DATE / /
PAGE NO.

## evaluation of $a_0$

Integrating both side over one period $(t_1, t_1+T)$ where $t_1$ is arbitrary

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \quad \text{—①}$$

$$\int_{t_1}^{t_1+T} f(t)\,dt = \int_{t_1}^{t_1+T} a_0\,dt + \int_{t_1}^{t_1+T} \Sigma (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)\,dt$$

$$\int_{t_1}^{t_1+T} f(t)\,dt = a_0 (t)_{t_1}^{t_1+T} + \sum_{n=1}^{\infty} \int_{t_1}^{t_1+T} a_n \cos n\omega_0 t\,dt + \int_{t_1}^{t_1+T} b_n \sin n\omega_0 t\,dt$$

$$\int_{t_1}^{t_1+T} f(t)\,dt = a_0 T + \sum_{n=1}^{\infty} \left[a_n \int_{t_1}^{t_1+T} \cos n\omega_0 t\,dt + b_n \int_{t_1}^{t_1+T} \sin n\omega_0 t\,dt\right.$$

$$\int_{t_1}^{t_1+T} f(t)\,dt = a_0 T$$

$$\boxed{a_0 = \frac{1}{T}\int_{t_1}^{t_1+T} f(t)\,dt}$$

we know that

$$\int_{t_1}^{t_1+T} \cos n\omega_0 t\,dt = 0$$

$$\int_{t_1}^{t_1+T} \sin n\omega_0 t\,dt = 0$$

## evaluation of $a_n$

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \quad \text{—①}$$

To evaluate $a_n$, the amplitude of $\cos n\omega_0 t$ component of $f(t)$, we multiply eq$^n$ ① by $\cos m\omega_0 t$ and integrate over one period $(t_1, t_1+T)$

$$f(t)\cos m\omega_0 = a_0 \cos m\omega_0 t + \sum_{n=1}^{\infty} a_n \cos m\omega_0 t \cdot \cos n\omega_0 t + b_n \sin n\omega_0 t \cdot \cos m\omega_0 t$$

$$\int_{t_1}^{t_1+T} f(t)\cos m\omega_0 t\,dt = \int_{t_1}^{t_1+T} a_0 \cos m\omega_0 t\,dt + \sum_{n=1}^{\infty} \int_{t_1}^{t_1+T} a_n \cos m\omega_0 t \cdot \cos n\omega_0 t\,dt + \int_{t_1}^{t_1+T} b_n \sin n\omega_0 t \cdot \cos m\omega_0 t\,dt$$

$$\int_{t_1}^{t_1+T} f(t)\cos m\omega_0 t\,dt = a_0 \int_{t_1}^{t_1+T} \cos m\omega_0 t\,dt + \sum_{n=1}^{\infty} a_n \int_{t_1}^{t_1+T} \cos m\omega_0 t \cdot \cos n\omega_0 t\,dt + b_n \int_{t_1}^{t_1+T} \sin n\omega_0 t \cdot \cos m\omega_0 t\,dt$$

DATE / /
PAGE No.

8

from formula

$$\int_{t_1}^{t_1+T} \cos m\omega_0 t\, dt = 0$$

$$\int_{t_1}^{t_1+T} \sin n\omega_0 t \cdot \cos m\omega_0 t\, dt = 0$$

$$\int_{t_1}^{t_1+T} \cos m\omega_0 t \cdot \cos n\omega_0 t\, dt = \begin{cases} 0 & m \neq n \\ \frac{T}{2} & m = n \end{cases}$$ from privious case (8)

Then, we can write,

$$\int_{t_1}^{t_1+T} f(t) \cos m\omega_0 t\, dt = \sum_{n=1}^{\infty} a_n \int_{t_1}^{t_1+T} \cos m\omega_0 t \cdot \cos n\omega_0 t\, dt$$

Case I $m \neq n$

That means

Right side part is zero.

Case II $m = n$

$$\int_{t_1}^{t_1+T} f(t) \cos m\omega_0 t\, dt = a_n \times \frac{T}{2}$$

$$\boxed{a_n = \frac{2}{T} \int_{t_1}^{t_1+T} f(t) \cos m\omega_0 t\, dt}$$

evaluation of $b_n$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

Multiplying both side by $\sin m\omega_0 t$

$$f(t) \sin m\omega_0 t = a_0 \sin m\omega_0 t + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t \cdot \sin m\omega_0 t + b_n \sin n\omega_0 t \cdot \sin m\omega_0 t)$$

Integrating both side over one period $(t_1, t_1+T)$

$$\int_{t_1}^{t_1+T} f(t) \sin m\omega_0 t\, dt = \int_{t_1}^{t_1+T} a_0 \sin m\omega_0 t\, dt + \sum_{n=1}^{\infty} \int_{t_1}^{t_1+T} a_n \cos n\omega_0 t \sin m\omega_0 t\, dt + \int_{t_1}^{t_1+T} b_n \sin n\omega_0 t \cdot \sin m\omega_0 t\, dt$$

DATE / /
PAGE No.

~~We know that~~

$$\int_{t_1}^{t_1+T} \sin m\omega_0 t \, dt = 0 \quad \text{from case ①}$$

$$\int_{t_1}^{t_1+T} \cos n\omega_0 t \cdot \sin m\omega_0 t \, dt = 0 \quad \text{from case ③}$$

Therefore,

$$\int_{t_1}^{t_1+T} f(t) \sin m\omega_0 t \, dt = b_n \int_{t_1}^{t_1+T} \sin n\omega_0 t \cdot \sin m\omega_0 t \, dt$$

from case ⑧

$$\int_{t_1}^{t_1+T} \sin m\omega_0 t \cdot \sin n \omega_0 t \, dt = \begin{cases} T/2 & m = n \neq 0 \\ 0 & m \neq n. \end{cases}$$

So, we can write.

$$\int_{t_1}^{t_1+T} \sin m\omega_0 t \cdot \sin n\omega_0 t \, dt = T/2$$

$$\int_{t_1}^{t_1+T} f(t) \sin m\omega_0 t \, dt = b_n \times \frac{T}{2}$$

$$\boxed{\therefore b_n = \frac{2}{T} \int_{t_1}^{t_1+T} f(t) \sin m\omega_0 t \, dt}$$

$$a_0 = \frac{1}{T} \int_{t_1}^{t_1+T} f(t) \, dt.$$

9

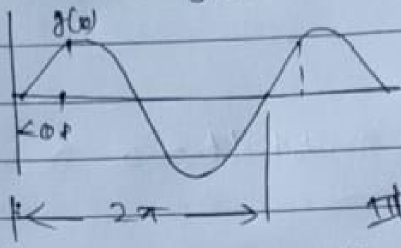
$$a_n = \frac{2}{T} \int_{t_1}^{t_1+T} f(t) \cos m\omega_0 t \, dt.$$

$$b_n = \frac{2}{T} \int_{t_1}^{t_1+T} f(t) \sin m\omega_0 t \, dt.$$

BAMBOOS

## Function of period $2\pi$

let $g(\theta)$ be a periodic function of period $2\pi$




$f(t) = f(t+T)$

10

$|||^{ly},\ g(\theta) = g(\theta + 2\pi)$

$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$

$\theta = \omega_0 t$

Similarly,

$g(\theta) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\theta + b_n \sin n\theta)$

$\theta = 2\pi f_0 t$

$= \frac{2\pi}{T} \cdot t$

$a_0 = \frac{1}{T} \int_{t_1}^{t_1+T} f(t)\,dt$

$d\theta = \frac{2\pi}{T}\,dt$

$\omega_0 = 2\pi f_0$

$= 2\pi \times \frac{1}{T_0}$

$= \frac{2\pi}{T_0}$

$a_0 = \frac{1}{T} \int_0^T f(t)\,dt$

$dt = \frac{T}{2\pi} \cdot d\theta$

$f(t) = g(\theta)$

suppose $t_1 = 0$

$t_1 + T \Rightarrow T$

$0 \to T$

$-\frac{T}{2} \to \frac{T}{2}$

$-\frac{T}{4} \to \frac{3T}{4}$

$a_0 = \frac{1}{T} \int_0^{2\pi} g(\theta) \cdot \frac{T}{2\pi} \cdot d\theta$

$$\boxed{a_0 = \frac{1}{2\pi} \int_0^{2\pi} g(\theta)\,d\theta}$$

$a_n = \frac{2}{T} \int_0^T f(t) \cos n\omega_0 t\,dt$

$f(t) = g(\theta),\ T \to 2\pi,\ \omega_0 t = \theta$

$dt = \frac{d\theta}{dt}$

$a_n = \frac{2}{T} \int_0^{2\pi} g(\theta) \cos n\theta \frac{T}{2\pi}\,d\theta$

$dt = \frac{T}{2\pi}\,d\theta$

$= \frac{\cancel{2}}{\cancel{T}} \times \frac{\cancel{T}}{\cancel{2}\pi} \int_0^{2\pi} g(\theta) \cos n\theta\,d\theta$

$$\boxed{a_n = \frac{1}{\pi} \int_0^{2\pi} g(\theta) \cos n\theta\,d\theta}$$

$b_n = \frac{2}{T} \int_0^T f(t) \sin n\omega_0 t\,dt$

Similarly

$$\boxed{b_n = \frac{1}{\pi} \int_0^{2\pi} g(\theta) \sin n\theta\,d\theta}$$