Jordan form :

$$T(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{(s-\lambda_1)^3 (s-\lambda_4)(s-\lambda_5) \cdots (s-\lambda_n)}$$

$$\frac{Y(s)}{U(s)} = b_0 + \frac{c_1}{(s-\lambda_1)^3} + \frac{c_4}{(s-\lambda_4)} + \cdots + \frac{c_n}{(s-\lambda_n)}$$

$$Y(s) = b_0 U(s) + \underbrace{\frac{c_1}{(s-\lambda_1)^3} U(s)}_{x_1} + \underbrace{\frac{c_2}{(s-\lambda_1)^2}}_{x_2} U(s) + \underbrace{\frac{c_3}{(s-\lambda_1)}}_{x_3} U(s) + \underbrace{\frac{c_4}{(s-\lambda_4)}}_{x_4} U(s) + \cdots + \underbrace{\frac{c_n}{(s-\lambda_n)}}_{x_n} U(s)$$

Define state variables

$$x_1(s) = \frac{1}{(s-\lambda_1)^3} U(s) \quad \text{— (1)}$$

$$x_2(s) = \frac{1}{(s-\lambda_1)^2} U(s) \quad \text{— (2)}$$

$$x_3(s) = \frac{1}{(s-\lambda_1)} U(s) \quad \text{— (3)}$$

$$x_4(s) = \frac{1}{(s-\lambda_4)} U(s) \quad \text{— (4)}$$

$$\vdots$$

$$x_n(s) = \frac{1}{(s-\lambda_n)} U(s) \quad \text{— (n)}$$

$$\frac{x_1(s)}{x_2(s)} = \frac{\frac{1}{(s-\lambda_1)^3}}{\frac{1}{(s-\lambda_1)^2}}$$

$$\frac{x_1(s)}{x_2(s)} = \frac{1}{(s-\lambda_1)^3} \times \frac{(s-\lambda_1)^2}{1} = \frac{1}{s-\lambda_1}$$

$$\frac{x_2}{x_3} = \frac{\frac{1}{(s-\lambda_1)^2}}{\frac{1}{(s-\lambda_1)}} = \frac{1}{(s-\lambda_1)^2} \times \frac{(s-\lambda_1)}{1} = \frac{1}{s-\lambda_1}$$

$$\frac{x_1(s)}{x_2(s)} = \frac{1}{(s-\lambda_1)} \Rightarrow s x_1(s) - \lambda_1 x_1(s) = x_2(s)$$

$$s x_1(s) = \lambda_1 x_1(s) + x_2(s) \quad \text{— (a)}$$

$$\frac{x_2(s)}{x_3(s)} = \frac{1}{(s-\lambda_1)} \Rightarrow s x_2(s) - \lambda_1 x_2(s) = x_3(s)$$

$$s x_2(s) = \lambda_1 x_2(s) + x_3(s) \quad \text{— (b)}$$

$$x_3(s) = \frac{1}{(s-\lambda_1)} U(s) \quad \text{III}^{ly}, \quad s x_3(s) = \lambda_1 x_3(s) + U(s) \quad \text{— (c)}$$

$$x_4(s) = \frac{1}{(s-\lambda_4)} U(s) \quad s x_4(s) = \lambda_4 x_4(s) + U(s) \quad \text{— (d)}$$

$$\vdots$$

$$s x_n(s) = \lambda_n x_n(s) + U(s) \quad \text{— (n)}$$

Using Inverse L.T

$$\left.\begin{aligned}
\dot{x}_1 &= \lambda_1 x_1 + x_2 \\
\dot{x}_2 &= \lambda_1 x_2 + x_3 \\
\dot{x}_3 &= \lambda_1 x_3 + u \\
\dot{x}_4 &= \lambda_4 x_4 + u \\
&\vdots \\
\dot{x}_n &= \lambda_n x_n + u
\end{aligned}\right\} \text{state eq}^n$$

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$$Y(s) = b_0 U(s) + c_1 x_1(s) + c_2 x_2(s) + c_3 x_3(s) - - + c_n x_n(s)$$

$$Y = b_0 u + c_1 x_1 + c_2 x_2 + c_3 x_3 + - - - + c_n x_n$$

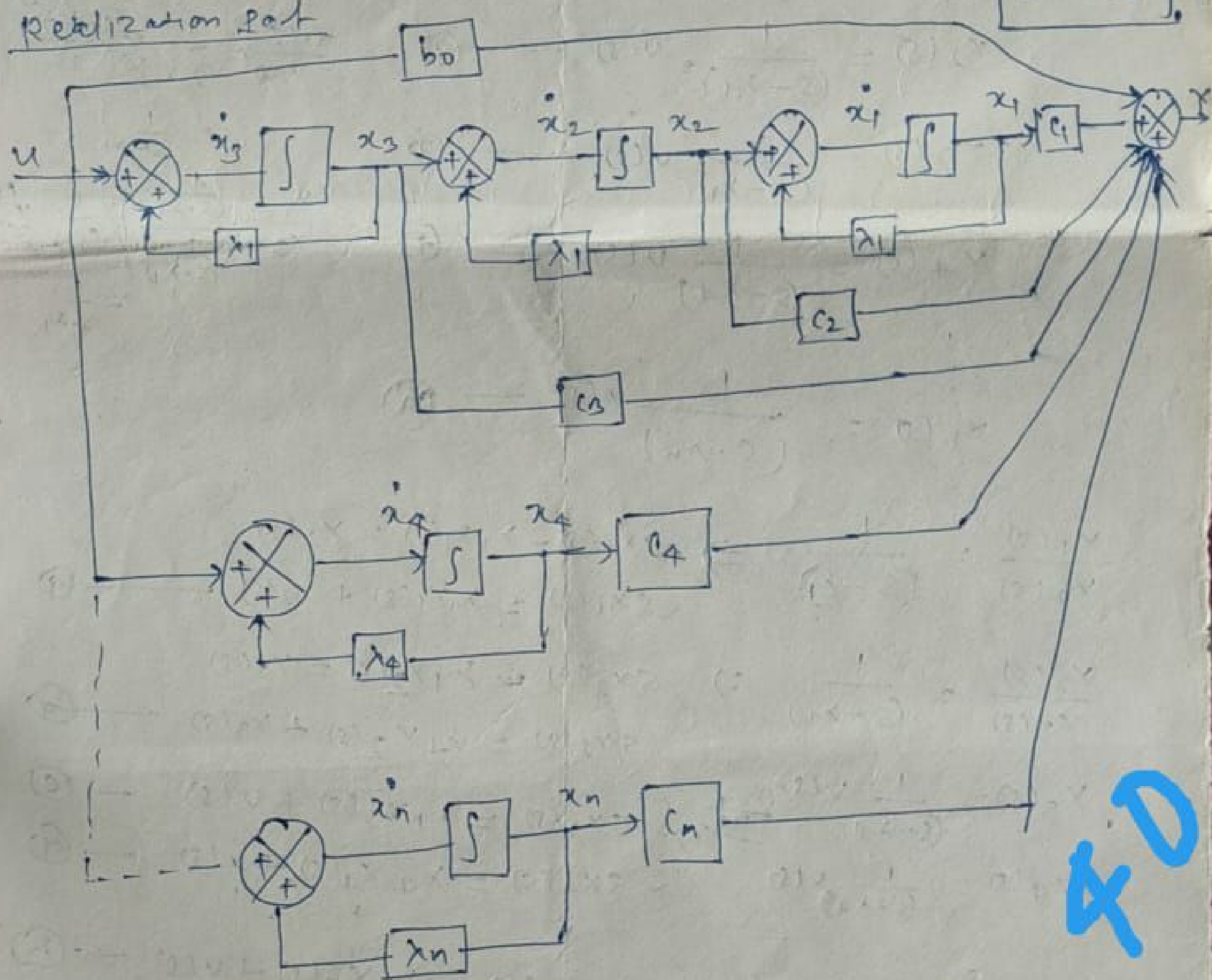
state space representation Jordan form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} \lambda_1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \lambda_1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \lambda_1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 & \cdots & 0 & 0 \\ \vdots & & & & & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & & \lambda_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} c_1 & c_2 & - - - & c_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + b_0 u$$

$$\boxed{\int \Rightarrow \frac{1}{s}}$$

Realization part



Prob: A control system has a T.F $H(s) = \frac{3(s+1)}{(s+5)(s+2)^2}$ obtain the state model and its realization

$$G(s) = \frac{3(s+1)}{(s+5)(s+2)^2}$$

$$G(s) = \frac{s+4}{s(s+1)^2(s+3)}$$

$$T(s) = \frac{Y(s)}{U(s)} = \frac{s+4}{s(s+1)^2(s+3)}$$

$$\frac{Y(s)}{U(s)} = \frac{c_1}{(s+1)^2} + \frac{c_2}{(s+1)^2} + \frac{c_3}{(s+3)} + \frac{c_4}{s}.$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{3(s+1)}{(s+2)^2(s+5)}$$

$$= \frac{c_1}{(s+2)^2} + \frac{c_2}{(s+2)} + \frac{c_3}{(s+5)}$$

where $c_1 = \cancel{(s+2)^2} \times \frac{3(s+1)}{\cancel{(s+2)^2}(s+5)} \Big|_{s=-2}$

$$= \frac{3(-2+1)}{(-2+5)} = \frac{3(-1)}{3} = -1. \quad \boxed{\therefore c_1 = -1}$$

$$c_2 = \frac{d}{ds}\left[\cancel{(s+2)^2}\frac{3(s+1)}{(s+5)\cancel{(s+2)^2}}\right]_{s=-2}$$

$$= \frac{d}{ds}\left[\frac{3(s+1)}{(s+5)}\right]_{s=-2}$$

$$= \frac{(s+5)\times 3 - 3(s+1)\cdot 1}{(s+5)^2}\Big|_{s=-2}$$

$$= \frac{\cancel{3s}+15-\cancel{3s}-3}{(s+5)^2}\Big|_{s=-2}$$

$$= \frac{12}{(-2+5)^2} = \frac{12}{(3)^2} = \frac{12}{9} = \frac{4}{3}. \quad \boxed{c_2 = \frac{4}{3}}$$

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$$c_3 = \cancel{(s+5)} \times \frac{3(s+1)}{(s+2)^2\cancel{(s+5)}}\Big|_{s=-5}$$

$$= \frac{3(-5+1)}{(-5+2)^2} = \frac{3\times -4}{(-3)^2} = \frac{-1\cancel{2}}{\cancel{9}} = \frac{-4}{3} \quad \boxed{c_3 = -\frac{4}{3}}$$

$$\therefore G(s) = \frac{Y(s)}{U(s)} = \frac{c_1}{(s+2)^2} + \frac{c_2}{(s+2)} + \frac{c_3}{(s+5)}$$

$$= \frac{-1}{(s+2)^2} + \frac{4/3}{(s+2)} + \frac{-\frac{4}{3}}{(s+5)}$$

$$Y(s) = -\underbrace{\frac{1}{(s+2)^2}U(s)}_{x_1(s)} + \frac{4}{3}\underbrace{\frac{U(s)}{(s+2)}}_{x_2(s)} - \frac{4}{3}\underbrace{\frac{U(s)}{(s+5)}}_{x_3(s)}$$

$Y(s) = -x_1(s) + \frac{4}{3}x_2 - \frac{4}{3}x_3.$

let us state variable be defined as follow

$$x_1(s) = \frac{U(s)}{(s+2)^2} \quad — ①$$

$$x_2(s) = \frac{U(s)}{(s+2)} \quad — ②$$

$$x_3(s) = \frac{U(s)}{(s+5)} \quad — ③$$

• Dividing ① by ② we get

$$\frac{x_1(s)}{x_2(s)} = \frac{1}{s+2}$$

$s x_1(s) + 2 x_1(s) = x_2(s)$

I.L.T of this eqn yield (initial cond^n to be zero)

$\dot{x}_1 + 2x_1 = x_2$

$\dot{x}_1 = -2x_1 + x_2$ — (4)

Also I.L.T of (2) yields

$s x_2(s) + 2 x_2(s) = U(s)$

$\dot{x}_2 = -2x_2 + U$ — (5)

III^{r} $x_3(s) = \dfrac{U(s)}{s+5}$

$s x_3(s) + 5 x_3(s) = U(s)$

I.L.T of the above

$\dot{x}_3 = -5x_3 + U$ — (6)

Also, $Y(s) = -1 \cdot x_1(s) + \frac{4}{3} x_2(s) - \frac{4}{3} x_3(s)$

I.L.T of the above eqn yields

$y = -x_1 + \frac{4}{3} x_2 - \frac{4}{3} x_3$

The state model as follow: (from eqn (4), (5) & (6))

state eqn,

$\dot{x}_1 = -2x_1 + x_2$

$\dot{x}_2 = -2x_2 + U$

$\dot{x}_3 = -5x_3 + U$

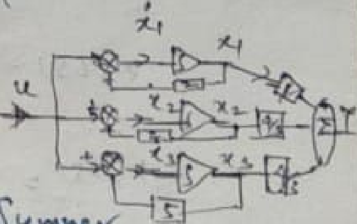
State eqn can be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u$$

The o/p eqn is written as

$y = -x_1 + \frac{4}{3} x_2 - \frac{4}{3} x_3$ → in place of c_1, c_2 and c_3 value are $-1, 4/3, -4/3$.

$$y = \begin{bmatrix} -1 & 4/3 & -\frac{4}{3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u$$



Realization Part