State Model of an nth order system in which the forcing function does not involve derivative terms.

If the Transfer function of the system has no zeros (i.e the differential eqn. of the system has no derivatives of the input) such a Transfer function has the form

$$T(s) = \frac{Y(s)}{U(s)} = \frac{b}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n} \quad -①$$

cross multiplication.

$$(s^n + a_1 s^{n-1} + \cdots + a_{n-1} + a_n) Y(s) = bU(s)$$

Taking ILT (Assuming all initial conditions are equal to zero), we have the differential eqns as

$$\frac{d^n y}{dt^n} + a\frac{d^{n-1} y}{dt^{n-1}} + a_2\frac{d^{n-2} y}{dt^{n-2}} + \cdots + a_{n-1}\frac{dy}{dt} + a_n y = bu \quad -②$$

This is an nth order system, so n state variables $x_1, x_2 --- x_n$ are required. let the First state variable be x_1 equal to the output variable y. let the 2nd state variable x_2 be equal to the First derivative of the output variable and so on

$$x_1 = y$$

$$x_2 = \frac{dy}{dt} = \dot{y}$$

$$x_3 = \frac{d^2y}{dt^2} = \ddot{y}$$

$$\vdots$$

$$x_n = \frac{d^{n-1}y}{dt^{n-1}} = y^{(n-1)}$$

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These eqn can be reduced to set of first order differential eqn given as follows.

Rewrite,

$$\left.\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
&\vdots \\
\dot{x}_{n-1} &= x_n \\
\dot{x}_n &= -a_1 x_1 - a_{n-1} x_2 - \cdots - a_1 x_n + bu
\end{aligned}\right\} \quad -(3)$$

eqn (3) can be writen as vector Matrix form

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b \end{bmatrix}}_{b} u \quad —(4)$$

i.e $\dot{x} = Ax + Bu$.

The output being $y = x_1$ the output eqn is given by

$$y = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

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$$y = Cx$$

Last row is comprised of the negative of the coefficients of the differential eqn and all other elements are zero. This form of matrix A is known as the companion form or bush form. All the elements in B except the last one zero.

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For the given Transfer function obtain a state Model and its relisation (Block diagram)
zero.

$$h(s) = \frac{Y(s)}{U(s)} = \frac{K}{s^3 + a_3 s^2 + a_2 s + a_1}$$

→ pole

Sol^n This transfer function has no zero.

$$\frac{Y(s)}{U(s)} = \frac{K}{s^3 + a_3 s^2 + a_2 s + a_1}$$

$$s^3 Y(s) + a_3 s^2 Y(s) + a_2 s Y(s) + a_1 Y(s) = K U(s)$$

Taking Inverse laplace Transform

$$\dddot{y}(t) + a_3 \ddot{y} + a_2 \dot{y}(t) + a_1 y(t) = K u(t)$$

$$\dddot{y}(t) = K u(t) - a_3 \ddot{y}(t) - a_2 \dot{y}(t) - a_1 y(t)$$

state variable as, 1st state variable as output

$$y(t) = x_1$$

$$\dot{y}(t) = \dot{x}_1 \Rightarrow x_2$$

$$\ddot{y}(t) = \dot{x}_2 \Rightarrow x_3$$

$$\dddot{y}(t) = \dot{x}_3 \Rightarrow$$

$$\dddot{y}(t) = K u(t) - a_3 x_3 - a_2 x_2 - a_1 x_1$$

Rewriting the eq^n or Rearragin the eq^n

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -a_3 x_3 - a_2 x_2 - a_1 x_1 + K U(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_1 & -a_2 & -a_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ K \end{bmatrix} u(t)$$

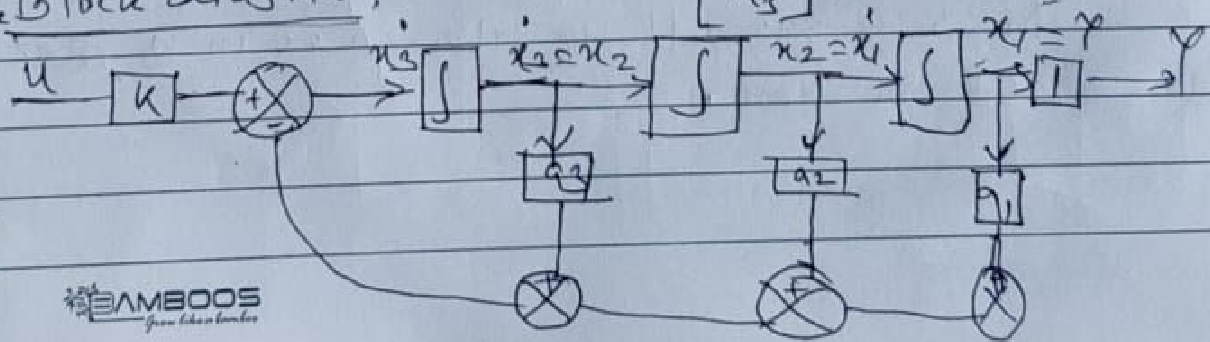
$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

u को K से गुणा करते पर $K U(t)$

$\dot{x}_3$ को Integrate करते पर x_3

x_3, x_2 के बराबर हैं इसलिए सबको समझता हैं/

Block diagram:



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