UNIT – 4

FORCES ON IMMERSED BODIES

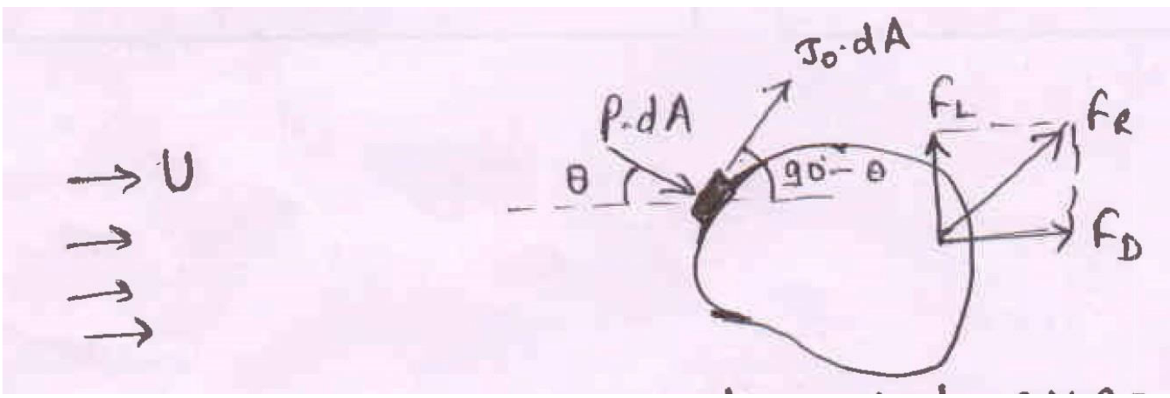
Lecture-01

Forces on immersed bodies

When a body is immersed in a real fluid, which is flowing at a uniform velocity U, the fluid will exert a force on the body. The total force (F_R) can be resolved in two components:

1. Drag (F_D): Component of the total force in the direction of motion of fluid.
2. Lift (F_L): Component of the total force in the perpendicular direction of the motion of fluid. It occurs only when the axis of the body is inclined to the direction of fluid flow. If the axis of the body is parallel to the fluid flow, lift force will be zero.

Expression for Drag & Lift



Forces acting on the small elemental area dA are:

i. Pressure force acting perpendicular to the surface i.e. p dA
ii. Shear force acting along the tangential direction to the surface i.e. τ_0dA

(a) <u>Drag force (F_D) :</u>

Drag force on elemental area = p dAcosθ + τ_0 dAcos(90 – θ) = p dAcosθ + τ_0dAsinθ

Hence Total drag (or profile drag) is given by,

$$F_D = \int p \ \cos\theta \ dA + \int \tau_0 \ \sin\theta \ dA$$

Where

$\int p \ \cos\theta \ dA$ = pressure drag or form drag, and

$\int \tau_0 \ \sin\theta \ dA$ = shear drag or friction drag or skin drag

(b) <u>Lift force (F_L) :</u>

Lift force on the elemental area = – p dAsinθ + τ_0 dA sin(90 – θ) = – p dAsinθ + τ_0dAcosθ

Hence, total lift is given by

$$F_L = \int \tau_0 \ \cos\theta \ dA - \int \mathrm{p} \sin\theta \ dA$$

The drag & lift for a body moving in a fluid of density ρ at a uniform velocity U are calculated mathematically as

$$F_D = C_D \rho A \frac{U^2}{2}$$

And

$$F_L = C_L \rho A \frac{U^2}{2}$$

Where A = projected area of the body or largest project area of the immersed body.

C_D & C_L = coefficient of drag & lift respectively. The equations for F_D & F_L are derived using dimensional analysis.

Then the resultant force is given by

$$F_R = \sqrt{F_D^2 + F_L^2}$$

Drag Coefficient (C_D)

- It is a dimensionless quantity used to quantify the drag or the resistance of an object in a fluid environment. Lower the C_D lesser will be the drag force.
- It is not an absolute constant for a given body shape. It varies with the speed of air flow (or more generally with the Reynold's number). e.g. a smooth sphere has a C_D that varies from high values for laminar flow to 0.47 for turbulent flow. Although the C_D decreases with increase in Reynold's number, the drag force increases.

Lift Coefficient (C_L)

- It is a dimensionless coefficient that relates the lift generated by a lifting body to the fluid density around the body, the fluid velocity and the associated projected area.
- It is a function of angle of attack (i.e. the angle between the axis of the body and the direction of flow), Reynold's number of the flow and Mach no.

S.No.	RGPV questions	Year	Marks
Q.1	Define total drag & Lift.	June 2015, Dec 2012	2, 2
Q.2	Define coefficient of drag & lift.	Dec 2014	2

Unit-04/Lecture-02

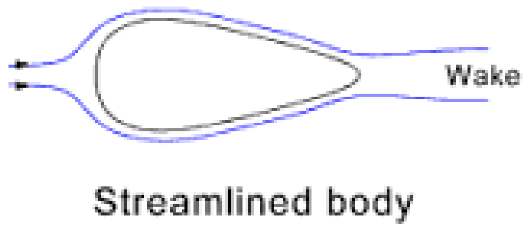
Types of drag

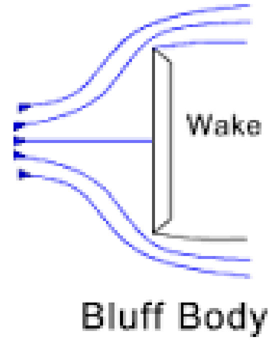
1. Pressure drag:
    - It is the drag which acts perpendicular to the surface of a body which is immersed in a fluid.
    - When a fluid flows through a *bluff body* whose surface is no-where parallel to the main stream flow, the fluid starts to leave the body surface & separation of flow occurs, due to which, a region of low pressure is developed on the downstream side of the body. This region is called "wake". The flow in this region forms a series of eddies. The pressure difference produces a drag on the body, known as pressure drag or form drag.
    - For a sphere, the pressure drag is one third of the total drag.
2. Friction drag:
    - It is the shear force acting along the tangential direction to the surface of a body immersed in a fluid.
    - When a fluid flows through a submerged body, a layer of fluid comes in contact with the surface of the body& a region of velocity gradient is formed, called boundary layer. This layer produces a shear stress on the surface of the body tangentially, which is called friction drag or shear drag or surface drag.
    - For a sphere, the friction drag is two-third of the total drag.
3. Deformation drag:
    - It occurs when viscous forces are more predominant than inertia forces or when Reynold's number is less than 0.1.
    - Due to viscosity of fluid, deformation of fluid particles is caused and certain forces are necessarily developed, which offer additional resistance to the motion of fluid. The component of the forces in the direction of the motion is called deformation drag.
    - It is usually negligible at higher reynold's number.

Streamlined body

- A body whose surface coincides with the streamlines, when the body is placed in a flow. E.g. airfoil.
- Behind a streamlined body, wake formation is very less & consequently, the pressure drag will be small. Then the total drag on the streamlined body will be due to friction drag only.
- A body may be streamlined when placed at low velocity but may not be streamlined at higher

velocity.





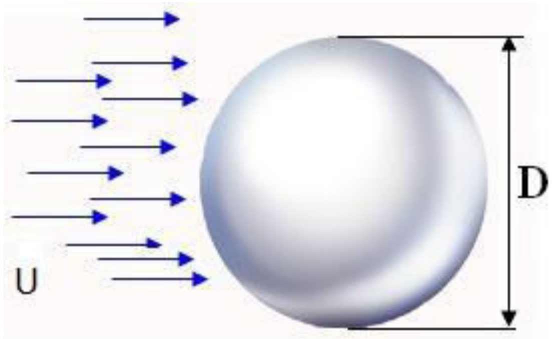
Bluff body

- A body whose surface does not coincide with the streamlines, when placed in a flow, called bluff body or blunt body. E.g. a flat plate placed perpendicular to the direction of flow.
- Formation of large wake zone (or eddies) occurs when a fluid flows through a bluff body, due to separation of flow.
- Pressure drag is very large as compared to friction drag in case of bluff bodies.

S.No.	RGPV questions	Year	Marks
Q.1	Differentiate between streamlined body & bluff body.	June 2015, Dec 2013	2, 4
Q.2	What is streamlining? What is its effect on the different types of drag?	Dec 2014, Dec 2011	2, 4
Q.3	Distinguish between the friction drag & pressure drag.	Dec 2013	4
Q.4	Define: (i) pressure drag, (ii) profile drag, (iii) Airfoil, (iv) wake or eddies	Dec 2012	4
Q.5	What are different types of drag?	Dec 2011, June 2009	6, 5
Q.6	Differentiate between form & surface drags.	Dec 2011	4

Unit-04/Lecture-03

Drag on a sphere



According to G.G. Stoke, up to Reynold's number 0.2, total drag on a smooth sphere is

$$\mathbf{F_D = 3\,\pi\,\mu\,D\,U}$$

Where μ = dynamic viscosity of fluid, D = diameter of sphere & U = velocity of flow.

Also, Skin friction drag is two-third of the total drag & Pressure drag is one-third of the total drag.

Expressions for C_D of a sphere:

1. **When $R_e < 0.2$**

   For sphere,

   $$F_D = 3\,\pi\,\mu\,D\,U \quad \text{... (i)}$$

   But

   $$F_D = C_D\,\rho\,A\,\frac{U^2}{2} = C_D\,\rho\,\frac{\pi}{4}D^2\frac{U^2}{2} \quad \text{... (ii)}$$

   Equating (i) & (ii), we get

   $$\mathbf{C_D = \frac{24\,\mu}{\rho\,U\,D}}$$

or

$$\mathbf{C_D = \frac{24}{R_e}}$$

Above equation is known as **Stoke's law**.

2. **When R_e is between 0.2& 5**

   $$C_D = \frac{24}{R_e}\left(1 + \frac{3}{16\,R_e}\right)$$

3. **When R_e is between 5 & 1000**

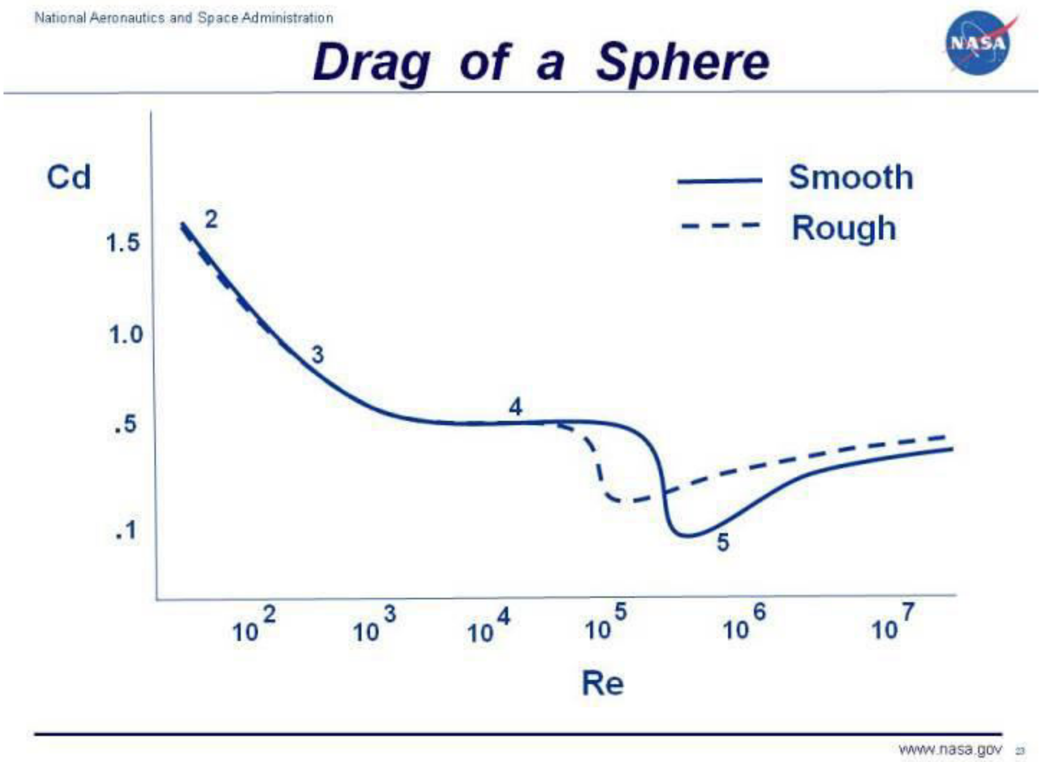
   $$C_D = 0.4$$

4. When R_e is between 1000 & 10^5

$$C_D = 0.5$$

5. When $R_e > 10^5$

$$C_D = 0.2$$



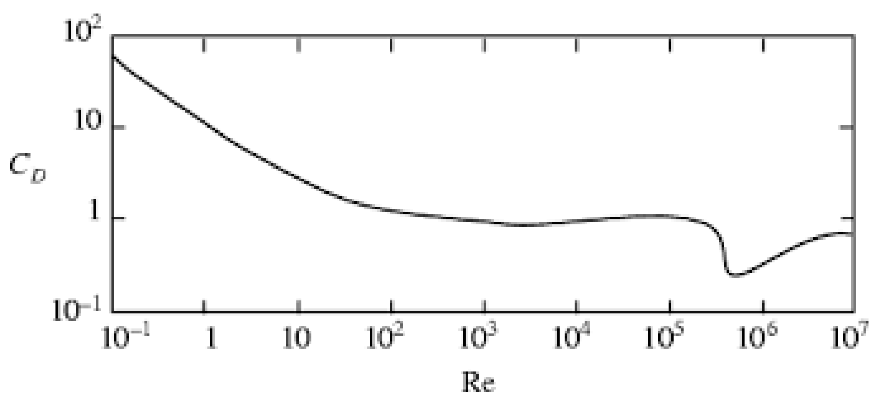
S.NO	RGPV QUESTIONS	Year	Marks
Q.1	Prove that the value of C_D for sphere for Reynold's number less than 0.2 is given by $C_D = 24/R_e$.	June 2015	7
Q.2	Explain with neat sketch the relationship between drag coefficient &reynold's number for smooth spheres.	Dec 2011	6

Unit-04/Lecture-04

Drag on a cylinder

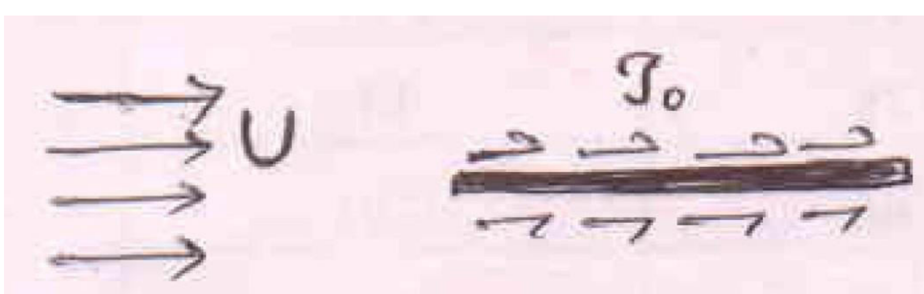
When a cylinder is placed in a fluid such that its length is perpendicular to the direction of flow, the drag force (F_D) depends upon the Reynold's number (R_e) of the flow. From experiments, it has been observed that

1. When $R_e < 1$, F_D is directly proportional to the velocity & hence C_D is inversely proportional to R_e.
2. With increase of Reynold's number from 1 to 2000, C_D decreases & reaches a minimum value of 0.95 at $R_e = 2000$.
3. With further increase of R_e from 2000 to 3×10^4, C_D increases & attains a maximum value of 1.2 at $R_e = 3 \times 10^4$.
4. When R_e is increased from 3×10^4 to 3×10^5, C_D further decreases. At 3×10^5, $C_D = 0.3$.
5. If R_e is increased beyond 3×10^6, C_D increases & it becomes equal to 0.7 in the end.



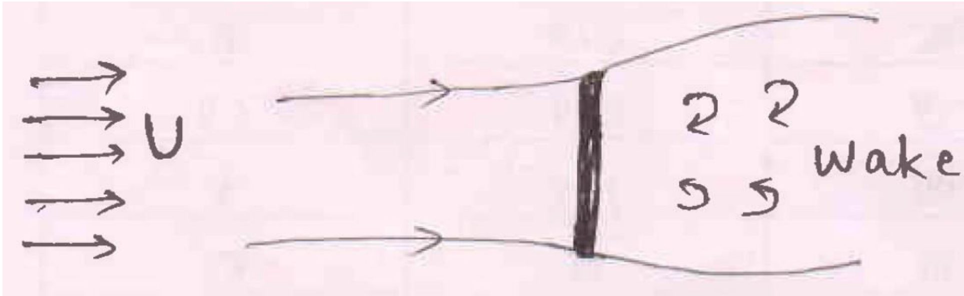
Drag on a flat plate:

1. **When plate is placed parallel to the direction of flow**



- $\int p \cos\theta \, dA = 0$
- total drag is equal to friction drag.

2. When plate is placed perpendicular to the direction of flow



- $\int \tau_0 \sin\theta \, dA = 0$
- Total drag is equal to pressure drag.

3. When plate is placed at an angle with the direction of flow other than 0^o& 90^o

- Total drag = pressure drag + shear drag

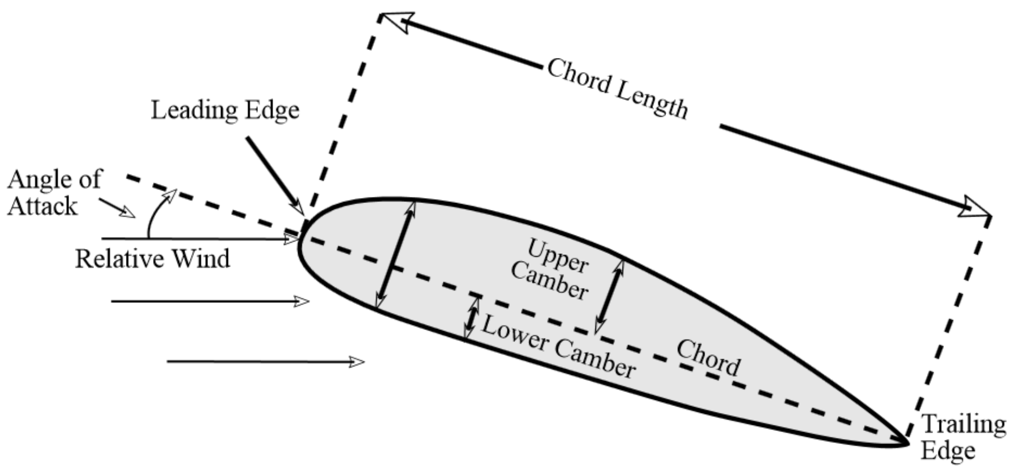
Unit-04/Lecture-05

Development of Lift of an airfoil

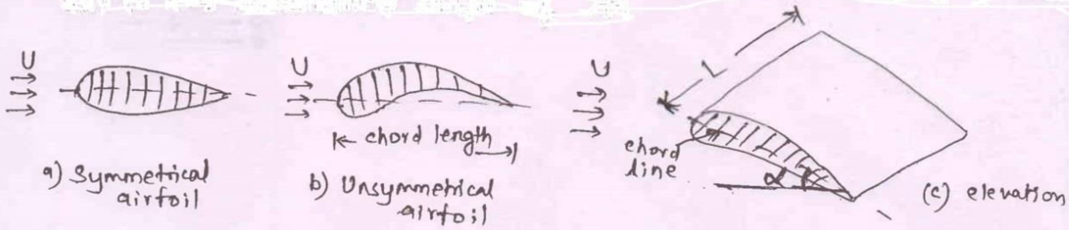
An airfoil or aerofoil is the shape of a wing or blade(of a propeller, rotor, orturbine) or sail as seen in cross-section. An airfoil-shaped body moved through a fluid produces an aerodynamic force. The component of this force perpendicular to the direction of motion is called lift.

Airfoils are the streamlined bodies which may be symmetrical or unsymmetrical in shapes. The airfoil is characterized by its

- chord length C,
- angle of attack α (which is the angle between the direction of the fluid flowing & chord line) &
- span L of the airfoil.



The lift of an airfoil is developed due to negative pressure created on the upper part of the airfoil. The drag force on an airfoil is always small due to the streamlined shape of it.



Circulation of an airfoil is given by, $\gamma = \pi \cdot c \cdot U \cdot \sin\alpha$ —— (1)

By Kutta-Joukowski, lift force is given by

$$F_L = \rho \cdot \gamma \cdot U \cdot L$$

$$= \rho(\pi \cdot c \cdot U \cdot \sin\alpha)\, U \cdot L \quad \text{(from (1))} \quad \text{—— (2)}$$

But $F_L = C_L \cdot \dfrac{\rho \cdot A \cdot U^2}{2}$ —— (3)

On equating (2) & (3), we get

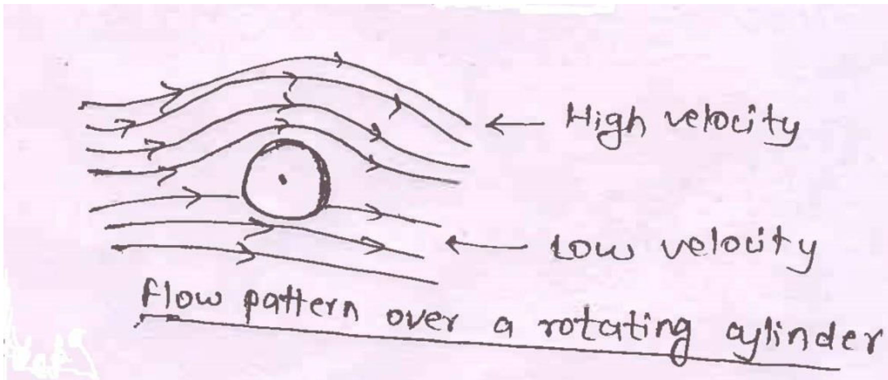
Lift coeff. of an airfoil $\boxed{C_L = 2\pi \sin\alpha}$

S.NO	RGPV QUESTIONS	Year	Marks
Q.1	What is an airfoil? On what factors does the total drag on an airfoil depends?	June 2015	3
Q.2	Define: (i) pressure drag, (ii) profile drag, (iii) Airfoil, (iv) wake or eddies	Dec 2012	4

UNIT 04/LECTURE 06

Magnus effect

- When a cylinder is rotated in a uniform flow, a lift force is produced on the cylinder. This phenomenon is called Magnus effect.
- This fact was investigated by a German physicist H.G. Magnus.
- Due to circulation, the flow is unsymmetrical on a cylinder as shown in the following figure.



- The velocity on the upper half is increased & on the lower half is decreased. Hence pressure on the upper half is decreased & on the lower half is increased. Hence, the lift force on the cylinder is produced when rotated in a uniform flow field.

S.No.	RGPV questions	Year	Marks
Q.1	What is magnus effect? Explain clearly.	Dec 2014, June 2009	3, 5
Q.2	Define Magnus effect.	Dec 2012	2

UNIT 4/ LECTURE 7

Numericals

Q.1 An aeroplane weighing 40 kN is flying in a horizontal direction at 340 km/h. The plane spans 15m & has wing surface area of 35 m². Calculate lift coefficient & the power required to drive the plane. Assume $c_D = 0.03$ & $\rho_{air} = 1.2\ kg/m^3$. [RGPV June 2009]

Soln Equilibrium condition: weight of the plane should be equal to lift force.

$F_L = W$

Now $W = F_L = C_L \cdot \dfrac{\rho \cdot A \cdot U^2}{2}$

$\Rightarrow \quad 40 \times 10^3 = \dfrac{C_L \cdot (1.2)(35)(94.44)^2}{2}$ $\quad [\because 340\ km/h = 94.44\ m/s = U]$

$\Rightarrow \quad \boxed{C_L = 0.214}$

Power required to drive the plane, $P = F_D \cdot U$

$= C_D \cdot \dfrac{\rho \cdot A \cdot U^2}{2} \cdot U$

$= \dfrac{(0.03)(1.2)(35)(94.44)^2}{2} \times (94.44)$

$\boxed{P = 530.64\ kW}$

Q.2 On a flat plate of 2m×1m experiments were conducted in a wind tunnel of 50 km/h. The plate is kept at such an angle that C_D & C_L are 0.18 & 0.9 respectively. Determine drag force, lift force, resultant force & power exerted by airstream on the plate. Take $\rho_{air} = 1.15\ kg/m^3$. [RGPV Dec 2013]

Soln: $U = 50\ km/h = 13.89\ m/s, \quad A = 2\times1 = 2\ m^2$

* $F_D = C_D \cdot \dfrac{\rho \cdot A \cdot U^2}{2} = \dfrac{(0.18)(1.15)(2)(13.89)^2}{2} = \underline{39.94\ N}$
* $F_L = C_L \cdot \dfrac{\rho \cdot A \cdot U^2}{2} = \dfrac{(0.9)(1.15)(2)(13.89)^2}{2} = \underline{199.68\ N}$
* $F_R = \sqrt{F_D^2 + F_L^2} = \sqrt{(39.94)^2 + (199.68)^2} = \underline{203.64\ N}$
* Power exerted, $P = F_D \cdot U = 39.94 \times 13.89 = \underline{554.77\ W}$

Q.3 An airplane flies at 250 km/h at an altitude of 4km. Determine lift & drag forces for an angle of attack of 8°. Also calculate power required to overcome drag. Take $C_L = 1.0$, $C_D = 0.065$, $\rho = 0.67\ kg/m^3$. The plane has a span of 11m & chord length of 1.83 m. [RGPV Dec. 2011]

Soln Given: $U = 250\ km/h = 69.44\ m/s$, $C = 1.83m$, $L = 11m$, $\alpha = 8°$

$A = L \times C = (11 \times 1.83) = 20.13$

Circulation, $\gamma = \pi \cdot C \cdot U \cdot \sin\alpha$

$= \pi (1.83)(69.44) \sin 8° = 55.56$

$\therefore$ Lift force, $F_L = \rho \cdot \gamma \cdot UL = (0.67)(55.56)(69.44)(11) = \underline{28434\ N}$

Drag force, $F_D = C_D \cdot \frac{\rho \cdot A \cdot U^2}{2} = \frac{(0.065)(0.67)(20.13)(69.44)^2}{2} = \underline{2113.6\ N}$

Power required, $P = F_D \times U = (2113.6)(69.44) = \underline{146.7\ kW}$

Q.4 Electrical transmission towers are stationed at 0.4 km intervals & a conducting cable 1.25 cm in diameter is stretched between them. If a wind of 100 km/h blows transversely across the wires, compute the total drag force on such a wire between two towers. The drag coeff. for $Re > 10^9$ may be taken as 1.20. [RGPV - Dec 2012]

Soln Projected area, $A = L \times D = 0.4 km \times 1.25 cm$

$= 400 m \times 0.0125 m = 5 m^2$

Assume $\rho_{air} = 1.2\ kg/m^3$.

Drag force, $F_D = \frac{C_D \cdot \rho \cdot A \cdot U^2}{2}$

$= \frac{1.2 \times 1.2 \times 5 \times (27.78)^2}{2}$ $[\because U = 100 km/h = 27.78\ m/s]$

$\underline{F_D = 2778.2\ N}$

Q.5: A kite 0.8 m × 0.8 m weighing 0.4 kgf assumes an angle of 12^o to the horizontal. The string attached to the kite makes an angle of 45^o to the horizontal. The pull on the string is 2.5 kgf when the wind is flowing at a speed of 30 km/h. Find the corresponding coefficient of drag and lift. Density of air is 1.25 kg/m^3.

Soln

Projected area of kite, A = 0.8 × 0.8 = 0.64 m^2

Weight of kite, W = 0.4 kgf = 0.4 × 9.81 = 3.924 N

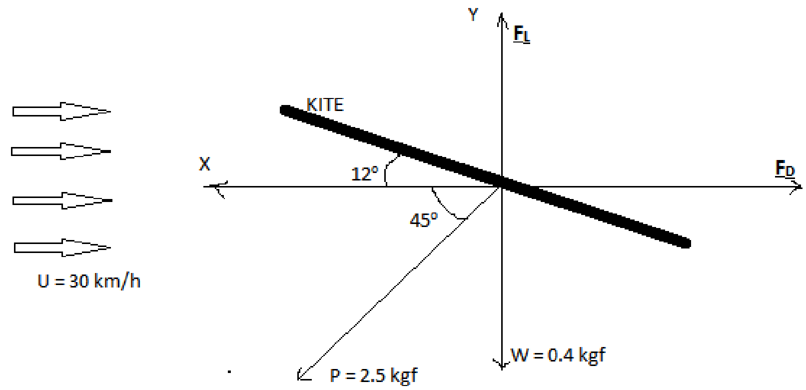
Pull on the string, P = 2.5 kgf = 2.5 × 9.81 = 24.525 N

Angle made by kite with horizontal, $\theta_1 = 12^o$

Angle made by stri ng with horizontal, $\theta_2 = 45^o$

Speed of wind, U = 30 km/h = 8.333 m/s

Density of air, ρ = 1.25 kg/m^3



Drag force, F_D = Force exerted by wind in the direction of motion (X-X direction) = component of pull, P along X-X direction = **P cos45°** = 24.525 cos 45° = **17.34 N**

Lift force, F_L = Force exerted by wind on the kite perpendicular to the direction of motion (Y-Y direction) = component of P in vertically downward direction + Weight of kite (W) = **P sin45° + W** = 24.525 sin45° + 3.924 N = **21.264 N**

Hence, **Coefficient of drag, C_D** = $(2 F_D) / (A \rho U^2) = (2 \times 17.34) / (0.64 \times 1.25 \times 8.333^2)$ = **0.624**

And

Similarly, **Coefficient of lift, C_L** = $(2 F_L) / (A \rho U^2) = (2 \times 21.264) / (0.64 \times 1.25 \times 8.333^2)$ = **0.765**

Assignment:

Q.6 A kite weighing 0.8 kgf has an effective area of 0.8 m^2. It is maintained in air at an angle of 10^o to the horizontal. The string attached to the kite makes an angle of 45^o to the horizontal and at this position the value of C_D and C_L are 0.6 and 0.8 respectively. Find the speed of the wind and tension in the string. Take density of air as 1.25 kg/m^3.

Terminal Velocity of a body

It is defined as the maximum constant velocity of a falling body (such as sphere or a composite body such as parachute together with man) with which the body will be travelling. When the body is allowed to fall from rest in the atmosphere, the velocity of the body increases due to acceleration of gravity. With increase of the velocity, the drag force, opposing the motion of the body, also increases. A stage is reached when the upward drag force acting on the body will be equal to the weight of the body. Then the net external force acting on the body will be zero and the body will be travelling at constant speed. This constant speed is called terminal velocity of the falling body.

If the body drops in a fluid, at the instant it has acquired terminal velocity; the net force acting on the body will be zero. The forces acting on the body at this state will be (1) weight of the body (W) acting downward, (2) Drag force (F_D) acting vertically upward and (3) Buoyant force (F_B) acting vertically upward.

The net force on the body should be zero, i.e. $W = F_D + F_B$

Q.7 A metallic ball of diameter 2 × 10^{-3} m drops in a fluid of specific gravity 0.95 and viscosity 15 poise. The density of the metallic ball is 12000 kg/m^3. Find (i) the drag force exerted by the fluid on the ball, (ii) the pressure drag and skin drag, (iii) the terminal velocity of the ball in the fluid.

Soln:

Diameter of metallic ball, D = 2 × 10^{-3}m

Density of ball = 12000 kg/m^3

Density of fluid = sp. gr. of fluid × 1000 = 0.95 × 1000 = 950 kg/m^3

Viscosity of the ball, μ = 15 poise = 1.5 Ns/m^2

Weight of the ball, W = density of ball × g × volume of ball = 12000 × 9.81 × (πD^3)/6 = 0.000493 N

Buoyant force, F_B = density of fluid × g × volume of ball = (0.95 × 1000) × 9.81 × (πD^3)/6 = 0.000039 N

When metallic ball reaches the terminal velocity, $W = F_D + F_B$

Or

(i) **Drag force, F_D** = W – F_B = 0.000493 – 0.000039 = **0.000454 N**

(ii) **Pressure drag = (1/3) F_D = 0.0001513 N**

Skin drag = (2/3) F_D = 0.0003028 N

(iii) **Terminal velocity of the ball, U** = F_D / (3πμD) = 0.000454 / (3 π × 1.5 × 2 × 10^{-3}) = **0.016 m/s**

Now checking the Reynold's number, R_e = (ρ U D)/ μ = (950 × 0.016 × 2 × 10^{-3}) / 1.5 = 0.02

Hence, since R_e < 0.2, then the expression F_D = 3πμDU for calculating terminal velocity of a sphere is valid.