Development of Surfaces 10

LEARNING OBJECTIVES

After reading this chapter, you will be able to understand the following:

- Purpose of development.
- Parallel line, radial line and triangulation method.
- Development of prism.
- Development of cylinder.
- Development of pyramid surface for both right angled and oblique solids.
- Development of surface of sphere.

10.1 Introduction

Development of surfaces means the true sizes and shapes of different surfaces spread out on a plane. The surface to be developed is flattened but without tearing or folding. Thus, the complete layout of a dimensional object on a plane surface is called **development.** For example, when a paper wrapped or folded along with a rectangular prism as shown in Fig. 1, is opened and spread out on the surface, the resulting sheet formed is called the development of surface of prism. The development can be visualized as a process of drawing successive surfaces of the object in their true shape and size with common edges joined. The object is obtained when the surface is folded along the fold lines. It is important that every line on the development is the true length of the corresponding edge on the surface.

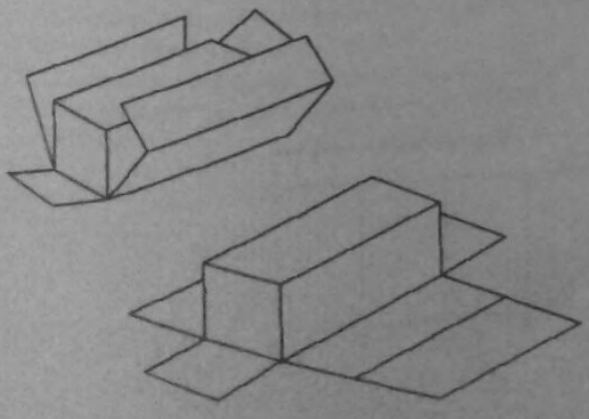


Figure 1

The knowledge of development of surfaces finds applications in many industries such as sheet metal work, packaging, aircraft and automobile design etc. These are classified into the following areas:

1. **Packaging industry:** Packaging uses the principles of surface development in a notable way. Most products we buy today are properly packaged in cardboard or plastic or metal containers. They range from a chocolate bars to shoes to refrigerators.
2. **Sheet metal development:** Sheet metal work is used for making hoppers, funnels, ducts, chimneys, boilers, etc. It involves bending, folding, rolling and cutting of metal sheets to make metal objects. For construction of these objects, metal sheets or plates are marked and cut according to the principles of development of surface, such that when folded the object of desired shape is obtained. The sheets that are marked for shaping or cutting to make all the outer surfaces of a specific object are called **development** or **pattern** of the piece. Suitable margins are left for joints.
3. **Other applications:** Development of surfaces has numerous other applications in
   (a) Revolving solids: prisms, pyramids, cones, cylinders.
   (b) Aircraft industry, ship building, etc.
   (c) Air-conditioning ducts, heat chambers, chimneys, ventilating pipes, etc.

Generally, the surfaces of polyhedra such as cubes, prisms and pyramids and curved surfaces such as cones and cylinder can be developed accurately. Double-curved (surface of revolution with only curved elements) and wrapped surfaces (generated by a moving line in a manner that its two adjacent positions do not lie in one plane) are theoretically

undevelopable. In this chapter, we will learn about various methods of development and apply those to development of lateral surfaces of solids like cubes, prisms, cylinders, pyramids, cones and spheres. We will also learn about the development of surfaces with cut and slots of above solids.

10.2 Methods of Development

The principle of development requires that the length of every line of development should be same as the corresponding line of the developed. The true length of the edge has thus to be found out first, and while drawing the development only the true length should be used. The length of the pattern is known as **stretchout** or **girth line** and is given by the perimeter of the object measured in a plane at right angle to the axis of the object. It is of importance in case of objects that have constant cross-section for the full-length.

10.2.1 Methods of Development of Lateral Surfaces (DLS)

In development of surfaces, generally lateral surfaces are developed and ends and bases are omitted. These can be incorporated if specifically required. In development of lateral surfaces, the staring and closing edges should be the same to obtain closed objects.

The following are the main methods for development of surfaces:

1. **Parallel line development**
   - Used to develop objects (or part of them) which have a constant cross-section throughout the length of the object, such as cubes, prisms, and cylinders.
   - Lines parallel to axis of detail are shown in the view that projects them as true lengths.
   - Prism and cylinder when stretched out result into rectangular sections.
   - For prism the corners of the details serve as surface line and for cylinders these are obtained by dividing the circumference into equal number of parts (generally twelve).
   - The development of a square prism and a cylinder by this method is illustrated in Fig. 2.
2. **Radial line development**
   - Used to develop right and oblique pyramids and cones.
   - Apex is taken as centre and true length of the slant edge or generator as radius (radial lines) is used for the development.
   - For cones, equally spaced radial lines are obtained by dividing the circular top view of the cone base into equal parts (generally twelve) (Fig. 3).

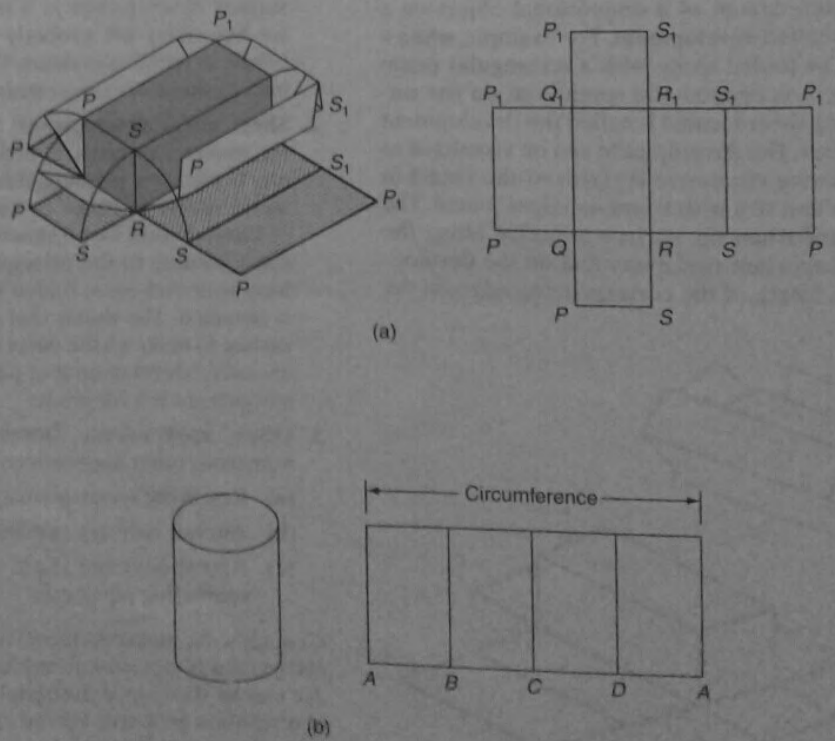


Figure 2 Development of lateral surface by parallel method for (a) prism and (b) cylinder.

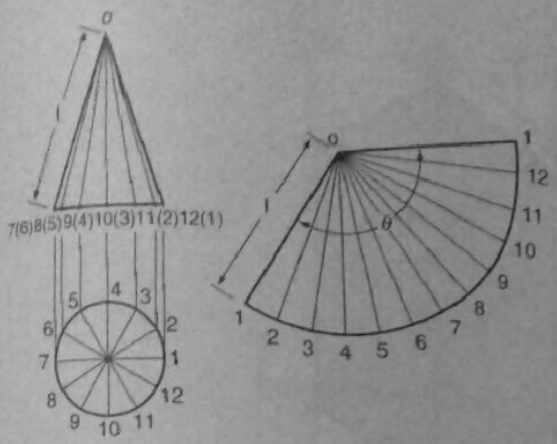


Figure 3 Development of right circular cone.

3. **Triangulation development**
   - Used to develop transition pieces; that is, solids with bottom and top bases of dissimilar shapes (Fig. 4).
   - For development by this method, the object is divided into a number of triangles, drawn side by side in order that generates the pattern.
   - The true sizes of the triangles are determined by first obtaining the true length of their sides.
4. **Approximate development**
   - Used to double-curved surfaces like spheres, as they are theoretically un-developable in a single plane.
   - Two standard methods used for development of spheres are: zone and Lune or Gore methods.

10.2.2 | Wiring, Hemming and Seaming

It is important to know about the processes of wiring, hemming and seaming in drawing developments.

1. **Wiring:** Reinforces open ends of an article by enclosing a wire in the edge.
2. **Hemming:** Hems are commonly used to reinforce an edge, hide rough edges, and improve appearance. Hemming involves single and double hemming of edges.
3. **Seaming:** One of the most important processes in sheet metal working is that of seaming. Seams of various kinds are used to join edges. Some of them are
   (a) *Lap seam* – used in the construction of small cylinders, square pipes, etc. which is usually soldered or riveted.
   (b) *Folded seam* – used in laying flat seam tin and copper roofing.
   (c) *Grooved seam* – is the universally used method of joining the edges of sheet metal for light material. It is used in joining two flat sheets of metal, making longitudinal seams in round and square pipes, and vertical side seams in sheet metal articles having a flaring or cylindrical surface.

10.3 Development of Prisms

10.3.1 | Development of Oblique Prisms

In an oblique prism, the axis of the solid is not at right angles to the base. The lateral surface of oblique prism is developed by parallel line method as in case of right prisms (Fig. 5). The following cases are possible:

1. Axis parallel to VP and inclined to HP: In this case the front view shows true length of all the lateral edges but the true size of the right section needs to be located. This can be done by the following steps:
   (i) Draw right section as an edge in the front view perpendicular to the fold lines.
   (ii) Draw auxiliary plane perpendicular to the axis or fold line.
   (iii) The top view of the edge drawn in step (i), in the auxiliary plane will give the true size of the right section.
2. Axis of the oblique plane inclined both to HP and VP: In this case true length and true size of the right section are obtained by the following two steps respectively:
   (i) Primary front view is drawn to obtain the true length of the lateral edges.
   (ii) Secondary top view is drawn on an auxiliary plane drawn perpendicular to to the axis of the primary front view.

These steps are illustrated by the following solved examples.

10.3.2 | Development of Right Prisms

If a square prism with axis height H and edge of base E is rolled for one complete revolution on a plane, it moves the distance equal to the perimeter of its base polygon, and area covered by it is equal to its development of lateral

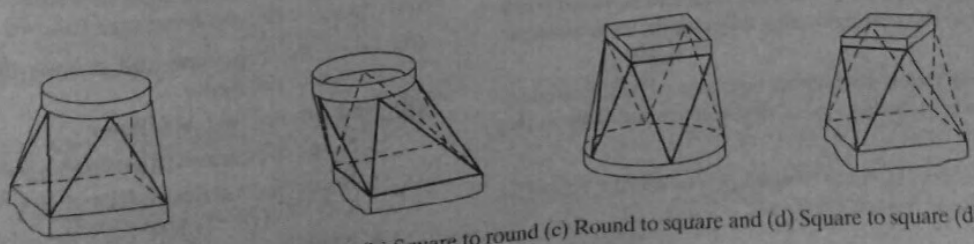


Figure 4 Transition objects: (a) Rectangular to round, (b) Square to round (c) Round to square and (d) Square to square (different size).

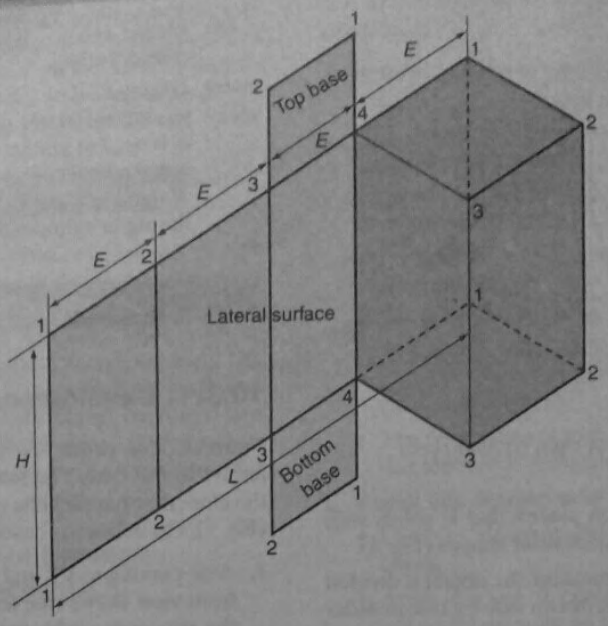


Figure 5 Development of a prism.

surface (DLS), which is equal to four times the area of a rectangular face of the square prism. Therefore, DLS of a prism is obtained by placing a number of rectangular faces one adjacent to the other, equal to the number of edges of base. Hence, DLS of the prism is represented as follows:

Rectangle of size = Height × Perimeter of base

$$= H \times L$$
$$= H \times n \times E$$

where H is the height of prism, n the number of edges of base, L the perimeter of base, E the length of the edge of base of prism.

In general, the development of lateral surface of a prism consists of the same number of rectangles in contact as the number of sides in the base of the prism. One side of the rectangle is equal to the length of the axis and the other side is equal to the length of the side of the base.

Some important points to remember in development of prisms are:

1. The development of any prism is always drawn in a direction perpendicular to the axis.
2. The true length of the base can be obtained in the top view.
3. Any long edge will be of true length in front view and on development surface, when the prism is kept on one of its base.
4. Any point on the longer or base edge can be transferred directly to the corresponding edge on the development surface.
5. Any point that is not on long and base edge is plotted on development surface by setting a secondary line in the principal view.

The steps in development of right prisms are illustrated in the examples in Solved Problems.

Solved Problems

1. *Draw the development of the lateral surfaces of a right square prism of edge base 25 mm and axis 60 mm long.*

Solution: See Fig. 6. The steps involved in development of surface are listed as follows.

(i) Draw the top and front views of the square prism.

(ii) Then the development is constructed by four equal rectangles of size 60 mm × 25 mm in contact and in sequence such that A_1A_1 = Perimeter of the prism base = Length of the front outline = $AA = AA_1$ = its height.

(iii) Divide AA and A_1A_1 into four equal divisions A_1B_1, B_1C_1, etc. of base 30 mm. Draw perpendicular line from B_1, C_1 and D_1 to get B, C, D.

(iv) Draw thick lines which give the development of the lateral surface of the square prism.

2. *A regular square prism, with a 25 mm base edge and a 50 mm axis height, is resting on base with axis perpendicular to HP, and two edges of base parallel to VP is to be wrapped by paper. Show development of lateral surface with fold mark on paper.*

Solution: Figure 7 shows DLS of a square prism. The procedure to solve this problem is as follows:

(i) Draw the top view and project the front view, as shown in the figure.

(ii) Name the points, as follows:

- Name the corners of prism as 1, 2, 3, 4 in the top view.
- Name the vertical edges of the front view as 1′-1′, 2′-2′, 3′-3′, 4′-4′, respectively.
- Name the vertical edges of DLS as 1-1, 2-2, 3-3, 4-4

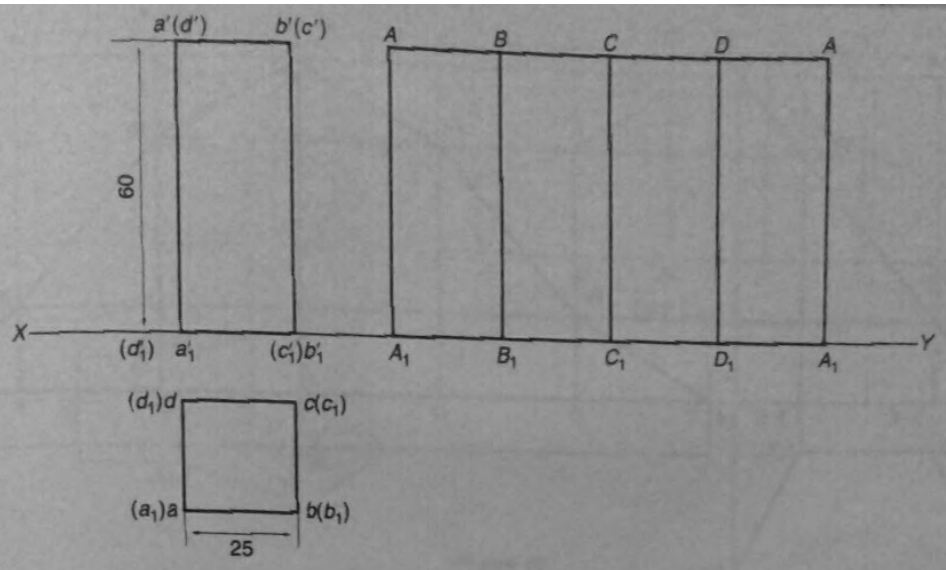


Figure 6

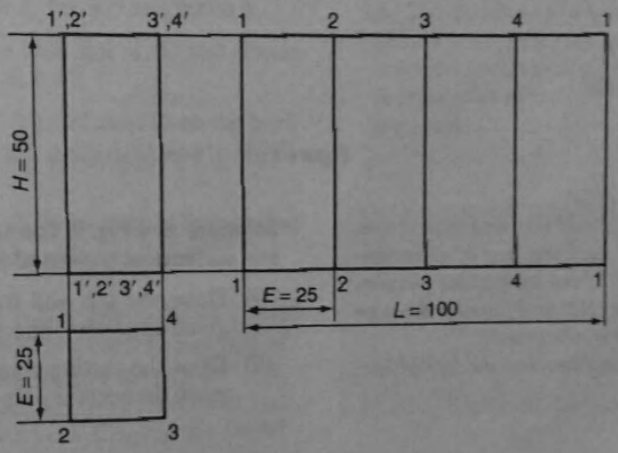


Figure 7

(iii) Draw a stretch line 1-1 equal to the length of perimeter (L) of square base for DLS directly in the line with the base of the front view; $L = E \times 4 = 25 \times 4 = 100$.

(iv) Divide the length 1-1 in four equal parts.

(v) Draw four equal rectangles in contact, each representing the rectangular face of prism.

3. *A hexagonal prism, having a 40 mm base side and a 90 mm axis height, has its two sides of base parallel to VP. A section plane cuts the prism such that the section plane passes through extreme left bottom corner to extreme right top corner of the prism. Show the DLS of the half-cut prism.*

Solution: Figure 8 shows DLS of a hexagonal prism. The steps in developing the lateral surface of the prism are listed as follows.

(i) Draw the top view and project the front view, as shown in the figure.

(ii) Draw DLS as discussed in Problem 2 (Initial construction line should be very thin).

(iii) Draw the cutting plane *S-S* passing through the extreme left bottom corner to the extreme right top corner.

(iv) Name the points, as follows:

- Name the corners of prism as 1, 2, 3, 4 in the top view, which represent the corners of the top base as well as of the bottom base.
- Name the vertical edges of the front view as 1′-1′, 2′-2′, 3′-3′, 4′-4′, 5′-5′, 6′-6′, respectively.
- Name the vertical edges of DLS as 1-1, 2-2, 3-3, 4-4, 5-5, 6-6.
- Name the point of intersection (common point) of cutting plane *S-S* and vertical edges in the front view as p_1', p_2', p_3', p_4', p_5', p_6', respectively. The cutting plane *S-S* cuts the vertical edges 1′-1′ at p_1'; 2′-2′ at p_2'; 3′-3′ at p_3'; 4′-4′ at p_4'; 5′-5′at p_5'; 6′-6′ at p_6' (vertical edge 1′-1′ will carry p_1'; 2′-2′ will carry p_2', and so on).
- Project p_1', p_2', p_3', p_4', p_5', p_6' horizontally and name the point of intersection as p_1, p_2, p_3, p_4, p_5, p_6, on respective vertical edges of DLS.

(v) Join all the points in proper sequence by straight line. Since prism is bound by rectangular plane surface, therefore, join all the points in sequence by straight line.

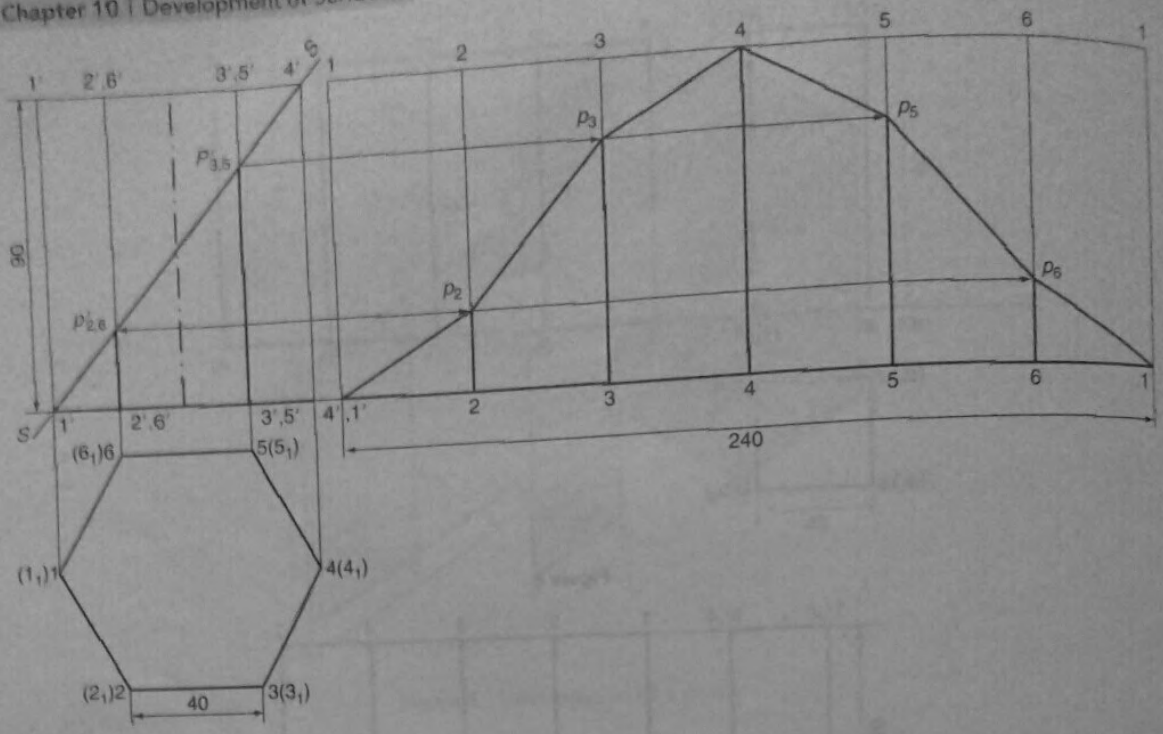


Figure 8

4. *A pentagonal prism, edge base 25 mm and axis 55 mm long, rests with its base on HP such that one of its rectangular faces is parallel to VP. It is cut by a plane perpendicular to VP inclined at 30° to HP and passing through the right corner of the top face of the prism.*
Draw the sectional top view and develop the lateral surfaces of the truncated prism.

Solution: See Fig. 9. The steps involved in developing lateral surfaces of truncated prisms are listed as follows.

(i) Draw the top and front views of the prism for the given dimension and position.

(ii) Draw the cutting plane *RS* inclined at 30° to HP and mark the section point 1′, 2′, 3′, (4′), (5′).

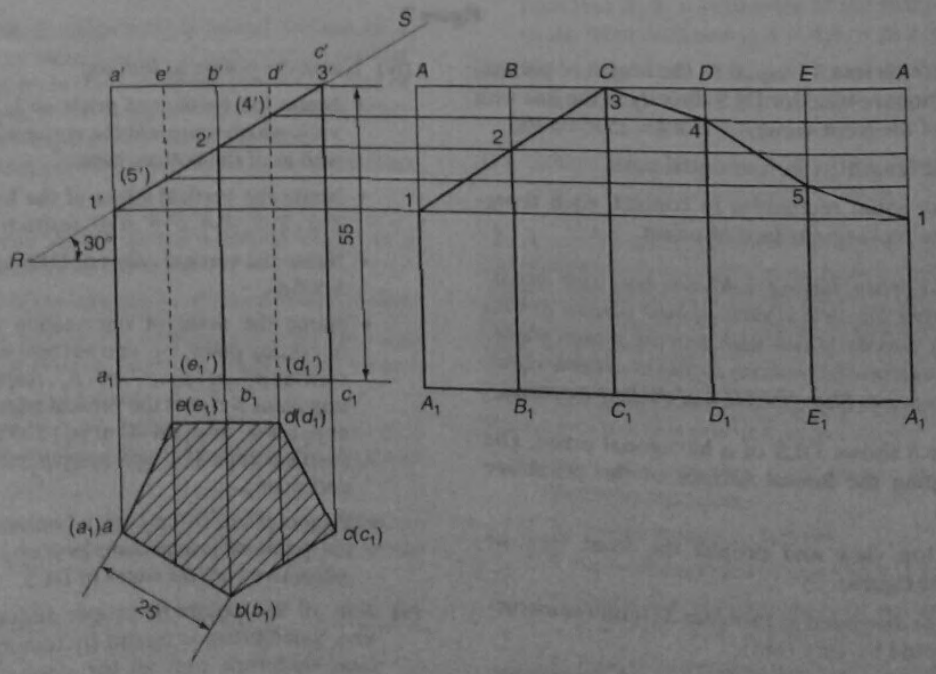


Figure 9

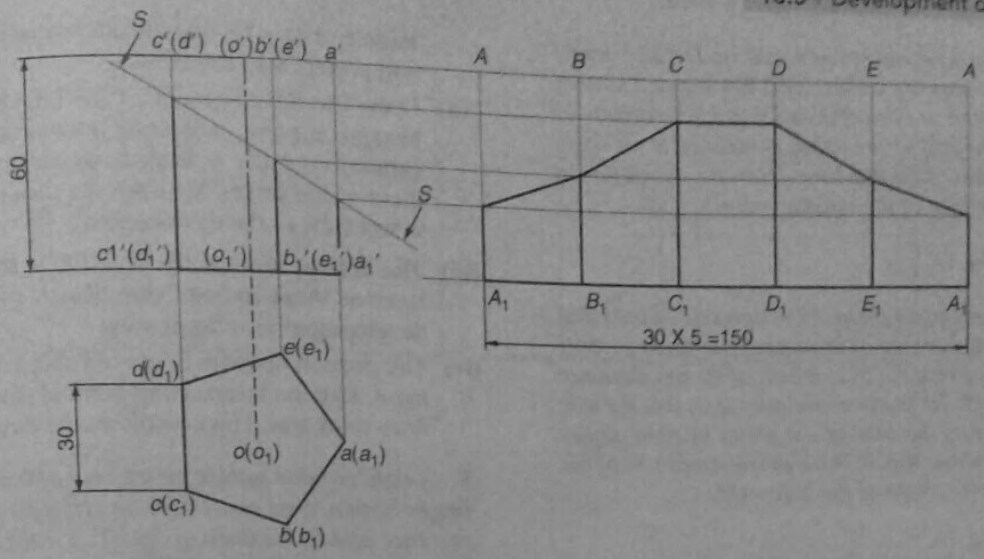


Figure 10

(iii) Draw the section top view 1, 2, 3, 4, 5 and hatch it.

(iv) Draw two stretched lines AA and A_1A_1 and divide into 5 equal parts of 25 mm each.

(v) From the section point 1′ in front view draw the horizontal line to meet AA_1 and mark 1. Similarly obtain the remaining points 2, ..., 5.

(vi) Join the points 12, 23, 34, 45, 51 in straight lines and so development is completed.

5. *A right regular pentagon prism of side 30 mm and 60 mm height is resting on its base on HP having one of its base edges perpendicular to the VP. An AIP inclined to HP at 30° and perpendicular to the VP cuts its axis at a distance of 36 mm from the base. Develop the lateral surface of the truncated prism.*

Solution: See Fig. 10.

6. *Two portions of a square prism have been removed as shown in its front view in Figure 11. Take side of the base = 40 mm and length of the axis = 70. The axis of the solid is inclined at 45° to HP. Develop the remaining portion of the solid.*

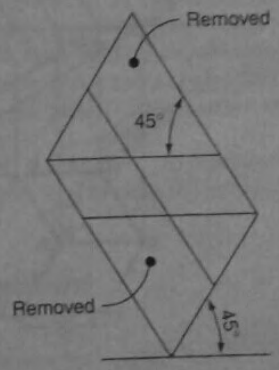


Figure 11

Solution: See Fig. 12.

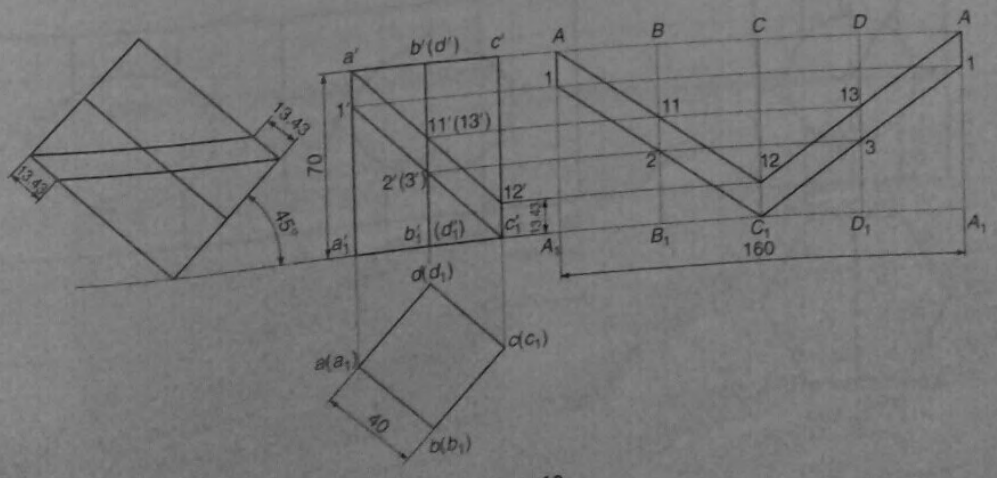


Figure 12

7. *An oblique hexagonal prism stands on HP and one of the edges of the base is parallel to VP. The top and bottom faces of the prism are parallel and regular hexagons of side 30 mm. The axis of the prism is inclined at an angle of 60° to the base and 60 mm long. Draw the development of the lateral surface of the oblique prism.*

Solution: See Fig. 13.

8. *A vertical hexagonal prism of 30 mm side of base and axis 65 mm long has one of its rectangular faces parallel to VP and nearer to it. A circular hole of 40 mm diameter is drilled through the prism completely such that the axis of the hole bisects the axis of the prism at right angles and is perpendicular to VP. Draw the development of the prism showing the shape of the hole on it.*

Solution: See Fig. 14.

(i) Draw the top and front views of hexagonal prism and draw a circle of 40 mm diameter in the front view middle of axis. Divide the circle into 4 or 8 or 12 parts and project it to top view.

(ii) Draw the development of the lateral surface of the hexagonal prism and draw horizontal lines form the generator of the hole in front view to the development of the prism. Measure the distance and transfer in top view to the development.

(iii) The hole portion of the face *FE* in the development has true shape in front view also. It will appear in the development as in front view.

(iv) The portion appears in curved shape in the development. Join the intersecting point in smooth curve and draw thick line. This completes the development.

9. *A right regular square prism, side of base 25 mm and height 50 mm, rests on its base on HP such that it vertical faces are equally inclined to the VP. A horizontal circular hole of diameter 30 mm drilled centrally through it such that the axis of the hole cuts the diagonally opposite vertically edges. Develop its lateral surface.*

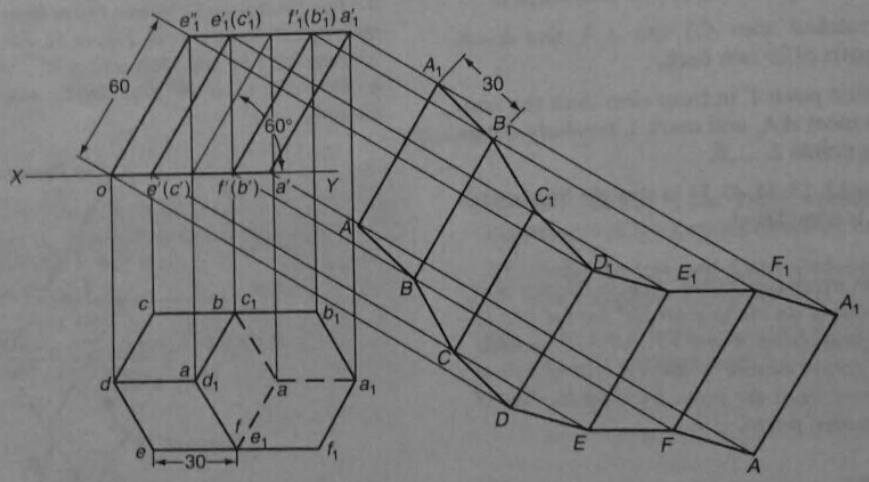


Figure 13

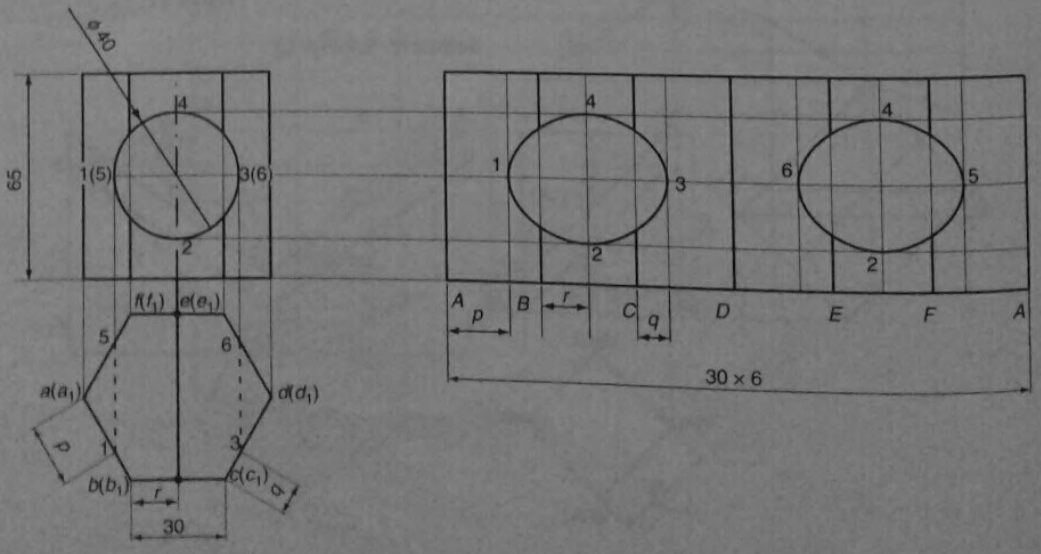


Figure 14

Solution: See Fig. 15.

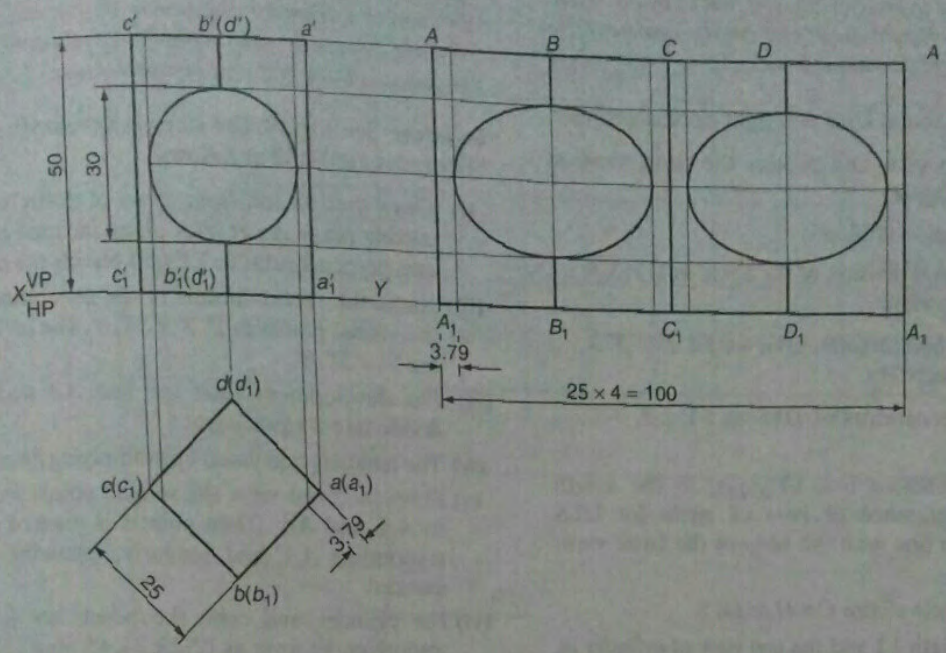


Figure 15

10.4 Development of Cylinders

10.4.1 | Development of Right Cylinders

If a right cylinder is rolled for one complete revolution on a plane, then it moves the distance equal to the circumference of its base circle (πD). The area covered by it in one revolution will be equal to its DLS, which is a rectangle of sides equal to the circumference (C) of its base circle and the height (H) of the cylinder, as shown in Fig. 16. In general, *the development of lateral surface of a cylinder is a rectangle having one side equal to the circumference of its base circle and the other equal to its length.*

Note that a cylinder has a smooth surface, so there are no fold lines. In development of surface by parallel line method, the elements are obtained by dividing the base into equal parts (generally 12) which are drawn on the surface. These serve same function as fold lines in the prism.

10.4.2 | Development of Oblique Cylinders

In an oblique cylinder, the axis of the solid is inclined at some angle to the plane of its circular base. So when projected, the base diameter is shown at an angle to the axis.

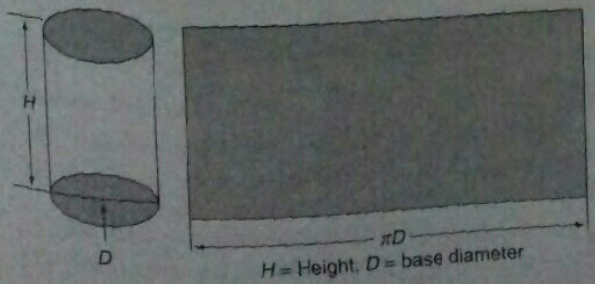


Figure 16

An oblique cylinder can be visualized as an oblique prism with infinite number of sides. Then the development of the lateral surface is done by the method same as that described for oblique prisms. Since there are no distinct fold lines for a cylinder, the end points of the surface lines are joined by a smooth curve.

The steps involved in development of right and oblique cylinders are illustrated by the following solved examples.

Solved Problems

10. *Draw the development of the cylinder of base 30 mm diameter and 70 mm length.*

Solution: See Fig. 17.

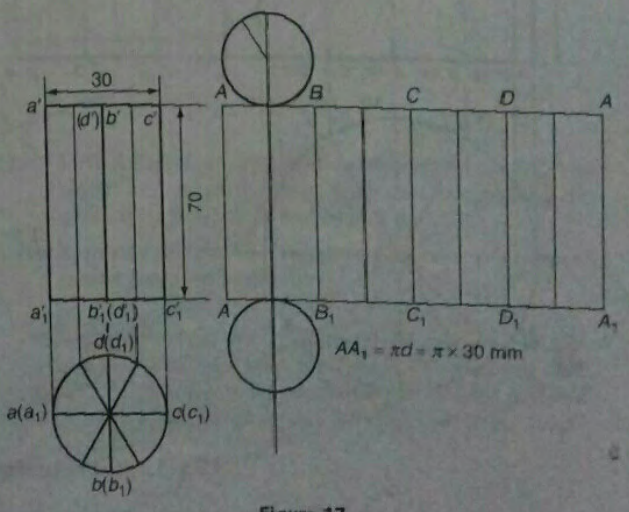


Figure 17

11. *A right circular cylinder, with a 50 mm base diameter and a 70 mm axis height, is resting on its base on HP. Draw DLS of a right circular cylinder.*

Solution: Figure 18 shows DLS of a right circular cylinder.

(i) Draw the top view and project the front view, as shown in the figure.

(ii) Name the points as follows:

- Name 12 equal division of the circle as 1, 2, 3, 4, ..., 12 in the top view.
- Name the generators of front view as 1′-1′, 2′-2′, 3′-3′, ..., 12′-12′, respectively.
- Name the generators of DLS as 1-1, 2-2, 3-3, ..., 12-12.

(iii) Draw the stretchout line 1-1 equal to the length of the circumference of base of circle for DLS directly in the line with the base of the front view; $C = 2\pi R$.

(iv) Draw a rectangle of size $C \times H$ as DLS.

(v) Divide the length 1-1 and the top view of cylinder in equal parts, say 12.

(vi) Project the generator in the front view.

12. *Draw the development of lateral surface of the lower portion of a cylinder of diameter 40 mm and axis 60 mm. A cutting plane is cut by a plane inclined at 45° to HP and perpendicular to VP and bisects the axis.*

Solution: See Fig. 19. The steps in development of surface of cylinder are listed as follows.

(i) Draw the top and front views of the cylinder and the cutting plane is cut by a plane inclined at 45° to HP and perpendicular to VP and bisects the axis.

(ii) Name the section points in the generation and mark the visible points as 1′, 2′, 3′, 4′, 5′. The invisible points are 5′, 6′, 7′, 8′.

(iii) For development start the line AA and A_1A_1 and divide into 8 equal parts.

(iv) The length is calculated by multiplying diameter with P.

(v) From the front view, the section points are projected to AA_1 and AA_1. Then point 1 is marked to the corresponding AA_1 and similarly remaining points are marked.

(vi) For cylinder and cone the points are joined with smooth curve form as 12, 23, 34, 45, 56, 67, 78, 81.

(vii) Thus the development of lateral surface of cylinder is developed.

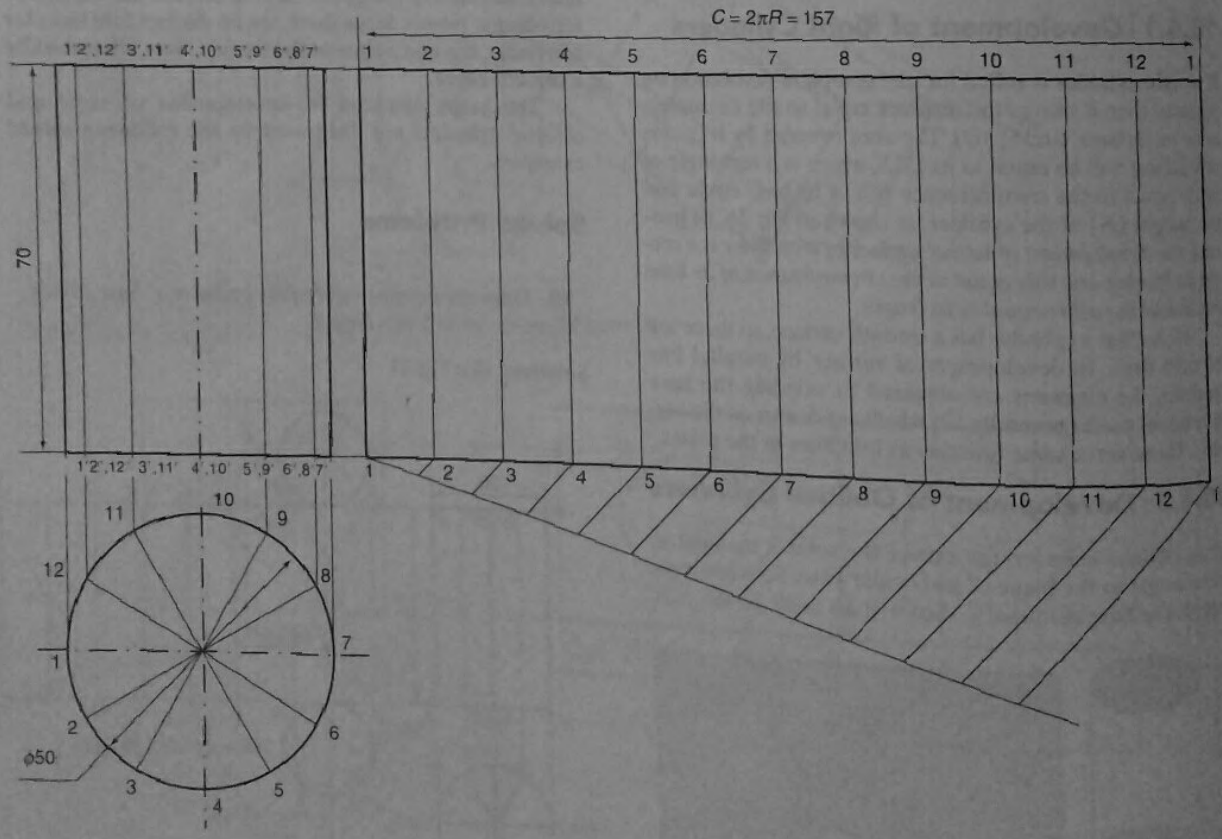


Figure 18

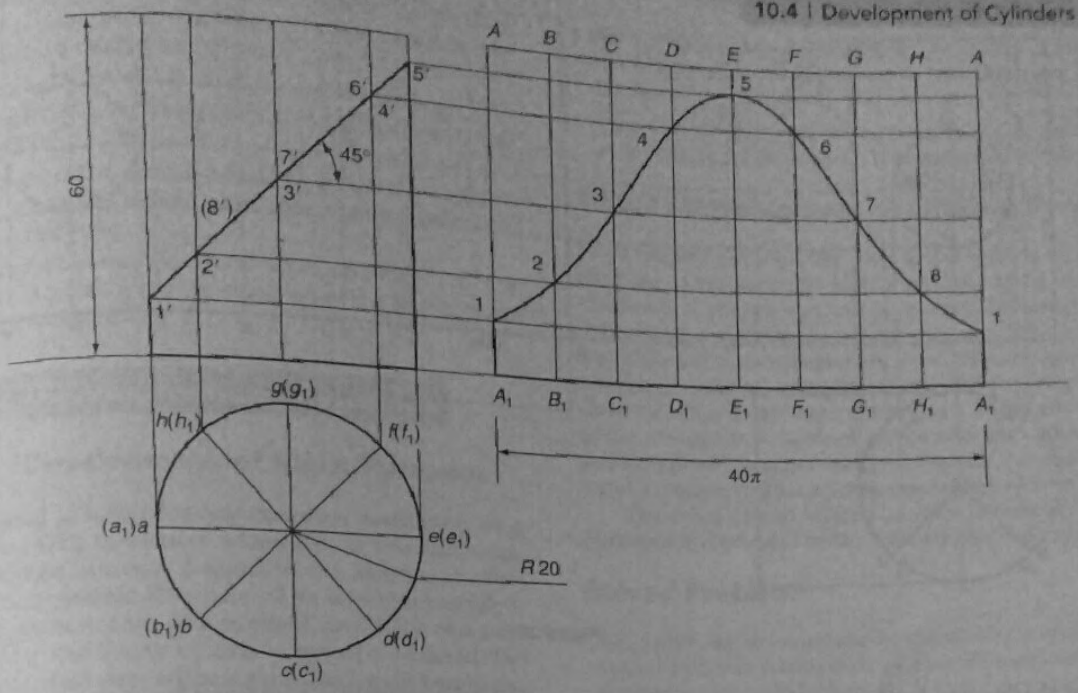


Figure 19

13. *A right cylinder of 30 mm diameter and 50 mm height of axis, is cut by a sectional plane inclined at 30° to HP and passes 20 mm from base along the axis. Draw the development of the truncated cylinder.*

Solution: See Fig. 20.

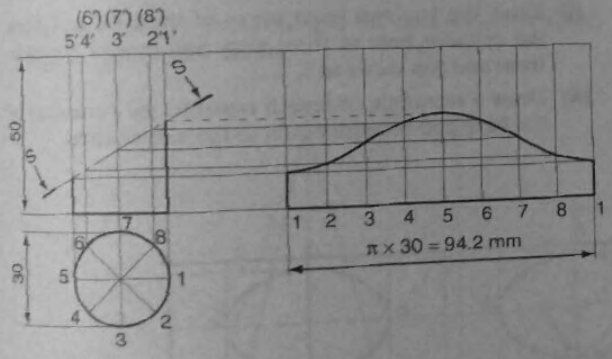


Figure 20

14. *The axis of a cylinder 70 mm long is inclined at 60° to the base. Draw the development of the oblique cylinder if the diameter of the base is 40 mm.*

Solution: See Fig. 21.

(i) Draw the top and front views of the given cylinder.

(ii) Divide the circle in the top view in 12 equal parts. Locate the corresponding generators in the front view and draw perpendiculars at the ends.

(iii) Select a line AA_1 parallel to $a'a'_1$.

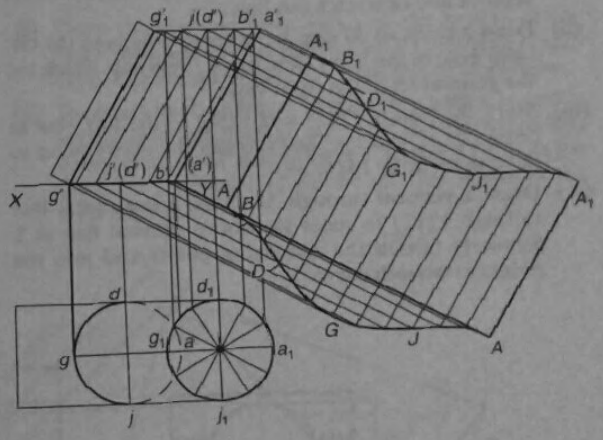


Figure 21

(iv) With A and A_1 as centres and radius equal to 1/12th the circumference of the base, draw arcs intersecting the generators through b' and b'_1 at BB_1.

(v) Similarly obtain the remaining points and join to obtain the development.

15. *A vertical cylinder of diameter 50 mm and height 80 mm is drilled by a hole of diameter 30 mm such that the axis of the hole is perpendicular to VP and parallel to HP. Draw the lateral surface development of the solid.*

Solution: See Fig. 22.

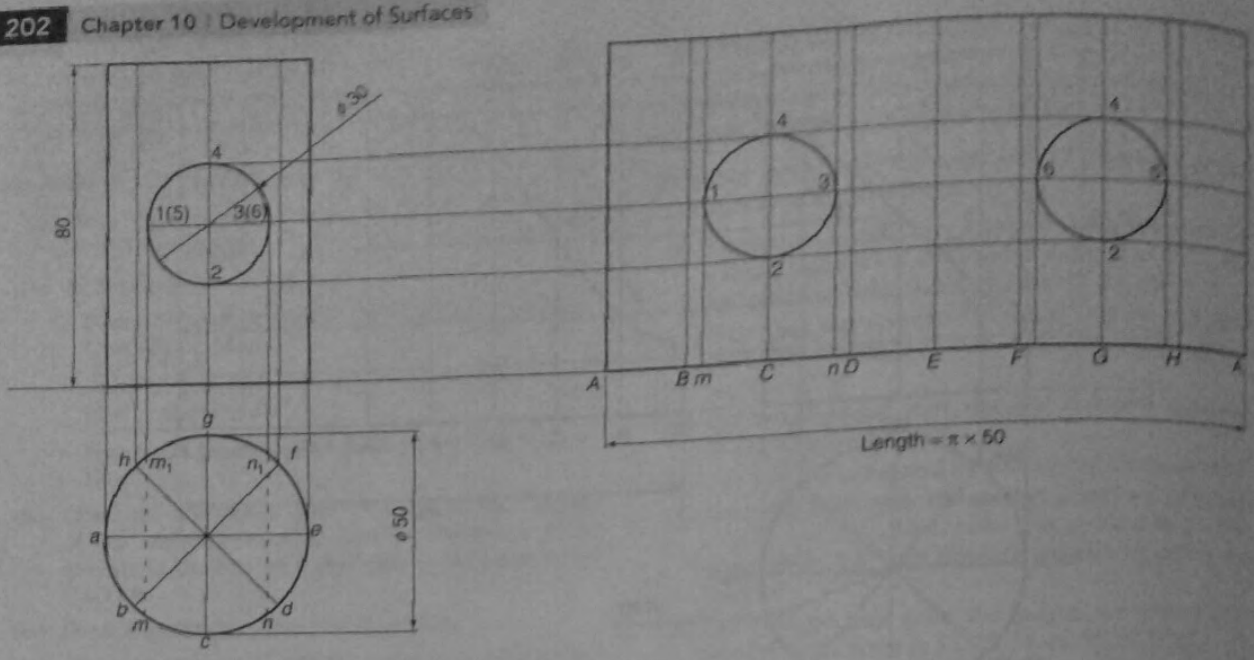


Figure 22

(i) Draw the top and front views of the cylinder 50 mm and height 80 mm and divide into 12 equal parts. Project to front view with equal division points. Draw vertical line of stretch out line.

(ii) Draw a circle of 30 mm diameter to represent the circular hole in the front view. Mark the points which cut the generators in the front view.

(iii) Mark the points $1'(1'_1)$ and $5'(5'_1)$ to the top view to obtain m_1m_1 and n_1n_1 and transfer the above point to development by $1M = 1m$.

(iv) Draw a vertical through M. Draw a horizontal line through $1'(1'_1)$ to meet the above vertical line at 1. Similarly obtain the remaining points and join the points in smooth curve.

16. *A cube of 30 mm edge stands on one of its faces on HP with a vertical face making 45° to VP. A cylinder of 25 mm diameter is holed centrally into the cube such that the hole passes through the opposite vertical edges of the cube. Draw the development of lateral surfaces of the cube which is having the hole.*

Solution: See Fig. 23. The steps in the development of surface are listed as follows.

(i) Draw the top and front views of the cube and draw the cylinder hole in front view and project points in front and top views as 1, 2, ...

(ii) Draw a rectangle of length equal to the perimeter of the base and breadth equal to the side of cube.

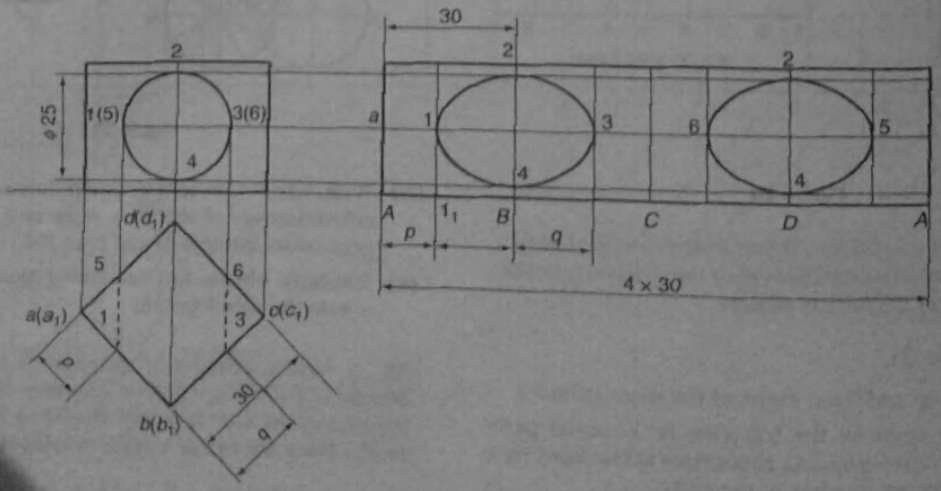


Figure 23

(iii) To draw development of cylindrical hole, the marked point 1'' is on the face of $a''b''b_1''a_1''$ which is not in true size since 1 cannot be obtained from front view.

(iv) Then mark 1_1 on AA such that $A1_1 = a1 = p$.

(v) A vertical line is drawn from 1_1 to cut horizontal line from 1' at 1.

(vi) Similarly, other point is obtained and a smooth curve is drawn along the point. The appearance of front side is like an ellipse.

(vii) Similarly, the rear side is drawn in smooth curve. The curve obtained is mirror reflection of the ellipse which is first obtained.

10.5 Development of Pyramids

10.5.1 | Development of Right Pyramids

If a pyramid is rolled for one complete revolution on a plane, then DLS consists of 'n' number of isosceles triangles in contact, where n is equal to the number of edges of base of a pyramid. The base of an isosceles triangle is equal to the base edge of a pyramid, and other two sides are equal to the length of slant edges of a pyramid. The true length of all slant edges is same for a right pyraid and is determined, based on the following:

1. The true length of the slant edge can be measured from the front view, if the top view of that edge is parallel to reference line XY.
2. True length can be measured from the top view if the front view of the slant edge is parallel to line XY.

For development of the lateral surface:

- With any point O as centre and the true length of slant edge as radius draw an arc.
- With radius equal to the length of the side of the base, mark same number of division as the number of sides in the base.
- Join the division points with each other in correct sequence and with point O, the centre of the arc.

10.5.2 | Development of Oblique Pyramids

In an oblique pyramid, the axis of the solid is inclined at an angle to the base. The lateral surface can be visualized to consist of triangles placed in contact. The length of all the slant edges is determined (as they are different in this case). The adjacent triangles are then laid out one by one in increasing order of slant edge length. The true lengths of all three side of a triangle are determined. Using anyone side of the triangle as base, draw an arc with the end as centre and length of the other side (with which the end is common) as radius, to intersect the third side.

The development of lateral surface of right and oblique pyramids is illustrated by the following solved examples.

Solved Problems

17. *Draw the development of the lateral surfaces of a square pyramid having side of base 30 mm and height 70 mm, resting with its base on HP and an edge of base is parallel to VP.*

Solution: See Fig. 24.

(i) Draw the top and front views of the square prism. Since the slant edge is parallel to XY the true length cannot be measured directly from top or front view.

(ii) To obtain the true length of the prism with o as centre and oa as radius, draw an arc to cut the horizontal line drawn from o at P_1 and project P_1 to the XY line to get the point P_1'. Now $o'P_1'$ will be the true length of the slant edge OA.

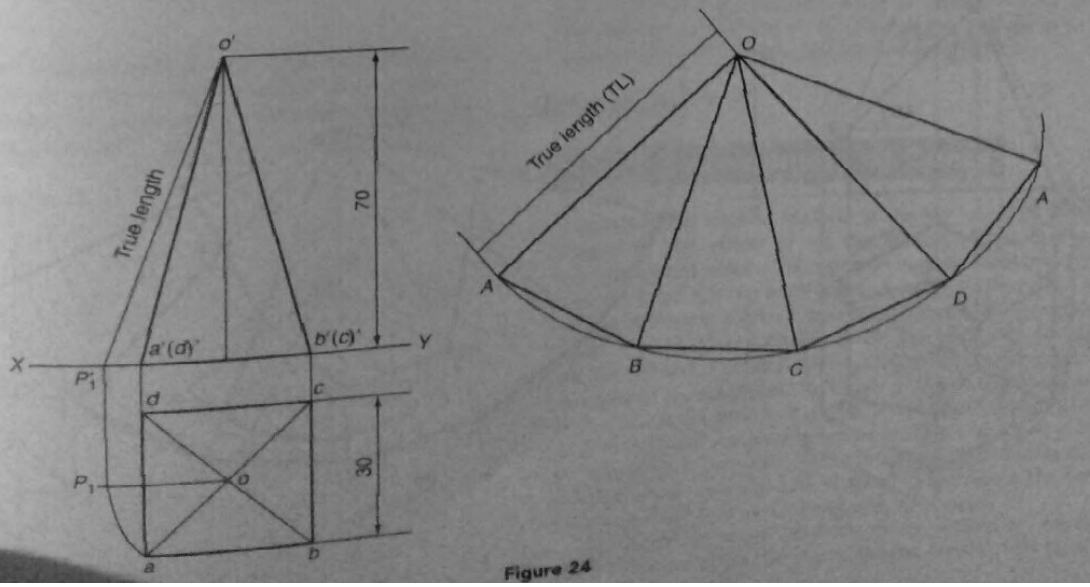


Figure 24

(iii) With o as centre $o'p_1'$ and as radius draw an arc. Divide the arc into 4 equal parts such that $AB = BC = CD = DA = 30$ mm.

(iv) Join the triangles OAB, OBC, OCD and ODA by thick lines, the development of lateral surface of the square pyramid is obtained.

18. *A hexagonal pyramid of base side 25 mm and height 55 mm is resting on its base on HP such that two sides of the triangle base are perpendicular to VP. A cutting plane is cut at an angle of 45° to HP and perpendicular to VP. The cutting plane passes through the middle of the axis of the pyramid. Obtain the development of the lateral surface of the truncated pyramid.*

Solution: See Fig. 25 The steps involved in development of truncated pyramid are listed as follows.

(i) Draw the top and front views of the given position.

(ii) Draw the cutting plane and mark the section points 1′, 2′, 3′, (4′), (5′), (6′) in the front view.

(iii) **To obtain true length:** With oa as radius and o as centre an arc is drawn to cut the horizontal line p and project p' to the XY. Join $p'o'$ to get true length of pyramid.

(iv) Draw the top view from the front view points.

(v) Then development of 1 on OA and 6 on OF such that 2 is on OB and 5 on OE. Similarly do for the points 3, 4.

(vi) Join the points by straight line 12, 23, ..., and complete the development of the truncated pyramid by thick lines.

19. *A square pyramid, having a 30 mm edge of base and a 50 mm axis length, rests vertically on its base with adjacent edges of base equally inclined to VP. It is cut by a cutting plane perpendicular to VP and inclined at 45° to HP, such that it bisects the axis. Draw DLS of the major part of the pyramid.*

Solution: Figure 26 shows DLS of the major side of truncated pyramid:

(i) Draw the top view and project the front view, as shown in the figure.

(ii) Draw the DLS by the usual method.

(iii) Draw the cutting plane S-S passing through the mid-point of axis at 45° to the XY line.

(iv) Name the points, as shown in the figure.

20. *A pentagonal pyramid, having a 40 mm base side and a 75 mm long axis, rests vertically on the HP with one of its inclined edges parallel to the VP. It is cut by a plane perpendicular to the VP and inclined at 45° to the HP, such that it bisects the axis. Major part of the edge, which is parallel to the VP, is retained. Draw DLS of the remaining portion of the pyramid.*

Solution: Figure 27 shows DLS of a pentagonal pyramid

(i) Draw the top view and project the front view, as shown in the figure.

(ii) Draw cutting plane S-S at 45° to XY and bisecting the axis.

(iii) Name the points, as shown in the figure.

(iv) Mark the points as $p_1', p_2', p_3', p_4', p_5'$ in the front view on respective slant edges where cutting plane cuts.

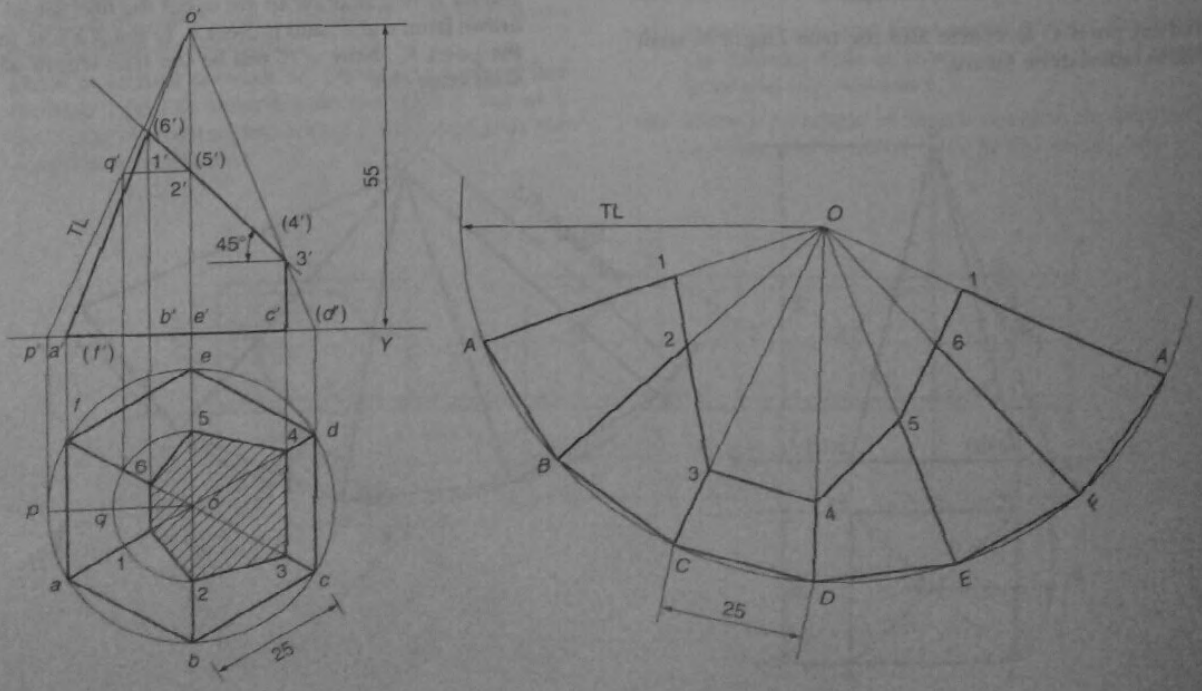


Figure 25

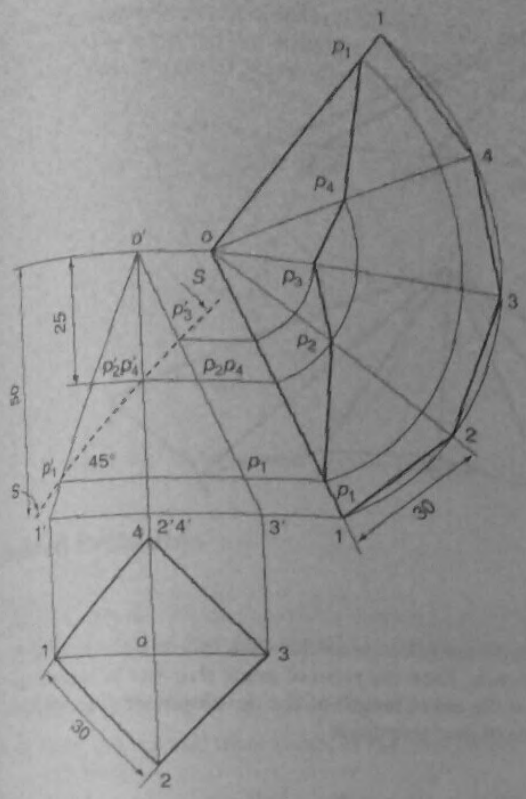


Figure 26

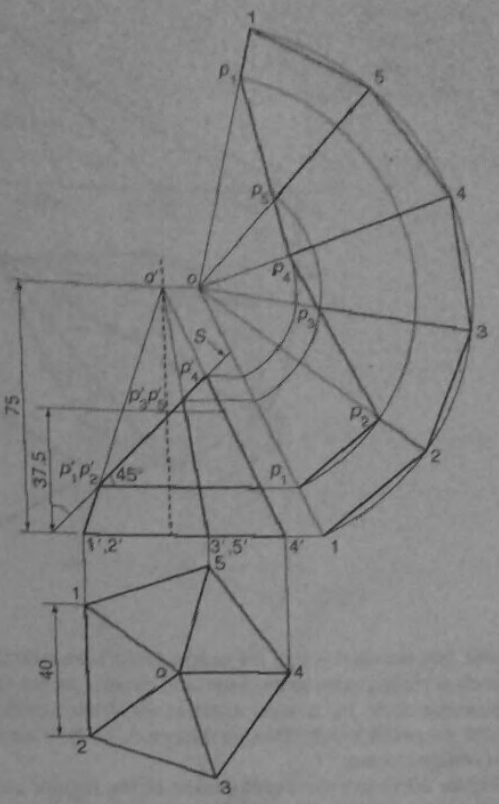


Figure 27

(v) Transfer these points to true length of slant edge and then carry the same to respective slant edge in DLS.

(vi) Join all the points in proper sequence by straight line in DLS.

21. *A square pyramid of base edge 25 mm and height of its axis 50 mm in resting in HP in such a manner that its base edge makes an angle of 45° with V.P. Develop the surface of the pyramid.*

Solution: See Fig. 28.

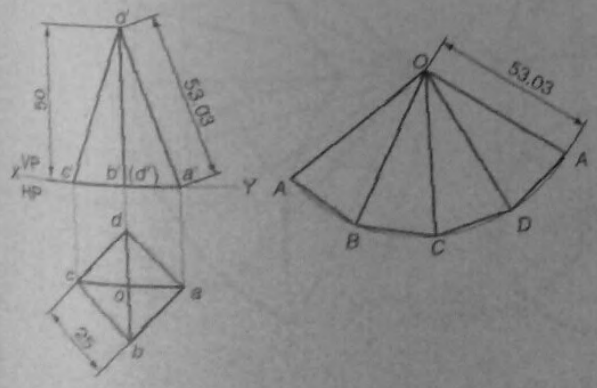


Figure 28

22. *A hexagonal pyramid with base side 40 mm and height 60 mm is resting on its base on HP such that one of the edges of the base is perpendicular to V.P. The front view of the axis is inclined at 45° to reference line XY and the top view is inclined at 30° in a manner that the apex is closer to line XY than to the base of the pyramid.*

Solution: See Fig. 29.

10.6 Development of Cones

In geometry, a conical surface is the unbounded surface formed by the union of all the straight lines that pass through a fixed point – the apex or vertex – and any point of some fixed space curve – the directrix – that does not contain the apex. Each of those lines is called a generatrix of the surface. Every conic surface is ruled and developable. In general, a conical surface consists of two congruent unbounded halves joined by the apex. Each half is called a nappe, and is the union of all the half-lines that start at the apex and pass through a point of some fixed space curve. (In some cases, however, the two nappes may intersect, or even coincide with the full surface.) Sometimes, the term conical surface is used to mean just one nappe.

In common usage in elementary geometry, regular cones are assumed to be right circular, where right means

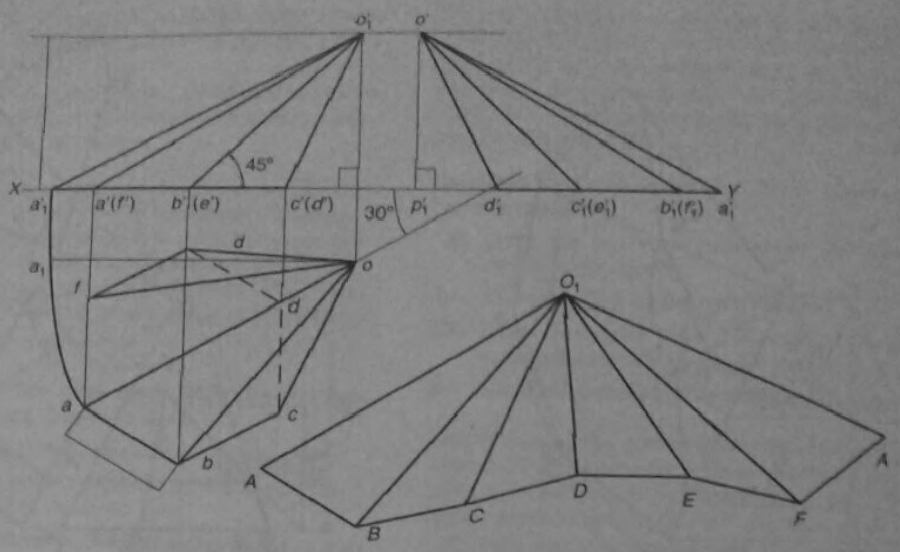


Figure 29

that the axis passes through the centre of the base (suitably defined) at right angles to its plane, and circular means that the base is a circle. In contrast, cones in which the axis does not pass perpendicularly through the centre of the base are called oblique cones.

Figure 30 shows the development of the regular cone by radial line method. Since all the surface radial lines are of the same true length, the development of a right cone is a sector of a circle. The angle (θ) subtended at the centre of the sector is determined by the radius of the base (circumference of base circle = $2\pi R$) and the slant height (L) of the cone. Then the ratio of angle θ to 360° is equal to the ratio of the curve length of the development to the circumference of the base circle

$$\frac{\theta}{360} = \frac{2\pi R}{2\pi L}$$

$$\theta = 360° \times \frac{R}{L}$$

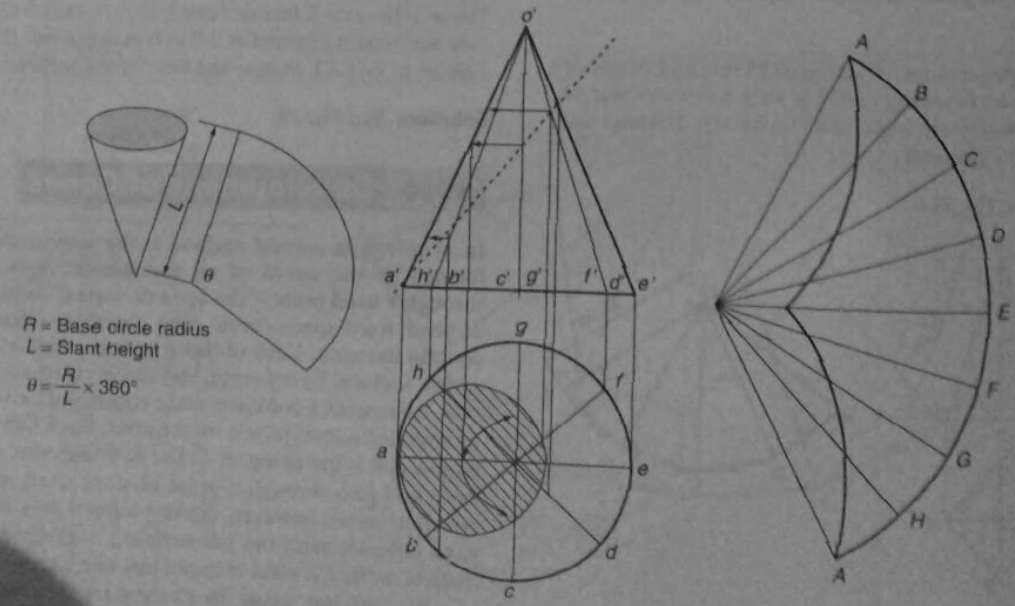


Figure 30 Development of regular conical object.

10.6.1 | Development of Frustum of a Cone

Frustum of a cone is the portion of a solid between the plane and the base that appears after cutting the cone by a plane parallel to its base. Figure 31 shows the frustum of a cone.

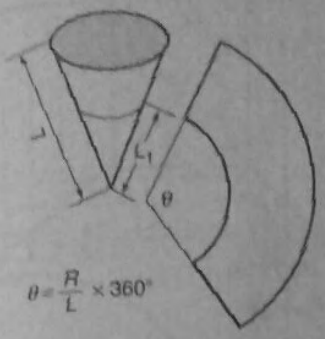


Figure 31 Development of cone frustum.

Solved Problems

23. *Draw the development of the lateral surface of a cone having a base diameter 40 mm and axis length 70 mm.*

Solution: See Fig. 32.

(i) Draw the top and front views of the cone of the given dimension.

(ii) The slant edge generator is the true length while considering the cone. Hence $o'a' = L$ = true length.

(iii) The lateral surface of the cone is a sector of circle of radius L. Circumference of the base circle = $2\pi r$. Angle subtended by the arc at centre is θ. So

$$L\theta = 2\pi r = 360° \times r$$

$$\theta = 360° \times \frac{r}{L}$$

(iv) Draw an arc with O as centre and L as radius. Finding θ measure the distance and divide into 4 equal parts. Thus the lateral surface of cone is obtained.

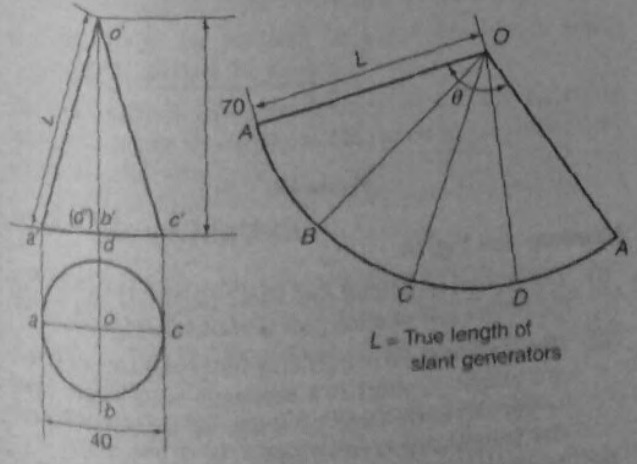
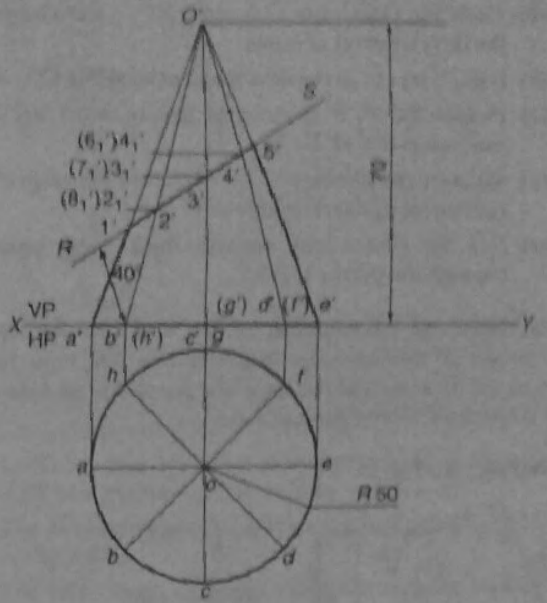


Figure 32

24. *A cone of base diameter 50 mm and height 70 mm is resting on its base on HP. It is cut by a plane perpendicular to VP and inclined at 40° to HP. The plane bisects the axis of the cone. Draw the development of its lateral surface.*

Solution: See Fig. 33.



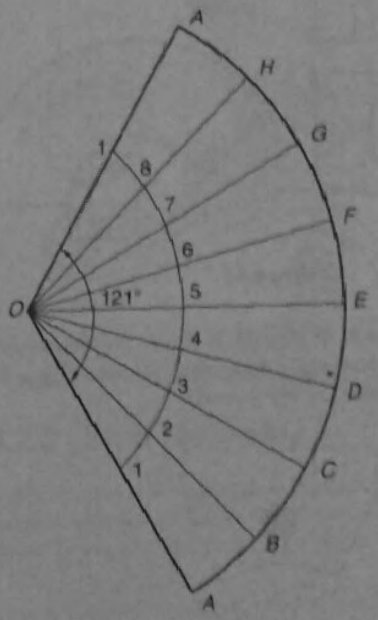


Figure 33

(i) Draw the top and front views of the cone for the given conditions.

(ii) Divide the circle in the top view into 8 equal parts.

(iii) Top view oa is parallel to XY. So $o'a' = OA$ = True length L of the generator.

(iv) To find θ:

Circumference of base circle = $2\pi R$

$L\theta = 2\pi R = 360° R$

$\theta = 360° \times R/L = 121°$

(v) The section points are marked on the front view and they are projected to the $o'a'$ line to measure the points $2'_1, 3'_1, \ldots$.

(vi) With O as centre and L as radius draw an arc and divide into 8 or 12 equals parts to subtend q at O.

(vii) Draw the radial line $OA, OB, OC, \ldots$ and complete the development of cone.

(viii) Name 1 on OA in the development such that $O1 = o'1'$.

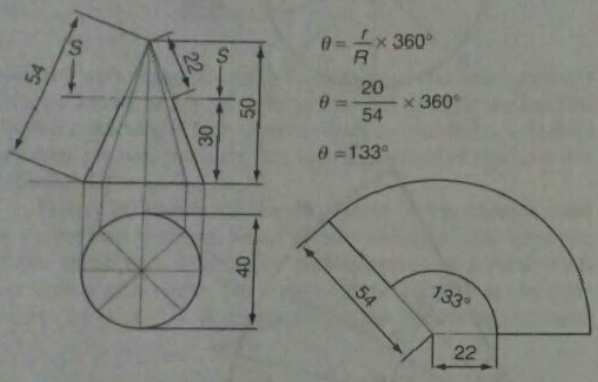
(ix) Project the 2′, 3′ horizontal line to meet the end generators $o'a'$ at $2'_1, 3'_1, \ldots$.

(x) Measure the distance $o'2'_1, o'3'_1, \ldots$ on the respective generators in the development.

(xi) Join the points with smooth thick curve passing through the points 1, 2, 3, ...

25. *Draw the development of the cone having base diameter 40 mm and axis height 50 mm. The cone has been cut by a straight section plane parallel to its base at a distance of 30 mm from its base.*

Solution: See Fig. 34.



$$\theta = \frac{r}{R} \times 360^\circ$$

$$\theta = \frac{20}{54} \times 360^\circ$$

$$\theta = 133^\circ$$

Figure 34

26. *A funnel shown in Fig. 35 is made by assembling the frustum of a cone and a truncated cylinder. Draw the complete development of the funnel.*

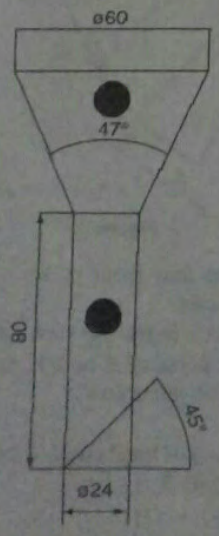
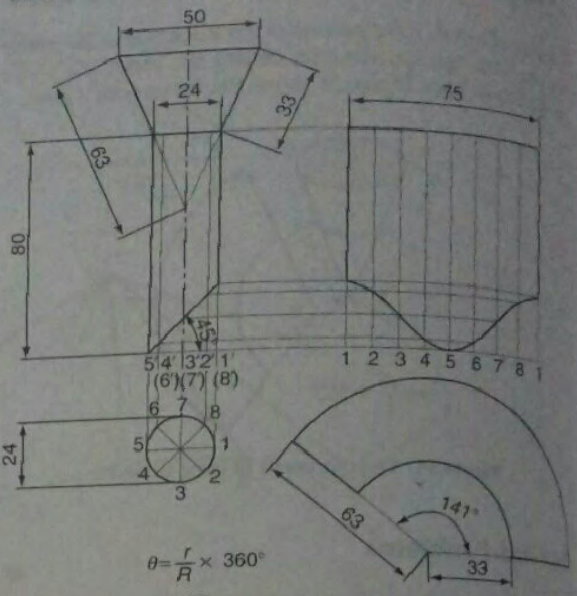


Figure 35

Solution: See Fig. 36.



$$\theta = \frac{r}{R} \times 360^\circ$$

$$\theta = \frac{25}{64} \times 360^\circ$$

$$\theta = 141^\circ$$

Figure 36

27. *Develop the lateral surface of the funnel shown in Fig. 37.*

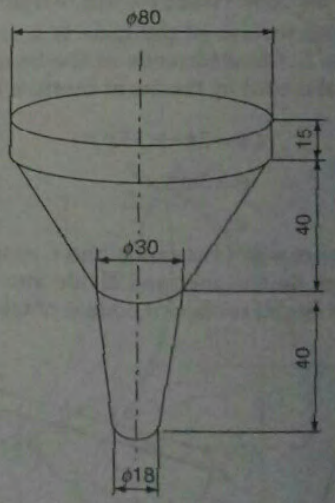


Figure 37

Solution: See Fig. 38.

(i) The funnel is divided into three parts I, II and III and the front view of each part is shown in Fig. 38(a).

(ii) The development of Part I (cylinder of diameter 80 mm and height 15 mm) is a rectangle with length equal to the circumference of circle of diameter 80 mm and breadth equal to 15 mm.

(iii) Part II is truncated cone of base diameter 30 mm and top circle diameter 80 mm. The development of Part II is obtained by the following steps:

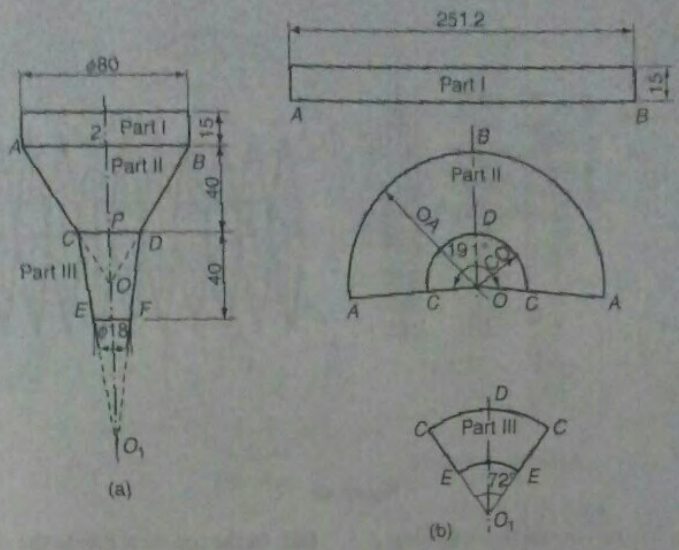


Figure 38

(a) Slant length of cone is equal to OA or OB. It can be obtained from equal ratio of side of two similar triangles OPD and OQB, and then applying Pythagoras theorem on triangle OQB.

(b) Sector angle is given by

$$\theta = \frac{\text{Circumference of top circle}}{\text{Slant length of the cone}} \times \frac{180}{\pi}$$

(c) Draw the sector of radius equal to slant length and sector angle θ.

(d) Draw an arc of radius OC with O as centre.

(iv) Part III is the lower part of the truncated cone and its development is drawn using the same steps as in development of Part II.

10.7 Development of Spheres

In development of spheres approximate method is used. The surface of the sphere is divided into a number of parts in two different ways as:

1. Zones: These are portions of the sphere enclosed between two planes perpendicular to the axis
2. Lunes: These are portions between two planes which contain the axis of the sphere.

The two methods for development of spheres based on ways of creating divisions of the surface of sphere are discussed as follows.

10.7.1 | Zone Method

In this method the sphere is divided into zones (defined above) and each zone is developed as frustum of a cone. The steps are:

1. Draw the top and front view of the sphere as shown in Fig. 39(a).
2. In the front view divide the circle into eight zones of equal width by chords AA_1, BB_1, etc. Each zone becomes a frustum of a cone, except the uppermost zone which is a cone of small altitude.
3. Consider one zone for development of surface (say AB). It is frustum of cone whose vertex is at P.
4. The development of its surface is a half sphere as shown in Fig. 39(b).
5. The same steps are followed to develop the surface of each zone.

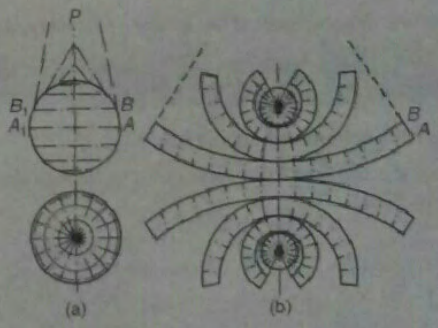


Figure 39 Development of sphere by zone method.

10.7.2 | Lune Method

In this method the sphere is cut into equal number of meridian sections or lunes which can be considered as sections of cylinders. The steps involved are:

1. Draw the top and front view of the sphere.
2. Divide the front view into eight equal parts with chords as shown in Fig. 40(a).
3. Consider one lune for development of the surface. It is a rectangle with length equal to half the perimeter of the circle and width equal to chord length Fig. 40(b). (Note that length is slightly less than half the perimeter because we are using chord length. Actually the length of arc AB should be used.)
4. The complete development of the sphere as shown in Fig. 40(c) can b+4e obtained by following the same steps for each lune.

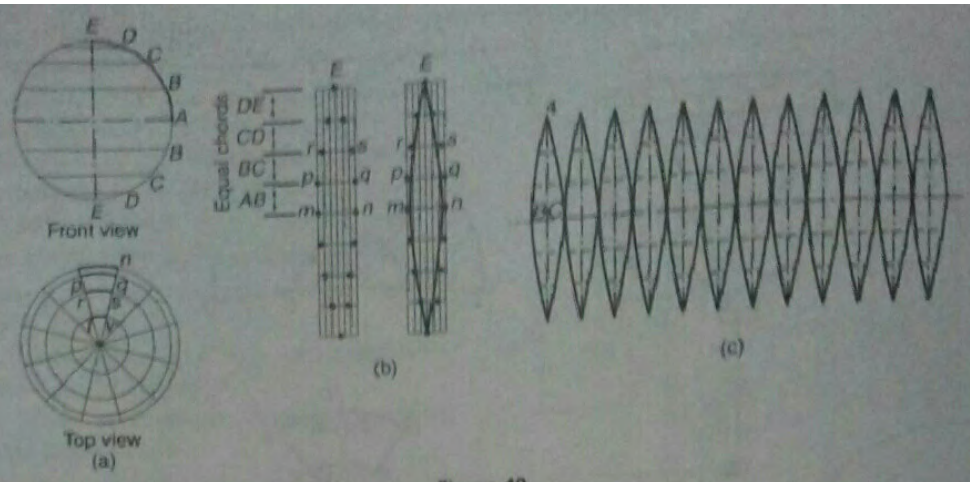


Figure 40

The development of spheres by the two methods is illustrated by the following solved examples.

Solved Problem

28. *Draw the development of a sphere of 50 mm diameter by the zone method.*

Solution: See Fig. 41:

29. *Draw the development of the sphere of 30 mm radius by Lune method.*

Solution: See Fig. 42. The steps involved are:

(i) Draw the sphere into twelve lunes.

(ii) Draw the top and front view of the sphere as shown in Fig. 42(a).

(iii) In the top view divide the circle into 8 parts as chord lengths $AB = BC = CD = DE$.

(iv) Take one lune for development and draw a rectangle with:

Length = 8 AB (as all chord are equal) = half the circumference of circle

Width = AB

(v) Divide the rectangle into eight equal part as shown in Fig. 42(b).

(vi) Mark chord lengths mn, pq, rs, tv in the development as shown in Fig. 42(b).

(vii) Join the points in proper sequence to get the development of the lune.

(viii) Repeat steps for each lune to obtain development of the sphere as shown in Fig. 42(c).

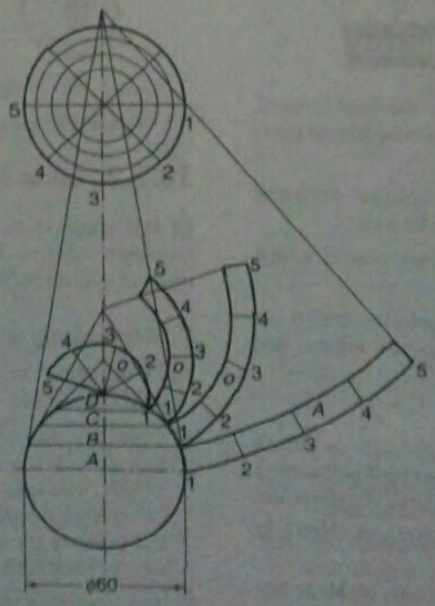


Figure 41 Development of half-zones

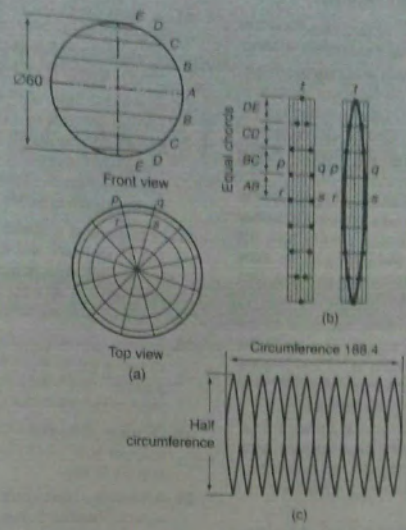


Figure 42

EXERCISES

Short Answer Questions

1. How do use the development in the practical life.
2. What do you mean by development? Explain its use?
3. Explain the method used for development of a right cone.
4. List the different methods of development of surfaces.
5. What is the principle of development of surfaces?
6. What is the shape obtained by development of the following:
   (a) Cone
   (b) Cylinder
   (c) Pyramid

Problems

7. A pentagonal prism of base side 30 mm and axis height 75 mm is resting on its base on HP with two of its lateral surfaces parallel to VP. It is cut by plane perpendicular to VP and inclined at 45° to HP, bisecting the axis. Draw the development of lateral surfaces of the lower portion of the prism.
8. A hexagonal prism of base side 30 mm and axis height 70 mm is resting on its base on HP with one of its faces parallel to VP. It is cut by plane perpendicular to VP and inclined at 35° to HP, meeting the axis at a distance of 40 mm from the base. Draw the development of lateral surfaces of the lower portion of the prism.
9. A pentagonal prism of base side 30 mm and height 60 mm is cut by a plane perpendicular to VP and 50° to HP and passing through the axis at a height of 35 mm above the base. Draw the development of the lower portion of the solid.
10. A hexagonal prism with side of base 25 mm and height 55 mm rests with its base on HP and one of its rectangular faces parallel to VP. A circular hole of 40 mm diameter is drilled through the prism such that the axis of the hole bisects the axis of the prism at right angles and is perpendicular to VP. Draw the development of the lateral surface of the prism with the hole.
11. A cylinder of diameter 45 mm and height 70 mm is resting vertically on one of its ends on the HP. It is cut by a plane perpendicular to VP and inclined at 45° to HP. The plane meets the axis at a point 35 mm above the base. Draw the development of the lateral surface of the lower portion of the truncated cylinder.
12. A vertical chimney of 60 m diameter joins a roof sloping at an angle of 35° with the horizontal. The shortest portion over the roof is 25 m. Determine the shape of the sheet metal from which the chimney can be fabricated. Take a scale of 1:20.
13. A right circular cone of base diameter 50 mm and height 75 mm is resting on its base on the ground. It is cut by a plane perpendicular to VP and inclined at 30° to HP.

The cutting plane bisects the axis of the cone. Draw the development of the lateral surface of the truncated cone.

14. Draw the projections of a cylinder of diameter 60 mm and height 75 mm which is resting on the HP. Draw a square of side 20 mm centrally with its sides equally inclined to HP in the front view of the cylinder. Draw the development of the remaining protion of the cylinder when the square portion is removed.

15. A cone of base diameter 50 mm and height 75 mm rests vertically on its base on the ground. A string is wound around the curved surface of the cone starting from the left extreme point on the base and ending at the same point. Find the shortest length of the string required. Also trace the path of the string in front and top views.

16. A hexagonal pyramid of base side 30 mm and height 65 mm rests on its base on the ground with a base edge parallel to VP. It is cut by a plane perpendicular to VP and inclined at 55° to HP and meets the axis at a height of 30 mm from the base. Draw the lateral surface development.

17. A square pyramid of base side 25 mm and altitude 50 mm rests on its base on the HP with two side of the base parallel to VP. It is cut by a plane bisecting the axis and inclined at 30° to the base. Draw the development of the lower part of the pyramid.

18. Draw the development of the lateral surfaces of the pyramid of height 4 cm. The pyramid is having hexagonal base of 2 cm each side length. The base is paralle to HP and two side of the base are parallel to the VP.

19. A pentagonal pyramid of base side 30 mm and height 70 mm is resting vertically on its base on the ground with one of its base edge parallel to VP. It is cut by a plane perpendicular to VP and parallel to HP at a distance of 35 mm above the base. Draw the development of the lateral surfaces of the frustum of pyramid. Also show the sectional plan view.

20. A pentagonal prism of base side 25 mm and height 60 mm stands on one of its ends on the HP with a rectangular face parallel to the VP. A hole of diameter 30 mm is drilled centrally through the prism in such a way that the axis of the hole bisects the axis of the prism at right angles. The axis of the hole is perpendicular to the VP. Draw the development of the lateral surfaces of the prism.

21. A square pyramid 25 mm side of base and 60 mm width is resting on its base. A section plane making an angle of 60° with the HP and cutting its axis at a height of 40 mm from the base. Develop the truncated pyramid.

22. A circular hole of diameter 30 mm is drilled through a vertical cylinder of diameter 50 mm and height 65 mm. The axis of the hole is perpendicular to the VP and meets the axis of the cylinder at right angles at a height of 30 mm above the base. Draw the development of the lateral surface of the cylinder.

23. A frustum of a square pyramid has its base 50 mm side, top 25 mm side and height 60 mm. It is resting with its base on HP, with two of its sides parallel to VP. Draw the projections of the frustum and show the development of its lateral surface.

24. A right cylinder of 30 mm diameter and 35 mm height of axis is cut by sectional plane inclined at 30° to HP and passed 18 mm from base along the axis. Draw the development of the truncated cylinder.

25. Develop the lateral surface of a cone whose axis is inclined at 60° to the base, having base diameter and altitude as 40 mm.

26. A cone of diameter 60 mm and height 80 mm is cut by a section plane such that the plane passes through the midpoint of the axis and tangential to the base circle. Draw the development of the lateral surface of the bottom portion of the cone.

27. A cone of base 50 mm diameter and axis 75 mm long, has a through hole of 25 mm diameter. The centre of the hole is 25 mm above the base. The axes of the cone and hole intersect each other. Draw the development of the cone with the hole in it.

28. A transition piece connects a square pipe of side 25 mm at the top and circular pipe of 50 mm diameter at the bottom, the axes of both the pipes being collinear. The height of the transition piece is 60 mm. Draw its development.

29. A pentagon prism of 25 mm base edges and 50 mm long, resting on its base with an edge of base at 45° to the VP. The prism is cut by a section plane V.T. inclined at 30° to the HP and passes through a point 25 mm from the base along its axis. Develop its lateral surface of the truncated prism.