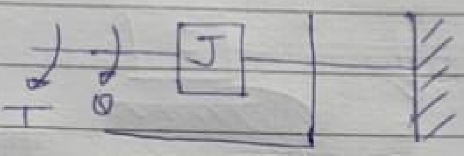
Rotational Motion

① Inertia (J) Damper (D) Spring (k)

(1) Inertia (J) :



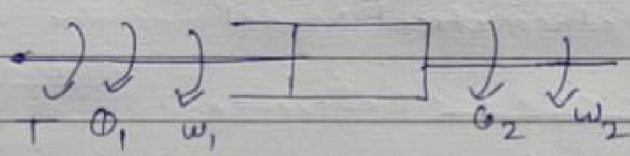
Inertia (J) is the property of an element that stores the kinetic energy of rotational motion. The inertia torque T is the product of moment of inertia and angular Accln.

Since inertia has only one displacement the relation is given by

$$T = J\frac{d\omega}{dt}$$

$$= J\frac{d^2\theta}{dt^2} \quad -① \left[\omega = \frac{d\theta}{dt}\right]$$

Damper (D) :



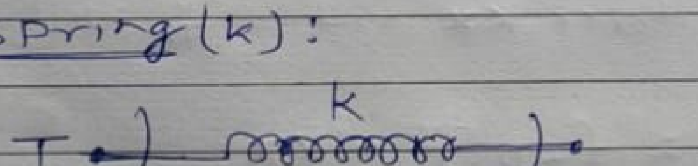
Damping Torque : When a Torque is applied to the damper, the damping torque for viscous friction is proportional to the difference in angular velocity of two ends of damper and is given by

The relation is given by

$$T = D\frac{d\theta}{dt}$$

$$T = D\frac{d}{dt}(\theta_1 - \theta_2)$$

Spring (k) :



Spring Torque : When a Torque is applied to the torsional spring as shown in fig.

Then the spring torque is produced which oppose the rotational spring and it is proportional to angular displacement

The relation is given by

$$T = k\theta$$

$$= k(\theta_1 - \theta_2)$$

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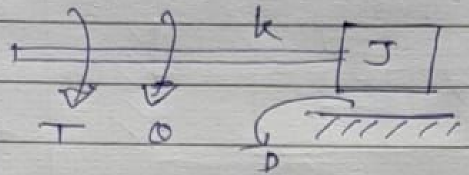
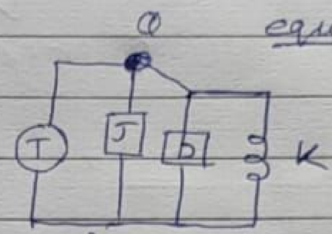
* Determin the torque eq^n of the system shown in fig.

Rotation example



equivalent ckt



इसमें θ (displacement) को Node मानना है बाकी सारे element को जोड़ देना है।

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let the Displacement be θ.

The torque eq^n of the system is given by

$$J\frac{d^2\theta}{dt^2} = T - D\frac{d\theta}{dt} - K\theta.$$

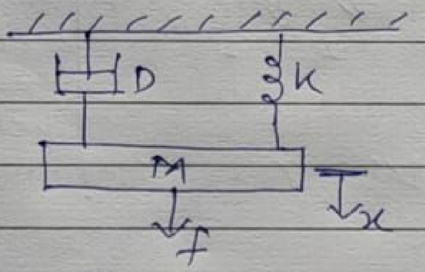
Because $D\frac{d\theta}{dt}$ and $K\theta$ opposes T

we can write.

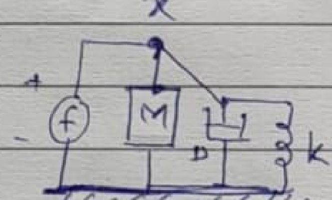
$$J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + K\theta = T \quad -(2)$$

* Determine the Mathematical Model for the system,

~~Torsional~~ Translational example



equivalent ckt



सबसे पहले displacement (x) को Node मानना है तब उससे जो connect है, मिला लो

The force f caused the displacement of Mass M this is opposed by spring as well as by the damper the force Kx and $D\frac{dx}{dt}$ will oppose f

$$M\frac{d^2x}{dt^2} = f - D\frac{dx}{dt} - Kx$$

$$M\frac{d^2x}{dt^2} + D\frac{dx}{dt} + Kx = f \quad -(1)$$

From Translation motion:

$$f = M\frac{d^2x}{dt^2} + D\frac{dx}{dt} + Kx \quad -(1)$$

From Rotation motion:

$$T = J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + K\theta \quad -(2)$$

Comparision of Translational and rotation motion:

Translation motion	Rotational motion
force (f)	Torque (T)
Mass (M)	Inertia (J)
Displacement (x)	Angular displacement (θ)
Damper (D)	Damper (D)
Spring (K)	Spring (K)
velocity $u = \frac{dx}{dt}$ or $v = \frac{dx}{dt}$	Angular velocity $\omega = \frac{d\theta}{dt}$

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D'Alembert's Principle:

For any body the algebraic sum of externally applied force and the force resisting motion any given direction is zero.

1st choose the reference direction all the force in the direction of reference direction consider as +ve and the forces opposite to the reference direction taken as −ve

According to D'Alembert's Principle.

$$f(t) + F_M(t) + F_D(t) + F_K(t) = 0$$

$$T(t) + T_J + T_D + T_K = 0$$

For any body, the algebraic sum of externally applied torque and the torque resisting rotation about any axis is zero.