Gear Train: It is possible to transmit mechanical energy from one part of a system to another via devices such as gear trains, timing belt over a pulley or lever.

Let us consider a gear train with N_1 being the number of teeth on the first gear, $N_2 \rightarrow$ The number of teeth on the second gear. T_1 The input torque on the 1st gear, θ_1 the angular displacement of the 1st gear. r_1 is the radius of the 1st gear, T_2 input torque of the ~~2nd~~ 2nd gear, θ_2 the angular displacement on the 2nd gear, r_2 radius of the 2nd gear. Moment of ~~inertia~~ inertia and viscous friction of motor of gear ① & ② are J_1, J_2 & D_1, D_2.

Rule ① The number N of teeth is proportional to the radius r of a gear

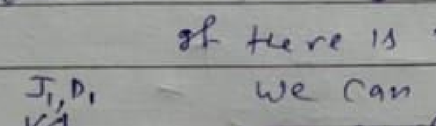
i.e. $r \propto N$

Therefore $r_1 \propto N_1$ and $r_2 \propto N_2$

$$\frac{r_1}{r_2} = \frac{N_1}{N_2} \quad — ①$$

$$r_1 N_2 = N_1 r_2 \quad — ②$$

Rule ② The distance travelled on a gear is equal



From fig. we can write

Distance travelled on a gear $r_1\theta_1 = r_2\theta_2$ — ③

Rule ③ Workdone $T\theta$ is analogous to $W = Fx$

If there is no loss,

we can write $T_1\theta_1 = T_2\theta_2$ — ④

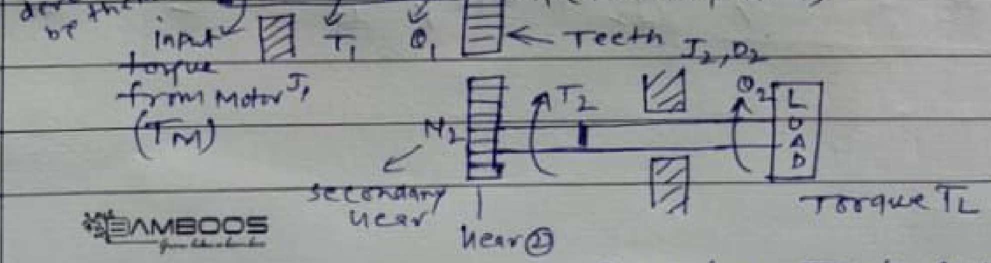


Fig: Gear Train system

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From eqn (3) and (4), we can write

$$\frac{T_1}{T_2} = \frac{r_1}{r_2} = \frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} \quad \text{--- (5)}$$

From fig. (2), we can write,

$$T_M = J_1 \frac{d^2\theta_1}{dt^2} + D_1 \frac{d\theta_1}{dt} + T_1 \quad \text{--- (6)}$$

where T_1 is transmitted towards load. Assuming no loss in the gear, we can write

$$T_2 = J_2 \frac{d^2\theta_2}{dt^2} + D_2 \frac{d\theta_2}{dt} + T_L \quad \text{--- (7)}$$

eqn (7), T_2 is the reflected torque on gear secondary and T_L is the load torque.

The relation betn T_1 & T_2 is given by

$$\frac{T_1}{T_2} = \frac{N_1}{N_2} \quad \text{--- (8)}$$

$$\therefore T_2 = \left(\frac{N_2}{N_1}\right) \cdot T_1 \quad \text{--- (9)} \quad \text{i.e. } T_2 = \left(\frac{N_2}{N_1}\right) \cdot T_1$$

Using eqn (9), eqn (7) can be written as

$$\left(\frac{N_2}{N_1}\right) T_1 = J_2 \frac{d^2\theta_2}{dt^2} + D_2 \frac{d\theta_2}{dt} + T_L$$

$$T_1 = \left(\frac{N_1}{N_2}\right)\left[J_2 \frac{d^2\theta_2}{dt^2} + D_2 \frac{d\theta_2}{dt} + T_L\right] \quad \text{--- (10)}$$

$$= \left(\frac{N_1}{N_2}\right)\left[J_2 \left(\frac{N_1}{N_2}\right) \frac{d^2\theta_1}{dt^2} + D_2 \left(\frac{N_1}{N_2}\right) \frac{d\theta_1}{dt} + T_L\right]$$

$$T_1 = J_2 \left(\frac{N_1}{N_2}\right)^2 \frac{d^2\theta_1}{dt^2} + D_2 \left(\frac{N_1}{N_2}\right)^2 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) T_L \quad \text{--- (11)}$$

From eqn (3)
$r_1\theta_1 = r_2\theta_2$
$\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1}$
$\frac{N_1}{N_2} = \frac{\theta_2}{\theta_1}$
$\theta_2 = \left(\frac{N_1}{N_2}\right)\theta_1$

Putting the value of T_1 in eqn (6)

$\dot{\theta}_1$
$\dot{\theta}_2 = \left(\frac{N_1}{N_2}\right) \frac{d\theta_1}{dt}$
$\frac{d\theta_2}{dt} = \frac{N_1}{N_2} \frac{d^2\theta_1}{dt^2}$
$\frac{d^2\theta_2}{dt^2} = \left(\frac{N_1}{N_2}\right)$

Using eqn (11), we can write eqn (6) as follows

$$T_M = \underbrace{\left[J_1 + J_2\left(\frac{N_1}{N_2}\right)^2\right]}\frac{d^2\theta_1}{dt^2} + \underbrace{\left[D_1 + D_2\left(\frac{N_1}{N_2}\right)^2\right]}\frac{d\theta_1}{dt} + \underbrace{\left(\frac{N_1}{N_2}\right)T_L}$$

$$T_M = J_{1eq} \frac{d^2\theta_1}{dt^2} + D_{1eq} \frac{d\theta_1}{dt} + T_L' \quad \text{--- (12)}$$

Where $J_{1eq} = J_1 + J_2\left(\frac{N_1}{N_2}\right)^2$ — (a)

$D_{1eq} = D_1 + D_2\left(\frac{N_1}{N_2}\right)^2$ — (b)

$T'_L = \left(\frac{N_1}{N_2}\right) T_L$ — (c)

eqn (12) shows the terms refered to primary
similarly, we can write

$$T_{2eq} = J_{2eq}\frac{d^2\theta_2}{dt^2} + D_{2eq}\frac{d\theta_2}{dt} + T_2$$

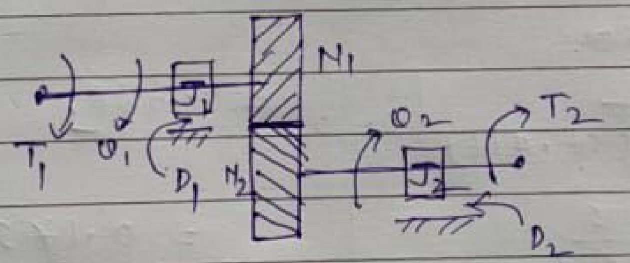
$$= J_{eq}\frac{d^2\theta_2}{dt^2} + D_{2eq}\frac{d\theta_2}{dt} + \left(\frac{N_2}{N_1}\right) T_M \text{ — (13)}$$

Where $J_{2eq} = J_2 + J_1\left(\frac{N_2}{N_1}\right)^2$ — (d)

$D_{2eq} = D_2 + D_1\left(\frac{N_2}{N_1}\right)^2$ — (e)

$T_{Meq} = \left(\frac{N_2}{N_1}\right) T_1$ — (f)

Prob: (1) Fig. show a gear train system find the displacement of the load on the shaft (side-2) if $T_1 = 20e^{-2t}$ is applied at side-1.



in fig.

$J_1 = 6\ kgm^2$, $J_2 = 48\ kgm^2$

$D_1 = 3\ Nm/rad/sec$

$D_2 = 12\ Nm/rad/sec$

$N_1 = 1$ & $N_2 = 4$

Soln. The equivalent J, D & T referring to side 2 are given by

$$J_{2eq} = J_2 + J_1\left(\frac{N_2}{N_1}\right)^2$$

$$D_{2eq} = D_2 + D_1\left(\frac{N_2}{N_1}\right)^2$$

$$T_2 = \left(\frac{N_2}{N_1}\right) T_1$$

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Again $T_2 = J_{eq} \frac{d^2\theta_2}{dt^2} + D_{eq} \frac{d\theta_2}{dt}$

Taking laplace transfer, we get

$$T_2(s) = s^2 J_{eq} \theta_2(s) + s D_{eq} \theta_2(s) \quad \text{— (1)}$$

$$J_{eq} = J_2 + J_1 \left(\frac{N_2}{N_1}\right)^2 = 48 + 6 \times \left(\frac{4}{1}\right)^2 = 144 \, kgm^2$$

$$D_{eq} = D_2 + D_1 \left(\frac{N_2}{N_1}\right)^2 = 12 + 3\left(\frac{4}{1}\right)^2 = 60 \, kgm^2$$

$$T_2 = T_1 \left(\frac{N_2}{N_1}\right) = \frac{4}{1} \times 20 e^{-2t} = 80 e^{-2t} = \frac{80}{s+2}$$

$$T_2(s) = s^2 J_{eq} \theta_2(s) + s D_{eq} \theta_2(s)$$

$$= (s^2 J_{eq} + s D_{eq}) \theta_2(s)$$

$$\theta_2(s) = \frac{T_2(s)}{s^2 J_{eq} + s D_{eq}} = \frac{\frac{80}{s+2}}{s(sJ_{eq} + D_{eq})}$$

Putting the value of J_{eq}, D_{eq} & T_2

$$\theta_2(s) = \frac{80}{s(s+2)(s \times 144 + 60)}$$

$$\theta_2(s) = \frac{80}{s(s+2)(144s+60)}$$

$$= \frac{0.556}{s(s+2)(s+0.416)}$$

Divided by 144 then 80 → .556, & .416

$$\theta_2(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+.416}$$

By Partial fraction:

$$A = \cancel{s} \cdot \frac{0.556}{\cancel{s}(s+2)(s+.416)}\Bigg|_{s=0} = 0.668$$

$$B = \cancel{(s+2)} \times \frac{0.556}{s\cancel{(s+2)}(s+.416)}\Bigg|_{s=-2} = 0.175$$

$$C = \cancel{(s+0.416)} \times \frac{0.556}{s(s+2)\cancel{(s+0.416)}}\Bigg|_{s=-0.416} = -0.843$$

$$\theta_2(s) = \frac{0.668}{s} + \frac{0.175}{s+2} - \frac{0.843}{s+0.416}$$

Taking ILT

$$\theta_2(t) = .668 + 0.175 e^{-2t} - 0.843 e^{-0.416t} \quad \text{Ans}$$

BAMBOOS
Grow like a bamboo