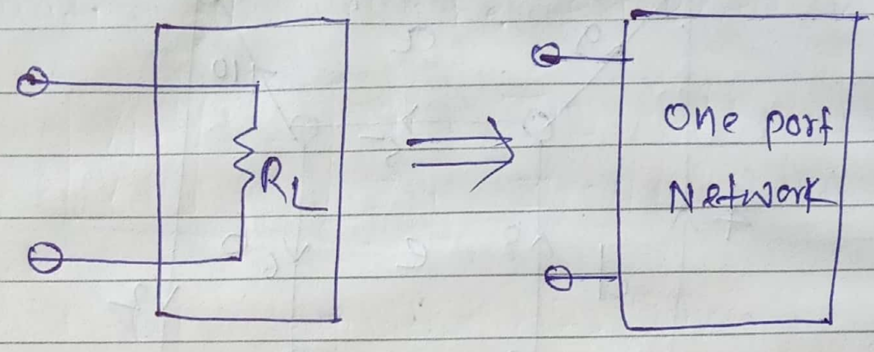
① Two part Network

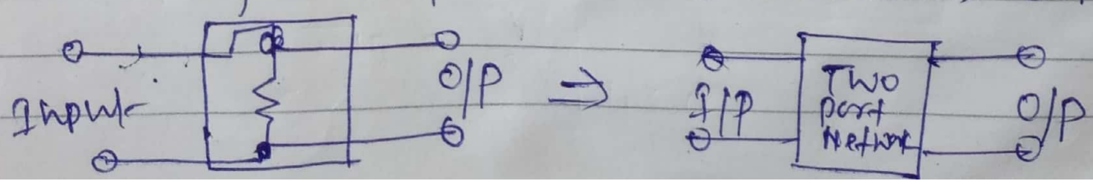
One part



Any active or passive network having only two terminals can be represented by an one port network. It is represent by a rectangular box. It must have two terminal. This rectangular box then represents one port or a two terminal network.

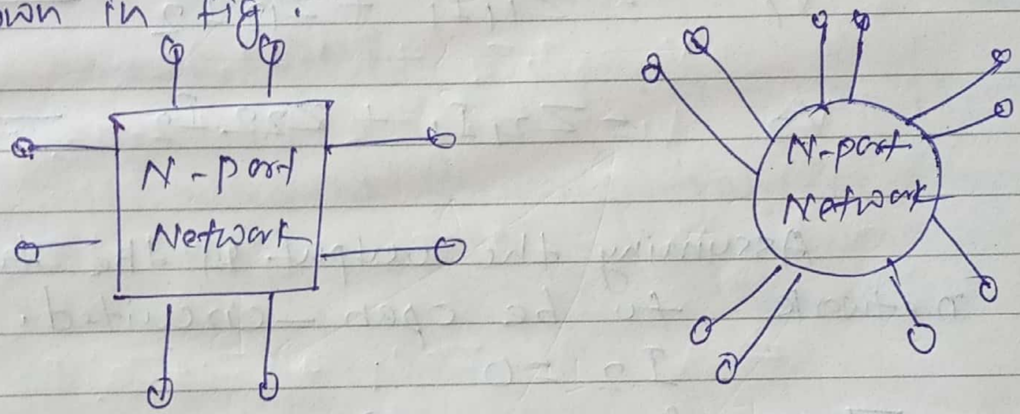
Two port:-

If the network box that represents a network consists of two pairs of terminal, where one pair of ~~terminal~~ terminals can be designated as input, the other pair being output it is called a two port network. Fig represent the diagramatic representation of a two port network. In such a network the driving source may be connected at the pair of input terminals while the other pair of terminals serves as output

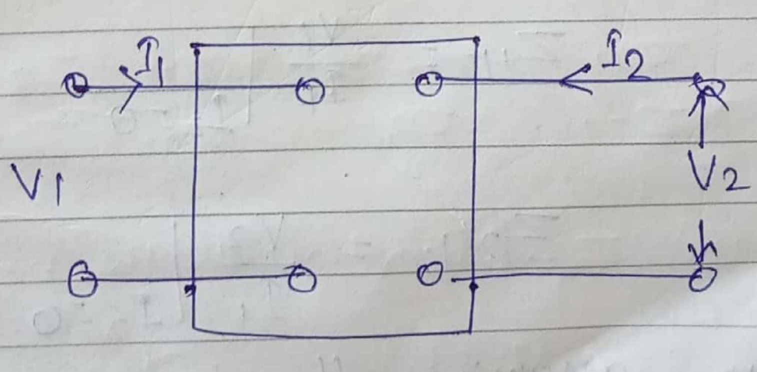


n-port network

If the network representation contains of 'n' - number of pairs of terminals, it is called a ~~n-2~~ n - port network. In a port network, the driving source may be connected in more than one number of pairs of terminals. The block diagram of an ~~network~~ n-port network has been shown in fig.



Z-parameters (open ckt Impedance parameters)



For the two port network

In this port V_1 & I_1 are the inputs and I_2 & V_2 are outputs. It can be expressed in terms of

$$V = [Z][I] \quad \text{---} \quad (1)$$

Where [Z] is the impedance matrix.
Equation ① can be expressed as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{--- (2)}$$

By equation ② gives

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (3)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (4)}$$

Assuming the output of the two-port network to be open-circuited.

$$I_2 = 0$$

Thus from eqn ③ with

$$I_2 = 0,$$

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

Again, assuming the input port of the same two port network to be open ckt.

$$I_1 = 0$$

From eqn ③

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

and from equation

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Then

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \& \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

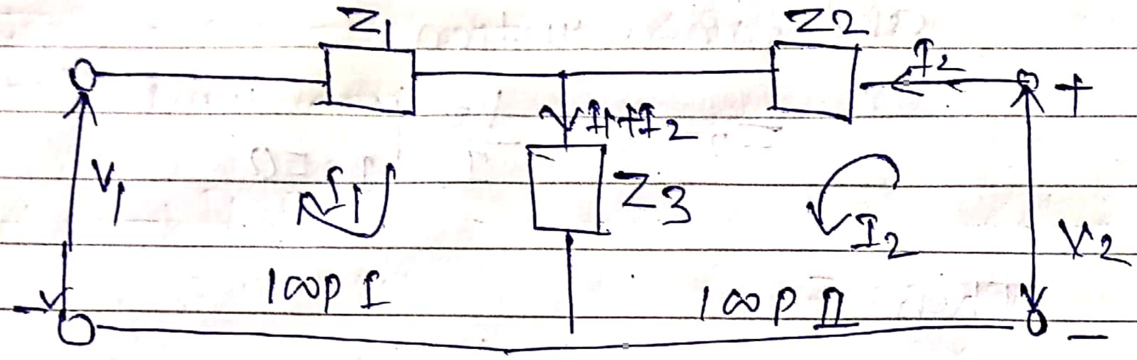
These are called input & o/p driving point impedance respectively

and $Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$ & $Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$

are called & forwork transfer & impedance

Z_{11}, Z_{12}, Z_{21} & Z_{22} are called impedance parameters or open-circuit parameters of the two port network.

(1) Find the Z-parameters for the following network



Soln: By KVL in the above fig, the loop eqns ~~be~~ can be written as

$$V_1 = I_1 Z_1 + (I_1 + I_2) Z_3$$

$$\Rightarrow V_1 = (Z_1 + Z_3) I_1 + Z_3 I_2 \quad \text{--- (I)}$$

and $$V_2 = I_2 Z_2 + (I_1 + I_2) Z_3$$

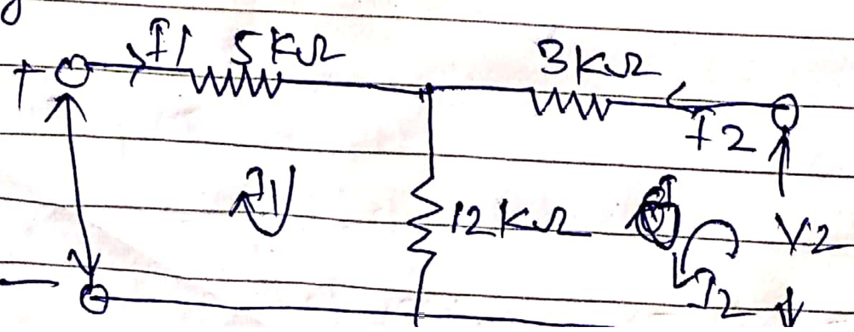
$$\Rightarrow V_2 = Z_3 I_1 + (Z_2 + Z_3) I_2 \quad \text{--- (II)}$$

Comparing eqns (I) & (II) with the standard Z-parameter equations

$Z_{11} = Z_1 + Z_3$; $Z_{12} = Z_3$

$Z_{21} = Z_3$; $Z_{22} = Z_2 + Z_3$

Q:- Find Z-parameters for the network shown in fig.



Using by KVL for loops I & II.

$$V_1 = 5\times10^3 I_1 + 12\times10^3 (I_1+I_2)$$

$$\Rightarrow V_1 = 17\times10^3 I_1 + 12\times10^3 I_2 \quad \text{--- (I)}$$

and

$$V_2 = 3\times10^3 I_2 + 12\times10^3 (I_1+I_2)$$

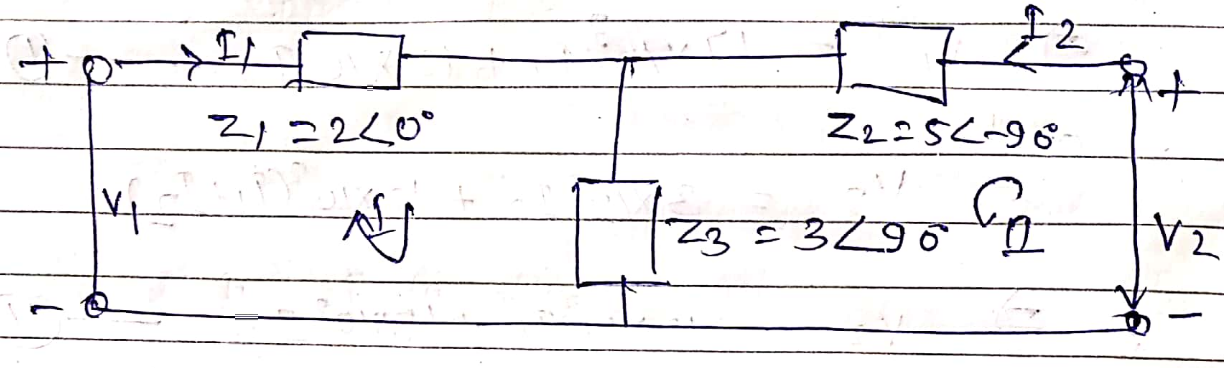
$$\Rightarrow V_2 = 12\times10^3 I_1 + 15\times10^3 I_2 \quad \text{--- (II)}$$

Comparing these two eqns (I) & (II) with Z-parameters.

$$Z_{11} = 17\times10^3\ \Omega$$
$$Z_{12} = 12\times10^3\ \Omega$$
$$Z_{21} = 12\times10^3\ \Omega$$
$$Z_{22} = 15\times10^3\ \Omega$$

(7)

Q:- In a T network of fig. $Z_1 = 2\angle 0°$, $Z_2 = 5\angle -90°$, $Z_3 = 3\angle 90°$, find the Z-parameters.



Soln

By KVL, the loop equations are

$$V_1 = 2\angle 0° I_1 + 3\angle 90° (I_1 + I_2)$$

$$\Rightarrow V_1 = [2\angle 0° + 3\angle 90°] I_1 + 3\angle 90° I_2 \quad \text{— (I)}$$

and $$V_2 = 5\angle -90° I_2 + 3\angle 90° (I_1 + I_2)$$

$$\Rightarrow V_2 = 3\angle 90° I_1 + [5\angle -90° + 3\angle 90°] I_2 \quad \text{— (II)}$$

From (I) & (II)

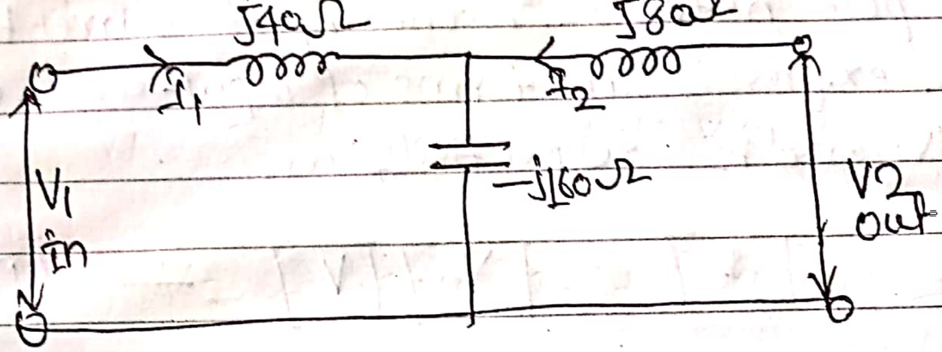
$$Z_{11} = 2\angle 0° + 3\angle 90° = 2+3j$$
$$= 3.605\angle +56° \,\Omega$$

$$Z_{12} = 3\angle 90° \,\Omega$$

$$Z_{21} = 3\angle 90° \,\Omega$$

$$Z_{22} = 5\angle -90° + 3\angle 90°$$
$$= -5j + 3j = -2j = 2\angle -90° \,\Omega$$

Q:- Find the open ckt parameter of the two port network shown in fig.



soln: Assuming open ckt at the output terminals

$$V_1 = I_1(j40 - 160j)\ V$$
$$= -I_1 j\ 120\Omega$$

Then $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = -j120\ \Omega$

and

since $V_2 = I_1(-j160$

$\therefore Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = -j160\ \Omega$

Similarly,

$V_2 = I_2 j(80 - 160) = -jI_2 80V$

that gives

$Z_{22}|_{I_1=0} = \frac{V_2}{I_2} = -j80V$

Y-parameters (short-ckt admittance parameters)

In a two port network, the input current I_1 and I_2 can be expressed in terms of input and output voltages V_1 and V_2 respectively as

$$[I] = [Y][V] \quad \text{—— (I)}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{—— (II)}$$

Equation (II) gives

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{—— (III)}$$

& $$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{—— (IV)}$$

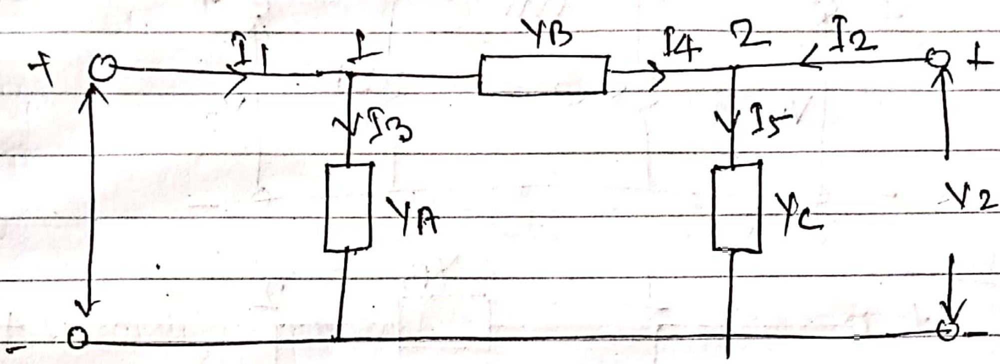
Assuming output voltage $V_2 = 0$

From eqn (III) & (IV)

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0}$$

~~Y_{11} & Y_{22} are~~

Q:- Find the Y-parameters of the following π-ckt ~~and draw~~



By using KCL at node 1

$I_1 = I_4 + I_3$

$\Rightarrow I_1 = V_1 Y_A + (V_1 - V_2) Y_B$

$\Rightarrow I_1 = V_1 (Y_A + Y_B) + (-Y_B) V_2$ —— (I)

Now, using KCL at node 2

$I_2 = I_5 - I_4$

$\Rightarrow I_2 = V_2 Y_c - (V_1 - V_2) Y_B$

$\Rightarrow I_2 = (-Y_B) V_1 + (Y_c + Y_B) V_2$ —— (II)

comparing eqn (I) & (II)

$Y_{11} = Y_A + Y_B$, $Y_{12} = -Y_B$

$Y_{21} = -Y_B$, $Y_{22} = Y_c + Y_B$

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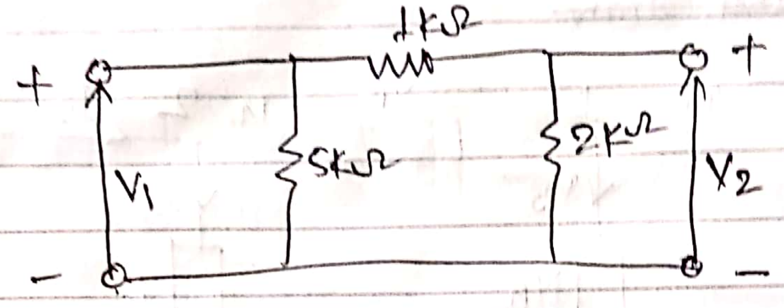
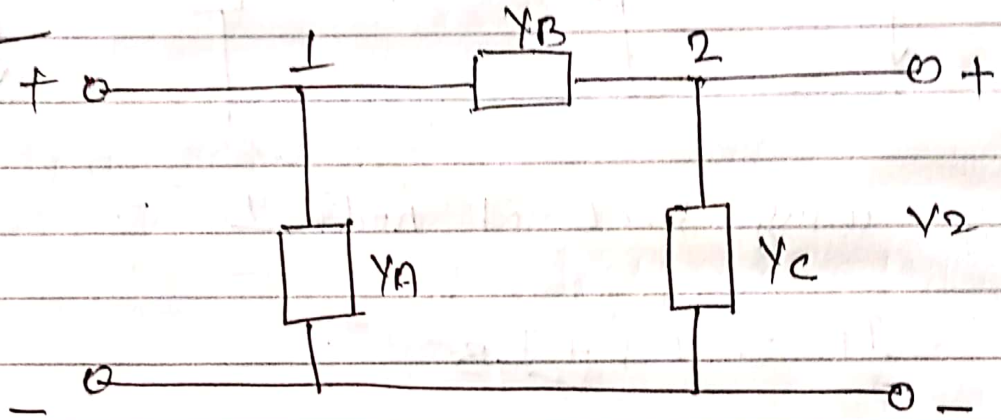
Q:- An π- attenuator has been shown in fig. Find the Y - parameters. ~~and~~



Soln.



$\therefore Y_A = \dfrac{1}{5k\Omega} = 0.2\times10^{-3}\ \mho$

$Y_B = \dfrac{1}{1k\Omega} = 10^{-3}\ \mho$

$Y_C = \dfrac{1}{2k\Omega} = 0.5\times10^{-3}\ \mho$

The y-parameters are

$Y_{11} = Y_A + Y_B = 1.2\times10^{-3}\ \mho$

$Y_{12} = -10^{-3}\ \mho$

$Y_{21} = -10^{-3}\ \mho$

$Y_{22} = Y_B + Y_C = 1.5\times10^{-3}\ \mho$

Hybrid parameter

As both short ckt and open ckt terminals conditions are utilized, hence this parameter representation is known as hybrid parameter representation. In this form of representation, the voltage of the input port and the current of the ~~put~~ output port are expressed in terms of the current of the input port and the voltage of the output port.

Here,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad \text{—— (1)}$$

$$\Rightarrow V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{—— (2)}$$

and

$$I_2 = h_{22} I_1 + h_{22} V_2 \quad \text{—— (3)}$$

Assuming short ckt conditions at the output $V_2 = 0$. This gives from eqn (2) & (3)

$$h_{11} = \frac{V_1}{I_1}$$

$$h_{21} = \frac{I_2}{I_1}$$

Here $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ & $h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$

This are called impedance and forward current gain. h_{11} is expressed in ohms while h_{21} is a unitless quantity.

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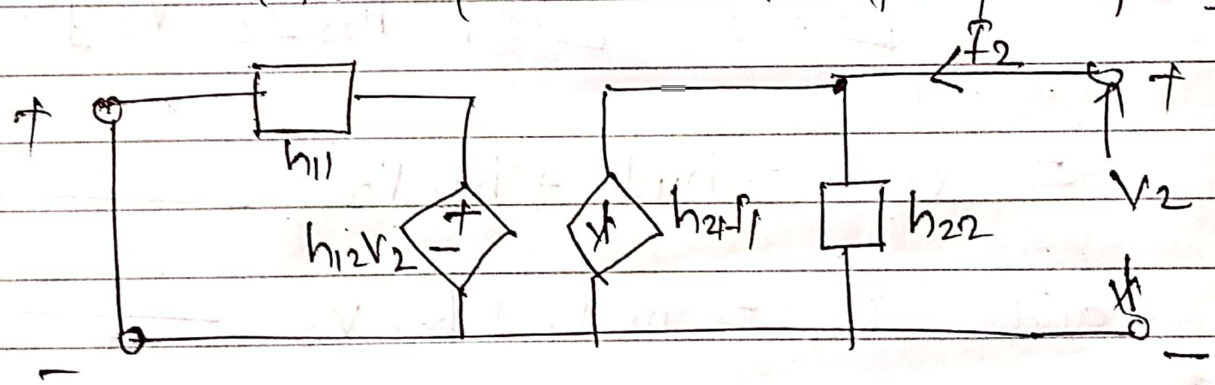
In similar way, from eqn (2) & (B) for, input ckt ~~condition~~ open-ckt condition with $I_1 = 0$

$$h_{12} = \left(\frac{V_1}{V_2}\right)\Big|_{I_1=0}$$

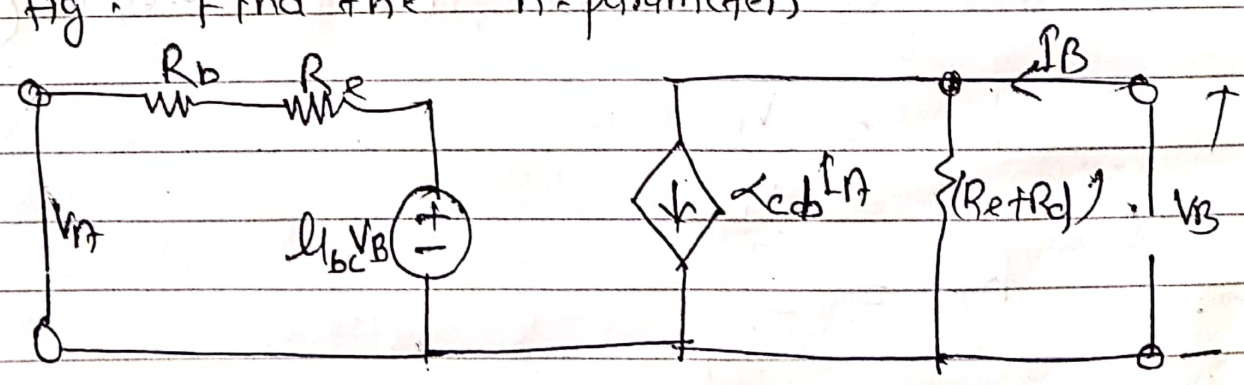
$$\& \; h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$$

These are called voltage gain and output admittance h_{12} has got no unit while h_{22} has unit of mho.

The equivalent ckt of hybrid param



Q:- CE transistor equivalent ckt has been shown in fig. Find the h-parameters



Apply KVL at the input port.

Apply KCL at output port loop.

$$I_B = \alpha_{cb} I_A + \frac{V_B}{R_e + R_d} \quad \text{———} \quad (2)$$

Write (1) & (2) in matrix form.

$$\begin{bmatrix} V_A \\ I_B \end{bmatrix} = \begin{bmatrix} R_b + R_e & \mu_{bc} \\ \alpha_{cb} & \frac{1}{R_e + R_d} \end{bmatrix} \begin{bmatrix} I_A \\ V_B \end{bmatrix}$$

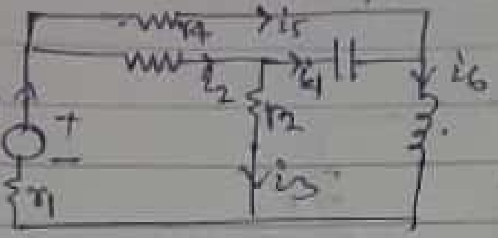
i.e $h_{11} = R_b + R_e$; $h_{12} = \mu_{bc}$

$h_{21} = \alpha_{cb}$; $h_{22} = \frac{1}{R_e + R_d}$

Assignment (1)

ECE, 6th sem., Network Theory

Q:-(1) Draw the oriented graph of the figure shown in fig. Also obtain the fundamental cut-set matrix (5)

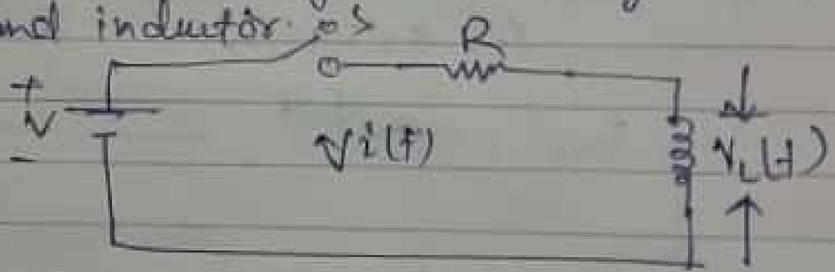


(2) The incidence matrix of a graph is given by

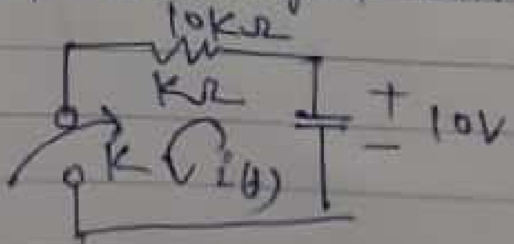
$$[A_i] = \begin{bmatrix} -1 & 0 & 0 & +1 & -1 & 0 \\ +1 & -1 & 0 & 0 & 0 & -1 \\ 0 & +1 & -1 & 0 & +1 & 0 \\ 0 & 0 & +1 & -1 & 0 & +1 \end{bmatrix}$$

Determine possible number of trees.

(3) consider a series R-L ckt as shown in figure The switch 's' is closed at time 't' = 0, Find the current i(t) through and voltage across the resistor and inductor



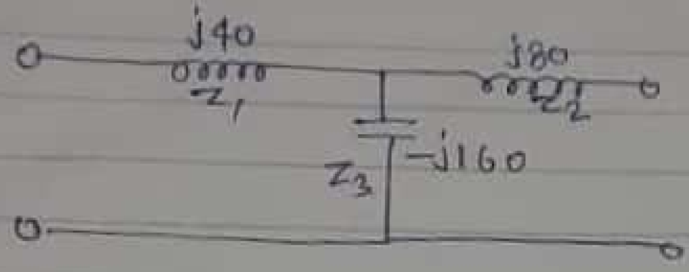
(4) A capacitor of 1μF being charged initially to 10V is connected to a resistance of 10kΩ and is allowed to discharged through it by switching K is closed. Find the ~~expressi~~ expression of discharged current



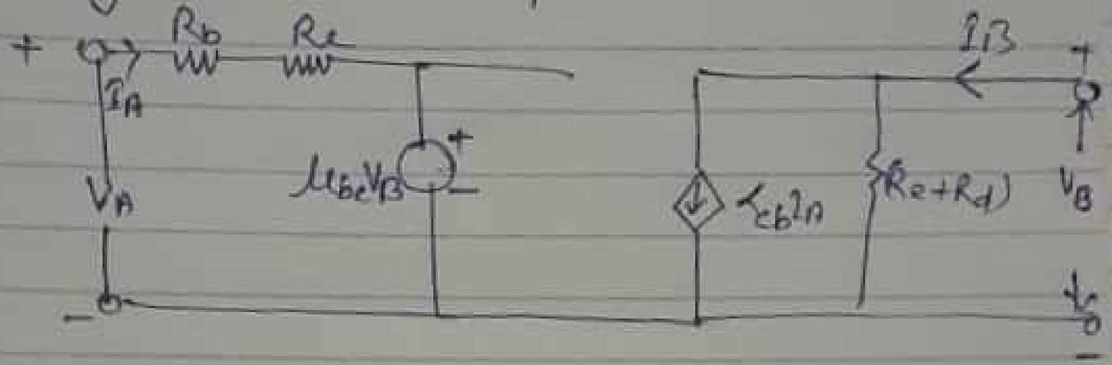
Assignment (2)

ECE, 6th sem, Network Theory.

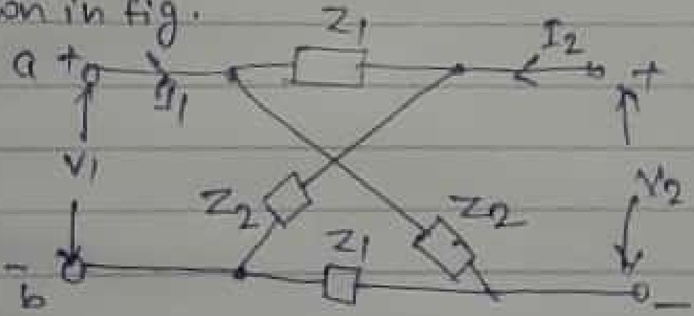
(5) Find Y-parameters of networks shown in fig from Z-parameters.



(6) CE transistor equivalent ckt has been shown in fig. Find the h-parameters.



(7) Find Z-parameters for the lattice network shown in fig.



(8) Obtain transmission parameter of the network shown in