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Alternative method :

State model of an nth order system in which the forcing functions involves derivative terms.

Consider an nth order system described by the differential eqn,

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1}y}{dt^{n-1}} + \cdots + a_{n-1}\frac{dy}{dt} + a_1 y =$$

$$b_0 \frac{d^n u}{dt^n} + b_1 \frac{d^{n-1}u}{dt^{n-1}} + \cdots + b_{n-1}\frac{du}{dt} + b_n u$$

Taking laplace Transform (Assuming all initial conditions are zero), we get

$$(s^n + a_1 s^{n-1} + \cdots a_{n-1} s + a_n) Y(s) = (b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n) U(s)$$

Transfer function is given by

$$T(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n} \quad \text{—①}$$

This is nth order system,

so n state variable $x_1, x_2 \cdots x_n$ are required.

$$x_1 = y - \beta_0 u$$

$$x_2 = \dot{y} - \beta_0 \dot{u} - \beta_1 u = \dot{x}_1 - \beta_1 u$$

$$x_3 = \ddot{y} - \beta_0 \ddot{u} - \beta_1 \dot{u} - \beta_2 u = \dot{x}_2 - \beta_2 u$$

$$x_4 = \dddot{y} - \beta_0 \dddot{u} - \beta_1 \ddot{u} - \beta_2 \dot{u} - \beta_3 u = \dot{x}_3 - \beta_3 u$$

$$\vdots$$

$$x_n = y^{n-1} - \beta_0 u^{n-1} - \beta_1 u^{n-2} - \cdots - \beta_{n-2}\dot{u} - \beta_{n-1} u$$

$$x_n = \dot{x}_{n-1} - \beta_{n-1} u$$

$$y = x_1 + \beta_0 u$$

$$\dot{x}_1 = x_2 + \beta_1 u$$

$$\dot{x}_2 = x_3 + \beta_2 u$$

$$\dot{x}_3 = x_4 + \beta_3 u$$

$$\vdots$$

$$\dot{x}_{n-1} = x_n + \beta_{n-1} u$$

$$\dot{x}_n = -a_n x_1 - a_{n-1} x_2 + \cdots - a_1 x_n + \beta_n u$$

$\beta_0 = b_0$ (let)

$\beta_1 = b_1 - a_1\beta_0$

$\beta_2 = b_2 - a_1\beta_1 - a_2\beta_0$

$\beta_3 = b_3 - a_1\beta_2 - a_2\beta_1 - a_3\beta_0$

$\vdots$

$\beta_n = b_n - a_1\beta_{n-1} - a_2\beta_{n-2} - \cdots - a_{n-1}\beta_1 - a_n\beta_0$

State space representation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & & \cdots & -a_1 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_{n-1} \\ \beta_n \end{bmatrix} u$$

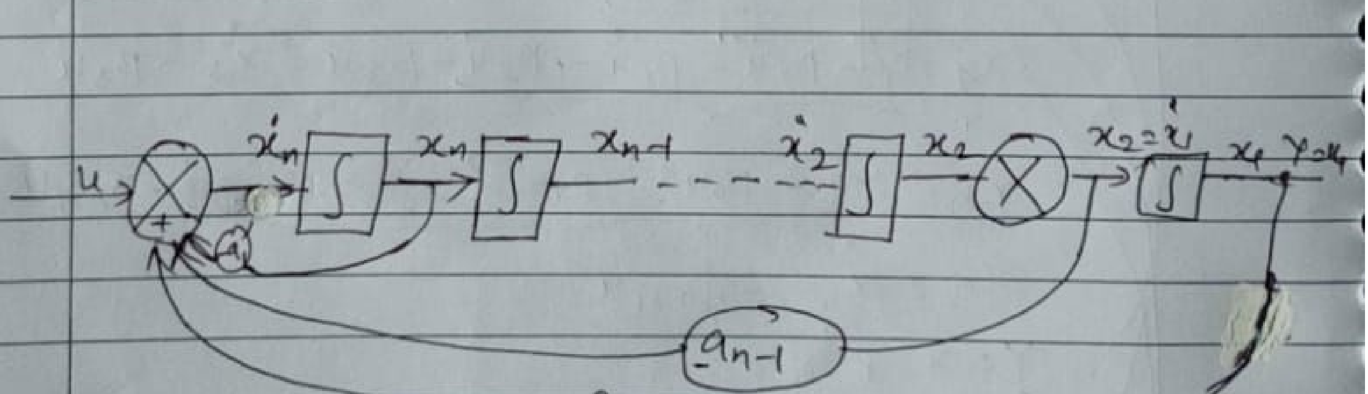
$$\dot{x} = Ax + Bu$$

$y = x_1 + \beta_0 u$ — o/p eq^n

$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \underbrace{[\beta_0]}_{D} u$$

$y = Cx + Du$ → output eq^n

Relization part



State Model of an nth order system in which the forcing function involves derivative terms.

Consider an the nth order system described by the differential equation,

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Taking laplace Transform (Assume all initial cond^n are zero) we get

$$(s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n)\, Y(s) = (b_0 s^n + b_1 s^{n-1} \cdots$$
$$\cdots + b_{n-1} s + b_n)\, U(s)$$

Transfer function is given by

$$T(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

$$T(s) = \frac{Y(s)}{U(s)} = \frac{X_1(s)}{U(s)} \cdot \frac{Y(s)}{X_1(s)}$$

where $\dfrac{X_1(s)}{U(s)} = \dfrac{1}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$

and $\dfrac{Y(s)}{X_1(s)} = b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n$

State space representation may be obtained by previous method.

Prob: ① A feedback system has a closed loop Transfer function $\frac{Y(s)}{U(s)} = \frac{10(s+4)}{s(s+1)(s+3)}$ construct three different state model for this system and give Block diagram representation for each model.

$$\frac{Y(s)}{U(s)} = \frac{10(s+4)}{s(s+1)(s+3)} \quad -①$$

Breaking the Transfer function in two parts.

$$\frac{Y(s)}{U(s)} = \frac{X_1(s)}{U(s)} \times \frac{Y(s)}{X_1(s)} \quad -②$$

let $$\frac{X_1(s)}{U(s)} = \frac{1}{s(s+1)(s+3)} \quad -③$$

and $$\frac{Y(s)}{X_1(s)} = 10(s+4) \quad -④$$

$$\frac{X_1(s)}{U(s)} = \frac{1}{s(s^2+4s+3)}$$

& $$\frac{X_1(s)}{U(s)} = \frac{1}{s^3+4s^2+3s}$$

Cross multiplication eqn.

$$s^3X_1(s) + 4s^2X_1(s) + 3sX_1(s) = U(s)$$

Taking inverse laplace Transform.

$$\dddot{X}_1 + 4\ddot{X}_1 + 3\dot{X}_1 = U$$

let $X_1 = x_1$

$\dot{X}_1 = x_2$

$\ddot{X}_1 = \dot{x}_2 = x_3$

$\therefore \dddot{X}_1 = \dot{x}_3 = U - 3\dot{X}_1 - 4\ddot{X}_1$

$= U - 3x_2 - 4x_3$

Consider the second part of Transfer fun from eqn ④

$$\frac{Y(s)}{X_1(s)} = 10(s+4)$$

$$\frac{Y(s)}{X_1(s)} = 10s+40$$

Cross multiplication.

$$Y(s) = 10 s X_1(s) + 40 X_1(s)$$

Taking inverse laplace.

$$Y = 10\dot{x}_1 + 40 x_1 \qquad \text{put } \dot{x}_1 = x_2$$

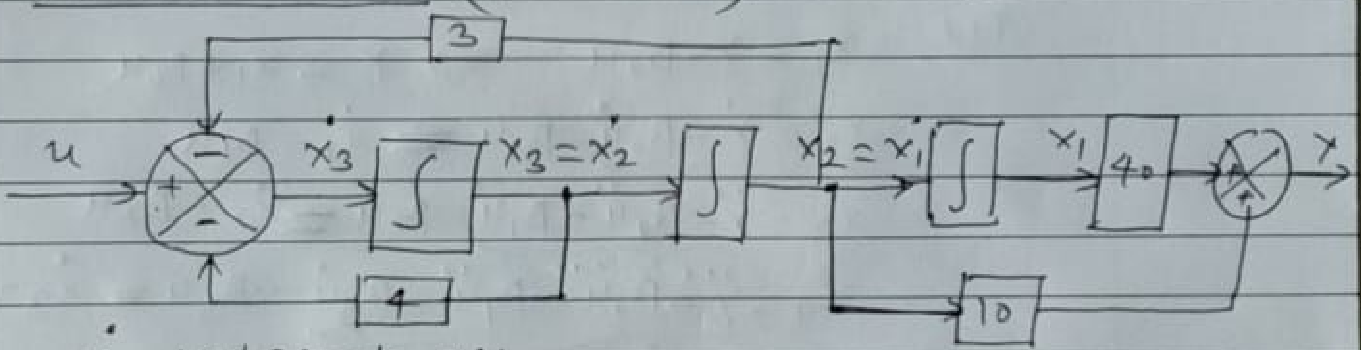
$$Y = 10 x_2 + 40 x_1 \qquad \& \ x_1 = x_1$$

The state model in the matrix form (state model)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 40 & 10 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Block diagram (Relisation) or design.



$\dot{x}_3$ integrate $= x_3$

x_3 equal $\dot{x}_2$

$\dot{x}_2$ integrate $= x_2$

$x_2 = \dot{x}_1$

$\dot{x}_1$ integrate $= x_1$

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② ~~Obtain phase variable representation for a system whose transfer function is given by~~

$$\frac{Y(s)}{U(s)} = \frac{6s^3+4s^2+3s+10}{s^3+8s^2+4s+20}$$

Solⁿ

Sol^n Here given $b_0=6,\ b_1=4,\ b_2=3,\ b_3=10$

$a_0=1,\ a_1=8,\ a_2=4,\ a_3=20$

$$\frac{Y(s)}{U(s)} = \frac{6s^3+4s^2+3s+10}{s^3+8s^2+4s+20}$$

Cross multiplication.

$$s^3Y(s)+8s^2Y(s)+4sY(s)+20Y(s) = 6s^3U(s) + 4s^2U(s)+3sU(s)+10U(s)$$

Taking Inverse laplace transform, we get

$$\dddot{y}(t)+8\ddot{y}(t)+4\dot{y}(t)+20y(t) = 6\dddot{u}(t)+4\ddot{u}(t) + 3\dot{u}(t)+10u(t) \quad -①$$

let $x_1 = y-\beta_0 u \Rightarrow y = x_1+\beta_0 u$

$x_2 = \underbrace{\dot{y}-\beta_0\dot{u}}-\beta_1 u = \dot{x}_1 - \beta_1 u$

$x_3 = \underbrace{\ddot{y}-\beta_0\ddot{u}-\beta_1\dot{u}}-\beta_2 u = \dot{x}_2-\beta_2 u$

$x_4 = \underbrace{\dddot{y}-\beta_0\dddot{u}-\beta_1\ddot{u}-\beta_2\dot{u}}-\beta_3 u = \dot{x}_3-\beta_3 u$

Rewriting these

$$\left.\begin{aligned}\dot{x}_1 &= x_2+\beta_1 u\\ \dot{x}_2 &= x_3+\beta_2 u\\ \dot{x}_3 &= -20x_1-4x_2-8x_3+\beta_3 u\end{aligned}\right\} -②$$

Now,

$\beta = b_0 = 6$

$\beta_1 = b_1 - a_1\beta_0$

$= 4-8\times 6$

$= 4-48 = -44$

$\therefore \beta_1 = -44.$

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$$\beta_2 = b_2 - a_1\beta_1 - a_2\beta_0$$
$$= 3 - 8\times(-44) - 4\times 6$$
$$= 3 + 352 - 24$$
$$\therefore \beta_2 = 331.$$

$$\beta_3 = b_3 - a_1\beta_2 - a_2\beta_1 - a_3\beta_0$$
$$= 10 - 8\times 331 - 4\times(-44) - 20\times 6$$
$$= 10 - 2648 + 176 - 120$$
$$= 186 - 2648 - 120$$
$$\beta_3 = -2582$$

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Again Rewriting:

$$\dot{x}_1 = x_2 - 44u$$
$$\dot{x}_2 = x_3 + 331u$$
$$\dot{x}_3 = -20x_1 - 4x_2 - 8x_3 - 2582u$$

Therefore state space representation is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -20 & -4 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -44 \\ 331 \\ -2582 \end{bmatrix} u$$

The output eqn is $Y = x_1 + \beta_0 u$
$$= x_1 + 6u$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 6[u]$$

Block diagram or Realisation or Design