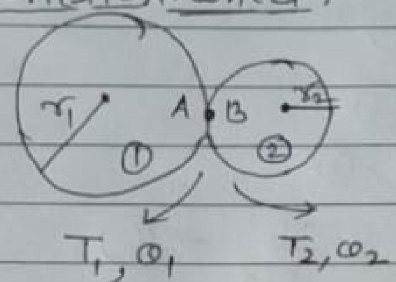
DATE / /
PAGE NO.

Mechanical coupling device (chain driver): Mechanical coupling device are (i) friction wheel (ii) ~~Gears~~ (iii) Lever etc. also here electrical analogous.

(i) **Friction wheel:**



Two wheels are mechanically coupled At point A and B wheels experience equal and opposite forces and the linear velocity at the points of contact will be same.

let ω_1 & ω_2 = Angular velocity
T_1 & T_2 = Torque of the wheel
r_1 & r_2 = Radius of the wheel.
θ_1 & θ_2 = Angular displacement of the wheel
N_1 & N_2 = No. of teeths on wheel.

An Ideal case work done by wheel ① will be equal to work done by wheel ②

work done by wheel ① $= T_1 \theta_1$
work done by wheel ② $= T_2 \theta_2$

$$T_1 \theta_1 = T_2 \theta_2$$

$$\frac{T_1}{T_2} = \frac{\theta_2}{T_1} \quad \text{—— ①}$$

Linear distance travelled along of the surface of wheel ① is $\theta_1 r_1$

Linear distance travelled along of the surface of wheel ② is $\theta_2 r_2$

The linear distance will be same.

$$\theta_1 r_1 = \theta_2 r_2$$

$$\frac{\theta_1}{\theta_2} = \frac{r_2}{r_1} \quad \text{—— ②}$$

combining eqn ① & ②

$$\boxed{\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{r_1}{r_2}} \quad \text{—— ③}$$

DATE / /

PAGE NO.

No. of teeth on wheels ∝ Radius of wheels.

$$N_1 \propto r_1 \quad \text{— (4)}$$

$$N_2 \propto r_2 \quad \text{— (5)}$$

combining eqn (3), (4) & (5), (3)

$$\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2} \quad \text{— (6)}$$

since, $\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2}$ $\quad \therefore \theta_2 = \frac{r_1}{r_2} \cdot \theta_1$ — (7)

Differentiating eqn (7).

$$\dot{\theta}_2 = \frac{r_1}{r_2} \cdot \dot{\theta}_1$$

But $\dot{\theta}_2 = \omega_2$

$\dot{\theta}_1 = \omega_1$

$$\frac{\omega_2}{\omega_1} = \frac{r_1}{r_2} \quad \text{— (8)}$$

18

combining eqn (6) & (8)

$$\boxed{\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2} = \frac{\omega_2}{\omega_1}} \quad \text{— (9)}$$

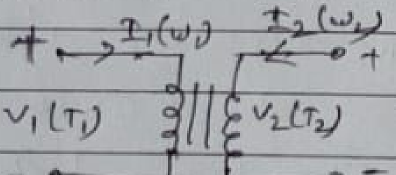
electrical analogous of friction wheels:

Let $\frac{r_1}{r_2}$ as the turn ratio $\frac{N_1}{N_2}$ of an ideal Transfer. Then torque will be analogous to velocity. and angular velocity will be analogous to current. This is force (Torque) - voltage (f-v) Analogy.

$r_1 : r_2 \rightarrow N_1 : N_2$

$\omega_1 : \omega_2 \rightarrow I_1 : I_2$

$T_1 : T_2 \rightarrow V_1 : V_2$



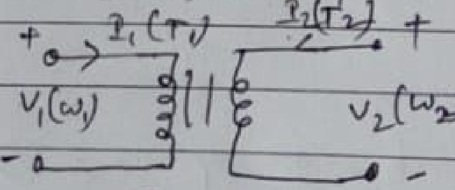
⇒ force current (F-I) Analogy

and of force (Torque) - current (f-I) Analogy

$r_2 : r_1 \rightarrow N_1 : N_2$

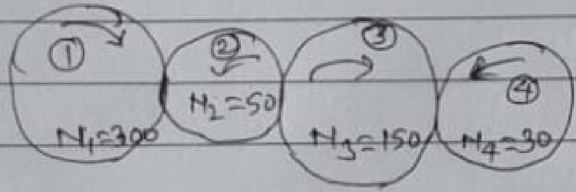
$\omega_1 : \omega_2 \rightarrow I_1 : I_2$

$T_1 : T_2 \rightarrow V_1 : V_2$



(f - I) Analogy

Prob: If $\theta_1 = 3$ radian (~~anticlock~~ clockwise), calculate the displacement of wheel 2, 3 and 4 (b) If $\omega_1 = 15$ rad/sec. calculate ω_3 and ω_4 (c) If $T_1 = 10$ Nm. calculate T_2 and T_3 (d) If angular accl^n ($\ddot{\theta}_1$) for wheel ① is 4 rad/sec^2. Calculate $\ddot{\theta}_3$ (e) find $r_1 : r_2$ & $r_1 : r_3$



Sol: (a)
$$\frac{\theta_2}{\theta_1} = \frac{N_1}{N_2}$$
$$\frac{\theta_2}{3} = \frac{300}{50}\ 6$$
$$\theta_2 = 18 \text{ rad}$$

$$\frac{\theta_2}{\theta_3} = \frac{N_3}{N_2}$$
$$\frac{18}{\theta_3} = \frac{150}{50}\ 3$$
$$\theta_3 = \frac{18}{3}\ 6 = 6 \text{ rad}$$

Similarly,
$$\frac{\theta_3}{\theta_4} = \frac{N_4}{N_3}$$
$$\frac{6}{\theta_4} = \frac{30}{150}\ 5$$
$$\theta_4 = 30 \text{ rad}.$$

(b)
$$\frac{\omega_3}{\omega_1} = \frac{N_1}{N_3}$$
$$\frac{\omega_3}{15} = \frac{300}{150}\ 2$$
$$\omega_3 = 30 \text{ rad/sec}$$

$$\frac{\omega_4}{\omega_1} = \frac{N_1}{N_4}$$
$$\frac{\omega_4}{15} = \frac{300}{30}\ 10$$
$$\omega_4 = 150 \text{ rad/sec}.$$

(c)
$$\frac{T_1}{T_2} = \frac{N_1}{N_2}$$
$$\frac{10}{T_2} = \frac{300}{50}\ 6$$
$$T_2 = \frac{10}{6}\frac{5}{3} \quad \therefore T_2 = \frac{5}{3} \text{ N-m}$$

$$\frac{T_1}{T_3} = \frac{N_1}{N_3}$$
$$\frac{10}{T_3} = \frac{300}{150}\ 2 \quad \therefore T_3 = \frac{10}{2}\ 5 = 5 \text{ N·m}$$

(d)
$$\frac{\ddot{\theta}_3}{\ddot{\theta}_1} = \frac{N_1}{N_3}$$
$$\frac{\ddot{\theta}_3}{4} = \frac{300}{150}\ 2 \quad \therefore \ddot{\theta}_3 = 8 \text{ rad/sec}^2$$

(e)
$$\frac{r_1}{r_2} = \frac{N_1}{N_2}$$
$$\frac{r_1}{r_2} = \frac{300}{50}\ \frac{6}{1}$$
$$r_1 : r_2 = 6 : 1$$

$$\frac{r_1}{r_3} = \frac{N_1}{N_3}$$
$$\frac{r_1}{r_3} = \frac{300}{150}\ \frac{2}{1}$$
$$r_1 : r_3 = 2 : 1$$