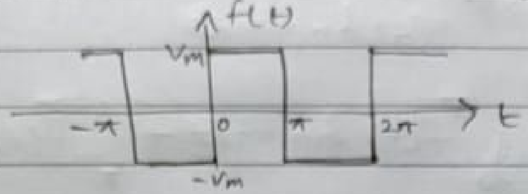
Prob ①

Determine the Fourier Series for the square wave voltage shown in fig ①



$$T = 2\pi \quad , \omega_0 = 1 \text{ (let)}$$

$$f(t) = V_m \qquad 0 < t < \pi$$

$$\quad = -V_m \qquad \pi < t < 2\pi$$

$$a_0 = \frac{1}{T}\int_{t_1}^{t_1+T} f(t)\,dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} f(t)\,dt$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} f(t)\,dt + \int_{\pi}^{2\pi} f(t)\,dt\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} V_m\,dt + \int_{\pi}^{2\pi} -V_m\,dt\right]$$

$$= \frac{1}{2\pi}\left[V_m\int_0^{\pi} dt - V_m\int_{\pi}^{2\pi} dt\right]$$

$$= \frac{1}{2\pi}\left[\cancel{V_m\pi} - \cancel{V_m\pi}\right]$$

$$= \frac{1}{2\pi}\times 0 = 0 \text{ Ans}$$

Angular frequency

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$$

~~[illegible]~~

in one cycle +ve area = -ve area

$$\underline{\underline{a_0 = 0}}$$

When time Period T Then

$$a_n = \frac{2}{T}\int_0^{T} f(t)\cos n\omega_0 t\,dt$$

$$= \frac{2}{2\pi}\int_0^{2\pi} f(t)\cos n\omega_0 t\,dt$$

$$= \frac{1}{\pi}\int_0^{2\pi} f(t)\cos n\omega_0 t\,dt$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\cos nt\,dt$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} f(t)\cos nt\,dt + \int_{\pi}^{2\pi} f(t)\cos nt\,dt\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} V_m\cos nt\,dt - V_m\int_{\pi}^{2\pi}\cos nt\,dt\right]$$

$$= \frac{1}{\pi}\left[V_m \times \left[\frac{\sin nt}{n}\right]_0^{\pi} - V_m\left[\frac{\sin nt}{n}\right]_{\pi}^{2\pi}\right]$$

$$= \frac{1}{\pi}\left[V_m(0-0) - V_m(0)\right]$$

$$a_n = 0. \text{ Ans}$$

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$t_1 = 0,\ t_1 + T = T$

$T = 2\pi,\ \therefore \omega_0 = 1$

$= \frac{2}{2\pi}\int_{t_1}^{t_1+T} f(t)\sin n\omega_0 t\,dt$

$$b_n = \frac{2}{T}\int_{t_1}^{t_1+T} f(t)\sin n\omega_0 t\,dt$$

Similarly $b_n = \frac{1}{2\pi}\int_0^{2\pi} g(t)\sin nt\,dt$

$$= \frac{1}{\pi}\left[\int_0^{\pi} f(t)\sin nt\,dt + \int_{\pi}^{2\pi} f(t)\sin nt\,dt\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} V_m \sin nt\,dt + \int_{\pi}^{2\pi} -V_m \sin nt\,dt\right]$$

$$= \frac{1}{\pi}\left[V_m\int_0^{\pi}\sin nt\,dt - V_m\int_{\pi}^{2\pi}\sin nt\,dt\right]$$

$$= \frac{1}{\pi}\left[V_m \times \left[-\frac{\cos nt}{n}\right]_0^{\pi} - V_m\left[-\frac{\cos nt}{n}\right]_{\pi}^{2\pi}\right]$$

$$= \frac{V_m}{\pi}\left[\left(\frac{-\cos n\pi + 1}{n}\right) - \left(\frac{-\cos 2n\pi + \cos n\pi}{n}\right)\right]$$

$$= \frac{V_m}{\pi}\left[\frac{1}{n} - \frac{\cos n\pi}{n} + \frac{\cos 2n\pi}{n} - \frac{\cos n\pi}{n}\right]$$

$$= \frac{V_m}{\pi}\left[\frac{1}{n} - \frac{2\cos n\pi}{n} + \frac{\cos 2n\pi}{n}\right]$$

when $\cos 2n\pi = 1$

$$= \frac{V_m}{\pi}\left[\frac{1}{n} - \frac{2\cos n\pi}{n} + \frac{1}{n}\right]$$

$$= \frac{V_m}{\pi}\left[\frac{2}{n} - \frac{2\cos n\pi}{n}\right]$$

$$b_n = \frac{V_m}{\pi}\left[\frac{2 - 2\cos n\pi}{n}\right]$$

$$b_n = \frac{2V_m}{n\pi}(1 - \cos n\pi) \quad \text{—— } (*)$$

$\cos\pi = -1$

$\cos 3\pi,\ \cos 5\pi = -1$

$n_0 = 1$	$b_1 = \frac{2V_m}{\pi}\times 2$	$b_1 = \frac{4V_m}{\pi}$
$n = 2$	$b_2 = \frac{2V_m}{2\pi}\times 0$	$b_2 = 0$
$n = 3$	$b_3 = \frac{2V_m}{3\pi}\times 2$	$b_3 = \frac{4V_m}{\pi}$
$n = 4$	$b_4 = \frac{2V_m}{4\pi}\times 0$	$b_4 = 0$
$n = 5$	$b_5 = \frac{2V_m}{5\pi}\times 2$	$b_5 = \frac{4V_m}{\pi}$

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$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right)$$

$$f(t) = a_0 + b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + b_3 \sin 3\omega_0 t + b_4 \sin 4\omega_0 t \ldots + b_n \sin n\omega_0 t$$

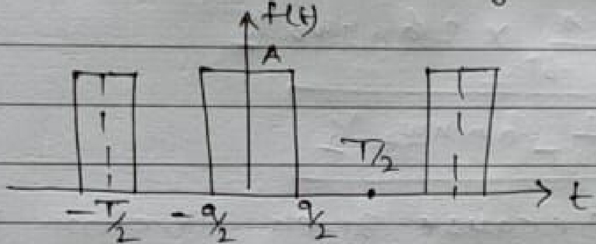
$$f(t) = 0 + \frac{4V_m}{\pi} \sin \omega_0 t + 0 + \frac{4V_m}{3\pi} \sin 3\omega_0 t + 0 + \frac{4V_m}{5\pi} \sin 5\omega_0 t$$

$$f(t) = \frac{4V_m}{\pi} \left(\sin \omega_0 t + \frac{1}{3} \sin 3\omega_0 t + \frac{1}{5} \sin 5\omega_0 t + \cdots \right)$$

$$\omega_0 = 1$$

$$f(t) = \frac{4V_m}{\pi} \left(\sin t + \frac{1}{3} \sin 3t + \frac{1}{5} \sin 5t + \cdots \right)$$

Prob 9 Obtain the Fourier series in trigonometric form for the periodic train of pulses of period T, pulse height A and width a shown in fig.



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Soln Here $f(t) = A \quad -\frac{a}{2} \leq t \leq \frac{a}{2}$

$\qquad = 0$ else where

$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} f(t)\,dt$$

$$= \frac{1}{T} \int_{-a/2}^{a/2} f(t)\,dt$$

$$= \frac{1}{T} \int_{-a/2}^{a/2} A\,dt$$

$$= \frac{A}{T} \left[t \right]_{-a/2}^{a/2}$$

$$= \frac{A}{T} \left[\frac{a}{2} - \left(-\frac{a}{2} \right) \right] = \frac{A}{T} \left[\frac{a}{2} + \frac{a}{2} \right] = \frac{A}{T} \times \frac{\cancel{2}a}{\cancel{2}}$$

$$\boxed{\therefore a_0 = \frac{Aa}{T}}$$

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$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\omega_0 t\,dt$$

$$= \frac{2}{T}\int_{-a/2}^{a/2} A\cos n\omega_0 t\,dt$$

$$= \frac{2A}{T}\int_{a/2}^{a/2} \cos n\omega_0 t\,dt$$

$$= \frac{2A}{T}\times\left[\frac{\sin n\omega_0 t}{n\omega_0}\right]_{-a/2}^{a/2}$$

$$= \frac{2A}{n\omega_0 T}\left[\sin n\omega_0 t\right]_{-a/2}^{a/2}$$

$$= \frac{2A}{n\omega_0 T}\left[\sin n\omega_0 \tfrac{a}{2} - (-\sin n\omega_0 \tfrac{a}{2})\right]$$

$$= \frac{2A}{n\omega_0 T}\left[\sin n\omega_0 \tfrac{a}{2} + \sin n\omega_0 \tfrac{a}{2}\right]$$

$$= \frac{2A}{n\omega_0 T}\times 2\sin n\omega_0 \tfrac{a}{2}$$

$$\boxed{a_n = \frac{4A}{n\omega_0 T}\sin n\omega_0 \tfrac{a}{2}}$$

~~$b_n = 0$, Because given waveform is even form~~ (even)

$$n\omega_0 T = n\times 2\pi f_0 \times T$$

$$= n\times 2\pi\times\frac{1}{\cancel{T}}\times\cancel{T}$$

$$= 2n\pi.$$

$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n\cos n\omega_0 t + b_n\sin n\omega_0 t)$$

$$= \frac{Aa}{T} + \sum_{n=1}^{\infty}\left(\frac{4A}{n\omega_0 T}\sin n\omega_0\frac{a}{2} + 0\right)$$

put $n = 1, 2, 3 - - -$

$$f(t) = \frac{Aa}{T} + \frac{4A}{2\pi}\sin\frac{\omega_0 a}{2} + - - - -$$ Ans.

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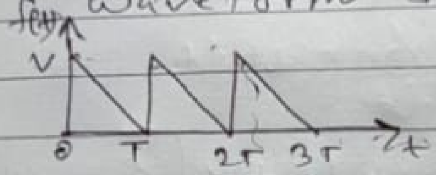
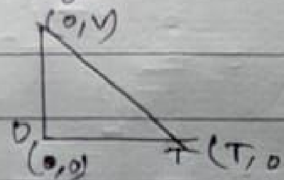
(3) Prob: Find the Fourier series representation of the waveform shown in fig.





$f(t) = -\frac{V}{T}t + V$ from slope eqn.

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$= \frac{1}{T}\int_0^T \left(-\frac{V}{T}t + V\right) dt$$

$$= \frac{1}{T}\left[-\frac{V}{T}\int_0^T t\,dt + V\int_0^T dt\right]$$

$$= \frac{1}{T}\left[-\frac{V}{\cancel{T}}\times\frac{T^{\cancel{2}}}{2} + VT\right]$$

$$= \frac{1}{\cancel{T}}\times\frac{V\cancel{T}}{2} = \frac{V}{2}$$

$$\boxed{a_0 = \frac{V}{2}}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\,dt$$

$$= \frac{2}{T}\int_0^T \left(-\frac{V}{T}t + V\right)\cos n\omega_0 t\,dt$$

$$= \frac{2}{T}\left[-\frac{V}{T}\int_0^T t\cos n\omega_0 t\,dt + V\int_0^T \cos n\omega_0 t\,dt\right]$$

$$= \frac{2}{T}\left[-\frac{V}{T}\left(t\frac{\sin n\omega_0 t}{n\omega_0} + \frac{\cos n\omega_0 t}{n^2\omega_0^2}\right)_0^T + V\left[\frac{\sin n\omega_0 t}{n\omega_0}\right]_0^T\right]$$

$$= \frac{2}{T}\left[-\frac{V}{T}\left(T\frac{\sin n\omega_0 T}{n\omega_0} + \frac{\cos n\omega_0 T}{n^2\omega_0^2} - \frac{1}{n^2\omega_0^2}\right)\right] \qquad \underline{\omega_0 T = 2\pi}$$

$$= -\frac{2V}{T^2}\left[\frac{\cos n\omega_0 T}{n^2\omega_0^2} - \frac{1}{n^2\omega_0^2}\right]$$

$$= -\frac{2V}{T^2}\left(\frac{\cos 2n\pi - 1}{n^2\omega_0^2}\right)$$

If $n =$ even, $\cos 2n\pi = 1$

$$\boxed{\therefore a_n = 0}$$

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$$b_n = \frac{2}{T}\int_0^T f(t)\sin n w_0 t \, dt$$

$$= \frac{2}{T}\int_0^T \left(-\frac{V}{T}t + V\right)\sin n w_0 t \, dt$$

$$= \frac{2}{T}\left[-\frac{V}{T}\int_0^T t \sin n w_0 t \, dt + V\int_0^T \sin n w_0 t \, dt\right]$$

$$= \frac{2}{T}\left[-\frac{V}{T}\left(-\frac{t\cos n w_0 t}{n w_0} + \frac{\sin n w_0 t}{n^2 w_0^2}\right)_0^T + V\left(-\frac{\cos n w_0 t}{n w_0}\right)_0^T\right]$$

$$b_n = \frac{2}{T}\left[-\frac{V}{T}\left(-\frac{T\cos n w_0 T}{n w_0} + \frac{\sin n w_0 T}{n^2 w_0^2}\right) + V\left(-\frac{\cos n w_0 T}{n w_0} + \frac{\cos 0}{n w_0}\right)\right]$$

Put. $w_0 T = 2\pi$ $\quad \cos 2n\pi = 1$

$\sin 2n\pi = 0$

$$= \frac{2}{T}\left(-\frac{V}{T}\times\left(\frac{-T}{n w_0}\right)\right) + 0$$

$$= \frac{2}{T}\left(\frac{V}{\cancel{T}}\times\frac{\cancel{T}}{n w_0}\right)$$

$$= \frac{2V}{n w_0 T} = \frac{\cancel{2}V}{n\times\cancel{2}\pi} = \frac{V}{n\pi}$$

Put $n = 1, 2, 3 \ldots$

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos n w_0 t + b_n \sin n w_0 t\right)$$

$$= a_0 + a_n + b_1 \sin w_0 t + b_2 \sin 2 w_0 t + b_3 \sin 3 w_0 t + \cdots + b_n \sin n w_0 t$$

$$= \frac{V}{2} + 0 + \frac{V}{\pi}\sin w_0 t + \frac{V}{2\pi}\sin 2 w_0 t + \frac{V}{3\pi}\sin 3 w_0 t + \cdots + \frac{V}{n\pi}\sin n w_0 t$$

Ans

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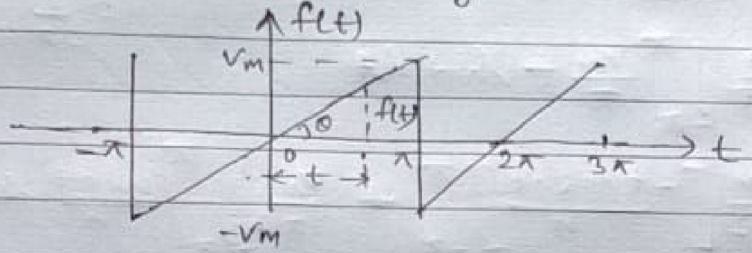
Prob: (4) Determine the Fourier series for the sawtooth wave shown in fig.



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~~we take the limit from $-\pi$ to π~~

Here $T = 2\pi$, $\omega_0 = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$

$\omega_0 = 1$

Slope eqn from fig. $f(t) = \frac{V_m}{\pi} t$ when $-\pi < t < \pi$

Determination of a_0

In one cycle, +ve area = Negative are

$a_0 = 0$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t) \cos nt\, dt$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{V_m}{\pi} t \cos nt\, dt$$

$$= \frac{V_m}{\pi^2}\int_{-\pi}^{\pi} t \cos nt\, dt$$

$$= \frac{V_m}{\pi^2}\left\{\left[\frac{t \sin nt}{n}\right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{\sin nt}{n} dt\right\}$$

$$= \frac{V_m}{n\pi^2}\left(\cancel{\pi \sin n\pi} - \cancel{\pi \sin n\pi}\right) + \frac{V_m}{n^2\pi^2}\left(\cos nt\right)_{-\pi}^{\pi}$$

$$= 0 + \frac{V_m}{n^2\pi^2}\left(\cancel{\cos n\pi} - \cancel{\cos n\pi}\right)$$

$$a_n = 0.$$

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$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin nt\, dt$$

$$b_n = \frac{2}{T}\int_{t_1}^{t_1+T} f(t)\sin n\omega_0 t\, dt$$
$$= \frac{2}{2\pi}\int_{t_1}^{t_1+T} f(t)\sin n\omega_0 t\, dt$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{V_m}{\pi} t \sin nt\, dt$$

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$$= \frac{V_m}{\pi^2}\left[\frac{-t\cos nt}{n}\right]_{-\pi}^{\pi} + \frac{V_m}{\pi^2}\int_{-\pi}^{\pi}\frac{\cos nt}{n}\, dt$$

$$= -\frac{V_m}{n\pi^2}\left(\pi\cos n\pi + \pi\cos n\pi\right) + \frac{V_m}{n^2\pi^2}\left[\sin nt\right]_{-\pi}^{\pi}$$

$$= -\frac{V_m}{n\pi^2}\times 2\pi\cos n\pi + \frac{V_m}{n^2\pi^2}\times 2\sin n\pi$$

But $\sin n\pi = 0$

$$b_n = -\frac{2V_m}{n\pi}\cos n\pi$$

when n is even $n = 2, 4, 6 \ldots$ $\cos n\pi = 1$.

$$b_1 = -\frac{2V_m}{n\pi}$$

$$b_2 = -\frac{2V_m}{2\pi}$$

$$b_4 = -\frac{2V_m}{4\pi}$$

$$b_6 = -\frac{2V_m}{6\pi}$$

when n is odd i.e $n = 1, 3, 5 \ldots$

$\cos n\pi = -1$

$$b_n = \frac{2V_m}{n\pi}$$

$$b_1 = \frac{2V_m}{\pi}, \quad b_3 = \frac{2V_m}{3\pi}, \quad b_5 = \frac{2V_m}{5\pi}$$

Thus, the Fourier series for the given saw tooth wave is

$$f(t) = \frac{2V_m}{\pi}\left(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t + \ldots\right)$$ Ans