Date / /
Page No.

Canonical form or canonical variable or (Diagonal):

$$T(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + b_2 s^{n-2} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n} \quad \text{— (1)}$$

$U \Rightarrow$ input, $Y =$ output.

$$T(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{(s-\lambda_1)(s-\lambda_2)\cdots(s-\lambda_n)}$$

where $\lambda_1, \lambda_2, \cdots \lambda_n$ (for distinct roots)

$$\frac{Y(s)}{U(s)} = b_0 + \frac{c_1}{(s-\lambda_1)} + \frac{c_2}{(s-\lambda_2)} + \cdots + \frac{c_n}{(s-\lambda_n)}$$

where $c_1, c_2 \cdots c_n$ are the residue of pole at $s = \lambda_i$

$$= b_0 + \sum_{i=1}^{n} \frac{c_i}{(s-\lambda_i)}$$

3^4

By cross multiplication:

$$Y(s) = b_0 U(s) + \frac{c_1}{(s-\lambda_1)} U(s) + \frac{c_2}{(s-\lambda_2)} U(s) + \frac{c_3}{(s-\lambda_3)} U(s) + \cdots + \frac{c_n}{(s-\lambda_n)} U(s) \quad \text{— (2)}$$

Define state variable

$$X_1(s) = \frac{1}{(s-\lambda_1)} U(s)$$

$$X_2(s) = \frac{1}{(s-\lambda_2)} U(s)$$

$\vdots$

$$X_n(s) = \frac{1}{(s-\lambda_n)} U(s)$$

$$X_1(s) = \frac{1}{(s-\lambda_1)} U(s)$$

$$\frac{c_1}{(s-\lambda_1)} U(s) = c_1 X_1(s)$$

Again cross multiplication

$$\left.\begin{aligned}
(s-\lambda_1) X_1(s) = U(s) &\Rightarrow s X_1(s) - \lambda_1 X_1(s) = U(s) \\
(s-\lambda_2) X_2(s) = U(s) &\Rightarrow s X_2(s) - \lambda_2 X_2(s) = U(s) \\
(s-\lambda_3) X_3(s) = U(s) &\Rightarrow s X_3(s) - \lambda_3 X_3(s) = U(s) \\
&\vdots \\
(s-\lambda_n) X_n(s) = U(s) &\Rightarrow s X_n(s) - \lambda_n X_n(s) = U(s)
\end{aligned}\right\} \text{— (3)}$$

Taking inverse laplace Transform of eqn ③

$$\dot{x}_1(t) - \lambda_1 x_1(t) = u(t)$$
$$\dot{x}_2(t) - \lambda_2 x_2(t) = u(t)$$
$$\dot{x}_3(t) - \lambda_3 x_3(t) = u(t)$$
$$\vdots$$
$$\dot{x}_n(t) - \lambda_n x_n(t) = u(t)$$

Rewriting,

$$\dot{x}_1 = \lambda_1 x_1 + u$$
$$\dot{x}_2 = \lambda_2 x_2 + u$$
$$\dot{x}_3 = \lambda_3 x_3 + u$$
$$\vdots$$
$$\dot{x}_n = \lambda_n x_n + u$$

State space representation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \underbrace{\begin{bmatrix} \lambda_1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2 & 0 & 0 & & 0 \\ 0 & 0 & \lambda_3 & 0 & \cdots & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & 0 & \cdots & \lambda_n \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}}_{B} u$$

$$\dot{x} = Ax + BU$$

From eqn ②

O/p ⟵ $Y(s) = b_0 U(s) + c_1 x_1(s) + c_2 x_2(s) + \cdots c_n x_n(s)$

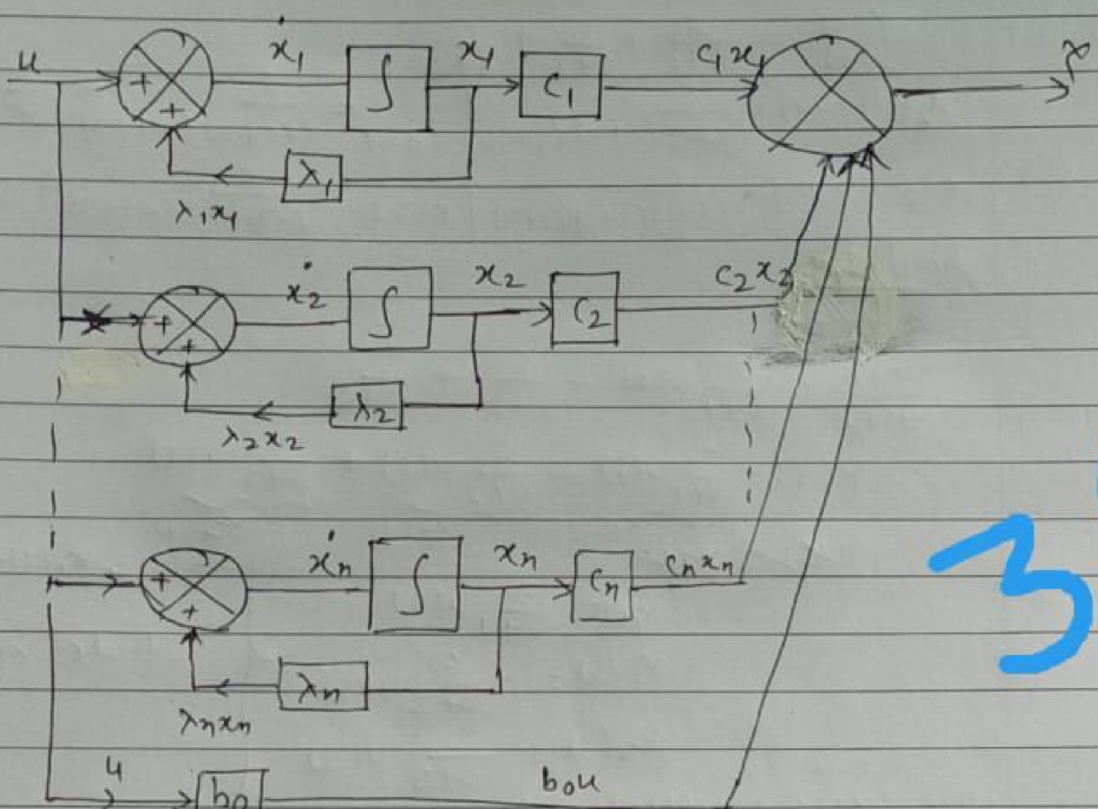
Taking Inverse laplace Transform

$$Y = c_1 x_1 + c_2 x_2 + \quad + c_n x_n + b_0 u$$

$$Y = \underbrace{\begin{bmatrix} c_1 & c_2 & c_3 & \cdots & c_n \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \underbrace{b_0}_{D} u$$

$$Y = CX + Du$$

Relisation (Block diagram or design)



3^6

Prob: Consider the system defined by

$$\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = 6u$$ where u is input

y is output obtained the state space representation using Partial fraction method on Canonical form

Soln Transfer function.

$$s^3 Y(s) + 6s^2 Y(s) + 11sY(s) + 6Y(s) = 6U(s)$$

$$T.F = \frac{Y(s)}{U(s)} = \frac{6}{s^3 + 6s^2 + 11s + 6}$$

$$s^3 + 6s^2 + 11s + 6 = 0$$

Trial and error method

$$-1 + 6 - 11 + 6 = 0 \quad \text{put } s = -1$$

$$s + 1 = 0$$

$$s^2(s+1) + 5s(s+1) + 6(s+1).$$

$$(s+1)[s^2 + 5s + 6] = 0$$

$$(s+1)[s^2 + 3s + 2s + 6] = 0$$

$$(s+1)[s(s+3) + 2(s+3)] = 0$$

$$(s+1)(s+2)(s+3) = 0$$

DATE / /
PAGE NO.

$$\frac{Y(s)}{U(s)} = \frac{6}{s^3+6s^2+11s+6}$$
$$= \frac{6}{(s+1)(s+2)(s+3)}$$

$$\frac{Y(s)}{U(s)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = (s+1) \times \frac{6}{(s+1)(s+2)(s+3)}\Big|_{s=-1} = \frac{6}{(-1+2)(-1+3)} = \frac{6}{2} = 3.$$

$$B = (s+2) \times \frac{6}{(s+1)(s+2)(s+3)}\Big|_{s=-2} = \frac{6}{(-2+1)(-2+3)} = \frac{6}{-1\times1} = -6.$$

Similarly

$C = 3.$

$$\frac{Y(s)}{U(s)} = \frac{3}{s+1} - \frac{6}{s+2} + \frac{3}{s+3}$$

$$\therefore Y(s) = \underbrace{\frac{3}{s+1}U(s)}_{x_1(s)} - \underbrace{\frac{6}{s+2}U(s)}_{x_2(s)} + \underbrace{\frac{3}{s+3}U(s)}_{x_3(s)}$$

$\rightarrow Y(s) = x_1(s) + x_2(s) + x_3(s)$

~~$Y(s) = x_1 L$~~

$$x_1(s) = \frac{3}{s+1}U(s)$$
$$x_2(s) = -\frac{6}{s+2}U(s)$$
$$x_3(s) = \frac{3}{s+3}U(s)$$

state variable

By cross multiplication.

$s x_1(s) + x_1(s) = 3U(s)$ or $\dot{x}_1 + x_1 = 3u$

$s x_2(s) + 2x_2(s) = -6U(s)$ $\quad \dot{x}_2 + 2x_2 = -6u$

$s x_3(s) + 3x_3(s) = 3U(s)$ $\quad \dot{x}_3 + 3x_3 = 3u$

~~$\dot{x}_1 + x_1 = 3u$~~

We can write

$$\dot{x}_1 = -x_1 + 3u$$
$$\dot{x}_2 = -2x_2 - 6u$$
$$\dot{x}_3 = -3x_3 + 3u$$

State space representation:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}3\\ -6\\ 3\end{bmatrix} u$$

37

$$Y = x_1 + x_2 + x_3$$

$$Y = \begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

$$Y = CX$$

BAMBOOS

Relization: