x_1}^{x_2} \partial x$$

$$\Rightarrow P_2 - P_1 = -\frac{32\mu V}{D^2}(x_2 - x_1)$$

$$\Rightarrow P_2 - P_1 = -\frac{32\mu V}{D^2} L$$

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$$\boxed{P_2 - P_1 = \frac{-32\mu V L}{D^2}}$$

$$\boxed{P_1 - P_2 = \frac{32\mu V L}{D^2}}$$

This is the required expression of pressure drop in a pipe of length L.

$$\frac{P_1 - P_2}{\omega} = h_L$$

$$P_1 - P_2 = \omega \cdot h_L$$

$$P_1 - P_2 = \rho g h_L$$

$$\frac{32\mu V L}{D^2} = \rho g h_L$$

$$\frac{32\mu V \cancel{L}}{D^{\cancel{2}}} = \rho \cancel{g} \cdot \frac{f \cancel{L} v^{\cancel{2}}}{2 \cancel{g} \cancel{D}}$$

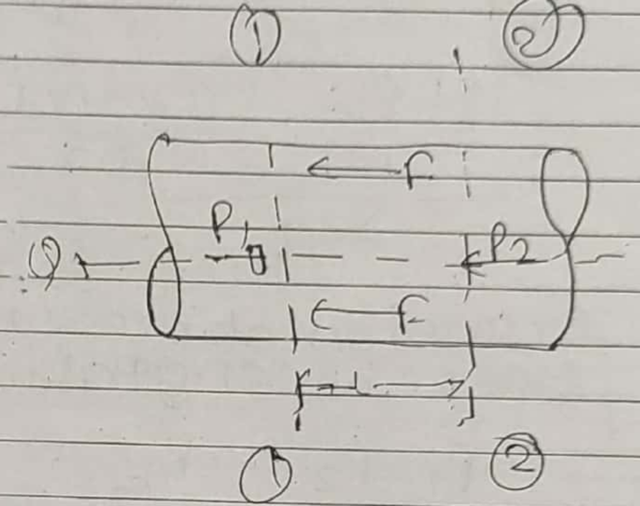
$$\frac{32\mu}{D} = \frac{\rho f V}{2}$$

$$\Rightarrow f = \frac{64\mu}{\rho V D}$$

$$f = \frac{64}{\left(\frac{\rho V D}{\mu}\right)}$$

$$\boxed{f = \frac{64}{Re}}$$

# Derivation for head loss due to friction.




consider a uniform horizontal circular pipe having a steady flow.

let

$P_1$ = pressure intensity at section 1-1

$P_2$ = pressure intensity at section 2-2

$V_1$ = velocity of flow 1-1

$V_2$ = velocity of flow 2-2

L = length of the pipe between 1-1 and 2-2

D = Diameter of the pipe

$f'$ = frictional resistance per unit ~~weight~~ wetted area per unit $(velo)^2$

$h_L$ = head loss due to friction

Now,

Total frictional resistance

$F = f' \times \text{wetted Area} \times (\text{velocity})^2$

$= f' \times \pi D L \times v^2$.

Now equating the forces we have

$P_1 A - P_2 A - F = 0$

$(P_1 - P_2) A = F$

$(P_1 - P_2) A = F$

$P_1 - P_2 = F/A$

$$= \frac{f' \times \cancel{\pi} \cancel{D} L v^2}{\frac{\cancel{\pi}}{4} D^{\cancel{2}}}$$

$$= \frac{4 f' L v^2}{D} \quad \text{——— (1)}$$

$$\frac{P_1 - P_2}{w} = h_L$$

$P_1 - P_2 = w h_L$.

$P_1 - P_2 = \rho g h_L$ ——— (2)

from (1) and (ii), we have.

$$\rho g h_L = \frac{4 f' L v^2}{D}$$

$$\boxed{h_L = \frac{4 f' L v^2}{\rho g D}}$$ This is the expression

$\frac{1}{\rho} = \frac{1}{2}$ then.

$h_L = 4\frac{f}{2}$

let $4f' = f$ then

$$\boxed{h_L = \frac{fLv^2}{\rho g D}}$$