Module
6
Shear, Bond, Anchorage, Development Length and Torsion

Lesson 16

Torsion in Beams - Limit State of Collapse

Instruction Objectives:

At the end of this lesson, the student should be able to:

- identify the beams and frames subjected to torsion,
- name and explain the two types of torsion,
- state the basis of approach of design for combined bending, shear and torsion as per IS 456,
- select the critical section for the design,
- determine the equivalent shear and moment from the given factored bending moment, shear and torsional moment,
- define equivalent nominal shear stress,
- state when do we provide minimum shear reinforcement in beams subjected to combined bending moment, shear and torsional moment,
- state when do we provide both longitudinal and transverse reinforcement in beams subjected to combined bending moment, shear and torsional moment,
- state when do we provide tensile, compressive and side face reinforcement, respectively, in beams subjected to combined bending shear and torsional moment,
- design the beams subjected to combined bending, shear and torsional moment as per IS 456.

6.16.1 Introduction

This lesson explains the presence of torsional moment along with bending moment and shear in reinforced concrete members with specific examples. The approach of design of such beams has been explained mentioning the critical section to be designed. Expressing the equivalent shear and bending moment, this lesson illustrates the step by step design procedure of beam under combined bending, shear and torsion. The requirements of IS 456 regarding the design are also explained. Numerical problems have been solved to explain the design of beams under combined bending, shear and torsion.

6.16.2 Torsion in Reinforced Concrete Members

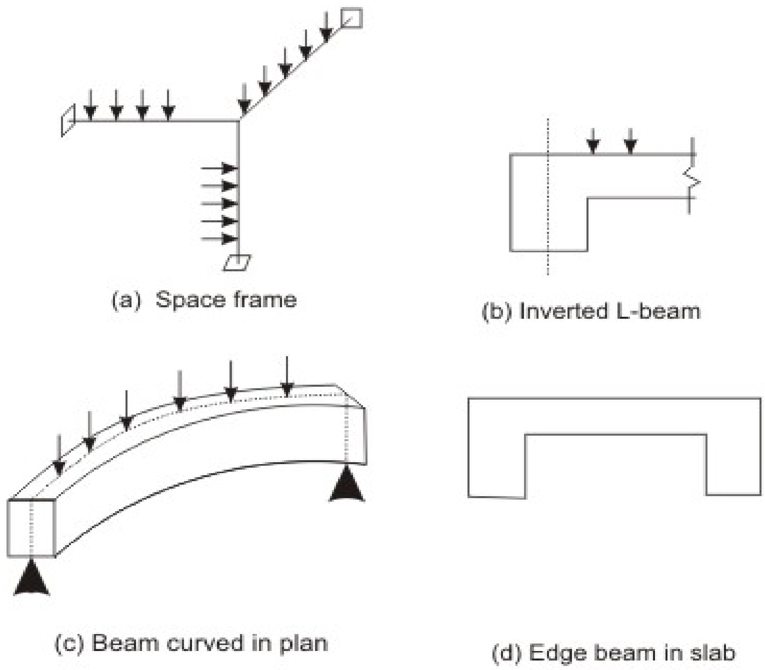

Fig. 6.16.1: Beams under combined bending, shear & torsion

On several situations beams and slabs are subjected to torsion in addition to bending moment and shear force. Loads acting normal to the plane of bending will cause bending moment and shear force. However, loads away from the plane of bending will induce torsional moment along with bending moment and shear. Space frames (Fig.6.16.1a), inverted *L*-beams as in supporting sunshades and canopies (Fig.6.16.1b), beams curved in plan (Fig.6.16.1c), edge beams of slabs (Fig.6.16.1d) are some of the examples where torsional moments are also present.

Skew bending theory, space-truss analogy are some of the theories developed to understand the behaviour of reinforced concrete under torsion combined with bending moment and shear. These torsional moments are of two types:

(i) Primary or equilibrium torsion, and
(ii) Secondary or compatibility torsion.

The primary torsion is required for the basic static equilibrium of most of the statically determinate structures. Accordingly, this torsional moment must be considered in the design as it is a major component.

The secondary torsion is required to satisfy the compatibility condition between members. However, statically indeterminate structures may have any of the two types of torsions. Minor torsional effects may be ignored in statically indeterminate structures due to the advantage of having more than one load path for the distribution of loads to maintain the equilibrium. This may produce minor cracks without causing failure. However, torsional moments should be taken into account in the statically indeterminate structures if they are of equilibrium type and where the torsional stiffness of the members has been considered in the structural analysis. It is worth mentioning that torsion must be considered in structures subjected to unsymmetrical loadings about axes.

Clause 41 of IS 456 stipulates the above stating that, "In structures, where torsion is required to maintain equilibrium, members shall be designed for torsion in accordance with 41.2, 41.3 and 41.4. However, for such indeterminate structures where torsion can be eliminated by releasing redundant restraints, no specific design for torsion is necessary, provided torsional stiffness is neglected in the calculation of internal forces. Adequate control of any torsional cracking is provided by the shear reinforcement as per cl. 40".

6.16.3 Analysis for Torsional Moment in a Member

The behaviour of members under the effects of combined bending, shear and torsion is still a subject of extensive research.

We know that the bending moments are distributed among the sharing members with the corresponding distribution factors proportional to their bending stiffness EI/L where E is the elastic constant, I is the moment of inertia and L is the effective span of the respective members. In a similar manner, the torsional moments are also distributed among the sharing members with the corresponding distribution factors proportional to their torsional stiffness GJ/L, where G is the elastic shear modulus, J is polar moment of inertia and L is the effective span (or length) of the respective members.

The exact analysis of reinforced concrete members subjected to torsional moments combined with bending moments and shear forces is beyond the scope here. However, the codal provisions of designing such members are discussed below.

6.16.4 Approach of Design for Combined Bending, Shear and Torsion as per IS 456

As per the stipulations of IS 456, the longitudinal and transverse reinforcements are determined taking into account the combined effects of bending moment, shear force and torsional moment. Two impirical relations of equivalent shear and equivalent bending moment are given. These fictitious shear force and bending moment, designated as equivalent shear and equivalent bending moment, are separate functions of actual shear and torsion, and actual bending moment and torsion, respectively. The total vertical reinforcement is designed to resist the equivalent shear V_e and the longitudinal reinforcement is designed to resist the equivalent bending moment M_{e1} and M_{e2}, as explained in secs. 6.16.6 and 6.16.7, respectively. These design rules are applicable to beams of solid rectangular cross-section. However, they may be applied to flanged beams by substituting b_w for b. IS 456 further suggests to refer to specialist literature for the flanged beams as the design adopting the code procedure is generally conservative.

6.16.5 Critical Section (cl. 41.2 of IS 456)

As per cl. 41.2 of IS 456, sections located less than a distance d from the face of the support is to be designed for the same torsion as computed at a distance d, where d is the effective depth of the beam.

6.16.6 Shear and Torsion

(a) The equivalent shear, a function of the actual shear and torsional moment is determined from the following impirical relation:

$$V_e = V_u + 1.6(T_u/b) \quad (6.22)$$

where V_e = equivalent shear,

V_u = actual shear,

T_u = actual torsional moment,

b = breadth of beam.

(b) The equivalent nominal shear stress τ_{ve} is determined from:

$$\tau_{ve} = (V_e / bd) \quad (6.23)$$

However, τ_{ve} shall not exceed $\tau_{c\max}$ given in Table 20 of IS 456 and Table 6.2 of Lesson 13.

(c) Minimum shear reinforcement is to be provided as per cl. 26.5.1.6 of IS 456, if the equivalent nominal shear stress τ_{ve} obtained from Eq.6.23 does not exceed τ_c given in Table 19 of IS 456 and Table 6.1 of Lesson 13. .

(d) Both longitudinal and transverse reinforcement shall be provided as per cl. 41.4 and explained below in sec. 6.16.7, if τ_{ve} exceeds τ_c given in Table 19 of IS 456 and Table 6.1 of Lesson 13 and is less than $\tau_{c\max}$, as mentioned in (b) above.

6.16.7 Reinforcement in Members subjected to Torsion

(a) Reinforcement for torsion shall consist of longitudinal and transverse reinforcement as mentioned in sec. 6.16.6(d).

(b) The longitudinal flexural tension reinforcement shall be determined to resist an equivalent bending moment M_{e1} as given below:

$$M_{e1} = M_u + M_t \tag{6.24}$$

where M_u = bending moment at the cross-section, and

$$M_t = (T_u/1.7)\{1 + (D/b)\} \tag{6.25}$$

where T_u = torsional moment,

D = overall depth of the beam, and

b = breadth of the beam.

(c) The longitudinal flexural compression reinforcement shall be provided if the numerical value of M_t as defined above in Eq.6.25 exceeds the numerical value of M_u. Such compression reinforcement should be able to resist an equivalent bending moment M_{e2} as given below:

$$M_{e2} = M_t - M_u \tag{6.26}$$

The M_{e2} will be considered as acting in the opposite sense to the moment M_u.

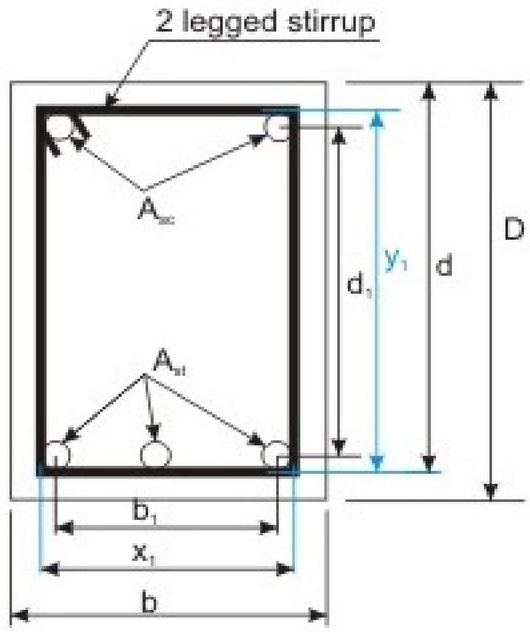

Fig. 6.16.2: Stirrups in beams

(d) The transverse reinforcement consisting of two legged closed loops (Fig.6.16.2) enclosing the corner longitudinal bars shall be provided having an area of cross-section A_{sv} given below:

$$A_{sv} = \frac{T_u \, s_v}{b_1 \, d_1 (0.87 \, f_y)} + \frac{V_u \, s_v}{2.5 \, d_1 (0.87 \, f_y)}$$

(6.27)

However, the total transverse reinforcement shall not be less than the following:

$$A_{sv} \geq (\tau_{ve} - \tau_c) \, b \, s_v \, /(0.87 \, f_y)$$

(6.28)

where T_u = torsional moment,

V_u = shear force,

s_v = spacing of the stirrup reinforcement,

b_1 = centre to centre distance between corner bars in the direction of the width,

d_1 = centre to centre distance between corner bars,

b = breadth of the member,

f_y = characteristic strength of the stirrup reinforcement,

τ_{ve} = equivalent shear stress as specified in Eqs.6.22 and 6.23, and

τ_c = shear strength of concrete as per Table 19 of IS 456 and Table 6.1 of Lesson 13.

6.16.8 Requirements of Reinforcement

Beams subjected to bending moment, shear and torsional moment should satisfy the following requirements:

(a) Tension reinforcement (cl. 26.5.1.1 of IS 456)

The minimum area of tension reinforcement should be governed by

$$A_s/(bd) = 0.85/f_y \tag{6.29}$$

where A_s = minimum area of tension reinforcement,

b = breadth of rectangular beam or breadth of web of T-beam,

d = effective depth of beam,

f_y = characteristic strength of reinforcement in N/mm^2.

The maximum area of tension reinforcement shall not exceed 0.04 bD, where D is the overall depth of the beam.

(b) Compression reinforcement (cl. 26.5.1.2 of IS 456)

The maximum area of compression reinforcement shall not exceed 0.04 bD. They shall be enclosed by stirrups for effective lateral restraint.

(c) Side face reinforcement (cls. 26.5.1.3 and 26.5.1.7b)

Beams exceeding the depth of 750 mm and subjected to bending moment and shear shall have side face reinforcement. However, if the beams are having torsional moment also, the side face reinforcement shall be provided for the overall depth exceeding 450 mm. The total area of side face reinforcement shall be at least 0.1 per cent of the web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness, whichever is less.

(d) Transverse reinforcement (cl. 26.5.1.4 of IS 456)

The transverse reinforcement shall be placed around the outer-most tension and compression bars. They should pass around longitudinal bars located close to the outer face of the flange in *T*- and *I*-beams.

(e) Maximum spacing of shear reinforcement (cl. 26.5.1.5 of IS 456)

The centre to centre spacing of shear reinforcement shall not be more than 0.75 d for vertical stirrups and d for inclined stirrups at 45^o, but not exceeding 300 mm, where d is the effective depth of the section.

(f) Minimum shear reinforcement (cl. 26.5.1.6 of IS 456)

This has been discussed in sec. 6.13.7 of Lesson 13 and the governing equation is Eq.6.3 of Lesson 13.

(g) Distribution of torsion reinforcement (cl. 26.5.1.7 of IS 456)

The transverse reinforcement shall consist of rectangular close stirrups placed perpendicular to the axis of the member. The spacing of stirrups shall not be more than the least of x_1, $(x_1 + y_1)/4$ and 300 mm, where x_1 and y_1 are the short and long dimensions of the stirrups (Fig.6.16.2).

Longitudinal reinforcements should be placed as close as possible to the corners of the cross-section.

(h) Reinforcement in flanges of *T*- and *L*-beams (cl. 26.5.1.8 of IS 456)

For flanges in tension, a part of the main tensile reinforcement shall be distributed over the effective flange width or a width equal to one-tenth of the span, whichever is smaller. For effective flange width greater than one-tenth of the span, nominal longitudinal reinforcement shall be provided to the outer portion of the flange.

6.16.9 Numerical Problems

Problem 1

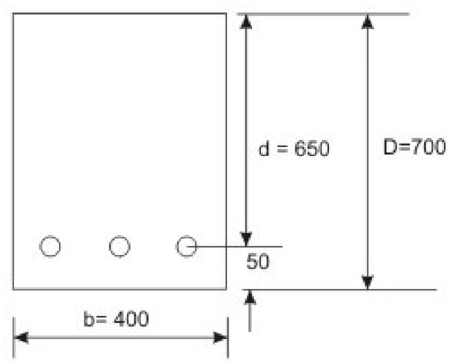

Fig. 6.16.3: Problem 1

Determine the reinforcement required of a ring beam (Fig.6.16.3) of b = 400 mm, d = 650 mm, D = 700 mm and subjected to factored M_u = 200 kNm, factored T_u = 50 kNm and factored V_u = 100 kN. Use M 20 and Fe 415 for the design.

Solution 1

The solution of the problem is illustrated in seven steps below.

Step 1: Check for the depth of the beam

From Eq.6.22, we have the equivalent shear

$$V_e = V_u + 1.6(T_u/b) = 100 + 1.6(50/0.4) = 300 \text{ kN}$$

From Eq.6.2.3, the equivalent shear stress

$$\tau_{ve} = (V_e/bd) = 300/(0.4)(0.65) = 1.154 \text{ N/mm}^2$$

From Table 6.2 of Lesson 13 (Table 20 of IS 456), $\tau_{c\max}$ = 2.8 N/mm^2. Hence, the section does not need any revision.

Step 2: Check if shear reinforcement shall be required.

Assuming percentage of tensile steel as 0.5, Table 6.1 of Lesson 13 (Table 19 of IS 456) gives τ_c = 0.48 N/mm^2 < τ_{ve} < $\tau_{c\max}$. So, both longitudinal and transverse reinforcement shall be required.

Step 3: Longitudinal tension reinforcement

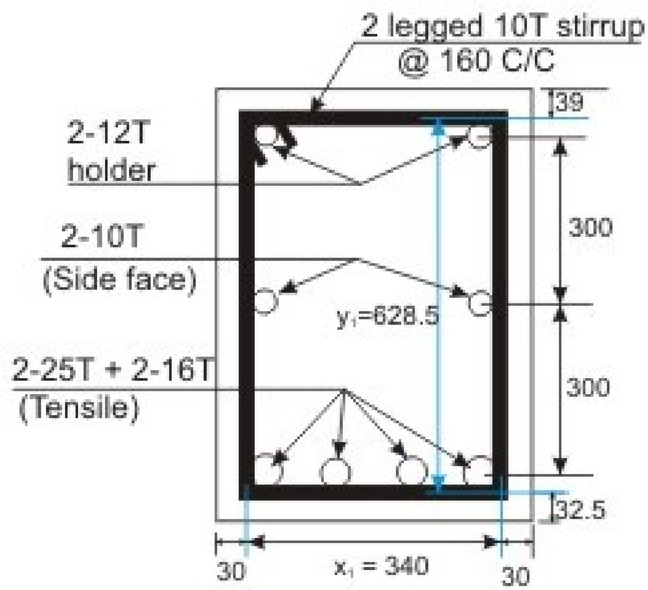

Fig. 6.16.4: Longitudinal (tension & side face) reinforcement of Prob. 1

From Eqs.6.24 and 6.25, we have,

$M_{e1} = M_u + M_t = M_u + (T_u/1.7)\{1 + (D/b)\}$

$= 200 + (50/1.7)\{1 + (700/400)\} = 200 + 80.88 = 280.88$ kNm

$M_{e1}/bd^2 = (280.88)(10^6)/(400)(650)(650) = 1.66$ N/mm^2

From Table 2 of SP-16, corresponding to $M_u/bd^2 = 1.66$ N/mm^2, we have by linear interpolation $p_t = 0.5156$. So,

$A_{st} = 0.5156(400)(650)/100 = 1340.56$ mm^2.

Provide 2-25T and 2-16T = 981 + 402 = 1383 mm^2. This gives percentage of tensile reinforcement = 0.532, for which τ_c from Table 6.1 of Lesson 13 is 0.488 N/mm^2.

From Eq.6.29, minimum percentage of tension reinforcement = $(0.85/f_y)(100)$ = 0.205 and from sec.6.16.8, the maximum percentage of tension reinforcement is 4.0. So, 2-25T and 2-16T bars satisfy the requirements (Fig. 6.16.4).

Step 4: Longitudinal compression reinforcement

Here, in this problem, the numerical value of M_t (= 80.88 kNm) is less than that of M_u (200 kNm). So, as per sec. 6.16.7c, longitudinal compression reinforcement shall not be required.

Step 5: Longitudinal side face reinforcement

Side face reinforcement shall be provided as the depth of the beam exceeds 450 mm. Providing 2-10 mm diameter bars (area = 157 mm^2) at the mid-depth of the beam and one on each face (Fig.6.16.4), the total area required as per sec.6.16.8c, 0.1(400)(300)/100 = 120 mm^2 < 157 mm^2. Hence o.k.

Step 6: Transverse reinforcement

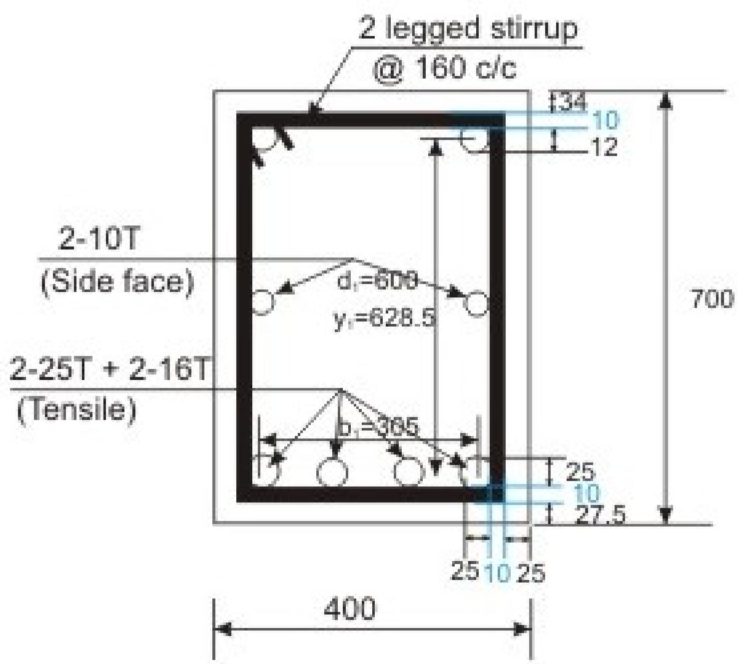

Fig. 6.16.5: Transverse reinforcement of Prob.1

Providing two legged, 10 mm diameter stirrups (area = 157 mm^2), we have (Fig.6.16.5)

d_1 = 700 - 50 - 50 = 600 mm

b_1 = 400 - 2(25 + 10 + 12.5) = 305 mm

From Eq.6.27, we have

$$0.87\, f_y\, A_{sv}/s_v \;=\; (T_u/\, b_1\, d_1) \;+\; (V_u/\, 2.5\, d_1)$$

Using the numerical values of T_u, b_1, d_1 and V_u, we have

$$0.87\, f_y\, A_{sv}/s_v \;=\; 339.89 \;\text{N/mm} \quad \ldots. \quad (1)$$

Again from Eq.6.28, we have

$$0.87\, f_y\, A_{sv}/s_v \;\geq\; (1.154 - 0.48)\, 400 \;\geq\; 269.6 \;\text{N/mm} \quad \ldots. \quad (2)$$

So, Eq.(1) is governing and we get for 2 legged 10 mm stirrups (A_{sv} = 157 mm^2),

s_v = 0.87(415)(157)/339.89 = 166.77 mm

Step 7: Check for s_v

Figure 6.16.4 shows the two legged 10 mm diameter stirrups for which x_1 = 340 mm and y_1 = 628.5 mm. The maximum spacing s_v should be the least of x_1, $(x_1 + y_1)/4$ and 300 mm (Figs. 6.16.4 and 5).

Here, x_1 = 340 mm, $(x_1 + y_1)/4$ = 242.12 mm. So, provide 2 legged 10 mm T stirrups @ 160 mm c/c.

6.16.10 Practice Questions and Problems with Answers

Q.1: Explain the situations when torsional moments remain present in beams and frames.

A.1: The first paragraph of sec. 6.16.2.

Q.2: Explain and differentiate between primary and secondary types of torsion.

A.2: Third, fourth and fifth paragraphs of sec. 6.16.2.

Q.3: Write expressions of equivalent shear and equivalent bending moment.

A.3: Eqs.6.22, 24 and 25.

Q.4: When do you provide minimum shear reinforcement in beam subjected to bending moment, shear and torsional moment?

A.4: Sec. 6.16.6(c)

Q.5: Explain the situations when do you provide longitudinal tension, compression and side face reinforcement in beam subjected to bending moment, shear and torsional moment.

A.5: Sec. 6.16.7a, b and c parts.

Q.6: Illustrate the steps of designing transverse reinforcement in beams subjected to bending moment, shear and torsional moment.

A.6: Sec. 6.16.7d.

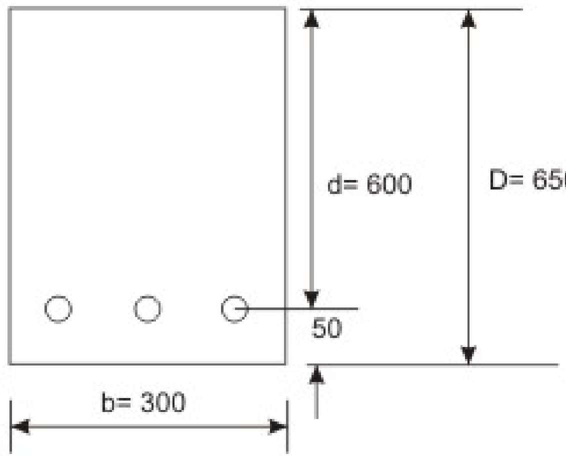

Fig. 6.16.6: Q. 7

Q.7: A reinforced concrete rectangular beam (Fig.6.16.6) of *b* = 300 mm, *d* = 600 mm and *D* = 650 mm is subjected to factored shear force V_u = 70 kN in one section. Assuming the percentage of tensile reinforcement as 0.5 in that section, determine the factored torsional moment that the section can resist if (a) no additional reinforcement for torsion is provided, (b) maximum steel for torsion is provided in that section, and (c) determine the reinforcement needed for the case (b). Assume M 30 concrete, Fe 500 for longitudinal and Fe 415 for transverse reinforcing steel bars.

A.7:

(a) When no additional reinforcement for torsion is provided in that section.

For M 30 concrete with 0.5 per cent tensile reinforcement, Table 6.1 of Lesson 13 (Table 19 of IS 456) gives τ_c = 0.5 N/mm^2 and Table 6.2 of Lesson 13 (Table 20 of IS 456 gives $\tau_{c\max}$ = 3.5 N/mm^2.

Since no additional reinforcement for torsion will be provided, we will take $\tau_{ve} = \tau_{c}$ = 0.5 N/mm^2. From Eq.6.23, we have

$$V_e = (\tau_{ve}) bd = 0.5(300)(600) \text{ N} = 90 \text{ kN}$$

From Eq.6.22, we have

$$T_u = (V_e - V_u)(b/1.6) = (90 - 70)(0.3/1.6) = 3.75 \text{ kNm}$$

So, that section of the beam can resist factored torsional moment of 3.75 kNm if no additional reinforcement is provided.

(b) When maximum steel is provided for torsion in that section.

In this case, $\tau_{ve} = \tau_{c\max}$ = 3.5 N/mm^2. Using this value in Eq.6.23, we get

$$V_e = (\tau_{ve}) bd = (3.5)(300)(600) \text{ N} = 630 \text{ kN}$$

From Eq.6.22, we get

$$T_u = (V_e - V_u)(b/1.6) = (630 - 70)(0.3/1.6) = 105 \text{ kNm}$$

This section, therefore, can resist a factored torsional moment of 105 kNm when maximum torsional reinforcement is provided.

(c) Determination of maximum torsional reinforcement for case (b)

Step 1: Determination of the M_u when 0.5 per cent tensile reinforcement is assumed.

From Table 4 of SP-16, for Fe 500 and M 30 with p_t = 0.5 per cent, we have M_u = (1.993)(300)(600)^2 Nmm = 215.244 kNm (using linear interpolation).

Step 2: Tension and compression reinforcement

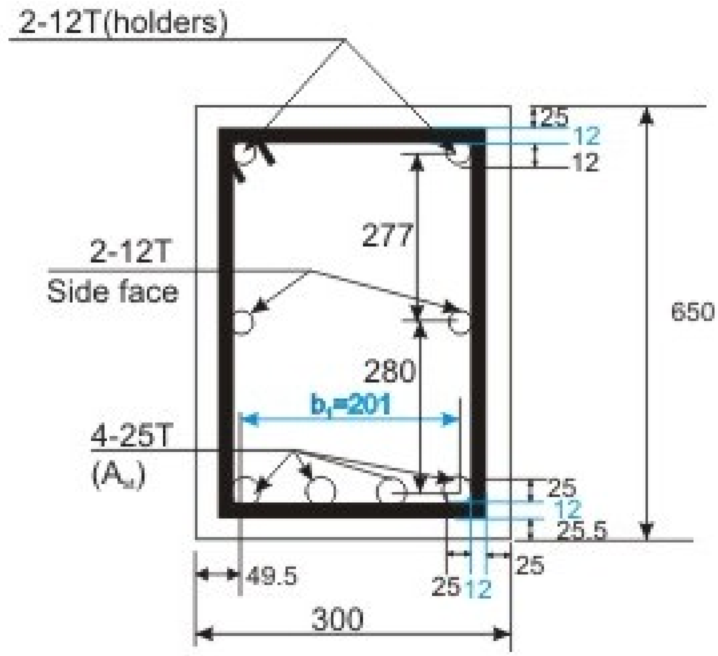

Fig. 6.16.7: Longitudinal (tension & side face) reinforcement of Q. 7

From Eqs.6.24 and 6.25, we get

M_{e1} = M_u + M_t = M_u + $(T_u/1.7)$ {1 + (D/b)}

= 215.244 + (105/1.7){1 + (650/300)}= 215.244 + 195.588 = 410.832 kNm

Here, the numerical value of M_t (= 195.588 kNm) is less than that of M_u (= 215.244 kNm). So, no compression reinforcement is needed.

Table 4 of SP-16 is used to determine tension reinforcement with M_u/bd^2 = 410.832/(0.3)(0.6)(0.6) = 3.804 N/mm. From Table 4, we get p_t = 1.064 by linear interpolation, which gives A_{st} = 1.064(300)(600)/100 = 1915.2 mm^2. Provide 4-25T (1963 mm^2) as shown in Fig.6.16.7. Now, p_t = 1963(100)/(300)(600) = 1.09, for which τ_c = 0.678 N/mm^2 (Table 6.1 of Lesson 13).

Step 3: Side face reinforcement

Since the depth of the beam exceeds 450 mm, we provide side face reinforcement with two 10 mm bars (area = 157 mm^2) near the mid-depth of the beam, one on each side to get the spacing of the bar 280 mm (Fig.6.16.7). Area required to satisfy = 0.1(300)(280)/(100) = 84 mm^2 < 157 mm^2, hence, o.k.

Step 4: Transverse reinforcement

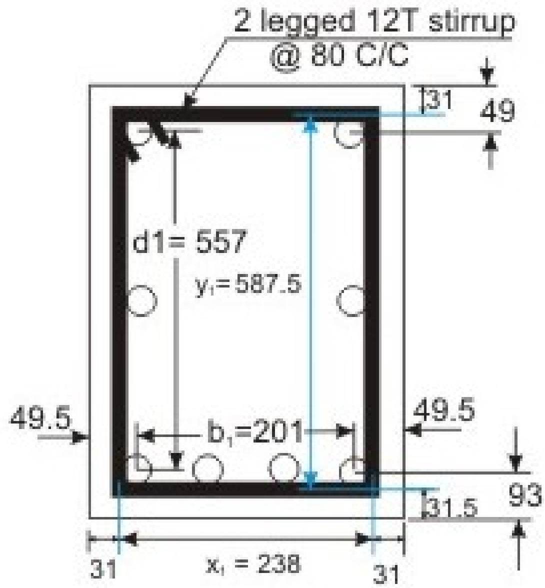

Fig. 6.16.8: Transverse reinforcement of Q.7

Assuming 2 legged 12 mm dia stirrup (area = 226 mm^2) of Fe 415, we have from Fig.6.16.8, d_1 = 557 mm and b_1 = 201 mm. From Eq.6.27,

$$\frac{0.87\, f_y\, A_{sv}}{s_v} = \frac{T_u}{b_1\, d_1} + \frac{V_u}{2.5\, d_1}$$

$$= \{105(10^6)/(201)(557)\} + \{70(10^3)/(2.5)(557)\}$$

$$= 988.13 \text{ N/mm}$$

From Eq.6.28 the least value of the above is $(\tau_{ve} - \tau_c)\, b$ = (3.5 - 0.678)(300) = 846.6 Nm^2. So, from 0.87 $f_y\, A_{sv}/s_v$ = 988.13 N/mm, we get

s_v = 0.87(415)(226)/988.13 = 82.57 mm

Step 5: Check for s_v

Figure 6.16.8 shows the stirrups of 12 mm diameter two legged for which x_1 = 238 mm and y_1 = 587.5 mm. The maximum spacing should be the least of x_1, $(x_1 + y_1)/4$ and 300 mm.

Here, x_1 = 238 mm, $(x_1 + y_1)/4$ = 206.375 mm and 300 mm. So, the spacing of 80 mm c/c is o.k. Provide 12 mm, 2 legged stirrups @ 80 mm c/c, as shown in Fig.6.16.8.

6.16.11 References

References:

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6.16.12 Test 16 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions. Each question carries five marks.

TQ.1: Explain and differentiate between primary and secondary types of torsion.

(10 marks)

A.TQ.1: Third, fourth and fifth paragraphs of sec. 6.16.2.

TQ.2: Explain the situations when do you provide longitudinal tension, compression and side face reinforcement in beam subjected to bending moment, shear and torsional moment.

(10 marks)

A.TQ.2: Sec. 6.16.7a, b and c parts.

TQ.3: Illustrate the steps of designing transverse reinforcement in beams subjected to bending moment, shear and torsional moment.

(10 marks)

A.TQ.3: Sec. 6.16.7d

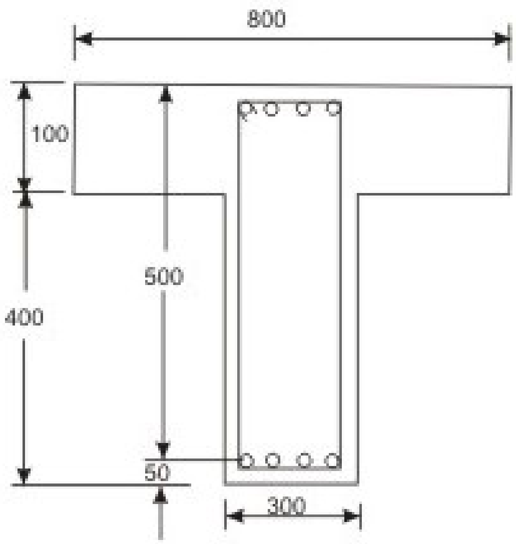

Fig. 6.16.9: Prob. TQ. 4

TQ.4: The beam of Fig.6.16.9 has factored bending moment M_u = 70 kNm, factored shear force V_u = 100 kN and factored torsional moment T_u = 60 kNm at one section. Design the reinforcement of that section assuming that the torsion is fully taken by the web. Assume M 30 concrete, Fe 500 for longitudinal and Fe 415 for transverse reinforcing steel bars.

(20 marks)

A.TQ.4:

Step 1: Checking of depth of the beam

From Eq.6.22, the equivalent shear

$$V_e = V_u + 1.6(T_u/b) = 100 + 1.6(60/0.3) = 420 \text{ kN}$$

From Eq.6.23, we get

$$\tau_{ve} = (V_e/bd) = 420/(0.3)(0.5) \text{ kN/m}^2 = 2.8 \text{ N/mm}^2$$

Table 6.2 of Lesson 13 gives $\tau_{c\max}$ = 3.5 N/mm^2 > τ_{ve}, So the depth is satisfying.

Step 2: Check if shear reinforcement is required

Assuming percentage of tensile steel as 0.5, Table 6.1 of Lesson 13 gives τ_c = 0.5 N/mm^2. Hence, both longitudinal and transverse reinforcements shall be provided.

Step 3: Longitudinal tension reinforcement

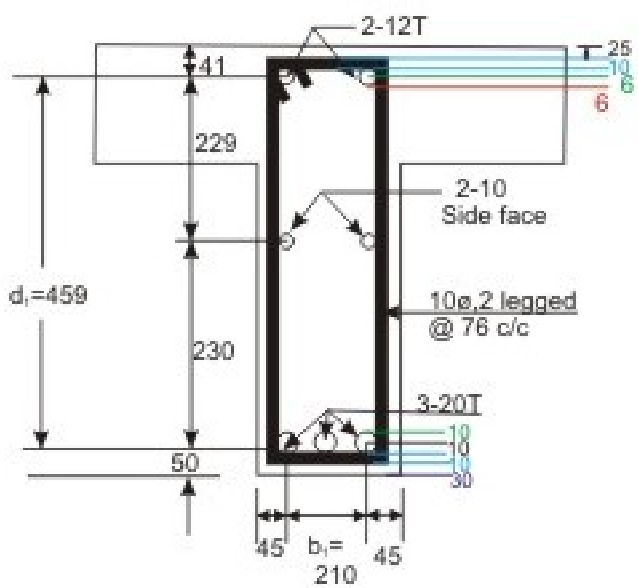

Fig. 6.16.10: Longitudinal (tension, compression & side face), and transverse reinforcement of TQ. 4

From Eqs.6.24 and 6.25, we have,

$$M_{e1} = M_u + M_t = M_u + (T_u/1.7)\{1 + (D/b)\}$$

$= 70 + (60/1.7)\{1 + (550/300)\} = 70 + 100 = 170$ kNm.

$M_{e1}/bd^2 = 170(10^6)/(300)(500)(500) = 2.267$ N/mm^2

Table 4 of SP-16 or M 30, Fe 500 and $M_u/bd^2 = 2.267$ N/mm^2, we have by linear interpolation $p_t = 0.577$.

$A_{st} = 0.577(300)(500)/100 = 865.5$ mm^2.

Minimum A_{st} (Eq.6.29) = 0.85 bd/f_y = 0.85(300)(500)/(500) = 255 mm^2.

Maximum A_{st} = 0.04 bD = 0.04(300)(550) = 6600 mm^2.

So, A_{st} = 865.5 mm^2 is acceptable.

Provide 3-20T (area = 942 mm^2) giving p_t = (942)(100)/(300)(500) = 0.628% (Fig. 6.16.10). Table 6.1 of Lesson 13 gives τ_c = 0.546 N/mm^2 (for p_t = 0.628%).

Step 4: Longitudinal compression reinforcement

Here, the numerical value of M_t (= 100 kNm) is greater than that of M_u (= 70 kNm), as computed in Step 3. So, compression reinforcement shall be provided for $M_{e2} = M_t - M_u = 100 - 70 = 30$ kNm. M_{e2}/bd^2 = 30(10^6)/(300)(500)(500) = 0.4 N/mm. Table 4 of SP-16 gives p_c = 0.093 to have compression steel reinforcement A_{sc} = 0.093(300)(500)/100 = 139.5 mm^2. Maximum area compression steel = 0.04 bD = 0.04(300)(550) = 6600 mm^2. Hence, A_{sc} = 139.5 mm^2 is satisfying. Provide 2-12T (area = 226 mm^2) as compression steel (Fig. 6.16.10).

Step 5: Side face reinforcement

Since the depth of the beam exceeds 450 mm, provide 2-10 mm T (area = 157 mm^2) near the mid-depth, one on each side of the beam with maximum spacing = 230 mm (Fig.6.16.9). Area required = 0.1(300)(230)/100 = 69 mm^2 < 157 mm^2. Hence, two 10 mmT bars as shown in Fig.6.16.10 is o.k.

Step 6: Transverse reinforcement

Using 10 mm T, 2 legged stirrups, we have d_1 = 459 mm and b_1 = 210 mm (Fig.6.16.10). From Eq.6.27, we have,

$$\frac{0.87\, f_y\, A_{sv}}{s_v} = \frac{T_u}{b_1\, d_1} + \frac{V_u}{2.5\, d_1}$$

$= 60(10^6)/(210)(459)\} + 100(10^3)/(2.5)(459)\}$

$= 709.61$ N/mm^2

However, Eq.6.28 gives the least value of $0.87\ f_y\ A_s/s_v = (\tau_{ve} - \tau_c)\ b$ = (2.8 - 0.546)(300) = 676.2 N/mm. So, $0.87\ f_y\ A_{sv}/s_v = 709.61$ gives

$$s_v = 0.87\ f_y\ A_{sv}/709.61 = 0.87(415)(157)/709.61 = 79.88 \text{ mm}$$

Step 7: Checking of s_v

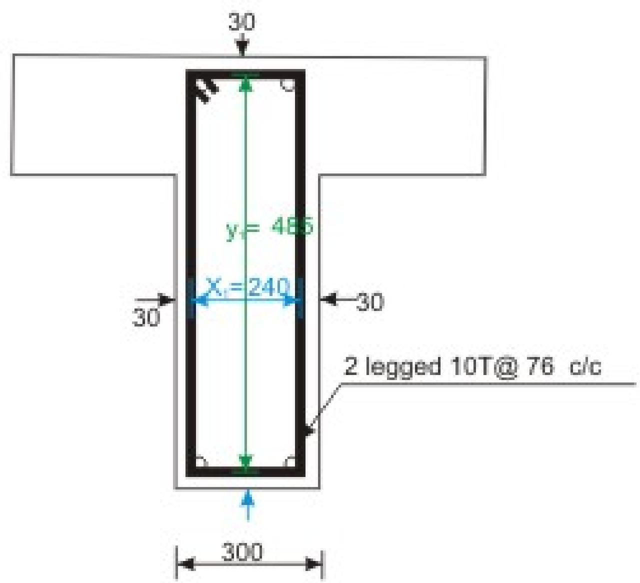

Fig. 6.16.11: Transverse reinforcement of TQ. 4

Figure 6.16.11 shows x_1 = 300 - 2(25 + 5) = 240 mm and y_1 = 550 - 30 - 35 = 485 mm. Accordingly, the maximum spacing is the least of x_1, $(x_1 + y_1)/4$ or 300 mm. Here, the least value is 181.25 mm. Hence s_v = 76 mm is acceptable (Fig.6.16.10).

6.16.13 Summary of this Lesson

This lesson explains the different situations when beams and frames are subjected to combined bending, shear and torsional moment. Briefly discussing about the analysis of such beams, design of beams as per IS 456 has been illustrated with numerical examples. The requirements of IS 456 have been specified. Solutions of practice and test problems will help the students in understanding the theory and designing the beams subjected to combined bending moment, shear and torsional moment.