# Module 5

# Couplings

# Lesson 2

# Design procedures for rigid and flexible rubber-bushed couplings

# Instructional Objectives

At the end of this lesson, the students should have the knowledge of

- Detailed design procedure of a typical rigid flange coupling.
- Detailed design procedure of a typical flexible rubber-bush coupling.

## 5.2.1 Rigid Flange Coupling

A typical rigid flange coupling is shown in **Figure- 5.1.2.1.4.2.**

If essentially consists of two cast iron flanges which are keyed to the shafts to be joined. The flanges are brought together and are bolted in the annular space between the hub and the protecting flange. The protective flange is provided to guard the projecting bolt heads and nuts. The bolts are placed equi-spaced on a bolt circle diameter and the number of bolt depends on the shaft diameter d. A spigot 'A' on one flange and a recess on the opposing face is provided for ease of assembly.

The design procedure is generally based on determining the shaft diameter d for a given torque transmission and then following empirical relations different dimensions of the coupling are obtained. Check for different failure modes can then be carried out. Design procedure is given in the following steps:

**(1)** Shaft diameter'd' based on torque transmission is given by

$$d = \left(\frac{16T}{\pi\tau_s}\right)^{1/3}$$

where T is the torque and $\tau_y$ is the yield stress in shear.

**(2)** Hub diameter $d_1$ =1.75d +6.5mm

**(3)** Hub length L = 1.5d

But the hub length also depends on the length of the key. Therefore this length L must be checked while finding the key dimension based on shear and crushing failure modes.

**(4)** Key dimensions:

If a square key of sides b is used then b is commonly taken as $\frac{d}{4}$ . In that case, for shear failure we have

$\left(\frac{d}{4}.L_k\right).\tau_y.\frac{d}{2}$ = T where $\tau_y$ is the yield stress in shear and $L_k$ is the key length.

This gives $L_k = \frac{8T}{d^2\tau_y}$

If $L_k$ determined here is less than hub length L we may assume the key length to be the same as hub length.

For crushing failure we have

$\left(\frac{d}{8}.L_k\right)\sigma_c.\frac{d}{2}$ = T where $\sigma_c$ is crushing stress induced in the key. This gives

$\sigma_c = \frac{16T}{L_k d^2}$

and if $\sigma_c < \sigma_{cy}$ , the bearing strength of the key material , the key dimensions chosen are in order.

**(5)** Bolt dimensions :

The bolts are subjected to shear and bearing stresses while transmitting torque.

Considering the shear failure mode we have

$$n.\frac{\pi}{4}d_b{}^2\tau_{yb}\frac{d_c}{2} = T$$

where n is the number of bolts, $d_b$ the nominal bolt diameter, T is the torque transmitted, $\tau_{yb}$ is the shear yield strength of the bolt material and $d_c$ is the bolt circle diameter. The bolt diameter may now be obtained if n is known. The number of bolts n is often given by the following empirical relation:

$$n = \frac{4}{150}d + 3$$

where d is the shaft diameter in mm. The bolt circle diameter must be such that it should provide clearance for socket wrench to be used for the bolts. The empirical relation takes care of this

Considering crushing failure we have

$$n.d_b t_2 \sigma_{cyb} \frac{d_c}{2} = T$$

where $t_2$ is the flange width over which the bolts make contact and $\sigma_{cyb}$ is the yield crushing strength of the bolt material. This gives $t_2$. Clearly the bolt length must be more than $2t_2$ and a suitable standard length for the bolt diameter may be chosen from hand book.

**(6)** A protecting flange is provided as a guard for bolt heads and nuts. The thickness $t_3$ is less than $t_2/2$. The corners of the flanges should be rounded.

**(7)** The spigot depth is usually taken between 2-3mm.

**(8)** Another check for the shear failure of the hub is to be carried out. For this failure mode we may write

$$\pi d_1 t_2 \tau_{yf} \frac{d_1}{2} = T$$

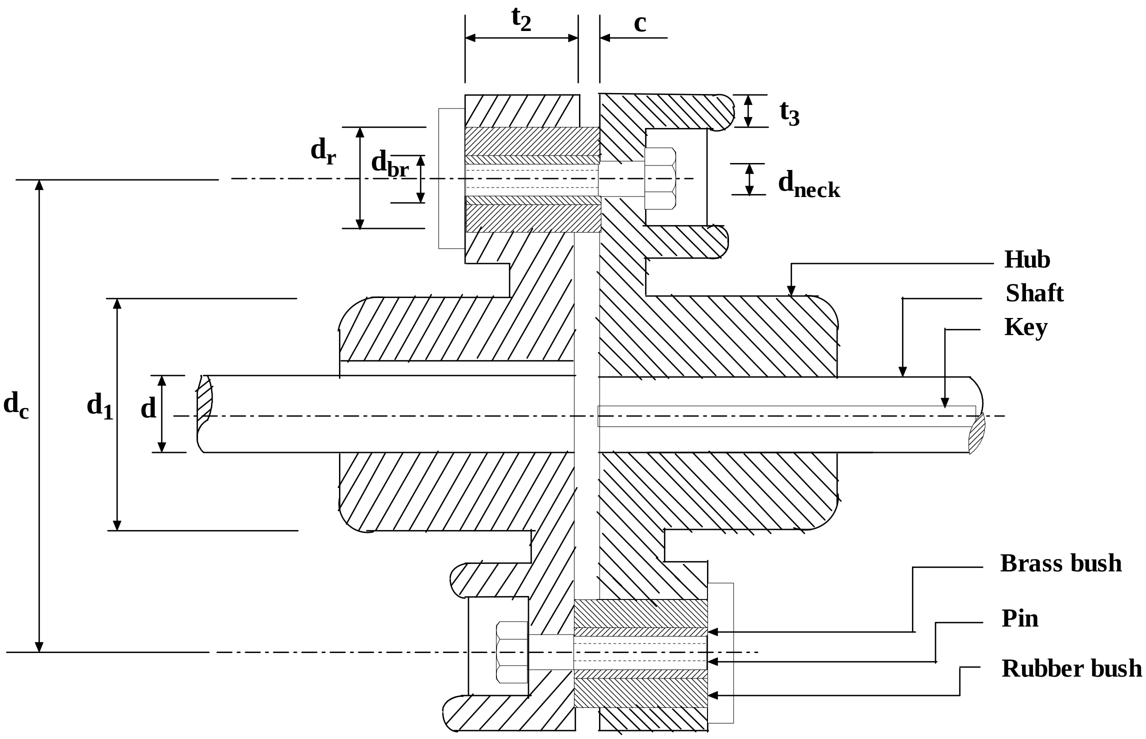
where $d_1$ is the hub diameter and $\tau_{yf}$ is the shear yield strength of the flange material.

Knowing $\tau_{yf}$ we may check if the chosen value of $t_2$ is satisfactory or not.

Finally, knowing hub diameter $d_1$, bolt diameter and protective thickness $t_2$ we may decide the overall diameter $d_3$.

## 5.2.2 Flexible rubber – bushed couplings

This is simplest type of flexible coupling and a typical coupling of this type is shown in **Figure-5.2.2.1.**




***5.2.2.1F**- A typical flexible coupling with rubber bushings.*

In a rigid coupling the torque is transmitted from one half of the coupling to the other through the bolts and in this arrangement shafts need be aligned very well.

However in the bushed coupling the rubber bushings over the pins (bolts) (as shown in **Figure-5.2.2.1**) provide flexibility and these coupling can accommodate some misalignment.

Because of the rubber bushing the design for pins should be considered carefully.

**(1) Bearing stress**

Rubber bushings are available for different inside and out side diameters. However rubber bushes are mostly available in thickness between 6 mm to 7.5mm for bores upto 25mm and 9mm thickness for larger bores. Brass sleeves are made to suit the requirements. However, brass sleeve thickness may be taken to be 1.5mm. The outside diameter of rubber bushing $d_r$ is given by

$d_r = d_b + 2\, t_{br} + 2\, t_r$

where $d_b$ is the diameter of the bolt or pin , $t_{br}$ is the thickness of the brass sleeve and $t_r$ is the thickness of rubber bushing. We may now write

$$n.d_r t_2 p_b \frac{d_c}{2} = T$$

where $d_c$ is the bolt circle diameter and $t_2$ the flange thickness over the bush contact area. A suitable bearing pressure for rubber is 0.035 N/mm$^2$ and the number of pin is given by $n = \frac{d}{25} + 3$ where d is in mm.

The $d_c$ here is different from what we had for rigid flange bearings. This must be judged considering the hub diameters, out side diameter of the bush and a suitable clearance. A rough drawing is often useful in this regard.

From the above torque equation we may obtain bearing pressure developed and compare this with the bearing pressure of rubber for safely.

(2) Shear stress

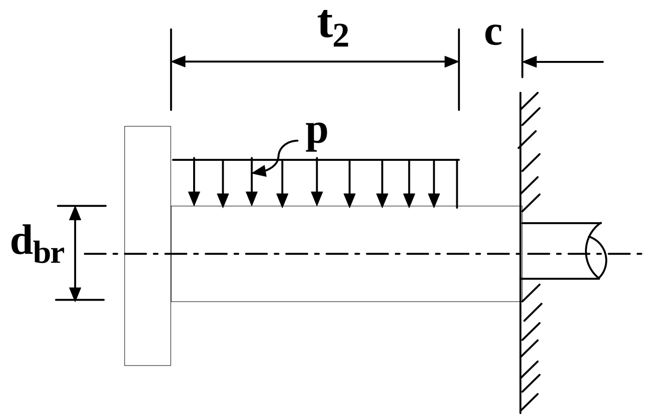
The pins in the coupling are subjected to shear and it is a good practice to ensure that the shear plane avoids the threaded portion of the bolt. Unlike the rigid coupling the shear stress due to torque transmission is given in terms of the tangential force F at the outside diameter of the rubber bush. Shear stress at the neck area is given by

$$\tau_b = \frac{p_b t_2 d_r}{\frac{\pi}{4} d_{neck}^2}$$

where $d_{neck}$ is bolt diameter at the neck i.e at the shear plane.

**Bending Stress**

The pin loading is shown in **Figure-5.2.2.2.**




***5.2.2.2F***- *Loading on a pin supporting the bushings.*

Clearly the bearing pressure that acts as distributed load on rubber bush would produce bending of the pin. Considering an equivalent concentrated load F= $pt_2d$ the bending stress is

$$\sigma_b = \frac{32F(t_2/2)}{\pi d_{br}^3}$$

Knowing the shear and bending stresses we may check the pin diameter for principal stresses using appropriate theories of failure.

We may also assume the following empirical relations:

Hub diameter = 2d

Hub length = 1.5d

Pin diameter at the neck $= \frac{0.5d}{\sqrt{n}}$

## 5.2.3 Problems with answers

**Q.1:** Design a typical rigid flange coupling for connecting a motor and a centrifugal pump shafts. The coupling needs to transmit 15 KW at 1000 rpm. The allowable shear stresses of the shaft, key and bolt materials are 60 MPa,50 MPa and 25 MPa respectively. The shear modulus of the shaft material may be taken as 84GPa. The angle of twist of the shaft should be limited to 1 degree in 20 times the shaft diameter.

**A.1:**

The shaft diameter based on strength may be given by

d = $\sqrt[3]{\frac{16T}{\pi\tau_y}}$ where T is the torque transmitted and $\tau_y$ is the allowable yield stress in shear.

Here T = Power/ $\left(\frac{2\pi N}{60}\right)$ = $\frac{15x10^3}{\left(\frac{2\pi x1000}{60}\right)} = 143Nm$

And substituting $\tau_y$ = 60x10$^6$Pa we have

d = $\left(\frac{16x143}{\pi x60x10^6}\right)^{\frac{1}{3}} = 2.29x10^{-2}\,m \simeq 23mm$.

**Let us consider a shaft of 25 mm which is a standard size.**

From the rigidity point of view

$$\frac{T}{J} = \frac{G\theta}{L}$$

Substituting T = 143Nm , J = $\frac{\pi}{32}(0.025)^4 = 38.3x10^{-9}\,m^4, G = 84x10^9\,Pa$

$$\frac{\theta}{L} = \frac{143}{38.3x10^{-9}\,x84x10^9}$$

= 0.044 radian per meter.

The limiting twist is 1 degree in 20 times the shaft diameter

which is $\frac{\frac{\pi}{180}}{20x0.025} = 0.035$ radian per meter

Therefore, the shaft diameter of 25mm is safe.

We now consider a typical rigid flange coupling as shown in **Figure 5.1.2.1.4.2F.**

**Hub-**

Using empirical relations

Hub diameter $d_1$ = 1.75d + 6.5 mm. This gives

$d_1$ = 1.75x25 + 6.5 = 50.25mm say $d_1$ = 51 mm

Hub length L=1.5d. This gives L = 1.5x25 = 37.5mm, say L= 38mm.

Hub thickness $t_1$= $\frac{d_1 - d}{2} = \frac{51-25}{2} = 13\text{mm}$

**Key** –

Now to avoid the shear failure of the key (refer to **Figure 5.1.2.1.1.2 F**)

$\left(\frac{d}{4}L_k\right).\tau_y.\frac{d}{2} = T$ where the key width w = $\frac{d}{4}$ and the key length is $L_k$

This gives $L_k$ = $\frac{8T}{(\tau_y d^2)}$ i.e. $\frac{8x143}{50x10^6 x(0.025)^2} = 0.0366 \text{ m} = 36.6 \text{ mm}$

The hub length is 37.5 mm. Therefore we take $L_k$ = 37.5mm.

To avoid crushing failure of the key (Ref to **Figure 5.1.2.1.1.2 F**)

$(\frac{d}{8}L_k)\sigma.\frac{d}{2}$ = T where $\sigma$ is the crushing stress developed in the key.

This gives $\sigma = \frac{16T}{L_k d^2}$

Substituting T = 143Nm, $L_k$ = 37.5 x $10^{-3}$ m and d = 0.025 m

$$\sigma = \frac{16x143x10^{-6}}{37.5x10^{-3}x(0.025)^2} = 97.62\text{MPa}$$

Assuming an allowable crushing stress for the key material to be 100MPa, the key design is safe. Therefore the key size may be taken as: a square key of 6.25 mm size and 37.5 mm long. However keeping in mind that for a shaft of diameter between 22mm and 30 mm a rectangular key of 8mm width, 7mm depth and length between 18mm and 90mm is recommended. We choose a standard key of 8mm width, 7mm depth and 38mm length which is safe for the present purpose.

**Bolts.**

To avoid shear failure of bolts

$$n\frac{\pi}{4}{d_b}^2\tau_{yb}\frac{d_c}{2} = T$$

where number of bolts n is given by the empirical relation

$n = \frac{4}{150}d + 3$ where d is the shaft diameter in mm.

which gives n=3.66 and we may take n=4 or more.

Here $\tau_{yb}$ is the allowable shear stress of the bolt and this is assumed to be 60 MPa.

$d_c$ is the bolt circle diameter and this may be assumed initially based on hub diameter $d_1$=51 mm and later the dimension must be justified

Let $d_c$ =65mm.

Substituting the values we have the bolt diameter $d_b$ as

$d_b = \left(\frac{8T}{n\pi\tau_{yb}d_c}\right)^{\frac{1}{2}}$ i.e. $\left(\frac{8x143}{4\pi x25x10^6 x65x10^{-3}}\right)^{\frac{1}{2}} = 7.48x10^{-3}$

which gives $d_b$ = 7.48 mm.

With higher factor of safety we may take $d_b$ = 10 mm which is a standard size.

We may now check for crushing failure as

$$nd_b t_2 \sigma_c \frac{d_c}{2} = T$$

Substituting n=4, $d_b$=10mm,$\sigma_c$=100MPa,$d_c$=65mm&T=143Nm and this gives $t_2$=2.2mm.

However empirically we have $t_2 = \frac{1}{2}t_1 + 6.5 = 13mm$

Therefore we take $t_2$=13mm which gives higher factor of safety.

**Protecting flange thickness.**

Protecting flange thickness $t_3$ is usually less than $\frac{1}{2}t_2$ we therefore take $t_3$ = 8mm since there is no direct load on this part.

**Spigot depth**

Spigot depth which is mainly provided for location may be taken as 2mm.

Check for the shear failure of the hub

To avoid shear failure of hub we have

$$\pi d_1 t_2 \tau_f \frac{d_1}{2} = T$$

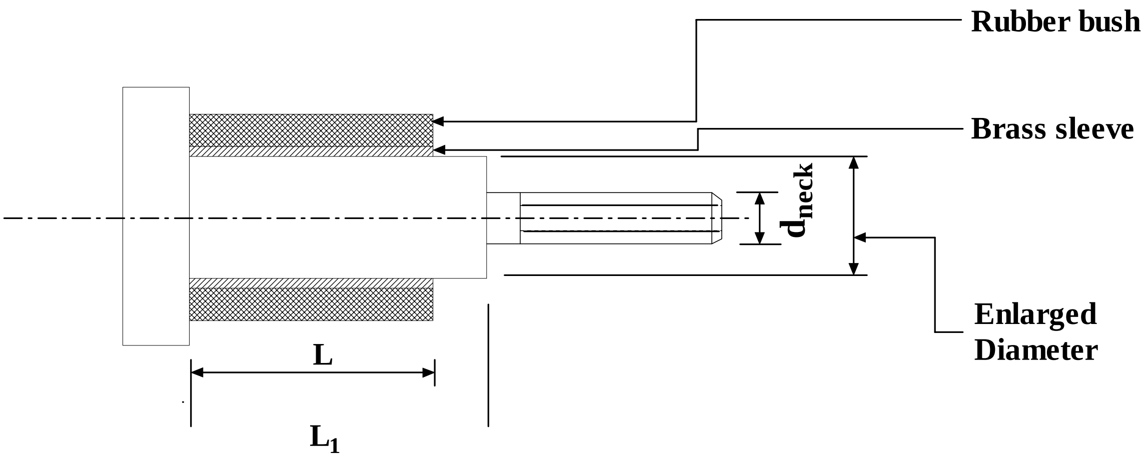
Substituting $d_1$=51mm, $t_2$=13mm and T = 143Nm, we have shear stress in flange $\tau_f$ as $\tau_f = \frac{2T}{(\pi d^2{}_1 t_2)}$

And this gives $\tau_f$ = 2.69 MPa which is much less than the yield shear value of flange material 60MPa.

**Q.2:** Determine the suitable dimensions of a rubber bush for a flexible coupling to connect of a motor and a pump. The motor is of 50 KW and runs at 300rpm. The shaft diameter is 50mm and the pins are on pitch circle diameter of 140mm. The bearing pressure on the bushes may be taken as 0.5MPa and the allowable shear and bearing stress of the pin materials are 25 MPa and 50 MPa respectively. The allowable shear yield strength of the shaft material may be taken as 60MPa.

**A.2:**

A typical pin in a bushed flexible coupling is as shown in **Figure-5.2.3.1.**




***5.2.3.1F***- *A typical pin for the bushings.*

There is an enlarged portion on which a flexible bush is fitted to absorb the misalignment. The threaded portion provided for a nut to tighten on the flange. Considering the whole pin there are three basic stresses developed in the pin in addition to the tightening stresses. There are (a) shear stresses at the unthreaded neck area (b) bending stress over the loaded portion (L) of the enlarged portion of the pin and (c) bearing stress.

However, before we consider the stresses we need to determine the pin diameter and length. Here the torque transmitted

$$T = \frac{50x10^3}{\left(\frac{2\pi x3000}{60}\right)} = 159Nm$$

Based on torsional shear the shaft diameter $d = \left(\frac{16T}{\pi\tau_y}\right)^{\frac{1}{3}}$

Substituting T=159Nm and $\tau_y$ = 60MPa, we have d = 23.8mm. Let the shaft diameter be 25mm. From empirical relations we have

Pin diameter at the neck $d_{neck}$ = $\frac{0.5d}{\sqrt{n}}$

where the number of pins n = $\frac{4d}{150} + 3$

Substituting d = 25 mm we have

n = 3.67 (say) 4

$d_{neck}$ = 6.25 (say) 8mm

On this basis the shear stress at the neck = $\dfrac{T}{\left[\frac{\pi}{4} d_{neck}^2 n \frac{d_c}{2}\right]}$ which gives 11.29 MPa and this is much less than yield stress of the pin material.

There is no specific recommendation for the enlarged diameter based on $d_{neck}$ but the enlarged diameters should be enough to provide a neck for tightening. We may choose

$d_{enlarged}$ = 16mm which is a standard size. Therefore we may determine the inner diameter of the rubber bush as

$d_{bush}$ = Enlarged diameter of the pin + 2x brass sleeve thickness.

A brass sleeve of 2mm thickness is sufficient and we have

$d_{bush}$ = 20mm

Rubber bush of core diameter up to 25mm are available in thickness of 6mm. Therefore we choose a bush of core diameter 20mm and thickness 6mm.

In order to determine the bush length we have

$$T = npLd_{bush}\frac{d_c}{2}$$

where p is the bearing pressure, ($Ld_{bush}$) is the projected area and $d_c$ is the pitch circle diameter. Substituting T= 159Nm, p = 0.5MPa, $d_{bush}$ = 0.02m and $d_c$ = 0.14m we have L = 56.78 mm.

The rubber bush chosen is therefore of 20mm bore size, 6mm wall thickness and 60 mm long.

## 5.2.4 Summary of this Lesson

Detailed design procedure of a rigid flange coupling has been discussed in which failure modes of different parts such as the shaft, key, bolts and protecting flange are described. Design details of a flexible coupling using rubber bushings have also been discussed. Here the failure modes of the flexible rubber bushings have been specially considered. Some typical problems have also been solved.

### 5.2.5 Reference for Module-5


1) A textbook of machine design by P.C.Sharma and D.K.Agarwal, S.K.Kataria and sons, 1998.
2) The elements of machine design by S.J.Berard and E.O.Waters, D.Van Nostrand Company, 1927.
3) Design of machine elements by M.F.Spotts, Prentice hall of India, 1991.
4) Mechanical engineering design by Joseph E. Shigley, McGraw Hill, 1986.
5) A text book of machine drawing by R. K. Dhawan, S. Chand and Co. Ltd., 1996.