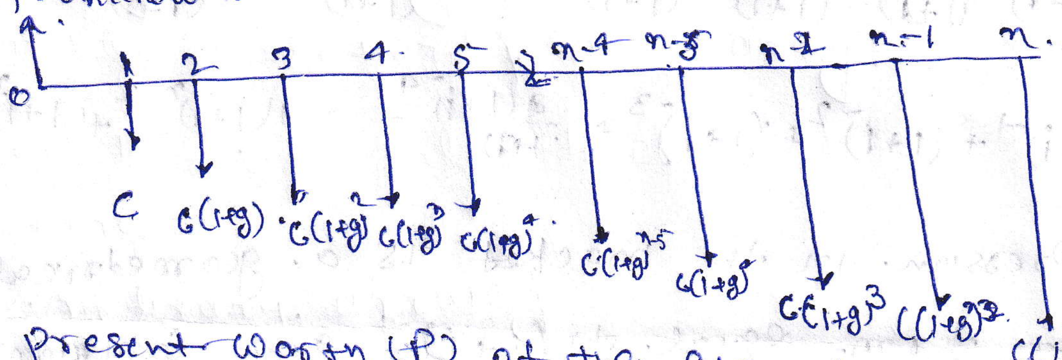
~~The at~~ by multiplying the uniform annual amount A with uniform series compound amount factor.

~~Linear~~ Cash flow involving geometric gradient series

Sometimes the cash flows may have ~~expens~~ expenses or ~~increase~~ incomes being increased by a constant percentage in the successive time periods i.e in successive years, such kind of cash flow is known as geometric gradient series with expense or receipt (C) at the end of year 1 and geometric percentage increase g %.



The present worth (P) of the geometric series can be calculated by considering each amount as the future worth and then taking sum a sum of these present worth values.

$$P = \left\{ \frac{C}{1+i} + \frac{C(1+g)}{(1+i)^2} + \frac{C(1+g)^2}{(1+i)^3} + \frac{C(1+g)^3}{(1+i)^4} + \text{---} + \frac{C(1+g)^{n-3}}{(1+i)^{n-3}} + \frac{C(1+g)^{n-2}}{(1+i)^{n-1}} + \frac{C(1+g)^{n-1}}{(1+i)^n} \right\} \quad (1)$$

above expression rewritten as follows

$$P = C\left\{ \frac{1}{(1+i)} + \frac{(1+g)}{(1+i)^2} + \frac{(1+g)^2}{(1+i)^3} + \frac{(1+g)^3}{(1+i)^4} + \text{---} + \frac{(1+g)^{n-3}}{(1+i)^{n-2}} + \frac{(1+g)^{n-2}}{(1+i)^{n-1}} + \frac{(1+g)^{n-1}}{(1+i)^n} \right. \quad (2)$$

$$\frac{P}{1+i} = C\left\{\frac{1}{(1+i)^2}+\frac{(1+g)}{(1+i)^3}+\frac{(1+g)^2}{(1+i)^4}+\frac{(1+g)^3}{(1+i)^5}+ \cdots \frac{(1+g)^{n-3}}{(1+i)^{n-1}}+\frac{(1+g)^{n}}{(1+i)^{n}} + \frac{(1+g)^{n-1}}{(1+i)^{n-1}}\right. \quad \text{--- (3)}$$

Now multiplying both side equation - 3 by $(1+g)$ resulting the following expression

$$\frac{P(1+g)}{1+i} = C\left\{\frac{(1+g)}{(1+i)^2}+\frac{(1+g)^2}{(1+i)^2}+\frac{(1+g)3}{(1+i)^3}+\frac{(1+g)^3}{(1+i)^4}+\frac{(1+g)^4}{(1+i)^5}+ \cdots + \frac{(1+g)^{n-2}}{(1+i)^{n-1}}+\frac{(1+g)^{n-1}}{(1+i)^{n}}+\frac{(1+g)^{n}}{(1+i)^{n+1}}\right\} \quad \text{--- (4)}$$

Subtracting equation (1) by eq (4)

$$\frac{P(g-i)}{1+i} = C\left\{\frac{(1+g)^n}{(1+i)^{n+1}} - \frac{1}{1+i}\right\} \quad \text{--- (5)}$$

equation 5 can be ~~writ~~ rewritten as.

$$\boxed{P = \frac{C\left\{\frac{(1+g)^n}{(1+i)^n} - \right\}}{(g-i)}} \quad \text{--- (6)}$$

The expression in eq - (6) is valid ~~eq~~ when $g \neq i$ when 'g' is equal to i then equation is

$$\boxed{P = \frac{Cn}{1+i}} \quad \text{--- (7)}$$

Now using the above expression one can easily calculate the present worth future worth and uniform annual worth of the cash equivalent ~~uniform~~ uniform annual worth of the cash flow involving either ~~expression~~ expenditure or income both increasing in the form of geometric gradient.

It may be noted here that the expression for Compound interest factors can also be obtained by using begining of year convention.

~~Irregular Series~~

Annuities

→ An annuities is a series of angular equally spaced payment over a define period of time ~~afte~~ often called them term of a constant rate of interest.

→ The payment may occur weekly fortnightly monthly Quarterly or yearly.

→ Annuties are classified in to 4 categories there are

1. ordinary annuity
2. Anuity due
3. deferred annuity.
4. Perpetuity

→ An ordinary annuity is an annuity where the regular payment are made ~~of~~ at the end of the successive time period.

→ Examples of annuities are

* Regular payment to saving accounts
* Periodic payment to a person from retirement fund
* Monthly home mortage payment
* Insurance Premium.