Nyquist plot:

① Determine the closed loop stability of a control system whose Open loop transfer function is

$$G(s)H(s) = \frac{K}{s(1+sT)}$$

Solution Given that

$$G(s)H(s) = \frac{K}{s(1+sT)} \quad \text{——— ①}$$

Put $s = j\omega$ in eqn ① then becomes

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(1+j\omega T)} \quad \text{——— ②}$$

~~But~~ Rationalizing the eqn ② and separating into real and Imagionary parts.

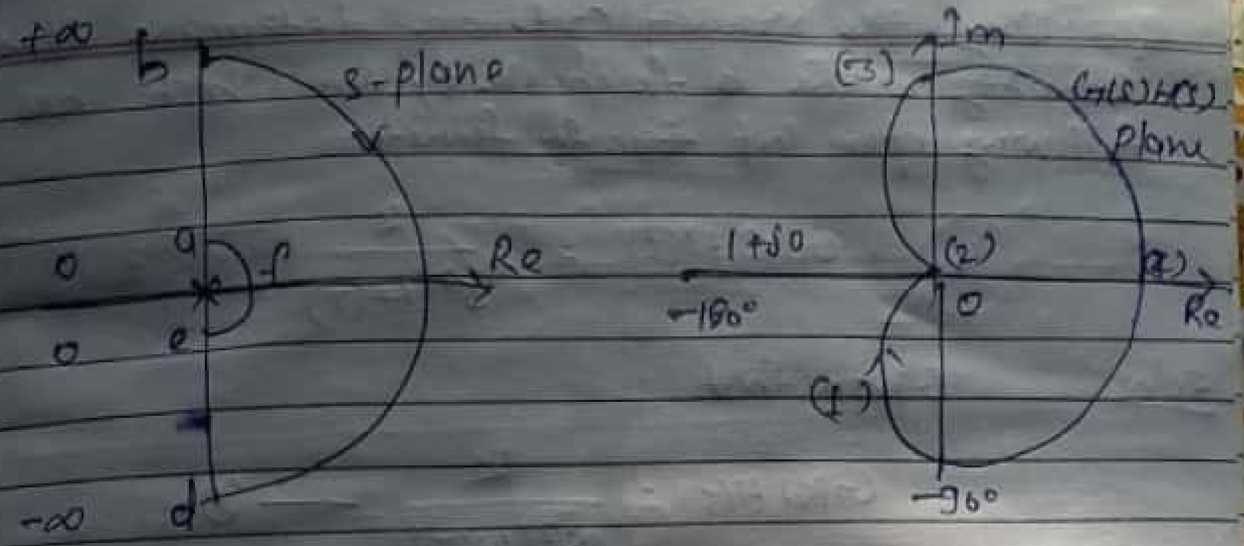
$$G(j\omega)H(j\omega) = -\frac{KT}{1+\omega^2T^2} - j\frac{K}{\omega(1+\omega^2T^2)} \quad \text{——— ③}$$

Take $\lim_{\omega \to 0} |G(j\omega)H(j\omega)| = \infty$

$$\lim_{\omega \to 0} \angle G(j\omega)H(j\omega) = -90^\circ$$

$$\lim_{\omega \to \infty} |G(j\omega)H(j\omega)| = 0$$

$$\lim_{\omega \to \infty} \angle G(j\omega)H(j\omega) = -180^\circ$$



Nyquist plot

The polar plot will lie in third quadrant. The Nyquist plot is shown in fig. The part of $0 < \omega < +\infty$ is drawn (1)(2) and for $-\infty < \omega < 0$ is shown in fig by the point (2), (3) which is mirror image of (1) and (2). The semicircular detour around the origin in s-plane is mapped into a semicircular path of infinite radius representing a charge of phase from $+\pi/2$ to $-\pi/2$.

As the point $(-1+j0)$ is not encircled by the plot, $\therefore N = 0$

$P = 0$

$\because N = Z - P \therefore Z = 0$

The no. of zeros or roots of the characteristic equation with positive real part is nil and hence the closed loop system is stable.

Q:- Using the Nyquist Criterion, determine the stability of the to. feedback system which has the following open loop transfer function

$$G(s)\,H(s) = \frac{K}{s^2(1+sT)}$$

Soln: Given that

$$G(s)\,H(s) = \frac{K}{s^2(1+sT)} \quad \text{--- (1)}$$

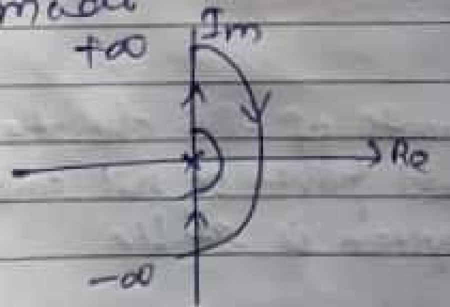
put $s = j\omega$ in eqn (1)

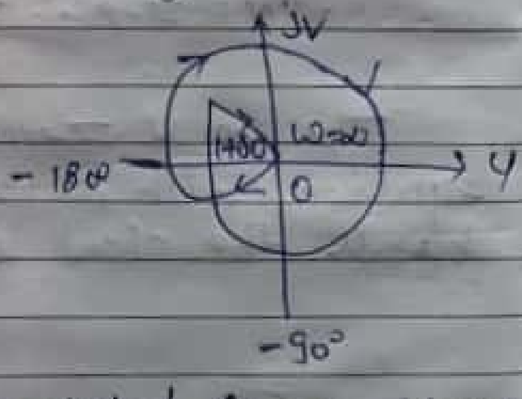
$$G(j\omega)\,H(j\omega) = \frac{K}{(j\omega)^2(1+j\omega T)} \quad \text{--- (2)}$$

Rationalizing the equation (2) and separating the real and imagionary part

$$G(j\omega)\,H(j\omega) = \frac{K}{-\omega^2(1+\omega^2T^2)} + j\frac{K}{\omega(1+\omega^2T^2)} \quad \text{--- (3)}$$

The Nyquist diagram is shown in the fig. Because of the double pole at $s=0$, a small semicircular detour at the origin should be made



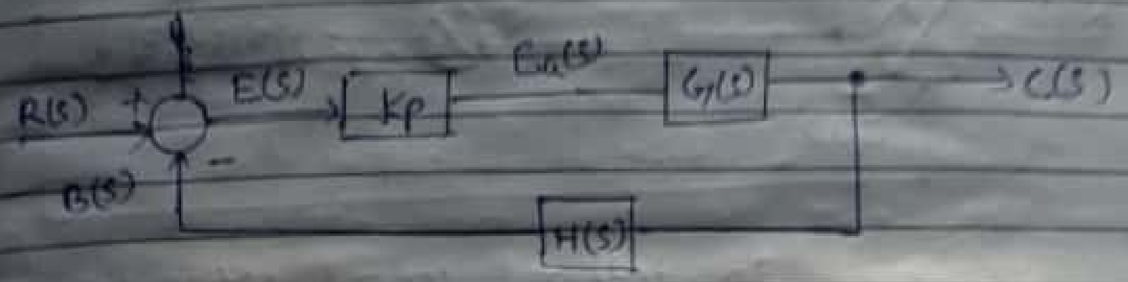


The point $(-1+j0)$ is encircled twice. Hence $N=2$, $P=0$, $Z=2$

Hence, the system is unstable.

Model-4 controll method

① P-control (proportional control)



The error signal is the difference of the reference input and feedback signal. then the error signal is given by

$$E(s) = R(s) - B(s)$$

In proportional control the actuating signal is proportional to the error signal E(s). Therefore, it is known as proportional control system. The proportional control action is shown by the block diagram

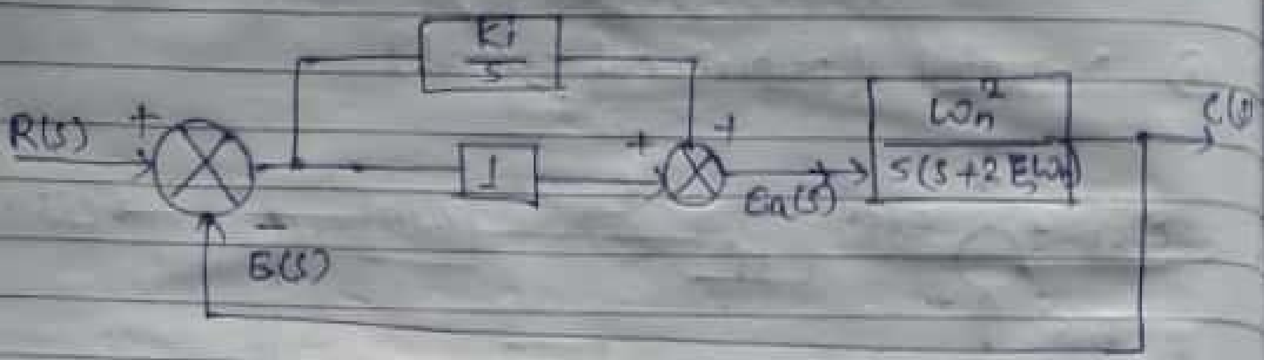
R(s) = Reference input
E(s) = Error signal
Ea(s) = Actuating signal
C(s) = output of the signal
B(s) = Feedback signal

For quick response, the control system should be underdamped. During the transient period in output the underdamped system has exponentially decaying oscillation. The sluggish (slow moving) overdamped response of a system can be made faster by increasing forward path gain of the system, the steady state error reduced but the maximum overshoot increase.

$$\frac{Ea(s)}{E(s)} = Kp$$

Where Kp is known as proportional ~~gain~~ gain.

PI controller



In integral control action the actuating signal consists of proportional error signal with integral of the error signal. The block diagram of integral control is shown in fig above.

From the block diagram the closed loop transfer function will be

$$\frac{C(s)}{R(s)} = \frac{\left(1+\frac{K_i}{s}\right)\left(\frac{\omega_n^2}{s(s+2\xi\omega_n)}\right)}{1+\left(1+\frac{K_i}{s}\right)\left(\frac{\omega_n^2}{s(s+2\xi\omega_n)}\right)} \quad \text{--- (I)}$$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{(s+K_i)\,\omega_n^2}{s^3+2\xi\omega_n s^2+\omega_n^2 s+K_i\omega_n^2} \quad \text{--- (II)}$$

The characteristic equation is the third order equation hence the system becomes third order system.

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)} = \frac{1}{1+\frac{(s+K_i)\omega_n^2}{s^2(s+2\xi\omega_n)}}$$

$$\Rightarrow E(s) = R(s)\,\frac{s^2(s+2\xi\omega_n)}{s^3+2\xi\omega_n s^2+\omega_n^2 s+K_i\omega_n^2} \quad \text{(III)}$$

From eqn (III) if R(s) is unit ~~step~~ ramp the steady state error will be zero.

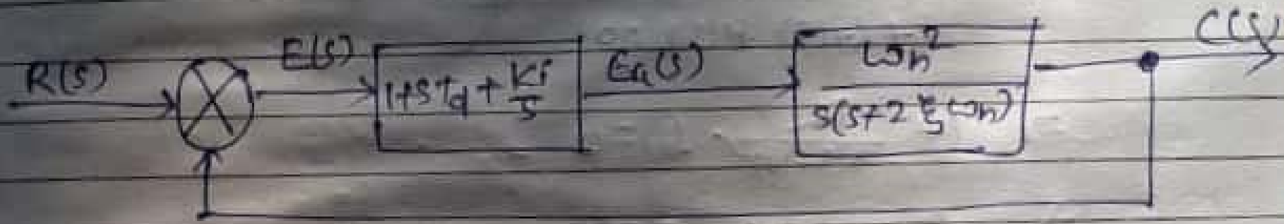
If the input is parabolic input i.e. $R(s)=\frac{1}{s^3}$ then we have steady state error.

$$E(s) = \frac{1}{s^3} \cdot \frac{s^2(s+2\xi\omega_n)}{s^3+2\xi\omega_n s^2+\omega_n^2 s+K_i\omega_n^2}$$

$$\Rightarrow e_{ss} = \lim_{s\to 0} sE(s) = \frac{2\xi\omega_n}{K_i\omega_n^2} = \frac{2\xi}{K_i\omega_n}.$$

PID controller

Proportional plus Derivative plus Integral control (PID)



In PID controller the actuating signal consists of proportional error signal added with derivative and integral of error signal.

The block diagram of PID control is shown in fig. above.

Q: A unity feedback system is characterised by an open loop transfer function

$$G(s) = \frac{K}{s(s+10)}$$

Determine the gain K so that the system will have a damping ratio of 0.5. For this value of K determine settling time, peak over shoot and peak time for a unit step input.

Soln:

The characteristic eqn

$$1+G(s)\cdot H(s)=0$$

$$\Rightarrow 1+\frac{K}{s(s+10)}\cdot 1=0$$

$$\Rightarrow s^2+10s+K=0$$

compare with $s^2+2\xi\omega_n s+\omega_n^2=0$

$\omega_n^2 = K$ $\therefore \omega_n = \sqrt{K}$

$2\xi\omega_n = 10$ $\therefore \omega_n = \sqrt{100} = 10$ rad/sec

$\Rightarrow 2\times 0.5\sqrt{K} = 10$

$\therefore K = 100$

Settling time, $t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.5\times 10} = 0.8$ sec

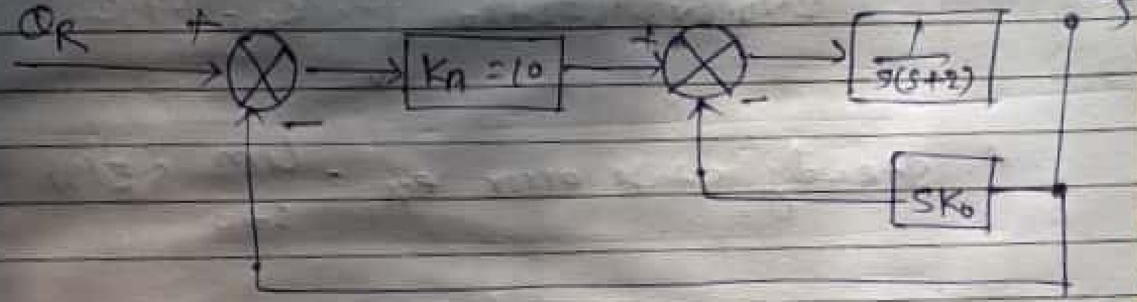
Maximum overshoot $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}\times 100$

$= e^{-\pi 0.5/\sqrt{1-(0.5)^2}}\times 100$

$= 16.3\%$

peak time, $t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{10(\sqrt{1-(0.5)^2})} = 0.363$ sec

Q:- A feedback system employing output-rate damping shown in fig. (a) In the absence of derivative feedback ($K_0 = 0$), determine the damping factor and natural frequency of the system. What is the steady error resulting from unit ramp input?
(b) Determine the derivative feedback constant K_0, which will increase the damping factor of the system to 0.6. What is the steady state error resulting from unit ramp input with this setting of the derivative feedback constant?



Soln:

(a) When $K_0 = 0$

$$G(s) = \frac{10}{s(s+2)}$$

its characteristic eqn is

$$1 + G(s).H(s) = 0$$

$$\Rightarrow 1 + \frac{10}{s(s+2)} \cdot 1 = 0 \qquad [H(s) = 1]$$

$\Rightarrow s^2 + 2s + 10 = 0$ comparing with $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$\therefore \omega_n^2 = 10$ | $\therefore \omega_n = 3.16$ rad/sec

$2\xi\omega_n = 2$

$2 \times \xi \times 3.16 = 2$ | $\xi = 0.316$

Steady state error $e_{ss} = \frac{2\xi}{\omega_n} = \frac{2 \times 0.316}{3.16} = 0.2$ rad

(b) The overall transfer function of given fig.

$$\frac{C(s)}{R(s)} = \frac{10}{s^2+(2+K_0)s+10}$$

it's characteristic eqn $s^2+(2+K_0)s+10=0$

compare with $s^2+2\xi\omega_n s+\omega_n^2=0$

$\therefore \omega_n^2=10$ | $\omega_n = 3.16$ rad/sec.

$2\xi\omega_n = 2+K_0$

$2\times0.6\times3.16 = 2+K_0$ | $\therefore K_0 = 1.792$

$\therefore$ steady state error $e_{ss} = \lim\limits_{s\to0} sE(s)$

But $E(s) = \frac{1}{1+G(s)H(s)} R(s)$

$$\therefore e_{ss} = \lim_{s\to0} s\cdot\frac{1}{s^2}\cdot\frac{1}{1+\frac{10}{s(s+2)+1.8s}}$$

$$\Rightarrow e_{ss} = \lim_{s\to0} s\cdot\frac{1}{s^2}\cdot\frac{s(s+2)+1.8s}{s(s+2)+1.8s+10}$$

$$\Rightarrow e_{ss} = \lim_{s\to0} \frac{s+2+1.8}{10} = 0.38$$

$\therefore e_{ss} = 0.38$ rad.