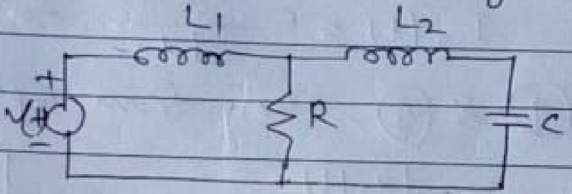
Signal & system

IV semester (electrical)

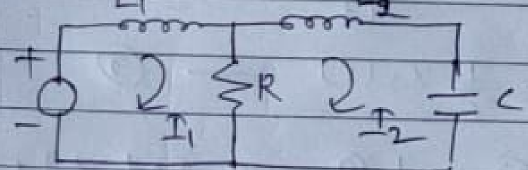
DATE / /

PAGE No.

(2) Obtain the state-space representation of the ckt as shown in fig



(Sol.)



By KVL, can can write, Loop (I1)

$$v(t) = L_1 \frac{dI_1(t)}{dt} + R[I_1(t) - I_2(t)]$$

$$v(t) = L_1 \frac{di_1(t)}{dt} + R[I_1(t) - I_2(t)] \quad \text{—} (1)$$

Loop (I_2)

$$L_2 \frac{di_2(t)}{dt} + \frac{1}{C}\int I_2(t)\,dt - R[I_1(t) - I_2(t)] = 0$$

$$L_2 \frac{di_2(t)}{dt} + v_c(t) - R[I_1(t) - I_2(t)] = 0 \quad \text{—} (2)$$

$$v_c(t) = \frac{1}{C}\int I_2(t)\,dt$$

let, $x_1(t)$ = current through $L_1 = I_1(t)$ — (3)

$x_2(t)$ = current through $L_2 = I_2(t)$ — (4)

$x_3(t)$ = voltage across $C = v_c(t)$ — (5)

using eqn (3) & (4), Then from eqn (1) we get

$$L_1 \frac{dx_1(t)}{dt} + R[x_1(t) - x_2(t)] - v(t) = 0$$

$$\dot{x}_1(t) = -\frac{R}{L_1}x_1(t) - \frac{R}{L_1}x_2(t) + \frac{v(t)}{R} \quad \text{—} (6)$$

$\dot{x}_1(t) = \frac{dx_1(t)}{dt}$

Divided by L_1

using eqn (3), (4) & (5) Then from eqn (2), We get

$$L_2 \frac{dx_2(t)}{dt} + x_3(t) - R[x_1(t) - x_2(t)] = 0$$

$$\dot{x}_2(t) = \frac{R}{L_2}x_1(t) - \frac{R}{L_2}x_2 - \frac{1}{L_2}x_3(t) \quad \text{—} (7)$$

$\dot{x}_2(t) = \frac{dx_2(t)}{dt}$

We have $v_c(t) = \frac{1}{c}\int I_2(t)\,dt$

$$I_2(t) = c\frac{dv_c(t)}{dt} \quad \text{— (8)}$$

Using eqn (4) & (5), then from eqn (8), we get

$$x_2(t) = c\frac{dx_3(t)}{dt}$$

$$\dot{x}_3(t) = \frac{dx_3(t)}{dt}$$

$$\dot{x}_3(t) = \frac{1}{c}x_2(t) \quad \text{— (9)}$$

From eqn (6), (7) and (9), we get state space representation as.

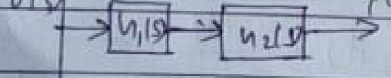
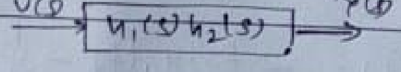
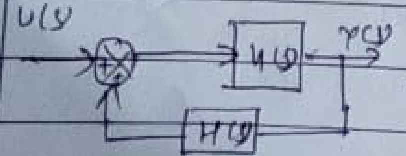
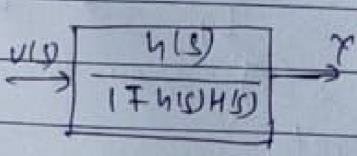
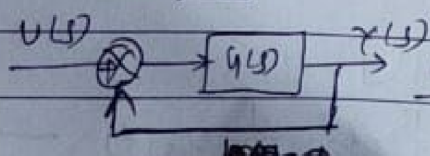
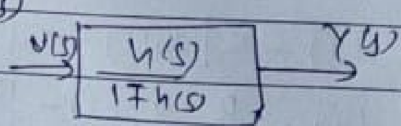
$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -\frac{R}{L_1} & \frac{R}{L} & 0 \\ \frac{R}{L_2} & -\frac{R}{L_2} & -\frac{1}{L_2} \\ 0 & \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} & 0 & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ 0 \\ 0 \end{bmatrix}$$

The output from eqn (5) is given by

$$Y(t) = x_3(t) = v_c(t)$$

$$= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

Block diagram for Realization

	Original ckt	equivalent ckt	Remarks
(1)			combining two block in cascade $\frac{Y(s)}{V(s)} = h_1(s)h_2(s)$
(2)			Eliminating a feed-back loop. $\frac{Y(s)}{V(s)} = \frac{h(s)}{1 \mp h(s)H(s)}$
(3)			Special case for $H=1$ $\frac{Y(s)}{V(s)} = \frac{h(s)}{1 \mp h(s)}$

(3) A system is given below, Find the state and output eqn of the system.

$\mathcal{L}\, u(t) = U(s)$
$\mathcal{L}\, y(t) = Y(s)$

$$u(t) \longrightarrow \boxed{\frac{K}{(s+1)(s+2)(s^2+1)}} \longrightarrow y(t)$$

Soln

$$\frac{Y(s)}{U(s)} = H(s)$$

$H(s) =$ Block function

$$H(s) = \frac{K}{(s+1)(s+2)(s^2+1)}$$

$$\frac{Y(s)}{U(s)} = \frac{K}{(s+1)(s+2)(s^2+1)}$$

$$\frac{Y(s)}{U(s)} = \frac{K}{s^4+3s^3+3s^2+3s+2}$$

By cross multiplication.

$$Y(s)\left[s^4+3s^3+3s^2+3s+2\right] = K\,U(s)$$

$$s^4Y(s)+3s^3Y(s)+3s^2Y(s)+3sY(s)+2Y(s) = K\,U(s)$$

Taking ILT

$$\ddddot{y}(t)+3\dddot{y}(t)+3\ddot{y}(t)+3\dot{y}(t)+2y(t) = K\,u(t)$$

let $y(t) = x_1$

$$\dot{y}(t) = \dot{x}_1 = x_2$$

$$\ddot{y}(t) = \dot{x}_2 = x_3$$

$$\dddot{y}(t) = \dot{x}_3 = x_4$$

$$\ddddot{y}(t) = \dot{x}_4 = K\,u(t) - 3x_4 - 3x_3 - 3x_2 - 2x_1$$

$$= K\,u(t) - 3\dot{x}_3 - 3\dot{x}_2 - 3\dot{x}_1 - 2x_1$$

Rewriting

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = K\,u(t) - 3x_4 - 3x_3 - 3x_2 - 2x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & -3 & -3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ K \end{bmatrix} u(t)$$

23

output $y(t) = x_1$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$ Ans