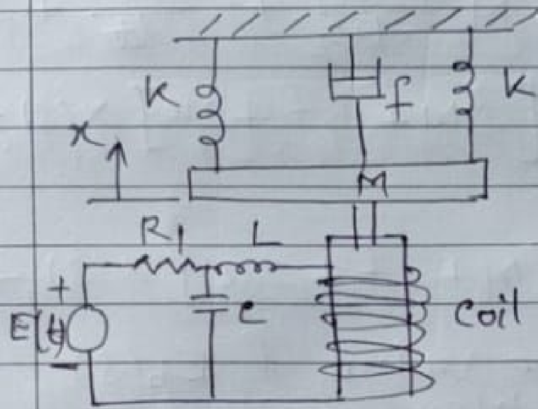
① Find the Transfer function $\frac{X(s)}{E(s)}$ for the electromechanical system shown in fig:-



(Sol^n) The given system consists of two parts
(a) electrical
(b) mechanical

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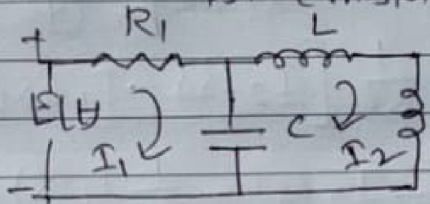
1st consider electrical system



Applying KVL in both mesh.

$$E(t) = R_1 I_1(t) + \frac{1}{C}\int[I_1(t) - I_2(t)]\,dt \quad -①$$

$$0 = \frac{1}{C}\left[I_2(t) - I_1(t)\right]dt + L\frac{di_2(t)}{dt} + e_b \quad -②$$

Taking Laplace Transform on both eq^n

$$E(s) = R_1 I_1(s) + \frac{1}{Cs}\left[I_1(s) - I_2(s)\right] \quad -③$$

$$0 = \frac{1}{sC}\left[I_2(s) - I_1(s)\right] + sL_2 I_2(s) + E_b(s) \quad -④$$

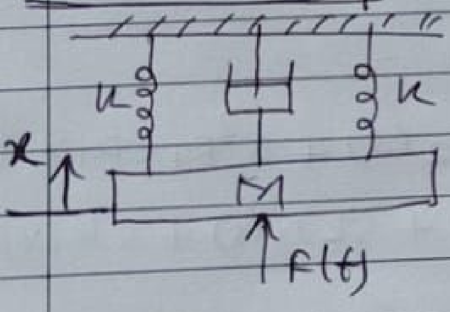
Now, $e_b \propto \frac{dx}{dt}$ f ∝ velocity

Back emf (counter emf) opposes the applied voltage

$e_b = k_1\frac{dx}{dt}$ where k_1 = back emf constt.

$E_b(s) = k_1 s X(s)$ — ⑤

Now consider the Mechanical system



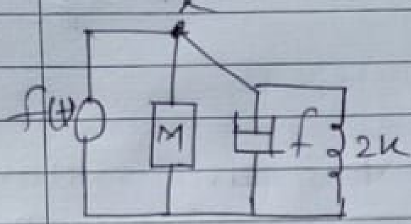
$F(t) \propto I_2(t)$

$F(t) = K_2 I_2(t)$

Taking laplace

$F(s) = K_2 I_2(s)$ — ⑥

equivalent ckt of mech system



$$f(t) = M\frac{d^2x(t)}{dt^2} + f\frac{dx(t)}{dt} + 2kx(t)$$

$$F(s) = [s^2M + sf + 2k]\,X(s) \quad —(7)$$

$$K_2 I_2(s) = [s^2M + sf + 2k]\,X(s) \quad —(8)$$

from eq^n (5) put the value of $E_b(s)$ in eq^n (4)

$$0 = \frac{1}{sC}[I_2(s) - I_1(s)] + sLI_2(s) + E_b(s)$$

$$= \frac{1}{sC}I_2(s) - \frac{1}{sC}I_1(s) + sLI_2(s) + E_b(s)$$

$$\frac{1}{sC}I_1(s) = \frac{1}{sC}I_2(s) + sLI_2(s) + K_1 s X(s)$$

$$I_1(s) = I_2(s) + s^2LC\,I_2(s) + s^2CK_1X(s) \quad —(9)$$

Now from eq^n (1)

$$E(s) = R_1I_1(s) + \frac{1}{sC}I_1(s) - \frac{1}{sC}I_2(s)$$

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$$= \left[R_1 + \frac{1}{sC}\right]I_1(s) - \frac{1}{sC}I_2(s)$$

putting the value of $I_1(s)$

$$E(s) = \left(R_1 + \frac{1}{sC}\right)\left[I_2(s) + s^2LC\,I_2(s) + s^2CK_1X(s)\right] - \frac{1}{sC}I_2(s)$$

$$E(s) = R_1I_2(s) + s^2R_1LC\,I_2(s) + s^2R_1CK_1X(s) + \cancel{\frac{1}{sC}I_2(s)} + sLI_2(s) + sK_1X(s) - \cancel{\frac{1}{sC}I_2(s)}.$$

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$$E(s) = \left[R_1 + s^2 R_1 LC + sL\right] I_2(s) + sk_1\left[1 + R_1 Cs\right] x(s)$$

$$E(s) - sk_1 x(s)(1 + R_1 Cs) = I_2(s)\left[R_1 + s^2 R_1 LC + sL\right]$$

$$I_2(s) = \frac{E(s) - sk_1(1 + R_1 Cs)x(s)}{R_1 + s^2 R_1 LC + sL} \quad \text{--- (10)}$$

Putting the value of $I_2(s)$ in eqn (8)

$$k_2 I_2(s) = \left[s^2 M + sf + 2k\right] x(s)$$

$$k_2\left[\frac{E(s) - sk_1 x(s)(1 + R_1 Cs)}{R_1 + s^2 R_1 LC + sL}\right] = (s^2 M + sf + 2k)x(s)$$

$$k_2 E(s) - sk_1 k_2(1 + R_1 Cs)x(s) = x(s)\left[(s^2 M + sf + 2k)\right]\left[R_1 + s^2 R_1 LC + sL\right].$$

$$k_2 E(s) - x(s) \cdot sk_1 k_2(1 + R_1 Cs) = x(s)\left[s^2 R_1 M + sR_1 f + 2kR_1 + s^4 M R_1 LC + s^3 R_1 LCf + 2s^2 R_1 LCk + s^3 ML + s^2 fL + 2ksL\right]$$

$$k_2 E(s) - x(s) \cdot sk_1 k_2(1 + R_1 Cs) = x(s)\left[s^4 M R_1 LC + s^3 L(R_1 Cf + M) + s^2 R_1 M + s^2 fL + 2s^2 R_1 LCk + 2kR_1 + sR_1 f + 2ksL\right]$$

$$k_2 E(s) = x(s)\left[s^4 M R_1 LC + s^3 L(R_1 Cf + M) + s^2(R_1 M + fL + 2R_1 LCk) + 2kR_1 + sR_1 f + 2ksL\right] + (sk_1 k_2 + s^2 k_1 k_2 R_1 C)x(s)$$

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$$K_2 E(s) = X(s)\Big[s^4 M R_1 L C + s^3 L(R_1 C f + M)$$
$$+ s^2(R_1 M + fL + 2R_1 L C k + k_1 k_2 R_1 C)$$
$$+ 2kR_1 + sR_1 f + 2ksL + s k_1 k_2\Big]$$

$$K_2 E(s) = x(s)\Big[s^4 M R_1 L C + s^3 L(R_1 C f + M)$$
$$+ s^2(R_1 M + fL + 2R_1 L C k + k_1 k_2 R_1 C)$$
$$+ s(R_1 f + 2kL + k_1 k_2) + 2kR_1\Big]$$

$$\frac{x(s)}{E(s)} = \frac{K_2}{s^4 M R_1 L C + s^3 L(R_1 C f + M) + s^2(R_1 M + fL + 2R_1 L C k + k_1 k_2 R_1 C) + s(R_1 f + 2kL + k_1 k_2) + 2kR_1}$$

This is required transfer function of the given electromechanical system

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