classmate

viscous flow incompressible

Reynolds No-(Re) :

It is defined as the ratio of inertia force to the viscous force ~~to the~~ and is given by reynolds no.

$$Re = \frac{\text{Inertia force}}{\text{viscous force}}$$

$$= \frac{\rho V L}{\mu}$$

where L = characteristic length.

→ In case of circular pipe the characteristic length is equal to the diameter of the pipe.

$$Re = \frac{\rho V D}{\mu}$$

$$Re = \frac{VD}{\frac{\mu}{\rho}}$$

$$\boxed{Re = \frac{VD}{\nu}}$$ where V = velocity

μ = is kinematic viscosity.

characteristic length of any cross section is given by $= \frac{4A}{P}$

$$= \frac{4 \times \text{Area}}{\text{Pressure}}$$

→ If $Re \le 2000$ the flow will be laminar.

→ If $Re > 4000$ the flow will be turbulent

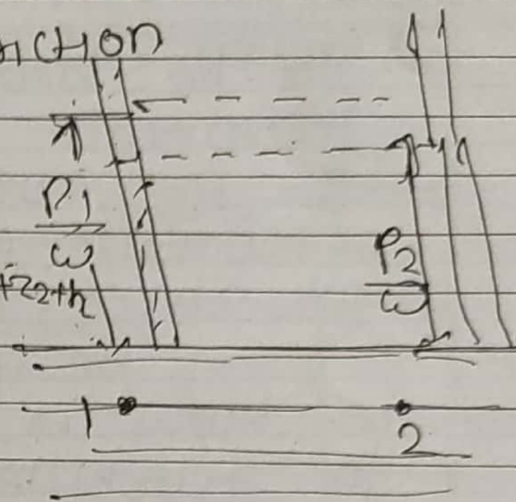
→ If $2000 < Re \le 4000$, the flow will be in transition zone

i.e. flow is converted from laminar to turbulent

→ Since it is the ratio of two same quantity so it is dimensionless and unitless number.

Head loss due to friction

Applying Bernoulli's (1) & (2) we have



$$\frac{P_1}{\omega} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\omega} + \frac{v_2^2}{2g} + z_2 + h_L$$

$$\because \; z_1 = z_2$$

$$v_1 = v_2$$

~~Applying Bernoulli (1) and~~

$$\frac{P_1}{\omega} = \frac{P_2}{\omega} + h_L$$

$$\frac{P_1}{\omega} - \frac{P_2}{\omega} = h_L$$

$$= \frac{P_1 - P_2}{\omega} = h_L$$

$$\boxed{\frac{P_1 - P_2}{\omega} = h_L}$$

In the dirn of flow the pressure decreases in order to overcome losses (h_L). This h_L is known as head loss due to friction.

Darcy-weisbach equation:- (Derive the expression of friction)

$$h_L = \frac{4f'lv^2}{2gd}$$

$$= \frac{flv^2}{2gd}$$

where

f' = friction coefficient

f = friction factor

l = length of the pipe

v = velocity of fluid in the pipe

g = Gravitational acceleration

d = diameter of the pipe.

This equation is used for finding out the head loss due to friction and is applicable for laminar flow and turbulent but the flow must be steady.

* Laminar flow through circular pipes: (Hagen-poiseulle flow)

let us suppose that a fluid is flowing through a circular pipe of radius 'R' with a flow rate 'Q'. consider an element of length ~~dx~~ dx and radius r. If the pipe is a horizontal the forces acting on this element along flow dir^n will be pressure force and shear force (due to viscosity) as shown in the figure.

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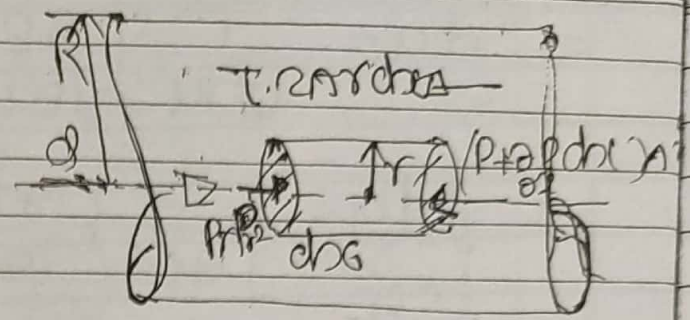


The forces acting on the fluid element are

(i) pressure force $= PA = P\pi r^2$ (in flow dirn)

(ii) pressure force $= P \times A = \left(P + \frac{\partial P}{\partial x}dx\right)\pi r^2$ (opposite to flow dirn)

surface

(iii) Shear force = shear stress x Area

$= \tau \times 2\pi r dx$ (opposite to flow dirn)

for equilibrium,

$$P.\pi r^2 - \left(P + \frac{\partial P}{\partial x}dx\right)\pi r^2 - \tau \times 2\pi r dx = 0$$

$$\Rightarrow \pi r\left[Pr - \left(P + \frac{\partial P}{\partial x}dx\right)r - \tau 2 dx\right] = 0$$

$$\Rightarrow Pr - \left(P + \frac{\partial P}{\partial x}.dx\right)r - 2\tau dx = 0$$

$$= \cancel{Pr} - \cancel{Pr} - \frac{\partial P}{\partial x}dx.r - 2\tau dx = 0$$

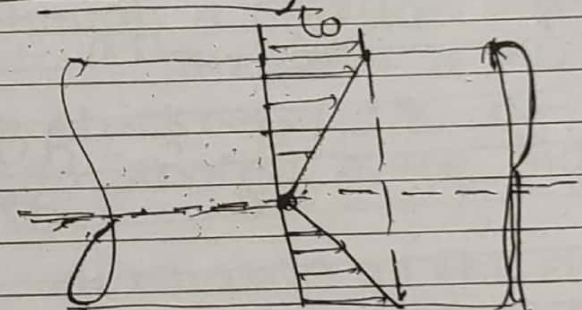
$$= -\frac{\partial P}{\partial x}\cancel{dx}.r = 2\tau \cancel{dx}$$

$$= -\frac{\partial P}{\partial x}\, r = 2\tau$$

$$\Rightarrow 2\tau = -\frac{\partial P}{\partial x} \cdot r$$

$$\Rightarrow \tau = -\frac{\partial P}{\partial x} \cdot \frac{r}{2}$$

$$\boxed{\tau = \left(-\frac{\partial P}{\partial x}\right)\frac{r}{2}} \quad (1)$$



$\tau \propto r$

$$\boxed{\tau_0 = \left(-\frac{\partial P}{\partial x}\right)\frac{R}{2}}$$

* shear stress varies linearly from 0 at the central line of the pipe to the maximum at the wall of the pipe.

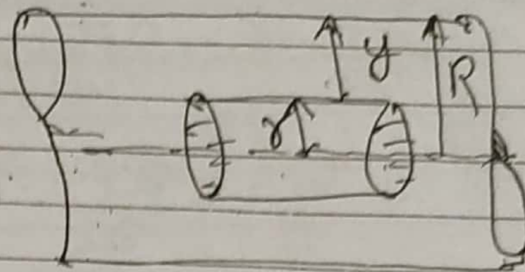
Velocity distribution :-

from eqn (1) we have

$$\tau = \left(-\frac{\partial P}{\partial x}\right)\frac{r}{2}$$

Again shear stress due to viscosity is given by

$$\tau = -\mu \frac{du}{dy}$$



$$R = r + y$$

$$dR = dr + dy$$

$$0 = dr + dy$$

$$\therefore \; dy = -dr$$

$$\frac{du}{dy} = -\frac{du}{dr}$$

$$\tau = -\mu \frac{du}{dr} \quad \text{——} \quad (ii)$$

from equation (i) and (ii)

$$\cancel{-}\mu \frac{du}{dr} = \cancel{-}\frac{\partial P}{\partial x} \cdot \frac{r}{2}$$

$$\Rightarrow \mu \frac{du}{dr} = \frac{\partial P}{\partial x} \cdot \frac{r}{2}$$

$$\Rightarrow \frac{du}{dr} = \frac{\partial P}{\partial x} \cdot \frac{r}{2\mu}$$

$$\Rightarrow \frac{du}{dr} = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) \cdot r$$

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$$\Rightarrow du = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) \cdot r\,dr$$

$$\Rightarrow \int du = \int \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) r\,dr$$

$$\therefore \int du = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) \int r\,dr$$

$$\Rightarrow u = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) \cdot \frac{r^2}{2} + C \quad \text{(iii)}$$

Applying boundry condition,
at $r = R$, $u = 0$
from (iii)

$$0 = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) \cdot \frac{R^2}{2} + C$$

$$0 = \frac{1}{4\mu}\frac{\partial P}{\partial x} R^2 + C$$

$$\boxed{C = -\frac{1}{4\mu}\frac{\partial P}{\partial x} R^2}$$

Put the value C in eqn (iii)

$$u = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) \cdot \frac{r^2}{2} - \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) \cdot \frac{R^2}{2}$$

$$= \frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right) r^2 - \frac{1}{4\mu}\frac{\partial P}{\partial x} R^2$$

$$= \frac{1}{4\mu} \cdot \left(\frac{\partial P}{\partial x}\right) \left[r^2 - R^2\right]$$

$$= -\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)\left[R^2 - r^2\right]$$

$$\boxed{u = -\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right) \cdot R^2\left[1 - \frac{r^2}{R^2}\right]}$$

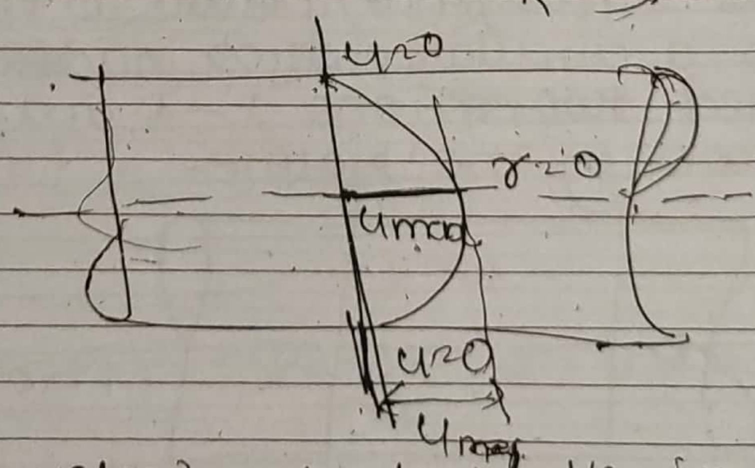
at the central line of the pipe,

$r = 0$, $u = u_{max}$

$$U_{max} = -\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right) R^2 \left[1 - 0\right]$$

$$U_{max} = -\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right) R^2$$

$$u = u_{max}\left[1 - \frac{r^2}{R^2}\right]$$



* The velocity of fluid varies parabolically from 0 at the surface of the pipe to u_{max} at the central line of the pipe.