4th Semester (ELECT)

Signal and System

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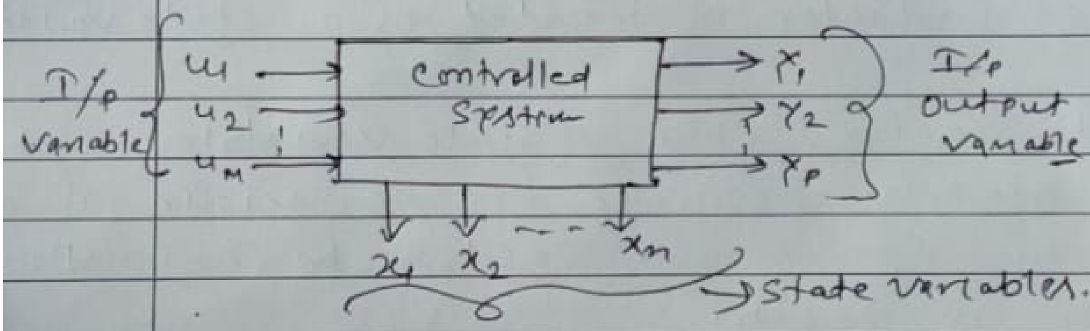
State space representation:

In the state space approach, Three types of variables that are involved in the Modelling of dynamic system: input variables represented by $u_1(t), u_2(t)$ ---- output variable represented by $y_1(t), y_2(t)$ ---- and state variables represented $x_1(t), x_2(t)$ ---- the state space representation ~~by~~ for a given system is not unique, except that the number of state variables is the same for any of the different state space representation of the same system.

The state-space representation in block diagram for is shown in fig.



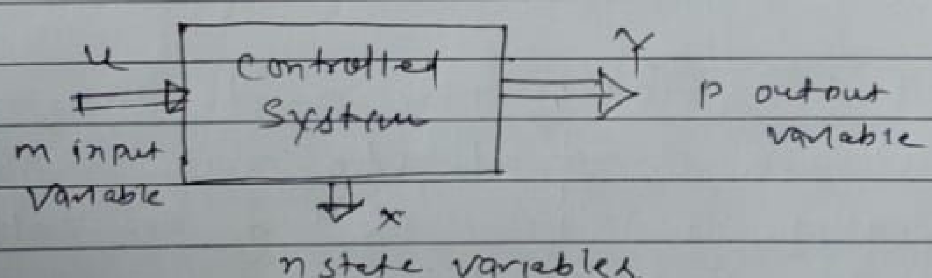


fig: state space representation of a system

(a) State-space representation of (SISO) system.

Consider the dynamics of a SISO system is described by the following differential eqn,

$$\frac{d y^n(t)}{dt^n} + a_1 \frac{d^{n-1} y(t)}{dt^{n-1}} + a_2 \frac{d^{n-2} y(t)}{dt^{n-2}} + \cdots$$

$$+ a_{n-1} \frac{d y(t)}{dt} + a_n y(t) = u(t) \quad - (1)$$

Assume, $y(t), \frac{dy(t)}{dt}, \cdots \frac{d^{n-1} y(t)}{dt^{n-1}}$ as a set of n-state variable

let $y = x_1$

$$\frac{dy}{dt} = \dot{x}_1 = x_2$$

$$\frac{d^2 y}{dt^2} = \dot{x}_2 = x_3$$

$$\vdots$$

$$\frac{d^{n-1} y}{dt^{n-1}} = \dot{x}_{n-1} = x_n$$

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Then from eqn (1) we have

$$x_n = -a_1 x_1 - a_{n-1} x_2 - \cdots - a_1 x_n + u \quad - (2)$$

we can write above set of eqn. in Matrix form as,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 1 \end{bmatrix} u \quad - (3)$$

and output $y = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad - (4)$

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eqn (3) and (4) can be written as

$$\dot{x} = Ax + Bu \quad \text{——— (5)}$$

and o/p $Y = Cx \quad \text{——— (6)}$

where, $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix}$, $A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & & -a_1 \end{bmatrix}$

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$B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}$

Hence, the dynamics of n order SISO (single input single output) system given by the eqn (5) & (6).

where $A \to$ is the system matrix of order $(n \times n)$

$B \to$ is the I/p coupling matrix of order $(n \times 1)$

$C \to$ is the o/p coupling matrix of order $(1 \times n)$

$x \to$ state vector of order $(n \times 1)$

$u \to$ scalar input of order (1×1)

$Y \to$ scalar output of order (1×1)

(b) State space representation of a multi-input multi-output (MIMO) systems

The state eqn of a linear time invariant system are a set of 1st order differential eqns where each 1st derivative of the state variable is a linear combination of system states and inputs

i.e

eqn (7) $\leftarrow \begin{cases} \dot{x}_1 = a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n + b_{11}u_1 + b_{12}u_2 + \cdots + b_{1m}u_m \\ \dot{x}_2 = a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n + b_{21}u_1 + b_{22}u_2 + \cdots + b_{2m}u_m \\ \vdots \\ \dot{x}_n = a_{n1}x_1 + a_{n2}x_2 + \cdots a_{nn}x_n + b_{n1}u_1 + b_{n2}u_2 + \cdots + b_{nm}u_m \end{cases}$

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Where the coefficient $a_{ij}'s$ and $b_{ij}'s$ are constt.
The eqn (7) can be written in vector Matrix form as

$$\dot{x} = Ax + Bu$$

where x is an $n \times 1$ state vector
u is a $m \times 1$ input vector
A is an $n \times n$ system Matrix
B is an $n \times m$ input Matrix defined by

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}, \quad A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nm} \end{bmatrix}$$

Similarly, the output variable at time t are linear combination of the input and state variables at time t.

eqn 8 ←
$$\begin{cases} y_1 = c_{11}x_1 + c_{12}x_2 + \cdots + c_{1n}x_n + d_{11}u_1 + d_{12}u_2 + \cdots + d_{1m}u_m \\ y_2 = c_{21}x_1 + c_{22}x_2 + \cdots + c_{2n}x_n + d_{21}u_1 + d_{22}u_2 + \cdots + d_{2m}u_m \\ \vdots \\ y_n = c_{p1}x_1 + c_{p2}x_2 + \cdots + c_{pn}x_n + d_{p1}u_1 + d_{p2}u_2 + \cdots + d_{pm}u_m \end{cases}$$

where the coefficients $c_{ij}'s$ and $d_{ij}'s$ are constt.
eqn (8) can be written as in vector Matrix form as

$$Y = Cx + Dy$$

where Y is a $P \times 1$ output vector
C is a $P \times n$ output Matrix
D is a $P \times m$ Transmission Matrix defined by

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix}, \quad C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & & \vdots \\ c_{p1} & c_{p2} & & c_{pn} \end{bmatrix}, \quad D = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1m} \\ d_{21} & d_{22} & \cdots & d_{2m} \\ \vdots & & & \\ d_{p1} & d_{p2} & \cdots & d_{pm} \end{bmatrix}$$

The state space representation of linear time invariant system is thus given by the following eqn.

$$\dot{x} = Ax + Bu \quad : \text{State eqn.}$$

$$y = Cx + Du \quad : \text{output eqn.}$$

Prob: nth order differential eqn. the system is described by

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 10y = 8u(t)$$

Where y is the output and u is the input to the system. obtain state space representation of the system

Soln ~~Select the state variable as~~

~~$x_1 = y$~~

~~$\frac{dx_1}{dt} = \dot{x} = x_2$~~

~~d~~

The given differential eqn in the format

$$\dddot{y} + 6\ddot{y} + 11\dot{y} + 10y = 8u(t)$$

Similarly, $y(\cancel{k+3}) + 6y(k+2) + 11(k+1) + 10k = 8u(t)$

let us choose the state variable's

output = y

$$x_1 = y \qquad \dot{x}_1 = \dot{y} = x_2$$

$$x_2 = \dot{y} \qquad \dot{x}_2 = \ddot{y} = x_3$$

$$x_3 = \ddot{y} \qquad \dot{x}_3 = \dddot{y} = 8u(t) - 6\ddot{y} - 11\dot{y} - 10y$$

For A value
x_1 & x_3 value is 0
x_1 & x_2 value are 0
output $y = x_1$
$= [1 \; 0 \; 0]$

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}}_{} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -11 & -6 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 8 \end{bmatrix}}_{B} u(t)$$

$$\dot{x} = Ax + Bu(t)$$

$$y = [1 \; 0 \; 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$