ECE - 6th semester
Signal and System

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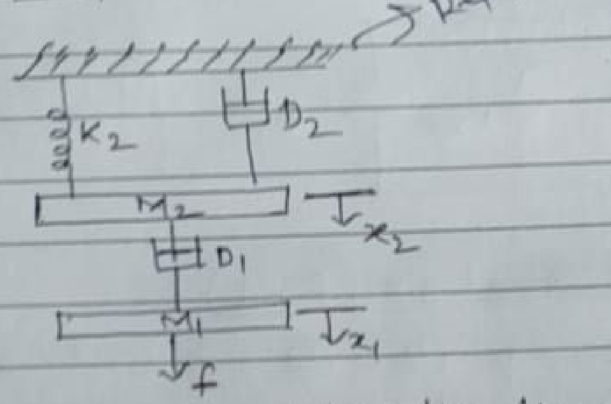
(1) Determine system eqn of the system shown in fig. Also Determine the force voltage (F-V) and force current (F-I) Analogy.

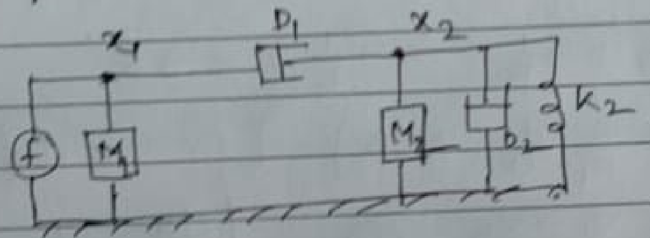


सबसे पहले displacement x_1 and x_2 को node मानेंगे। और x_1 node में जो element connect होंगे उसे जोड़ देंगे। साथ ही साथ node x_2 में जो element connect उसे जोड़ देंगे।

Mass, Damper and spring को जिससे denote करेगा question में उसी के अनुसार Answer देना है। जैसे - spring को D से denote कर सकता है इस case में D लिख देता है लेकिन spring का expression लिखता है उसी तरह damper के बदले F भी दे सकता है ठीक उसी तरह D F लिखकर damper expression लिख देता है।

Soln. Using by nodal analysis the system representation is shown in fig.

equivalent ckt :



At node x_1

$$f = M_1\frac{d^2x_1}{dt^2} + D_1\frac{d}{dt}(x_1 - x_2) \quad -①$$

Because spring is not connected in node x_1.

At node x_2

$$D_1\frac{d}{dt}(x_2 - x_1) + M_2\frac{d^2x_2}{dt^2} + D_2\frac{dx_2}{dt} + K_2x_2 = 0$$

$$M_2\frac{d^2x_2}{dt^2} + D\frac{dx_2}{dt} + K_2x_2 = D_1\frac{d}{dx}(x_1 - x_2)$$

$$D_1\frac{d}{dx}(x_1 - x_2) = M_2\frac{d^2x_2}{dt^2} + D\frac{dx_2}{dt} + K_2x_2 \quad -②$$

(F - V) [Force voltage] Analogy :

$f \to V,\ M \to L,\ D \to R,\ K \to \frac{1}{C},\ x \to q$

from eqn ① we get

$$V = L_1\frac{d^2q_1}{dt^2} + R_1\frac{d}{dt}(q_1 - q_2)$$

$$V = L\frac{di}{dt} + R_1(i_1 - i_2) \quad -③$$

$\boxed{\frac{dq_1}{dt} = i_1 \qquad \frac{dq_2}{dt} = i_2}$

from eqn ②, we have

$$R_1\frac{d}{dt}(q_1 - q_2) = L_2\frac{d^2q_2}{dt^2} + R_2\frac{dq_2}{dt} + \frac{1}{C_2}q_2$$

$$R_1(i_1 - i_2) = L_2\frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt \quad -④$$

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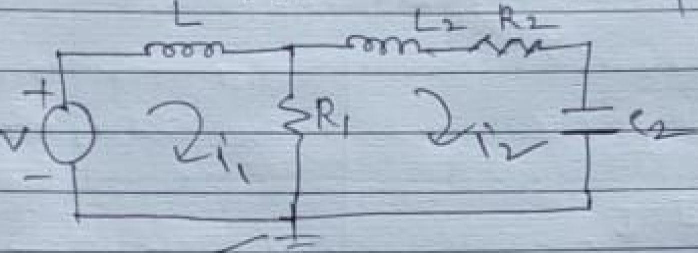
From eq^n (3) & (4) we can write.(draw)

Force-voltage Analogy



ckt को eq^n (3) एवं (4) से मिलाने पर मिल जाता है।

reference

Force - current Analogy (F-I) :

Again eq^n (1) & (2) to write

$$f = M_1\frac{d^2x_1}{dt^2} + D\frac{d}{dt}(x_1-x_2) \quad \text{— (1)}$$

$$D\frac{d}{dt}(x_1-x_2) = M_2\frac{d^2x_2}{dt^2} + D_2\frac{dx_2}{dt} + K_2x_2 \quad \text{— (2)}$$

(F-I) Analogy :

$$f \to i,\ M \to c,\ D \to \frac{1}{R},\ K \to \frac{1}{L},\ x \to \phi$$

$$i = C_1\frac{d^2\phi_1}{dt^2} + \frac{1}{R_1}\frac{d}{dt}(\phi_1-\phi_2) \quad \text{[From eq}^n\text{ (1)]}$$

$$i = C_1\frac{dv_1}{dt} + \frac{V_1-V_2}{R_1} \quad \text{— (5)}$$

$$\frac{d\phi_1}{dt} = V_1$$

$$\frac{d\phi_2}{dt} = V_2$$

$$\frac{1}{R_1}\frac{d}{dt}(\phi_1-\phi_2) = C_2\frac{d^2\phi_2}{dt^2} + \frac{1}{R_2}\frac{d\phi_2}{dt} + \frac{1}{L_2}\phi_2$$

$$\frac{V_1-V_2}{R_1} = C_2\frac{dv_2}{dt} + \frac{V_2}{R_2} + \frac{1}{L_2}\int V_2\,dt \quad \text{— (6)}$$

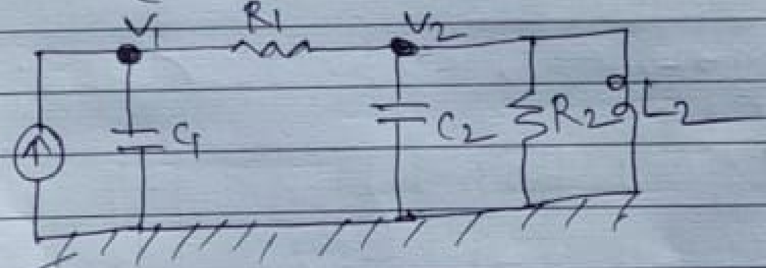
$$\frac{d\phi_2}{dt} = V_2$$

$$d\phi_2 = V_2\,dt$$

$$\int d\phi_2 = \int V_2\,dt$$

From eq^n (5) and (6).

(F-I) Analogy is shown in fig.



eq^n (5) एवं (6) को मिलाने पर मिल जाता है

~~model~~ Analysis

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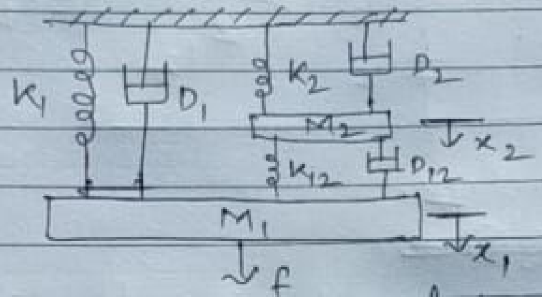
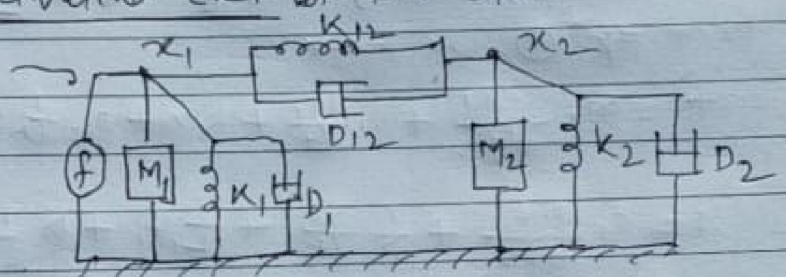
(2) Find the system eqn of the system shown in fig. Also determine the F-V and F-I Analogy.



सबसे पहले Mass eqn, Damper eqn तब Spring eqn लिखना है। क्योंकि Mass → accln. Damper → velocity eqn, Spring → displacement

Soln equivalent ckt of the system



At node x_1,

$$f = M_1\frac{d^2x_1}{dt^2} + D_1\frac{dx_1}{dt} + K_1x_1 + D_{12}\frac{d}{dt}(x_1-x_2) + K_{12}(x_1-x_2) \quad -(1)$$

At node x_2

$$0 = M_2\frac{d^2x_2}{dt^2} + D_2\frac{dx_2}{dt} + K_2x_2 + D_{12}\frac{d}{dt}(x_2-x_1) + K_{12}(x_2-x_1)$$

$$D_{12}\frac{d}{dt}(x_1-x_2) + K_{12}(x_1-x_2) = M_2\frac{d^2x_2}{dt^2} + D_2\frac{dx_2}{dt} + K_2x_2 \quad -(2)$$

eqn (1) & (2) represent system eqn

Force voltage Analogy (F-V):

$f \to v$, $M \to L$, $D \to R$, $K \to \frac{1}{C}$, $x \to q$, $u \to i$

~~From eqn (1)~~

$$\cancel{v = L_1\frac{d^2q_1}{dt^2} + R_1\frac{dq_1}{dt} + \frac{1}{C}\int i_1\,dt + R_{12}(i_1-i_2) + \frac{1}{C_{12}}\int(i_1-i_2)\,dt}$$

$\cancel{\frac{dq_1}{dt} = i_1}$

$$v = L_1\frac{d^2q_1}{dt^2} + R_1\frac{dq_1}{dt} + \frac{1}{C}q_1 + R_{12}\frac{d}{dt}(q_1-q_2) + \frac{1}{C_{12}}(q_1-q_2)$$

$$v = L_1\frac{di_1}{dt} + R_1i_1 + \frac{1}{C}\int i_1\,dt + R_{12}(i_1-i_2) + \frac{1}{C_{12}}\int(i_1-i_2)\,dt \quad -(3)$$

~~from eqn (2) we have~~

$$\cancel{R_{12}\frac{d}{dt}(q_1-q_2) + \frac{1}{C_{12}}(q_1-q_2) = L_2\frac{d^2q_2}{dt^2} + R_2\frac{dq_2}{dt} + \frac{1}{C_2}q_2}$$

$$R_{12}(i_1-i_2) + \frac{1}{C_{12}}\int(i_1-i_2)\,dt = L_2\frac{di_2}{dt} + R_2i_2 + \frac{1}{C_2}\int i_2\,dt \quad -(4)$$

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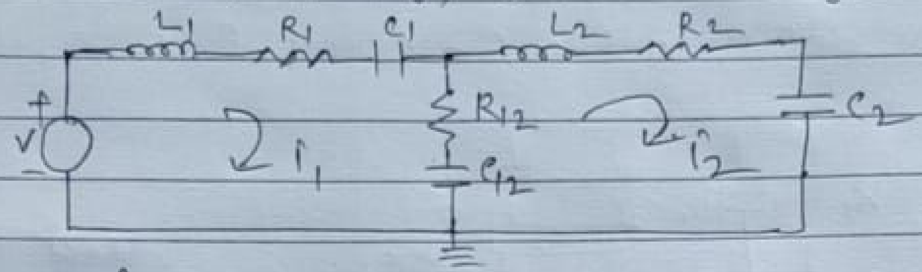
The (f-v) Analogy is shown in fig.



For (F-I) analogy.

~~Again eqn ①, & ② are~~

$f \to i,\ M \to C,\ D \to \frac{1}{R},\ k \to \frac{1}{L},\ x \to \psi$

from eqn ①, we have

$$f = M_1 \frac{d^2 x_1}{dt^2} + D_1 \frac{dx_1}{dt} + k_1 x_1 + D_{12} \frac{d}{dt}(x_1 - x_2) + k_{12}(x_1 - x_2)$$

$$i = C_1 \frac{d^2 \psi_1}{dt^2} + \frac{1}{R_1}\frac{d\psi}{dt} + \frac{1}{L_1}\psi_1 + \frac{1}{R_{12}}\frac{d}{dt}(\psi_1 - \psi_2) + \frac{1}{L_{12}}(\psi_1 - \psi_2)$$

$$i = C_1 \frac{dv_1}{dt} + \frac{v_1}{R_1} + \frac{1}{L_1}\int v_1 dt + \frac{v_1 - v_2}{R_{12}} + \frac{1}{L_{12}}\int (v_1 - v_2)\,dt. \quad \text{— ⑤}$$

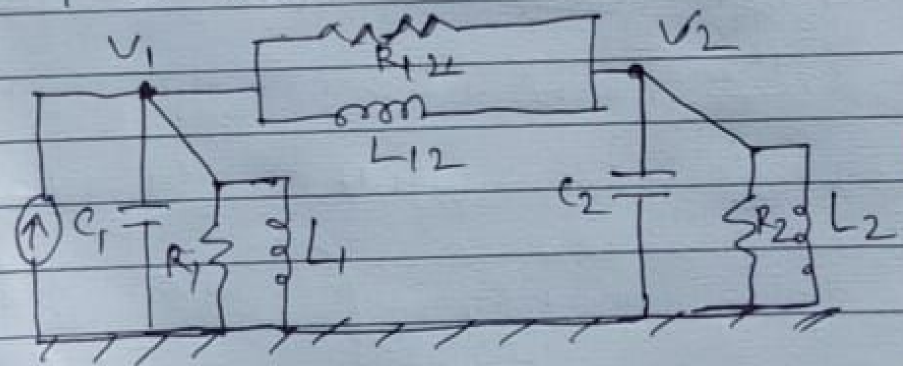
~~from eqn ②, we have~~

$$D_{12}\frac{d}{dt}(x_1 - x_2) + k_{12}(x_1 - x_2) = M_2 \frac{d^2 x_2}{dt^2} + D_2 \frac{dx_2}{dt} + k_2 x_2$$

$$\frac{1}{R_{12}}\frac{d}{dt}(\psi_1 - \psi_2) + \frac{1}{L_{12}}(\psi_1 - \psi_2) = C_2 \frac{d^2 \psi_2}{dt^2} + \frac{1}{R_2}\frac{d\psi}{dt} + \frac{1}{L_2}\psi_2$$

$$\frac{v_1 - v_2}{R_{12}} + \frac{1}{L_{12}}\int (v_1 - v_2)\,dt = C_2 \frac{d}{dt}(v_2) + \frac{v_2}{R_2} + \frac{1}{L_2}\int v_2\,dt \quad \longrightarrow ⑥$$

from eqn ⑤ & ⑥, (F-I) Analogy is shown in fig.



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