UNIT 7 SLOPE DEFLECTION METHOD

Structure

7.1 INTRODUCTION

This unit is useful to analyse indeterminate structures, like continuous beams & plane frames. This method was presented by Prof. G. A. Maney in 1915 as a general method for analysis of rigid-jointed structures. The unknowns in this method are *degrees of freedom*, i.e. displacements. Therefore, it is one of the *displacement methods*. It is a classical method on which the moment distribution method, Kani's method and matrix stiffness method are based. The prerequisite for this is the knowledge of *fixed end moments* computation.

Objectives

After studying this unit, you should be able to carry out analysis of continuous beams and rigid-jointed plane frames. This involves,

- formulation of equilibrium equations,
- computation of primary unknowns, i.e. *displacements* by solving linear simultaneous equations,
- computation of final member *end moments* at joints, and
- drawing shear force (SF) and bending moment (BM) diagrams.

7.2 BASIC CONCEPTS

7.2.1 Sign Convention

Clockwise moments and clockwise rotations are considered as positive.

Here, we must recognise the distinction between the words, "moment" and "bending moment". The former is a physical concept and the sign depends upon whether it is clockwise (positive) or anti-clockwise (negative). The latter is an internal stress resultant of a structural member, and is assumed positive when sagging or considered negative when hogging. Thus, a positive (clockwise) moment at the right hand end of a beam causes negative (hogging) bending moment there; whereas at the left hand, it causes a positive (sagging) bending moment. This must be clearly kept in mind when writing down the equations, and while drawing bending moment diagrams (BMDs).

7.2.2 Fixed End Actions

Any span of a beam with both ends fixed is subjected to different types of loading. The formulae for fixed end actions are shown in Table 7.1, which are listed as case nos. 1 to 6. Case nos. 7 to 9 give fixed end actions produced due to either rotation or lateral translation of joint. The fixed end moments are useful in formulation of slope deflection equations.

We recall that these fixed end moments are derived by method of consistent deformation. The displacements are computed by using moment area theorem or Macaulay's method.

The calculation of fixed end moments is required in various units of this course-topic. You are advised to remember simple cases by heart and practise the use of various formulae given in the Table 7.1.

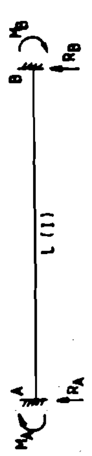
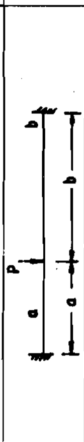
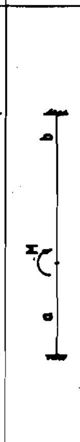
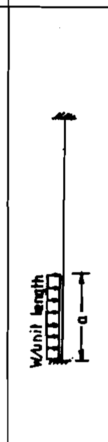
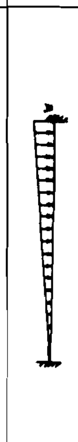
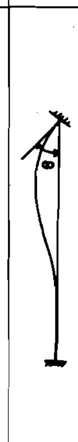
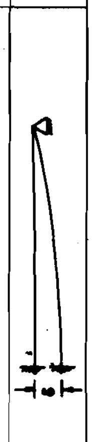


Table 7.1 : Fixed End Actions

Case No.	Beam	M_A	M_B	R_A	R_B
1.		$-\dfrac{P\,ab^2}{L^2}$	$\dfrac{P\,ba^2}{L^2}$	$\dfrac{Pb^3}{L^3}(3a+b)$	$\dfrac{Pa^3}{L^3}(a+3b)$
2.		$\dfrac{Mb\,(2a-b)}{L^2}$	$\dfrac{Ma\,(2b-a)}{L^2}$	$-\dfrac{8\,Mab}{L^3}$	$\dfrac{8\,Mab}{L^3}$
3.		$-\dfrac{Pa\,(L-a)}{L}$	$\dfrac{Pa\,(L-a)}{L}$	P	P
4.		$-\dfrac{wL^2}{12}$	$\dfrac{wL^2}{12}$	$\dfrac{wL}{2}$	$\dfrac{wL}{2}$
5.		$-\dfrac{wa^2}{12\,L^2}(8L^2-8aL+8a^2)$	$\dfrac{wa^3}{12\,L^2}(4L-3a)$	$\dfrac{wa}{2}L^3\,(2L^3-2a^3L+a^3)$	$\dfrac{wa^3}{2L^3}(2L-a)$
6.		$-\dfrac{wL^2}{30}$	$\dfrac{wL^3}{20}$	$\dfrac{3\,wL}{20}$	$\dfrac{7\,wL}{20}$
7.		$\dfrac{6\,EI\,\delta}{L^2}$	$\dfrac{6\,EI\,\delta}{L^2}$	$-\dfrac{12\,EI\,\delta}{L^3}$	$\dfrac{12\,EI\,\delta}{L^3}$
8.		$\dfrac{2\,EI\,\theta}{L}$	$\dfrac{4\,EI\,\theta}{L}$	$-\dfrac{6\,EI\,\theta}{L^2}$	$\dfrac{6\,EI\,\theta}{L^2}$
9.		—	$\dfrac{3\,EI\,\theta}{L}$	$-\dfrac{3\,EI\,\theta}{L}$	$\dfrac{3\,EI\,\theta}{L}$
10.		$\dfrac{3\,EI\,\delta}{L^3}$	—	$-\dfrac{3\,EI\,\delta}{L^3}$	$\dfrac{3\,EI\,\delta}{L^3}$

7.2.3 Rotational Stiffness

Consider a fixed beam AB of span L and flexural rigidity EI as shown in Figure 7.1 (c) in which end A is rotated by θ_A.

The moment induced at end A is

$$M_{AB} = 4\,EI \frac{\theta_A}{L} = k_{AA}\,\theta_A \quad \text{(Refer Table 7.1, Case 8)}$$

where $k_{AA} = 4EI/L$ is called *rotational stiffness* which can also be defined as the moment required at A to produce unit rotation at A.

And, the moment induced at B is

$$M_{BA} = 2\,EI \frac{\theta_A}{L} = k_{BA}\,\theta_A \quad \text{(Refer Table 7.1, Case 8)}$$

where k_{BA} can be defined as the moment produced at B due to the application of unit rotation at A.

7.2.4 Lateral Displacement Factor

Consider a fixed beam AB of span L and flexural rigidity EI as shown in Figure 7.1 (e). End A is displaced lateraly by δ downward with respect to end B, keeping both ends restrained against rotation. Due to this displacement δ.

$$M_{AB} = 6\,EI \frac{\delta}{L^2} = k_{A\delta} \times \delta \quad \text{(Refer Table 7.1, Case 7)}$$

where $k_{A\delta} = 6EI/L^2$ is called **lateral displacement factor** which can be defined as the moment produced at A due to unit lateral displacement at A with respect to B.

Similarly, $$M_{BA} = 6\,EI \frac{\delta}{L^2} = k_{B\delta} \times \delta$$

Therefore, $$k_{A\delta} = k_{B\delta} = \frac{6\,EI}{L^2}$$

where $k_{A\delta}$ is the moment produced at A due to unit displacement at A and $k_{B\delta}$ is the moment produced at B due to unit displacement at A.

7.3 STEPS FOR ANALYSIS

(a) *Computation of fixed end moments* : The formulae for fixed end actions for various load cases are given in Table 7.1. The sign convention followed is that for moments, i.e. clockwise – positive and anticlockwise – negative.

(b) *Relate member end moments to joint displacements* : The end moments are as follows (derived in Section 7.4) :

$$M_{AB} = \frac{2\,EI}{L}\left(2\,\theta_A + \theta_B + \frac{3\,\delta}{L}\right) + FM_{AB}$$

$$M_{BA} = \frac{2\,EI}{L}\left(\theta_A + 2\,\theta_B + \frac{3\,\delta}{L}\right) + FM_{BA}$$

Compute the end moments for each member using the expressions. These are also known as **slope deflection equations**.

(c) *Formulate equilibrium equations* : These are obtained by making algebraic sum of moments at each joint as zero. In case of frames with sway, additional equations are obtained considering shear condition. Details are explained in Sections 7.5 and 7.9.1.

(d) *Solve the equations* : This will give displacements (primary unknowns), i.e. θ_A, θ_B, δ etc.

(e) *Back-substitution* : In the expressions for end moments formed in step (b), substitute values of known displacements as obtained in step (d). This gives final end moments for each member. From this, the support reactions can also be calculated.

(f) Sketch shear force diagram (SFD) and bending moment diagram (BMD).

7.4 SLOPE DEFLECTION EQUATION

The equation relates '*member end moments*' to '*joint displacements*'. The relation is derived as follows. Let us consider member *AB* of any plane frame or beam as shown in Figure 7.1 (a). *A* and *B* are ends of the member. Due to given loading, end *A* laterally translates downward by δ with respect to end *B* and rotation at *A* and *B* are θ_A and θ_B respectively. We observe that Figure 7.1 (a) can be considered as superposition of four figures, i.e. Figures 7.1 (b) to (e).

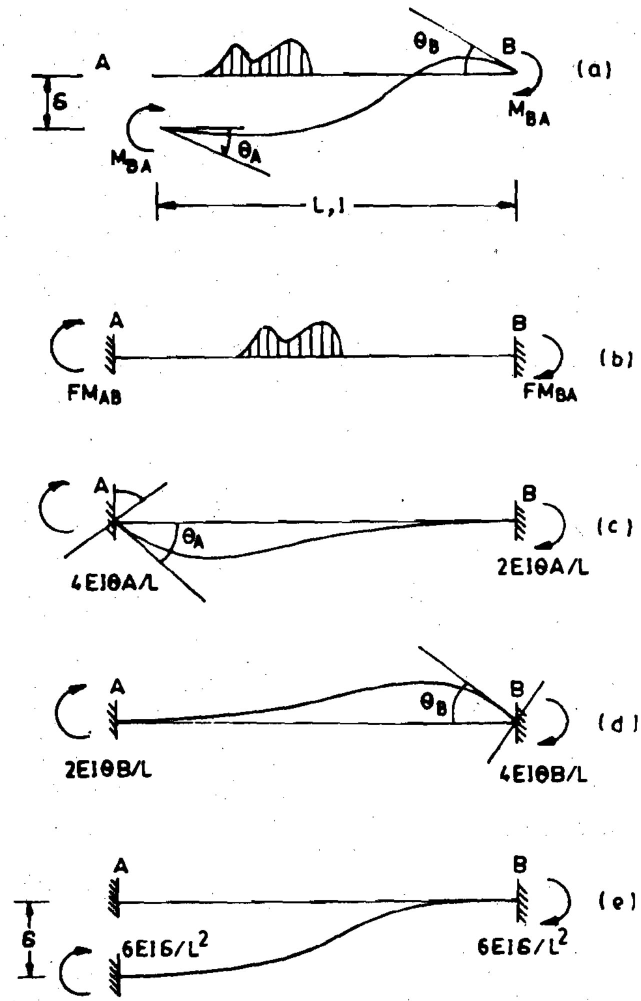


Figure 7.1

In Figure 7.1 (b), *A* and *B* are restrained and FM_{AB} & FM_{BA} are resulting fixed end moments (due to the load action). The Table 7.1 gives the fixed end moments due to different actions or support settlement/rotation etc.

In Figure 7.1 (c), end *A* is rotated by θ_A clockwise while *B* is kept restrained. Due to this rotation θ_A, the end moments are $M_{AB} = k_{AA}\,\theta_A = 4EI\,\theta_A/L$ and $M_{BA} = k_{BA}\,\theta_A = 2EI\,\theta_A/L$ as explained in Section 7.2.3.

In Figure 7.1 (d), end *B* is rotated by θ_B while *A* is kept restrained. Due to this rotation θ_B, the end moments are $M_{AB} = k_{AB}\,\theta_B = 2EI\,\theta_B/L$ and $M_{BA} = k_{BB}\,\theta_B = 4EI\,\theta_B/L$.

In Figure 7.1(e), end *A* settles down by θ with respect to end *B* while both ends are restrained against rotation. Due to this settlement, the end moments will be

$M_{AB} = k_{A\delta}.\delta = M_{AB} = k_{B\delta}.\delta = (6EI\delta/L^2)$ at both ends *A* and *B* as explained in Section 7.2.4.

The end moments M_{AB} and M_{BA} are the sumation of the above four cases as shown in Figures 7.1 (b) to (e). The final moments at ends *A* and *B* respectively are as follows :

$$M_{AB} = FM_{AB} + \frac{4EI\,\theta_A}{L} + \frac{2EI\,\theta_B}{L} + \frac{6EI\,\delta}{L^2} \quad (7.1)$$

$$M_{BA} = FM_{BA} \frac{2EI\,\theta_A}{L} + \frac{4EI\,\theta_B}{L} + \frac{6EI\,\delta}{L^2} \quad (7.2)$$

which can be remembered as

$$M_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B + \frac{3\delta}{L}\right) + FM_{AB} \quad (7.3)$$

$$M_{BA} = \frac{2EI}{L}\left(\theta_A + 2\theta_B + \frac{3\delta}{L}\right) + FM_{BA} \quad (7.4)$$

When there is no lateral translation δ,

$$M_{AB} = 2EI\left(\frac{2\theta_A + \theta_B}{L}\right) + FM_{AB} \quad (7.5)$$

$$M_{BA} = 2EI\left(\frac{\theta_A + 2\theta_B}{L}\right) + FM_{BA} \quad (7.6)$$

Note : Here, it should be noted that a lateral translation δ which causes the beam to rotate anti-clockwise as shown in Figure 7.1 (e), is considered positive. Since, it induces clockwise (positive) end moments, for the opposite case δ will have to be taken as negative.

7.5 EQUILIBRIUM EQUATIONS

In this method, displacements, i.e. rotation and translations at the joints are the unknowns. The unknown displacements are also termed as degrees of freedom or degree of kinematic indeterminacy. If there are n degrees of freedom (i.e. unknowns) for a given structure, n equations are required to be formulated using equilibrium conditions.

The equilibrium equations for end moments of all members meeting at a joint can be obtained as explained in the following paragraphs.

If there are n number of joints in any structure, for each joint the following procedure is adopted. Let us consider joint i, the sumation of all member end moments for all members meeting at a joint must be equal to external applied moment at that joint

At joint i,

$$\sum_{j=1}^{j=m} M_{ij} = M_i \quad (7.7)$$

where,

M_i = external moment applied at i,

j = far ends for all members with $j = 1$ to m, and

m = number of members meeting at joint i. Usually, external moment applied at joints are zero. In such a case

$$\sum_{j=1}^{j=m} M_{ij} = 0 \quad (7.8)$$

Similar equations are formed for each joint.

7.6 ANALYSIS OF CONTINUOUS BEAMS

As slope deflection method is a displacement method the unknowns are displacements. Axial deformations are neglected in this method and lateral displacement (if any) are known values, in case of continuous beams. Therefore, the unknown degree of freedom are only rotation of joints. Recall the steps for analysis. Here the slope deflection equations are formed for each span separately. Then at each joint, moment equilibrium conditions are applied to form linear simultaneous equations having unknown rotations. By solving these equations, the unknown rotations are found out.

Example 7.1

Analyse the continuous beam shown in Figure 7.2 (a).

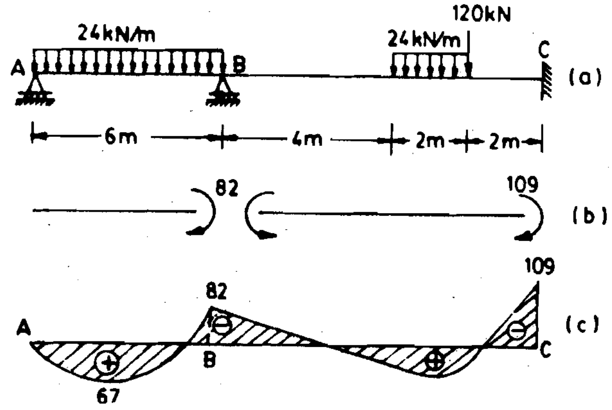


Figure 7.2

Solution

Unknowns : Slope at A, i.e θ_A and slope at B, i.e. θ_B

Step 1 : *Fixed end moments*

For span AB : (due to uniformely distributed load)

$$FM_{AB} = -\frac{24 \times 6^2}{12} = -72 \text{ kN m}\,; \; FM_{BA} = \frac{24 \times 6^2}{12} = 72 \text{ kN m}$$

For span BC : FM_{BC} and FM_{CB} are due to part u.d.l and due to point load. Due to point load

$$FM'_{BC} = \frac{-60 \times 2^2 \times 6}{8^2} = -22.5 \text{ kN m}$$

$$FM'_{CB} = \frac{60 \times 2 \times 6^2}{8^2} = +67.5 \text{ kN m}$$

For computing fixed end moments due to part u.d.l, let us refer Table 7.1 (Case 5) and applying method of superposition for downward load for $a = 6$ m and upward load for $a' = 4$ m

$$FM''_{BC} = -w \times a^2 \left(\frac{6L^2 - 8La + 3a^2}{12\, L^2} \right) + w \times a'^2 \left(\frac{6L^2 - 8La' + 3a'^2}{12\, L^2} \right)$$

$$FM''_{BC} = -\left[\frac{(24 \times 6^2)\ (6 \times 8^2 - 8 \times 8 \times 6 + 3 \times 6^2)}{(12 \times 8^2)} \right] + \left[\frac{(24 \times 4^2)\ (6 \times 8^2 - 8 \times 8 \times 4 + 3 \times 4^2)}{(12 \times 8^2)} \right]$$

$$= -33.5 \text{ kN m}$$

$$FM''_{CB} = wa^3 \frac{(4L - 3a)}{12L^2} - wa'^3 \frac{(4L - 3a')}{12L^2}$$

$$= \left[24 \times 6^3 \frac{(4 \times 8 - 3 \times 6)}{(12 \times 8^2)} \right] - \left[24 \times 4^3 \frac{(4 \times 8 - 3 \times 4)}{(12 \times 8^2)} \right]$$

$$= 54.5 \text{ kN m}$$

Thus, $FM_{BC} = FM'_{BC} + FM''_{BC} = -22.5 - 33.5 = -56.0$ kN m

$FM_{CB} = FM'_{CB} + FM''_{CB}$, $+67.5 + 54.5 = +122.0$ kN m

Step 2 : *Relate displacements (slopes) to end moments*

$$M_{AB} = \left[\frac{2EI\,(2\theta_A + \theta_B)}{6}\right] - 72\,; \quad M_{BA} = \left[\frac{2EI\,(\theta_A + 2\,\theta_B)}{6}\right] + 72$$

$$M_{BC} = \left[\frac{2EI\,(2\theta_B + \theta_C)}{8}\right] - 56\,; \quad M_{CB} = \left[\frac{2EI\,(\theta_B + 2\theta_C)}{8}\right] + 122$$

Step 3 : *Equilibrium equations*

As C is fixed end, $\theta_C = 0$.

Joint A : At hinged joint, $M_{AB} = 0$

Hence, $\left[\dfrac{EI\,(2\theta_A + \theta_B)}{3}\right] - 72 = 0$ (from Step 2)

$$\therefore \quad 2\theta_A + \theta_B = \frac{216}{EI} \qquad \text{... (I)}$$

Joint B : $M_{BA} + M_{BC} = 0$

Thus, $$\left[\frac{EI\,(\theta_A + 2\theta_B)}{3} + 72\right] + \left[\frac{EI\,(2\theta_B + 0)}{4} - 56\right] = 0.$$

$$\left[\frac{(\theta_A + 2\,\theta_B)}{3}\right] + \frac{(\theta_B)}{2} = \frac{-16}{EI}$$

$$2\theta_A + 7\theta_B = \frac{-96}{EI} \qquad \text{... (II)}$$

Step 4 : *Solution of equations*

On solving the two equations (I) and (II), we get

$\theta_A = \dfrac{134}{EI}$ positive (indicates clockwise rotation)

$\theta_B = \dfrac{-52}{EI}$ negative (indicates anticlockwise rotation)

Step 5 : *Final moments* (can be obtained by substituting the values of θ_A and θ_B in Step 2 above).

$$M_{AB} = \frac{(2 \times 134 - 52)}{3} - 72 = 0$$

$M_{BA} = \dfrac{(134 - 2 \times 52)}{3} + 72 = 82$ kN m (clockwise moment)

$M_{BC} = \dfrac{(-2 \times 52)}{4} - 56 = -82$ kN m (anticlockwise moment)

$M_{CB} = \dfrac{(-52)}{4} + 122 = 109$ kN m (clockwise moment)

Step 6 : *Bending moment diagram*

Let us recall here that clockwise moment is considered positive. From previous step, mark the final end moments for each member with appropriate sign, i.e. clockwise as positive or anticlockwise as negative. These are shown in Figure 7.2 (b).

At this stage, we revert back to our usual *bending moments sign conventions*, i.e. sagging bending moment as positive and hogging bending moment as negative as given in Section 7.2.1. Now the bending moment diagram is easily drawn which is shown in Figure 7.2 (c). Note that the ordinates for BM are drawn on the side where bending tension is developed.

SAQ 1

Analyse the continuous beam shown in Figure 7.3. Take EI constant throughout.

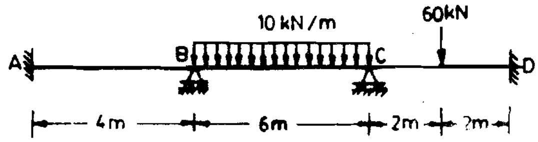


Figure 7.3

Example 7.2

For continuous beam ABC shown in Figure 7.4 (a), if support B settles by 4 mm, find bending moment at A, B and C.

Take EI as constant = 36×10^3 kN m^2.

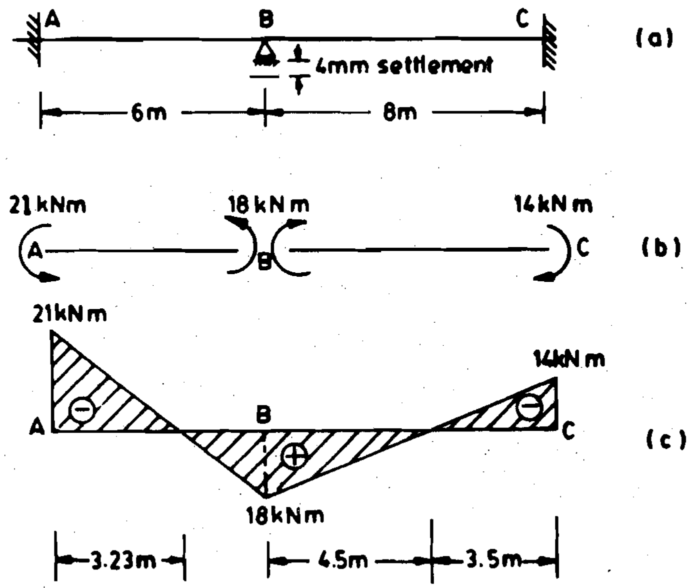


Figure 7.4

Solution

Unknowns : Slope at B, i.e. θ_B is the only unknown; since $\theta_A = \theta_C = 0$

Fixed end moments

$FM_{AB} = FM_{BA} = FM_{BC} = FM_{CB} = 0$ (as no external load acting on beam)

δ for $AB = -4$ mm $= -0.004$ m (induces anticlockwise moments)

δ for $BC = 4$ mm $= 0.004$ m (induces clockwise moments)

End moments & slopes relation

Span AB : $\theta_A = 0$

$$M_{AB} = \frac{2\,EI}{6}\left[\theta_B + \frac{3\,(-0.004)}{6}\right] \tag{I}$$

$$M_{BA} = \frac{2\,EI}{6}\left[2\,\theta_B + \frac{3\,(-0.004)}{6}\right] \tag{II}$$

Span $BC : \theta_C = 0$

$$M_{BC} = \frac{2\,EI}{8}\left[2\,\theta_B + \frac{3 \times 0.004}{8}\right] \qquad \text{(III)}$$

$$M_{CB} = \frac{2\,EI}{8}\left[\theta_B + \frac{3 \times 0.004)}{8}\right] \qquad \text{(IV)}$$

Equilibrium equation

At joint $B : M_{BA} + M_{BC} = 0$

$$\frac{2\,EI}{6}\,[2\,\theta_B - 0.002] + \frac{2\,EI}{8}\,[2\,\theta_B + 0.0015] = 0$$

$$\text{giving } \theta_B = 0.25 \times 10^{-3} \text{ radians (clockwise)}$$

On substituting back in the Eqs. (I) to (IV) and putting $EI = 36 \times 10^3$ kN m^2, we get,

Final moments

$M_{AB} = -21$ kN m (anticlockwise) ; $M_{BA} = -18$ kN m (anticlockwise)

$M_{BC} = 18$ kN m (clockwise) ; $M_{CB} = 14$ kN m (clockwise)

Bending moment diagram

Figure 7.4 (b) is free body diagram showing final moments acting at the joints and Figure 7.4 (c) shows bending moment diagram (BMD).

You must take note how the clockwise/anti-clockwise sign of the moments have been converted into sagging/hogging signs for the bending moment.

Example 7.3

Attempt Example 7.1 with same loading, if support B sinks by 8 mm and $EI = 72 \times 10^3$ kN m^2 uniform throughout.

Solution

Unknowns : θ_A and θ_B as fixed end slope $\theta_c = 0$

Span AB : right support B settles, $\delta = -0.008$ m

Slope deflection equations are as follows :

$$M_{AB} = \frac{2EI}{6}\left(2\theta_A + \theta_B + \frac{3\,(-0.008)}{6}\right) - 72$$

and $$M_{BA} = \frac{2EI}{6}\left(\theta_A + 2\theta_B + \frac{3\,(-0.008)}{6}\right) + 72$$

Span BC : left support B settles, $\delta = 0.008$ m

Slope deflection equations are as follows :

$$M_{BC} = \frac{2EI}{8}\left(2\theta_B + \theta_C + \frac{3 \times 0.008}{8}\right) - 56 = 0$$

$$= \frac{2EI}{8}\left[2\theta_B + 0.003\right] - 56 \quad (\text{since } \theta_C = 0)$$

Equilibrium equations

Joint $A : M_{AB} = 0$

$$EI\,(2\theta_A + \theta_B) - 0{:}004\,EI - 72 \times 3 = 0$$

On putting values,

$$EI\,(2\theta_A + \theta_B) = 0.004 \times 72 \times 10^3 + 72 \times 3$$

$$\therefore \quad 2\theta_A + \theta_B = \frac{504}{EI} \qquad \text{(I)}$$

Joint B : $M_{BA} + M_{BC} = 0$

Thus, we get

$$\left[EI\ \frac{(\theta_A + 2\,\theta_B)}{3} - \frac{EI}{6} \times 0.008 + 72\right] + \left[\frac{EI\,\theta_B}{2} + \frac{EI \times 0.003}{4} - 56\right] = 0$$

On substituting $EI = 72 \times 10^3$ kN m^2, we get

$$\left[\frac{EI\,(\theta_A + 2\,\theta_B)}{3} + \frac{EI\,\theta_B}{2}\right] - 24 - 2 = 0$$

On simplification, it gives $$2\,\theta_A + 7\,\theta_B = \frac{156}{EI} \qquad \text{(II)}$$

Solution of equations (I) and (II) gives

$$\theta_B = \frac{-58}{EI} \quad \text{and} \quad \theta_A = \frac{281}{EI}$$

On substituting these values in the moment equations, we get final moments

$$M_{BA} = \left[\frac{(281 - 2 \times 58)}{3}\right] - 24 = 31 \text{ kN m}$$

$$M_{BC} = \left[\frac{(-2 \times 58)}{4}\right] - 2 = -31 \text{ kN m}$$

$$M_{CB} = \frac{2EI}{8}\left(\theta_B + \frac{3 \times 0.008}{8}\right) + 122 = 161.5 \text{ kN m}$$

Example 7.4

Analyse the continuous beam shown in Figure 7.5 (a). EI is constant throughout.

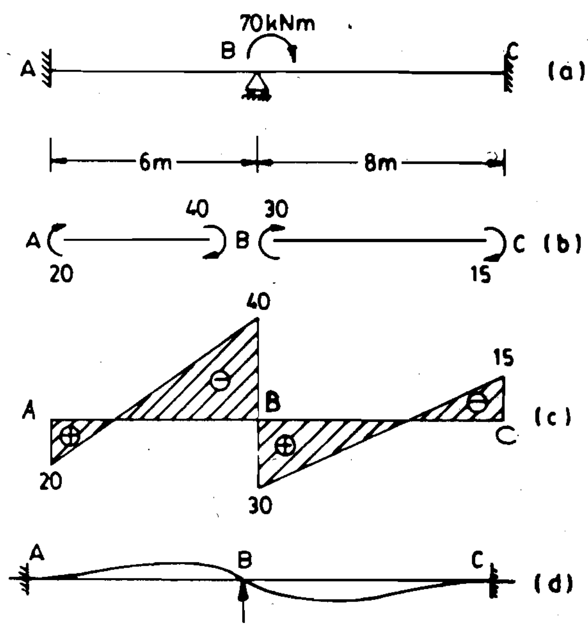


Figure 7.5

Solution

Unknowns : θ_B only as $\theta_A = \theta_C = 0$

Fixed end moments

As there are no transverse load on the beam, we have

$$FM_{AB} = FM_{BA} = FM_{BC} = FM_{CB} = 0$$

Member end moment and slope relation

For span *AB*

$$M_{AB} = \left(\frac{2EI}{6}\right)(\theta_B)\ ; \qquad M_{BA} = \left(\frac{2EI}{6}\right)(2\theta_B)$$

For span *BC*

$$M_{BC} = \left(\frac{2EI}{8}\right)(2\theta_B)\ : \qquad M_{CB} = \left(\frac{2EI}{8}\right)(\theta_B)$$

Equilibrium equation

$M_{BA} + M_{BC}$ = external moment at B = 70 kN m (Positive, since clockwise)

$$EI\ \theta_B\left(\frac{2}{3}+\frac{1}{2}\right) = 70 \qquad \therefore\ \theta_B = \frac{60}{EI}\text{(Clockwise)}$$

Final moments (after back substitution)

$$M_{AB} = 20\text{ kN m} \qquad M_{BA} = 40\text{ kN m}$$

$$M_{BC} = 30\text{ kN m} \qquad M_{CB} = 15\text{ kN m}$$

Figure 7.5 (b) is a free body diagram showing moments at joints and Figure 7.5 (c) shows bending moment diagram drawn on tension side. Figure 7.5 (d) shows deflected shape of the beam.

Again, you are advised to note the sign of all the joint rotation, moments and bending moments.

SAQ 2

Analyse the continuous beam shown in Figure 7.6 for (a) only with external loads, (b) with external load and support settlement and rotation. The support B settles by 40 mm and end C rotates by 1/100 radians clockwise

Take $EI = 800$ kN m^2 throughout.

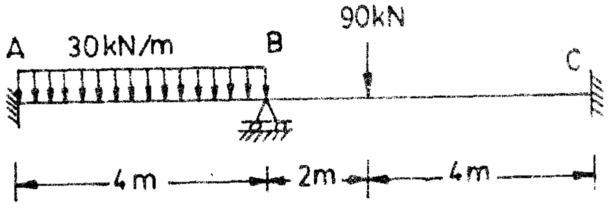


Figure 7.6

7.7 ANALYSIS OF PLANE FRAMES WITHOUT SWAY

In general, frames will have lateral displacement. Specific cases in which *sway* will not occur are

- A restraint in horizontal direction in the form of fixed, hinge or roller support is provided on vertical face to prevent horizontal displacement at storey level.
- The plane frame is symmetric about vertical axis with respect to geometry, flexural rigidity, end conditions and displacement of joints and it should be symmetrically loaded.

When there is no horizontal translation of joints, the sway δ in horizontal direction is zero. Deflection in vertical direction at columns is taken as zero because axial deformations in the columns (vertical members) are neglected. Therefore, the only unknowns are rotations at the supports and joints.

Example 7.5

Analyse the bent shown in Figure 7.7 (a).

Solution

Here, there is no horizontal sway as member BC is fixed at C. (fixed hinge)

Thus, degrees of freedom = 2, i.e. θ_B and θ_C while $\theta_A = 0$,

Fixed end moments

$$FM_{AB} = \frac{-PL}{8} = -30 \times \frac{4}{8} = -15 \text{ kN m} \qquad FM_{BA} = 15 \text{ kN m}$$

$$FM_{BC} = -120 \times \frac{4}{8} = -60 \text{ kN m} \qquad FM_{CB} = 60 \text{ kN m}$$

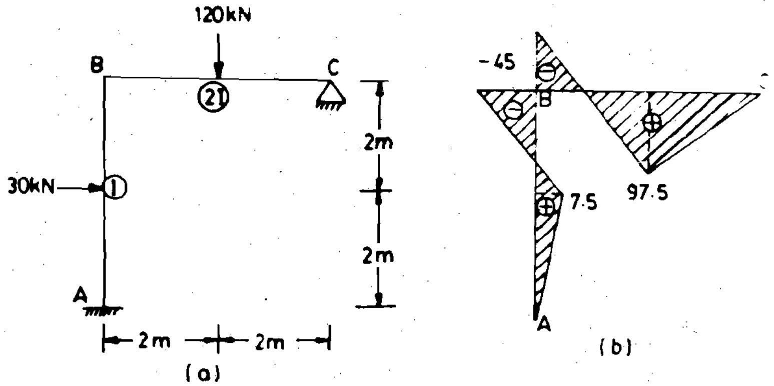


Figure 7.7

Slope deflection equations

$$M_{AB} = \left[\frac{2EI\,(2\theta_A + \theta_B)}{4}\right] - 15\,; \quad M_{BA} = \left[\frac{2EI\,(\theta_A + 2\,\theta_B)}{4}\right] + 15$$

$$M_{BC} = \left[\frac{2 \times 2EI\,(2\theta_B + \theta_C)}{4}\right] - 60\,; \quad M_{CB} = \left[\frac{2 \times 2EI\,(\theta_B + 2\theta_C)}{4}\right] + 60$$

Equilibrium equations

At hinge, $\quad M_{CB} = 0$, giving $\theta_B + 2\,\theta_C = \dfrac{-60}{EI}$ (I)

At joint B, $\quad M_{BA} + M_{BC} = 0$ gives

$$EI\,\theta_B + 15 + \frac{2\,EI}{\theta_B} + EI\,\theta_C - 60 = 0$$

On simplification, we get, $\quad 3\,\theta_B + \theta_C = \dfrac{45}{EI}$ (II)

Solution of equations

On solving equations (I) and (II), we get

$$\theta_B = \frac{30}{EI} \text{ (clockwise) and } \theta_C = \frac{-45}{EI} \text{ (anticlockwise)}$$

On substitution these values, final moments are as follows :

$$M_{AB} = \left[\frac{2EI\left(\frac{1}{EI}\right)(30)}{4}\right] - 15 = 0$$

$$M_{BA} = \left[\frac{2EI\left(\frac{1}{EI}\right)(2 \times 30)}{4}\right] + 15 = 45 \text{ kN m}$$

$$M_{BC} = \left[\frac{2 \times 2EI\left(\frac{1}{EI}\right)(2 \times 30 - 45)}{4}\right] - 60 = -45 \text{ kN m}$$

$$M_{CB} = \left[\frac{4EI\left(\frac{1}{EI}\right)(30 - 2 \times 45)}{4}\right] + 60 = 0$$

Here, although A is fixed, incidentally $M_{AB} = 0$. Bending moment diagram is shown in Figure 7.7 (b).

Note :

In case of plain frames, the bending moment sign convention is as follows : Bending moments causing inside tension positive, those causing tension in outer fibers is negative.

Example 7.6

Analyse the plane frame shown in Figure 7.8 (a).

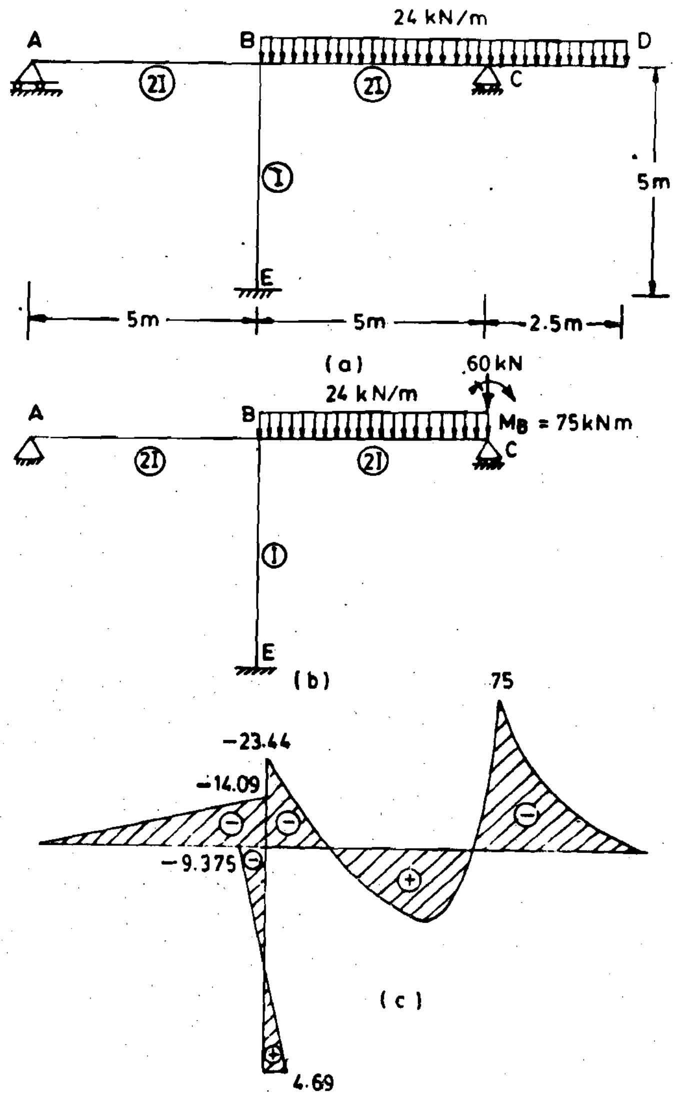


Figure 7.8

Solution

As CD is overhanging portion, it is statically determinate. Thus,

$$M_{CD} = \frac{-24 \times 2.5^2}{2} = -75 \text{ kN m (anticlockwise)}$$

The overhanging loaded cantilever is replaced by a downward load of 60 kN and a clockwise moment of 75 kN m acting at joint C.

The equivalent frame is shown in Figure 7.8 (b). The 60 kN force at support C is not causing any bending moment.

Unknowns : θ_A, θ_B, and θ_C as $\theta_E = 0$

Hence, degrees of freedom = 3

Fixed end moments

$$FM_{AB} = FM_{BA} = FM_{BE} = FM_{EB} = 0$$

$$FM_{BC} = \frac{-24 \times 5^2}{12} = -50 \text{ kN m, and } FM_{CB} = 50 \text{ kN m}$$

Slope deflection equations

$$M_{AB} = \frac{2E \times 2I\,(2\theta_A + \theta_B)}{5}$$

$$M_{BA} = \frac{2E \times 2I\,(\theta_A + 2\theta_B)}{5} = \frac{4\,EI\,(1.5\theta_B)}{5} = 1.2\,EI\,\theta_B$$

$$M_{BC} = \left[\frac{2E \times 2I\,(2\theta_B + \theta_C)}{5}\right] - 50 = 1.6\,EI\,\theta_B + 0.8\,EI\,\theta_C - 50$$

$$M_{CB} = \left[\frac{2E \times 2I\,(\theta_B + 2\,\theta_C)}{5}\right] + 50 = 0.8\,EI\,\theta_B + 1.6\,EI\,\theta_C + 50$$

$$M_{BE} = \frac{2EI\,(2\theta_B + \theta_E)}{5} = 0.8\,EI\,\theta_B$$

$$M_{EB} = \frac{2E\,(2\theta_E + \theta_B)}{5} = 0.4\,EI\,\theta_B$$

Equilibrium equations

Joint A :

$$M_{AB} = 0; \quad \text{giving } \theta_A = \frac{-\theta_B}{2}$$

Joint B :

$$M_{BA} + M_{BC} + M_{BE} = 0$$

$$0.8\,EI\,(\theta_A + 2\theta_B) + [1.6\,EI\,\theta_B + 0.8\,EI\,\theta_C - 50] + 0.8\,EI\,\theta_B = 0$$

$$1.2\,EI\,\theta_B + 1.6\,EI\,\theta_B + 0.8\,EI\,\theta_C + 0.8\,EI\,\theta_B = 50$$

$$9\theta_B + 2\theta_C = \frac{125}{EI} \qquad \text{(I)}$$

Joint C :

$$M_{CB} = 75 \text{ kN m}$$

$$0.8\,EI\,\theta_B + 1.6\,EI\,\theta_C + 50 = 75$$

$$\theta_B + 2\,\theta_C = \frac{125}{4EI} \qquad \text{(II)}$$

Subtracting Eq. (II) from Eq. (I), we get

$$8\theta_B = \frac{375}{4EI}; \quad \theta_B = \frac{375}{32EI}$$

$$\theta_C = \frac{625}{64EI}; \quad \theta_A = \frac{-375}{64EI}$$

On substitution of these values, we get

$$M_{BA} = 1.2\, EI\, \theta_B = \frac{225}{16} = 14.06 \text{ kN m}$$

$$M_{BC} = 1.6 \times \frac{375}{32} + 0.8 \times \frac{625}{64} - 50 = -23.44 \text{ kN m}$$

$$M_{BE} = 0.8\, EI\, \theta_B = \frac{75}{8} = +9.375 \text{ kN m}$$

$$M_{EB} = 0.4\, EI\, \theta_B = +\frac{75}{16} = 4.69 \text{ kN m}$$

Bending moment diagram is shown in Figure 7.8 (c). Here, carefully note the sign of the bending moment.

SAQ 3

Analyse the bent shown in Figure 7.7 (a) of Example 7.5 replacing fixed support *A* by a hinged one.

SAQ 4

Analyse the frame shown in Figure 7.9. Take *EI* as constant throughout.

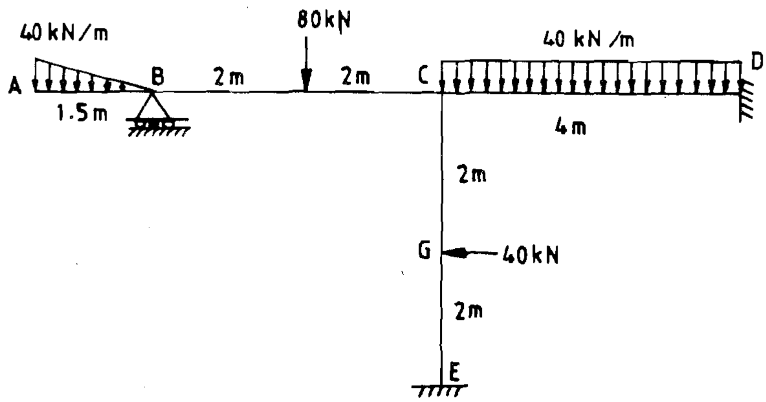


Figure 7.9

7.8 SYMMETRICAL STRUCTURES

When a two-span beam or two-bay plane frame is symmetric with respect to geometry and loading, the axis of symmetry is passing through central support and hence, the rotation at this joint is zero. In case of odd number of spans/bays, the axis of symmetry passes through the mid point of central beam where there is maximum vertical deflection and hence, rotation is zero. Therefore, while taking advantage of symmetry, only half the structure is considered with appropriate boundary condition at midspan/mid point.

Example 7.7

Analyse the single storey two bay portal frame shown in Figure 7.10 (a).

Solution

Here, the degrees of freedom is three, i.e. the rotations θ_B, θ_C and θ_D. But due to symmetry, we have $\theta_B = -\theta_D$, and $\theta_C = 0$. For analysis, the frame as shown in Figure 7.10 (b) as half of the given structure is considered and $\theta_C = 0$. Therefore, θ_B is the only unknown.

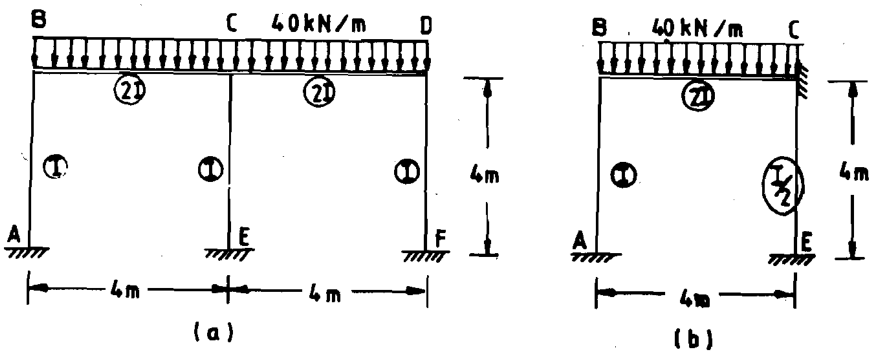


Figure 7.10

Fixed end moments

$$FM_{BC} = -18 \times \frac{4^2}{12} = -24 \text{ kN m} \quad FM_{CB} = +24 \text{ kN m}$$

Slope deflection equations

$$M_{AB} = \frac{2EI\,(2\theta_A + \theta_B)}{4}; \qquad M_{BA} = \frac{2EI\,(\theta_A + 2\,\theta_B)}{4}$$

$$M_{BC} = \left[\frac{4EI\,(2\theta_B + \theta_C)}{4}\right] - 24; \quad M_{CB} = \left[\frac{4EI\,(\theta_B + 2\theta_C)}{4}\right] + 24$$

Equilibrium condition

At joint B:

$$M_{BA} + M_{BC} = 0, \text{ with } \theta_A = \theta_C = 0$$

$$\theta_B + 2\theta_B - 24/EI = 0, \text{ giving } \theta_B = 8/EI$$

Final moments

$$M_{AB} = \frac{8}{2} = 4 \text{ kN m}; \qquad M_{BA} = 8 \text{ kN m}$$

$$M_{BC} = 2 \times 8 - 24 = -8 \text{ kN m}; \quad M_{CB} = 1 \times 8 + 24 = 32 \text{ kN m}$$

From the principles of symmetry, for the right half of the structure, we have

$M_{FD} = -4$ kN m; $M_{DF} = -8$ kN m;

$M_{DC} = 8$ kN m; $M_{CD} = -32$ kN m;

For practise, you should draw the bending moment diagram.

SAQ 5

Analyse the portal frame as shown in Figure 7.11, taking advantage of symmetry. Take EI as constant throughout.

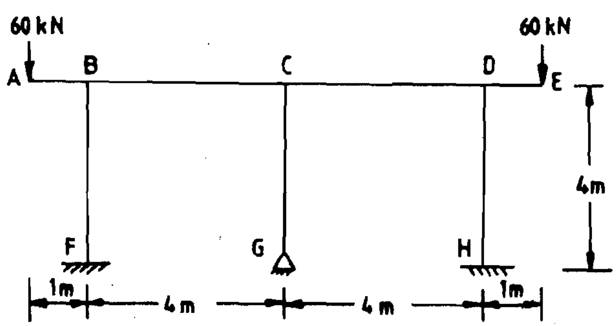


Figure 7.11

SAQ 6

Analyse the portal frame shown in Figure 7.12 (a).

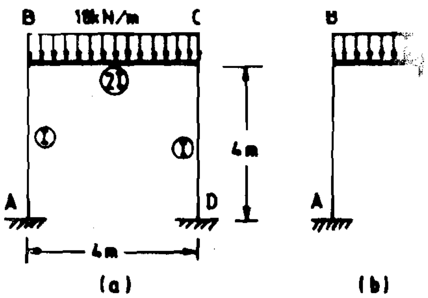


Figure 7.12

7.9 ANALYSIS OF PLANE FRAMES WITH SWAY

When the portal frame is likely to sway, i.e it will displace the vertical members in the horizontal direction, it cannot be neglected. In such a case, it is to be included for column members in slope deflection equations much in the same way as support settlements are considered for beam elements. The plane frames sway in the following situations :

- Unsymmetrical frame even though the load is symmetrical.
- Unsymmetrical loading on the plane frame (even though the frame is geometrically symmetrical).
- Different end conditions of columns and unequal column heights.
- The flexural rigidity *EI* is not symmetric about axis of symmetry.
- Horizontal loading on columns.
- Settlement of supports.
- Any combination of above.

7.9.1 Storey-shear Equation

As discussed earlier, the equilibrium equations consist of equilibrium of moments at joints in case of frame without sway. If there is sway, the additional equation is furnished by *equilibrium of horizontal shear.* This can be explained by the following illustration.

Consider the single-storey, double-bay portal frame subjected to horizontal loads as shown in Figure 7.13 (a). The free body diagrams of three columns are shown in Figure 7.13 (b).

The horizontal reactive forces at each ends of column members will be due to :

- moments at column ends, and
- horizontal external forces.

End moments are assumed clockwise to be positive. The corresponding couple, will be formed due to reactive forces, which is anticlockwise. Therefore, H_A, H_C and H_E are assumed acting towards right while H_B, H_D and H_F towards left.

Although the reactive forces are transferred at the top of the columns (i.e. at *B*, *D*, and *F*), those are rigid end, not restrained in horizontal direction. Therefore, sway will occur. Let the propping force (*P*) preventing the sway be assumed acting towards left. This can be predicted in following two ways :

- since H_B, H_D and H_F are towards left, and
- from the deflected shape at top as tension side is on left.

The additional equilibrium condition is available from summation of horizontal forces as zero ($\sum F_x = 0$) for entire frame.

Considering forces acting towards right as positive, we get

$$H_A + H_C + H_E - P + w \times L_1 = 0 \quad \text{(I)}$$

In above equation, H_A, H_C and H_E are again unknowns. Hence, they are to be related to end moments.

Let us consider equilibrium of each column to find horizontal reactions at bottom.

Their free body diagrams are shown in Figure 7.13 (b).

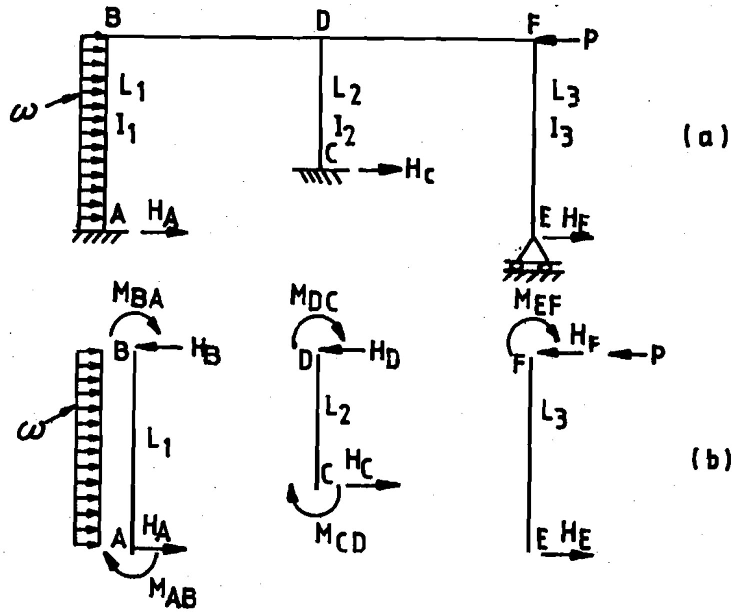


Figure 7.13 : Frames with Sway

For column AB : Taking moment about B,

$$M_{AB} + M_{BA} - H_A \times L_1 - \frac{wL_1^2}{2} = 0$$

$$H_A = \left(\frac{M_{AB} + M_{BA} - \frac{wL_1^2}{2}}{L_1} \right) \quad \text{(II)}$$

Similarly, for column CD : $H_C = \dfrac{M_{CD} + M_{DC}}{L_2}$ (III)

for column EF : $H_E = \dfrac{M_F}{L_3}$ (as $M_{EF} = 0$ due to hinge at E) (IV)

From Eqs. (I), (II), (III) and (IV), the shear equation is

$$\left(\frac{M_{AB} + M_{BA} - \frac{wL_1^2}{L_1}}{2} \right) + \frac{(M_{CD} + M_{DC})}{L_2} + \frac{M_{FE}}{L_3} - P + w\,L_1 = 0 \quad \text{(V)}$$

If there are n storeys, n such shear equations will be available. This is illustrated in Examples 7.8 and 7.9.

7.9.2 Applications

Example 7.8

Analyse the plane frame shown in Figure 7.14 (a).

Solution

Here, the degrees of freedom is three, namely rotations θ_B, θ_c and unknown (horizontal) sway δ. Horizontal displacement of roller C is also horizontal displacement of end B, as the axial deformation (of beam BC) is neglected.

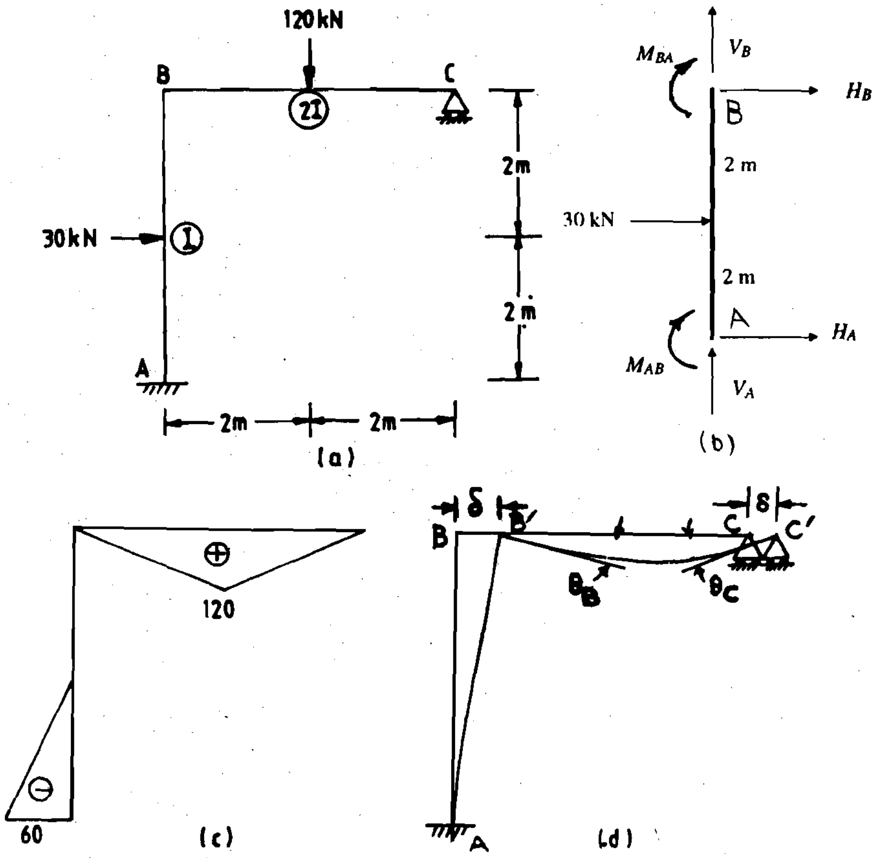


Figure 7.14

Slope deflection equations

These equations are modified for column AB only.

$$M_{AB} = \frac{2EI}{4}\left(2\theta_A + \theta_B + \frac{3\delta}{4}\right) - 15$$

$$M_{BA} = \frac{2EI}{4}\left(\theta_A + 2\,\theta_B + \frac{3\delta}{4}\right) + 15$$

(Here, $\theta_A = 0$)

$$M_{BC} = \frac{2E\,(2I)}{4}\left[2\theta_B + \theta_C\right] - 60$$

$$M_{CB} = \frac{2E\,(2I)}{4}\left[\theta_B + 2\,\theta_C\right] + 60$$

Equilibrium conditions

Joint C,

$$M_{CB} = 0 \quad \text{gives} \qquad \theta_B + 2\,\theta_C = \frac{-60}{EI} \tag{I}$$

Joint B,

$$M_{BA} + M_{BC} = 0$$

or

$$EI\,\theta_B + \left(\frac{3}{8} \times EI\,\delta\right) + 15 + 2EI\,\theta_B + EI\,\theta_C - 60 = 0$$

It gives $$24\,\theta_B + 8\,\theta_C + 3\,\delta = \frac{360}{EI} \qquad \text{(II)}$$

Shear condition for column AB is additionally required, which is illustrated in Figure 7.14 (c).

Taking moments about B

$$M_{AB} - H_A \times 4 - 30 \times 2 + M_{BA} = 0$$

$$\therefore \quad H_A = \left[\frac{M_{AB} + M_{BA} - 60}{4}\right] = \left[\frac{\frac{2EI}{4}\left(3\theta_A + 3\theta_B + 6\,\frac{\delta}{4}\right) - 60}{4}\right]$$

where $\theta_A = 0$.

As $H_C = 0$, and considering $\sum F_x = 0$ for entire bent, we get

$H_A + 30 = 0$, $\quad H_A = -30$ kN, i.e. leftward.

Putting this value in expression of H_A in terms of θ_A and θ_B

$$60 - (30 \times 4) = \frac{2EI}{4} \times \left(3\theta_B + \frac{3\delta}{2}\right)$$

Therefore, $$2\theta_B + \delta = \frac{-80}{EI} \qquad \text{(III)}$$

Using elimination technique,

Equation & Operation	$\theta_B EI$	$\theta_C EI$	δEI	Constant
(I)	1	2	0	− 60
(II)	24	8	3	+ 360
(III)	2	0	1	− 80
(IV) = (II) − 4 × (I)	20	0	3	600
(V) = (IV) − 3 × (III)	14	0	0	840

On solving Eqs. (IV) and (V), we get

$$\theta_B = \frac{+60}{EI} \text{ (clockwise)}$$

On putting this value in Eq. (I), $\qquad \theta_C = \frac{-60}{EI}$ (anticlockwise)

On putting these values in Eq. (III) $\qquad \delta = \frac{-200}{EI}$ (rightward)

On substitution of these values in slope deflection equations, we get

$M_{AB} = -60$ kN m; $\qquad M_{BA} = 0$

$M_{BC} = (2 \times 60) - 60 - 60 = 0$; $\qquad M_{CB} = (60 - 2 \times 60) + 60 = 0$

Figure 7.14 (c) gives bending moment diagram (BMD) and Figure 7.14 (d) gives the deflected shape (elastic curve) of the frame.

Example 7.9

Analyse the plane frame given in Figure 7.15 (a). When there is a clockwise rotational slip of 0.002 radians and a vertical downward displacement of 12 mm occuring at joint A. Take $E = 200$ GPa, $I = 400 \times 10^6$ mm^4

Solution

Here, $\qquad E = 200 \text{ GPa} = 200 \times 10^9 \text{ N/m}^2$,

$\qquad I = 400 \times 10^6 \text{ mm}^4 = 400 \times 10^6 \times 10^{-12} \text{ m}^4$

Let us consider EI in kN m^2

$$EI = 200 \times 10^9 \times 400 \times 10^6 \times (10^{-3})^4 \text{ Nm}^2$$
$$= 80 \times 10^6 \text{ Nm}^2 = 80 \times 10^3 \text{ kN m}^2$$

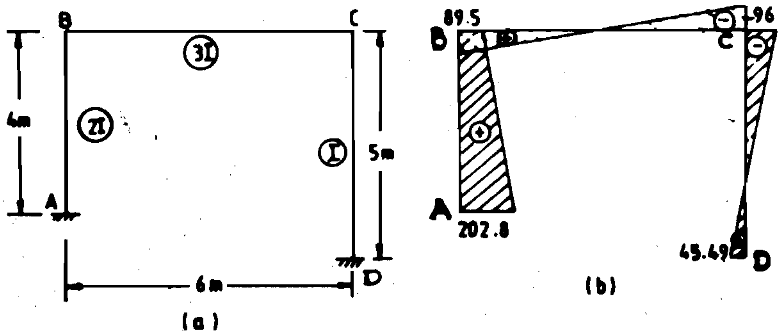


Figure 7.15

Degree of freedom

Here, the degrees of freedom are three, namely θ_B, θ_C and sway δ assumed towards right. The sway δ has negative sign since it causes anticlockwise fixing moments in columns *AB* and *DC*.

We have, $\theta_A = 0.002$ radian clockwise, $\theta_D = 0$; vertical settlement at *B* with respect to $A = 12$ mm $= 0.012$ m. The settlement, δ is positive for member *BC* as it causes clockwise fixing moments at its ends.

Slope deflection equations

$$M_{AB} = \frac{2E \times 2I\left(2\theta_A + \theta_B - \frac{3\delta}{4}\right)}{4} = EI\,(\theta_B - 0.75\,\delta + 0.004)$$

$$M_{BA} = \frac{2E \times 2I\left(\theta_A + 2\,\theta_B - \frac{3\delta}{4}\right)}{4} = EI\,(2\theta_B - 0.75\,\delta + 0.002)$$

$$M_{BC} = 2E \times 3I\left(\frac{2\theta_B + \theta_C + \left(\frac{3 \times 0.012}{6}\right)}{6}\right) = EI\,(2\theta_B + \theta_C + 0.006)$$

$$M_{CB} = \frac{2E \times 3I\,(\theta_B + 2\theta_C + 0.006)}{6}$$

$$M_{CD} = \frac{2EI\left(2\theta_C - \frac{3\delta}{5}\right)}{5} = EI\;(0.8\,\theta_C - 0.24\,\delta)$$

$$M_{DE} = \frac{2EI\left(\theta_C - \frac{3\delta}{5}\right)}{5} = EI\,(0.4\,\theta_C - 0.24\,\delta)$$

Equilibrium conditions

Joint *B*

$$M_{BA} + M_{BC} = 0$$

$$4\theta_B + \theta_C + 0.008 - 0.75\,\delta = 0 \qquad \text{(I)}$$

Joint *C*

$$M_{CB} + M_{CD} = 0$$

$$\theta_B + 2.8\,\theta_C - 0.24\,\delta + 0.006 = 0$$

Shear condition

$$H_A + H_D = 0 \text{ (no horizontal load)}$$

Now, $H_A = \dfrac{M_{AB} + M_{BA}}{4}$; $H_D = \dfrac{M_{CD} + M_{DC}}{5}$

$$\therefore \quad \left[\frac{(M_{AB} + M_{BA})}{4}\right] + \left[\frac{(M_{CD} + M_{DC})}{5}\right] = 0$$

On substitution, $5\theta_B + 1.6\,\theta_C - 3.14\,\delta + 0.001 = 0$ (III)

On solving above equaions (I), (II) and (III), we get

$\delta = -0.0002525$ m (to left)

$\theta_C = -0.001573$ radian (anticlockwise)

$\theta_B = -0.001654$ radian (anticlockwise)

Final moments

$$M_{AB} = EI\,(\theta_B - 0.75\,\delta + 0.004)$$

$$= 80 \times 10^3\,(-0.001654 + 0.0001893 + 0.004)$$

$$= 202.8 \text{ kN m}$$

Similarly,

$M_{BA} = -89.5$ kN m

$M_{BC} = 89.5$ kN m $\qquad M_{CB} = 96$ kN m

$M_{CD} = -96$ kN m $\qquad M_{DC} = 45.49$ kN m

Figure 7.15 (b) shows the bending moment diagram on tension side.

SAQ 7

Analyse the plane frame shown in Figure 7.16. Take EI as constant for all the members.

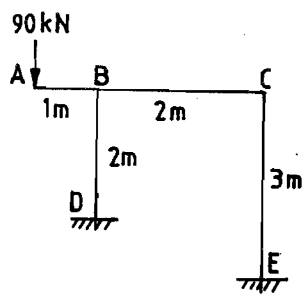


Figure 7.16

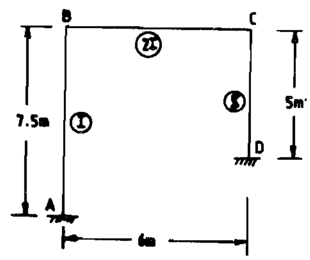


Figure 7.17

SAQ 8

Analyse the portal frame shown in Figure 7.17 above. There is rotational slip of 0.002 radian clockwise and vertical settlement of 10 mm at joint D.

Take $E = 200$ kN/mm^2 and $I = 3 \times 10^5$ mm^4.

7.10 SUMMARY

- *Slope deflection equations*

  The slope deflection equations for a member can be expressed as

$$M_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B + \frac{3\delta}{L}\right) + FM_{AB}$$

  and

$$M_{BA} = \frac{2EI}{L}\left(\theta_A + 2\theta_B + \frac{3\delta}{L}\right) + FM_{BA}$$

  where FM_{AB} and FM_{BA} are fixed end moments due to loads. If there is no lateral translation, $\delta = 0$ in the above expression. θ_A and θ_B are rotations at ends A and B respectively. Rotation is zero for fixed end. Moment is zero for a hinged or roller support.

- *Equilibrium equation*

  (a) If there are m members meeting at a joint then algebraic sum of moments at a joint is zero. Thus, equilibrium condition for joint i is

$$\sum_{i=1}^{m} M_i = 0$$

  (b) For any plane frame, the algebraic sum of horizontal reactions at supports and external horizontal loads on columns must be equal to zero. This is known as shear condition. Shear equations are required to be formed, in addition to equilibrium equations for moments at joints, in case of plane frames with sway.

- The solution of equilibrium equations will give displacements or primary unknowns. Final moments are then obtained by substituting these displacements into slope deflection equations.

- When the frame is symmetrical with respect to geometry and loading, the advantage of symmetry is taken by selecting half the structure for analysis with appropriate boundary condition.

- Causes of side sway for plane frames are as follows :

  (a) unsymmetrical loading,

  (b) geometric asymmetry of frame such as different end conditions, or different EI, or different heights of columns,

  (c) horizontal loading, and

  (d) settlement of supports, and any combination of these.

7.11 KEY WORDS

Initial Fixed End Moments : Moments at the ends of restrained member due to loads or settlement of support.

Sinking of Support or Settlement of Support : Lateral displacement of one end of the member with respect to the other. This causes moments at both ends of a member.

Side Sway : Lateral displacement of end of the column, which induces moments at both ends depending upon boundary condition.

Slope Deflection Equations : Equations relating final moments at end of member with initial moments due to load and displacements, i.e. rotations at two ends of members and translation of the supports.

Equilibrium Equations : Algebraic sum of moments at a joint equals to zero should be satisfied which is a statical condition of equilibrium.

Shear Condition : This is an additional condition to compute unknowns in case of sway which states that summation of horizontal forces, i.e. horizontal reactions at supports and lateral loads on any particular column level is zero.

7.12 ANSWERS TO SAQs

SAQ 1

Unknowns : θ_B and θ_C

Equations : $7\theta_B + 2\theta_C = \dfrac{90}{EI}$ and $2\theta_B + 7\theta_C = 0$

Displacements : $\theta_B = \dfrac{14}{EI}$ (clockwise) and $\theta_C = -\dfrac{4}{EI}$ (anticlockwise)

End moments : $M_{AB} = 7$ kN m $\quad M_{BA} = 14$ kN m $= - M_{BC}$

$M_{DC} = - 28$ kN m $\quad M_{CB} = 34$ kN m $= - M_{CD}$

SAQ 2

Unknowns : θ_B in both cases is $\dfrac{20}{EI}$ (clockwise) [it has become incidentally].

End moments (all hogging and in kN m) are as given below :

	M_A	M_B	M_C
Case (a)	30	60	50
Case (b)	42	48	66

SAQ 3

Unknowns θ_A, θ_B and θ_C

Repeat the procedure similar to Example 7.6.

$\theta_A = 0, \theta_B = \dfrac{30}{EI}$ and $\theta_C = -\dfrac{45}{EI}$ can be obtained

Incidentally, the bending moment diagram will be same as Figure 7.7 (b).

SAQ 4

$$\theta_C = \frac{10.303}{EI} \text{ and}$$

$$\theta_B = \frac{4.849}{EI}$$

End moments are shown in bending moment diagram in Figure 7.18.

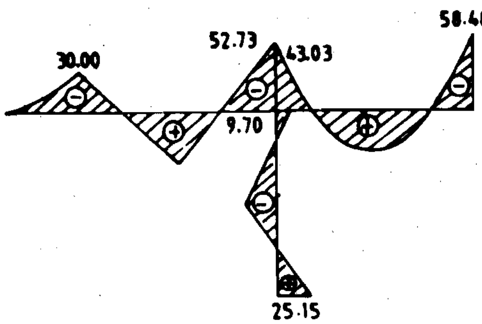


Figure 7.18

SAQ 5

Unknowns : $\theta_B = \theta_D = -\dfrac{30}{EI}$

$M_{BF} = M_{BC} = - M_{DH} = - M_{DC} = - 30$ kN m $= 2M_{FB} = 2M_{HD}$

SAQ 6

Here, the degrees of freedom is two, i.e. θ_B and θ_C but $\theta_B = \theta_C$.

The modified half structure is shown in Figure 7.12 (b). The unknown displacements will be θ_B and deflection at centre line. Hence, in this case there is no advantage in considering symmetry. So, let us analyse the whole structure and take on symmetry as $\theta_B = -\theta_C$.

Fixed end moments

$$FM_{CB} = 24 \text{ kN m} = -FM_{BC} \text{ (as found in Example 7.1)}$$

Slope deflection equations

As $\theta_A = \theta_D = 0$

We get $M_{AB} = \dfrac{2EI(\theta_B)}{4} = \dfrac{EI\,\theta_B}{2}$

$$M_{BA} = \frac{2EI(\theta_B)}{4} = EI\,\theta_B$$

Similarly, $M_{BC} = EI\,\theta_B - 24$

Equilibrium conditions

At joint B, $M_{BA} + M_{BC} = 0$

$$EI\,\theta_B + (EI\,\theta_B - 24) = 0$$

Thus, $EI\,\theta_B = 12$

Final moments

$M_{AB} = 6$ kN m $\quad M_{BA} = 12$ kN m

$M_{BC} = -12$ kN m $\quad M_{CB} = 12$ kN m

$M_{CD} = -12$ kN m $\quad M_{DC} = -6$ kN m

SAQ 7

Unknowns : θ_B, θ_C and δ (horizontal displacements of B and C)

Displacements : $EI\,\theta_B = -32.54$ kN m^2

$EI\,\theta_C = 5.092$ kN m^2

$EI\delta = -23.35$ kN m^3

where EI is in kN m^2 and θ_B & θ_C are in radians.

End moments (in kN m) (clockwise as positive)

$M_{AB} = 0 \quad M_{BA} = 90$

$M_{BC} = -60 \quad M_{CB} = -22.36$

$M_{BD} = -30 \quad M_{DB} = 2.49$

$M_{CE} = 22.36 \quad M_{EC} = 18.96.$

SAQ 8

Moments in kN m are as follows :

$M_{AB} = -275.091 \quad M_{BA} = 42.652 = -M_{BC}$

$M_{DC} = 87.28 \quad M_{CD} = -68.44 = -M_{CD}$