STRUCTURAL ANALYSIS - II

SYLLABUS

UNIT-I: MOMENT DISTRIBUTION METHOD

Analysis of Single Bay Single Storey Portal Frame including side Sway. Analysis of inclined Frames. Kani's Method: Analysis of Continuous Beams including settlement of supports. Analysis of single bay single storey and single bay two Storey Frames by Kani's Method including side Sway. Shear Force Bending Moment Diagrams. Elastic curves.

UNIT-II: SLOPE DEFLECTION METHOD

Analysis of Single Bay Single Storey Portal Frames by Slope Deflection Method including side Sway – Shear Force Bending Moment Diagrams. Elastic curve. Two Hinged Arches: Introduction-Classification of Two hinged Arches-Analysis of Two Hinged Parabolic Arches due to temperature and elastic shortening of rib.

UNIT-III: APPROXIMATE METHODS OF ANALYSIS

Introduction – Analysis of multi-storey frames for lateral loads: Portal Method , Cantilever method and Factor method. Analysis of multi-storey frames for gravity (vertical) loads. Substitute Frame method Analysis of Mill bents.

UNIT-IV: MATRIX METHOD OF ANALYSIS

Introduction –Static and Kinematic Indeterminacy-Analysis of Continuous beams including settlement of supports, using Stiffness Method. Analysis of Pin –jointed determinate plane frames using Stiffness method-Analysis of Single Bay Single Storey Portal Frame including side Sway using Stiffness Method. Analysis of Continuous Beams upto three degree of indeterminacy using Flexibility Method. Shear Force Bending Moment Diagrams. Elastic curves.

UNIT-V: INFLUENCE LINES FOR INDETERMINATE BEAMS

Introduction –ILD for two span continuous beam with constant and variable moments of inertia. ILD for Propped Cantilever beams. Indeterminate Trusses: Determination of Static and Kinematic Indeterminacies - Analysis of trusses having single and two degrees of internal and external indeterminacies-Castigliano's second Theorem.

UNIT - 1

MOMENT DISTRIBUTION METHOD

MDM - Analysis of Single Bay Single Storey Portal Frames including side Sway. Analysis of inclined frames.

MDM - The method was first introduced by "Prof. Hardy Cross" in 1930.

The method could be used for the analysis of all types of statically indeterminate beams or rigid frames.

Stiffness : It is denoted by 'k'. The moment 'k' is also known as absolute stiffness or simply stiffness.

The stiffness of a member is a moment required to rotate the end under consideration through unit angle.

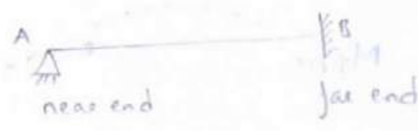
It is the amount of force required to produce unit deflection or displacement is called stiffness.

Proposition - 1 : The moment 'k' required to rotate the near end of a prismatic beam through a unit angle, without translation, the far end being freely supported is :

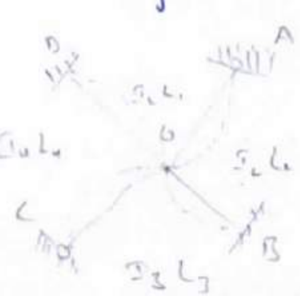
$$k = \frac{3EI}{L} \quad \frac{3}{4}\frac{I}{L}$$

Proposition - 2 : The moment 'k' required to rotate the near end of a prismatic beam through unit angle, without translation, the far end is fixed :

$$k = \frac{4EI}{L} = \frac{I}{L}$$



Proposition - 3 : A moment which it rotates the near end of beam without translation, the farend being fixed, induces at the far end a moment of one half its magnitude & in the same direction.



$$\therefore \; M_1 = \frac{3E_1I_1}{L_1}\theta = k_1\theta$$

$$M_2 = \frac{4E_2I_2}{L_2}\theta = k_2\theta$$

$$M_3 = \frac{3E_3I_3}{L_3}\theta = k_3\theta$$

$$M_4 = \frac{4E_4I_4}{\ldots} = k_4\theta$$

$$M = M_1 + M_2 + M_3 + M_4 \quad -(i)$$

$$M_1 : M_2 : M_3 : M_4 :: k_1 : k_2 : k_3 : k_4 \quad -(ii)$$

$$\therefore M_1 = \frac{k_1}{k_1 + k_2 + k_3 + k_4} \cdot M = \frac{k_1}{\Sigma k} M$$

$$M_2 = \frac{k_2}{k_1 + k_2 + k_3 + k_4} \cdot M = \frac{k_2}{\Sigma k} \cdot M$$

$$'' \quad = \frac{k_3}{\Sigma k} \cdot M$$

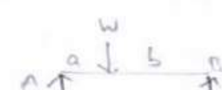
$$'' \quad = \frac{k_4}{\Sigma k} \cdot M$$

Preposition-4 : A moment which tends to rotate a joint without translation, will be divided amongst the connecting members at the joint in proportion to their "stiffness".

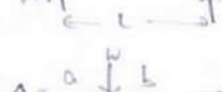
The quantities $\frac{k_1}{\Sigma k}$, $\frac{k_2}{\Sigma k}$, $\frac{k_3}{\Sigma k}$, $\frac{k_4}{\Sigma k}$ are called distribution factors. The moments M_1, M_2, M_3, M_4 are called as distribution moments.

Fixed End Moments :

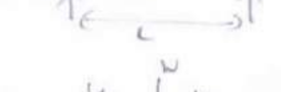
$$M_{FAB} = -\frac{Wab^2}{l^2}$$



$$M_{FBA} = \frac{Wa^2b}{l^2}$$



$$M_{FAB} = -\frac{WL}{8}$$

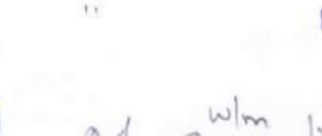


$$M_{FBA} = \frac{WL}{8}$$

$$M_{FAB} = -\frac{wl^2}{12}$$

$$M_{FBA} = \frac{wl^2}{12}$$

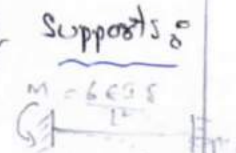
$$M_{FAB} = \frac{M \times b(3a - l)}{l^2} \ (\rightarrow)$$



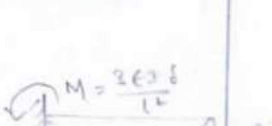
$$M_{FBA} = \frac{Ma(3b - l)}{l^2} \ (\leftarrow)$$

Sinking of Supports :

$$M = \frac{6EI\delta}{l^2}$$



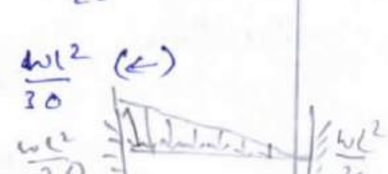
$$M = \frac{3EI\delta}{l^2}$$



Sinking of propped cantilever.

$$M_{FAB} = -\frac{wl^2}{20} (\rightarrow)$$

$$M_{FBA} = \frac{wl^2}{30} (\leftarrow)$$



Portal frames with sideway :

Sway, one of the degrees of freedom.

In portal frames, the amount of 'sawy' or joint movement is not known & the analysis is done by assuming some arbitary fixed moments.

These assumed fixed moments due to side sway are then distributed & the reactions are found.

The algebraic sum of the horizontal reactions due to the assumed sway moments must be equal to the sawy force.

Causes of Side sway :

1) Eccentric or unsymmetrical loading on the portal frame.
2) Unsymmetrical out-line of portal frame.
3) Different end conditions of the columns of the portal frame.
4) Non-uniform section of the members of the frame
5) Horizontal loading on the columns of the frame.
6) Settlement of the supports of the frame
7) Combination of all above.

Method of Analysis :

Analysis of portal frames with side sway is done as following

Step 1:- a) Hold the joints against side sway by applying a force 'P'. calculate the fixed end moments due to external loads & distribute the moments.

b) Calculate the horizontal & vertical reactions. The algebraic sum of the two horizontal reactions at the column bases will give the value of the restraining or holding force 'P'. The sway force S will be in the opposite direction & of the magnitude of 'P'.

Step-2 : a) Remove the holding force P & permit the joints to sway. This will cause a set of fixed end moments. To start with, assume suitable sway moments at the four joints, A,B,C & D of the frame, in proportion.

b) Calculate the horizontal & vertical reactions due to the assumed sway moments. The algebraic sum of the horizontal reactions of two column bases must be equal to the sway force S. If not, reduce the sway moments assumed proportionately. The sway moments must be of such magnitude that the algebraic sum of the horizontal reactions due to sway is equal to the sway force S.

Let H_1 & H_2 be the horizontal reactions.

Let $c(H_1 + H_2) = S$

Then, actual sway moments = c × Assumed Sway moments

Thus the actual sway moments are known.

Step-3 : a) The final moments at each joint will be equal to the algebraic sum of the moments due to initial moments.
[a) (as obtained in step 1(a) & the moments due to actual sway (as obtained in step (2))]

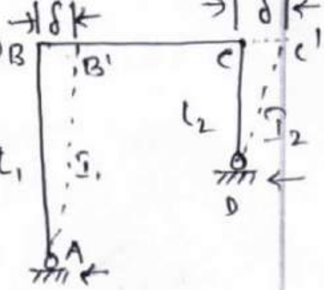
b) The final reactions will be equal to the algebraic sum of those found in 1(b) & 2(b).

Ratio of Sway Moments at Column Heads :

When the joints sway, a set of moments are introduced at the two column heads (and bases) of a portal frame. The ratio of the sway moments at the two column heads [i.e $M_{BA} : M_{CD}$] will depend upon the end conditions. Let us now take different end conditions to derive the standard expressions for the ratio of the sway moments.

Case - I : Both ends hinged :

Consider a portal frame with dimensions. Let a force P cause the frame to sway, so that the joint B moves to B' through a horizontal distance δ. Considering no change in the length of BC joint C will move to C' through distance δ.



(a)

The fixed End moment due to movement or settlement of the support when beam hinged @ one end & fixed at other end is :-

$$M_{BA} = \frac{3EI_1\delta}{L_1^2} \quad -① \qquad \text{Dividing ① \& ②}$$

$$\text{||ly} \quad M_{CD} = \frac{3EI_2\delta}{L_2^2} \quad -② \qquad \boxed{\frac{M_{BA}}{M_{CD}} = \frac{I_1/L_1^2}{I_2/L_2^2}}$$

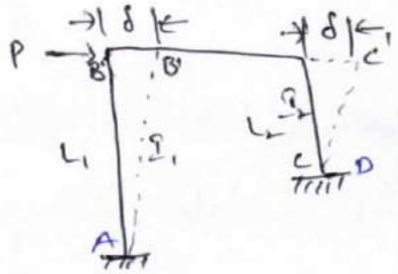
$\therefore$ both the columns rotate in same direction, the moments M_{BA} & M_{CD} will be either both positive or negative.

for fig @ the moments are -ve since they rotate, clock-wise direction.

Case - II : Both ends fixed :

movement $BB' = CC' = \delta$

Fixed at ends A & B.



$$M_{BA} = M_{AB} = \frac{6EI_1\delta}{L_1^2}$$

$$M_{CD} = M_{DC} = \frac{6EI_2\delta}{L_2^2}$$

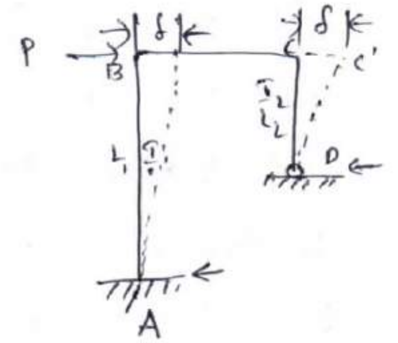
$$\therefore \boxed{\frac{M_{BA}}{M_{CD}} = \frac{I_1/L_1^2}{I_2/L_2^2}}$$

clockwise rotation, the sway moments will be -ve.

anticlockwise " , the " " " " +ve.

Case. III - One end fixed and other end hinged :

$BB' = CC' = \delta$



$$M_{BA} = M_{AB} = \frac{6EI_1\delta}{L_1^2} \quad -(1)$$

$$M_{CD} = \frac{3EI\delta}{L_2^2} \quad -(2)$$

$$\therefore \frac{M_{BA}}{M_{CD}} = \frac{\cancel{6}^{2}EI_1\delta}{L_1^2} \Big/ \frac{\cancel{3}EI\delta}{L_2^2} = \frac{2I_1/L_1^2}{I_2/L_2^2}$$

$$\therefore \boxed{\frac{M_{BA}}{M_{CD}} = \frac{2I_1/L_1^2}{I_2/L_2^2}}$$

$$\frac{M_{BA}}{M_{CD}} = \frac{I_1/L_1^2}{I_2/L_2^2} \; ; \quad \frac{M_{BA}}{M_{CD}} = \frac{I_1/L_1^2}{I_2\, L_2^2} \; ; \quad \frac{M_{BA}}{M_{CD}} = \frac{2I_1/L_1^2}{I_2/L_2^2}$$

are the ratio of the moments induced at the column heads due to side sway, for various end conditions.

Note : Relative stiffness (k) :

The absolute stiffness (k) of a member when far end is fixed - $\frac{4EI}{L}$ [when several members meeting at a joint]

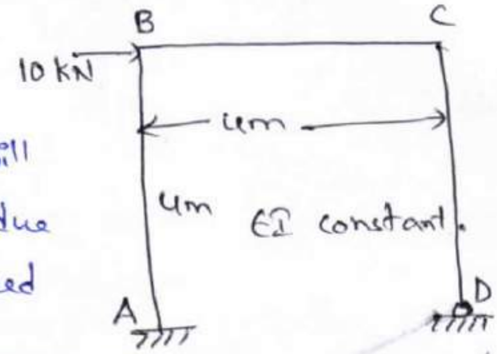
The member meeting at a joint are of same material & are fixed at the far end, the stiffness of each member is $\frac{I}{L}$.

The member meeting at a joint are freely supported at the other end, the stiffness is $\frac{3}{4} \cdot \frac{I}{L}$.

Problem :

1. Analyse the portal frames shown below. The end A is fixed and D is hinged. The joints B & C are rigid. Draw the B.M.D & sketch the deflected shape of the frame.

Sol:- Distribution factors :



∴ The load acting at the joint, there will be no fixed end moments. Therefore due to side sway, moments will be induced at joint A, B & C.

Joint	Member	Relative Stiffness K	Sum ΣK	D. F (K/ΣK)
B	BA	$I/L = I/4$	$\frac{2I}{4}$	0.5
	BC	$I/L = I/4$		0.5
C	CB	$I/L = I/4$	$\frac{7I}{16}$	0.57
	CD	$\frac{3}{4}\times\frac{I}{L} = \frac{3}{4}\times\frac{I}{4} = \frac{3I}{16}$		0.43

$\frac{1}{4}+\frac{3}{16}$
$\frac{4+3}{16}$
$=\frac{7}{16}$

Side Sway :

The side sway will be clockwise in direction; at the columns AB & CD. Thus −ve moment will be induced at A, B & C. [One end fixed & other hinged]

$$\therefore \frac{M_{BA}}{M_{CD}} = \frac{2\cdot I_1/L_1^2}{I_2/L_2^2} = \frac{\frac{2I}{4^2}}{\frac{I}{4^2}} = \frac{2}{1}$$

$$\therefore \frac{M_{BA}}{\ } = \frac{2}{1}$$

$M_{BA} = M_{AB}$

Let assume arbitrary moments :

$M_{CD} = -5 \text{ kN·m}$

$M_{AB} = M_{BA} = -10 \text{ kN·m}$

A	B	B	C	C	D	
	0.5	0.5	0.57	0.43		
-10.0	-10.0	—	—	-5.0	0	F.E.M
—	+5	+5	+2.86	+2.14	—	Bal
+2.50	—	+1.43	+2.50	—	—	C.O
—	-0.715	-0.715	-1.425	-1.075	—	Bal
-0.357	—	-0.712	-0.357	—	—	C.O
—	0.356	0.356	0.203	0.153	—	Bal
0.178	—	0.1015	0.178	—	—	C.O
—	-0.050	-0.050	-0.1014	-0.076	—	Bal
-0.025	—	-0.0507	-0.025	—	~~-0.038~~	C.O
—	0.025	0.025	0.0142	0.0107	—	Bal
~~-0.0125~~			—	—		
-7.70 kN·m	-5.384	5.384	3.845	-3.847	0	F.M

Final Moments : $M_{AB} = -7.70 \text{ kN·m}$

$M_{BA} = -5.384 \text{ kN·m}$

$M_{BC} = 5.384 \text{ kN·m}$

$M_{CB} = 3.845 \text{ kN·m}$

$M_{CD} = -3.847 \text{ kN·m}$

Reactions :

Horizontal Reaction at $A = \frac{M_{AB} + M_{BA}}{L}$

$= \frac{-(7.70 + 5.38)}{4}$

$= \frac{+13.08}{4} = 3.27$ kN($\leftarrow$)

$= -3.27 (\leftarrow)$

Reaction A = 3.27 kN ($\leftarrow$)

Horizontal Reaction at D $= \frac{M_{DC} + M_{CD}}{4}$

$= \frac{-(3.85 + 0)}{4} = -0.963$

$= -0.963 (\leftarrow)$

Reaction D = 0.963 kN ($\leftarrow$)

∴ The sway force causing the assumed moments

$= 3.27 + 0.963$

$= 4.233$ kN ($\rightarrow$)

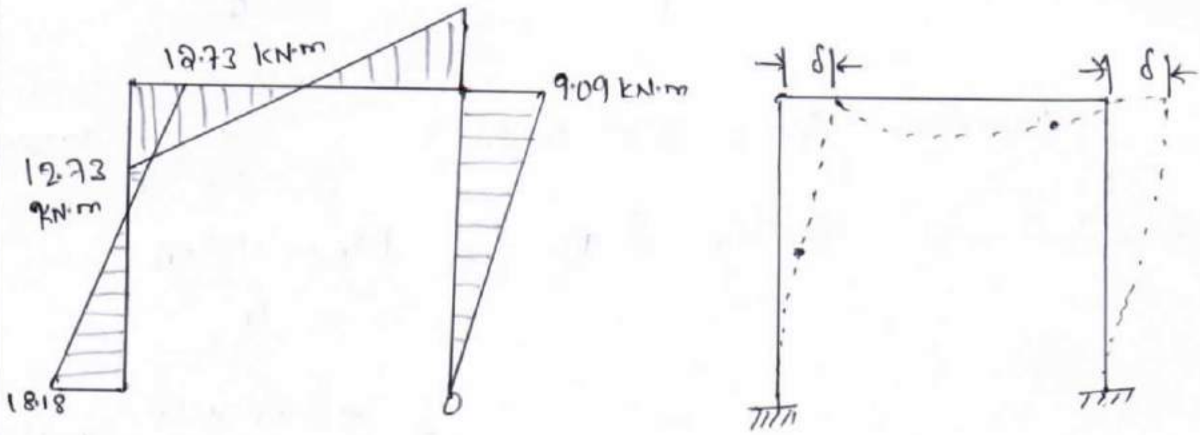
∵ The actual sway force = 10 kN

∴ The increased moments proportionately in the ratio of $\frac{10}{4.233} = (2.36)$

	A	B		C		D
Sway = 4.233 kN	-7.70	-5.38	+5.38	+3.85	-3.85	0
Sway = 10 kN	18.172	-12.69	12.69	9.062	-9.062	0

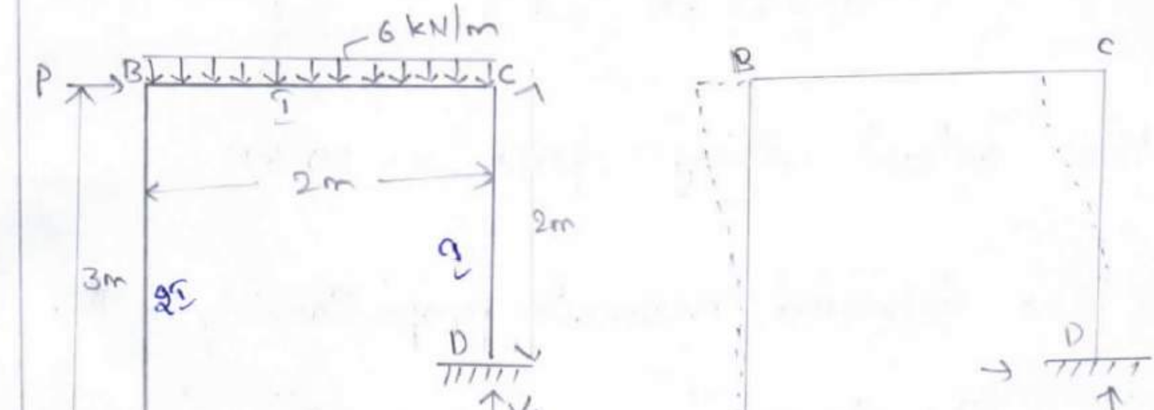
The horizontal reaction @ A $= \dfrac{3.27}{4.233} \times 10 = 7.72$ kN $(\leftarrow)$

The horizontal reaction @ D $= \dfrac{0.963}{4.233} \times 10 = 2.28$ kN $(\leftarrow)$



ⓐ B.M.D ⓑ Deflected shape

Prb: Draw the B.M.D & sketch the deflected shape of the frame shown in fig : The ends A & B are fixed & BC is loaded with U.D.L of 6 kN/m.



Sol:- Fixed End Moments :

$$M_{FBC} = M_{FCB} = \mp \frac{w \times l^2}{12} = \mp \frac{6 \times 2^2}{12}$$

$$= \pm 2.0 \text{ kN.m}$$

$M_{FBC} = -2.0$ kN.m ; $M_{FCB} = +2.0$ kN.m

b) Distribution Factors :

Joint	Member	Relative (k) Stiffness	Sum ($k/\Sigma k$)	D.F ($k/\Sigma k$)
B	BA	$\frac{2I}{3} = \frac{2I}{3}$	$\frac{4I+3I}{6} = \frac{7I}{6}$	0.57
	BC	$\frac{I}{L} = \frac{I}{2} =$		0.43
C	CB	$\frac{I}{L} = \frac{I}{2}$	$\frac{2I}{2} = I$	0.50
	CD	$I/L = I/2$		0.50

c) Moment Distribution Table :

A	B		C		D	
	0.57	0.43	0.5	0.5		
0	0	−2.0	+2.0	0	0	F.E.M
—	+ 1.14	+0.86	−1.0	−1.0	—	Bal
+0.57	—	−0.50	+0.43	—	−0.50	C.O
—	+0.29	+0.21	−0.21	0.22	—	Bal
+0.15	—	−0.11	+0.10	—	−0.11	C.O
—	+0.06	+0.05	−0.05	−0.05	—	Bal
+0.03	—	−0.03	+0.03	—	−0.03	C.O
+0.01	+0.02	+0.01	−0.02	−0.01	—	B & C.O
+0.76	+1.51	−1.51	+1.28	−1.28	−0.64	Final moment.

d) <u>Reactions</u> :

$$= \frac{M_{AB} + M_{BA}}{L}$$

Horizontal Reaction @ A = $h_a = \frac{0.76 + 1.51}{3}$

$h_a = 2.27/3 = 0.76$ kN ($\rightarrow$)

& horizontal reaction @ D, $h_d = -\left(\frac{1.28+0.64}{2}\right) = \frac{1.92}{2}$

$= -0.96$

$h_d = 0.96$ kN ($\leftarrow$)

The horizontal force $P = 0.96 - 0.76 = 0.20$ kN ($\rightarrow$)

$P = 0.20$ kN ($\rightarrow$)

The side sway value of P

The value of P preventing side sway $= 0.20$ kN $\rightarrow$

e)

Side Sway: Now let a side sway $S = 0.20$ kN $\leftarrow$ be applied at 'C'. ∴ The columns AB & DC rotates in anticlockwise direction & thus clockwise moments will be induced at 'column heads'.

$$\frac{M_{BA}}{M_{CD}} = \frac{I_1/L_1^2}{I_2/L_2^2} = \frac{2I/9}{I/4} = \frac{8}{9}$$

Let the arbitary moments be:

$M_{BA} = M_{AB} = +8$ kN·m

$M_{CD} = M_{DC} = +9$ kN·m

g) Reactions:

Horizontal Reaction at A $= \frac{6.26 + 4.53}{3} = 3.60$ kN($\rightarrow$)

Horizontal Reaction at D $= \frac{5.15 + 7.08}{2} = 6.12$ kN($\rightarrow$)

f) Distribution of Correcting Moments :

A	B	B	C	C	D	
	0.57	0.43	0.5	0.5		
+8.0	+8.0	–	–	+9.0	+9.0	FEM
–	-4.57	-3.43	-4.50	-4.50	–	Bal
-2.29	–	-2.25	-1.72	–	-2.25	CO
–	+1.29	+0.96	+0.86	+0.86	–	Bal
+0.64	–	+0.43	+0.48	–	+0.43	CO
–	-0.25	-0.18	-0.24	-0.24	–	Bal
-0.13	–	-0.12	-0.09	–	-0.12	CO
–	+0.07	+0.05	+0.05	+0.04	–	Bal
+0.04	–	+0.02	+0.03	–	+0.02	CO
–	-0.01	-0.01	-0.02	-0.01	–	Bal
+6.26	+4.53	-4.53	-5.15	+5.15	+7.08	Final Moments

Actual sway moments : 0.20 kN (←)

The sway force which induces the assumed moments

= 3.6 + 6.12 = 9.7k (←)

The corrected sway moment : Table.

		A	B	B	C	C	D	
S.M	1. Sway = 9.72 kN	+6.26	+4.53	-4.53	-5.15	+5.15	+7.08	(f)
	2. Sway = 0.20 kN p=	+0.12	+0.09	-0.09	-0.10	+0.10	+0.15	
F.S.M	3. Non-Sway	+0.76	+1.51	-1.51	+1.28	-1.28	+0.64	(c)
	4. Final Moment	+0.88	+1.60	-1.60	+1.18	-1.18	-0.49	

The final Reactions are as:

Horizontal Reaction at A $= \frac{0.88 + 1.60}{3} = 0.83$ kN ($\rightarrow$)

Horizontal Reaction at D $= \frac{-1.18 - 0.49}{2} = 0.83$ kN ($\leftarrow$)

Vertical Reaction @ A $= \left.\begin{matrix} \\ \cancel{\Sigma M_D = 0} \end{matrix}\right\} \Sigma M_D = 0$

UDL × Distance   F.M@A   F.M@D

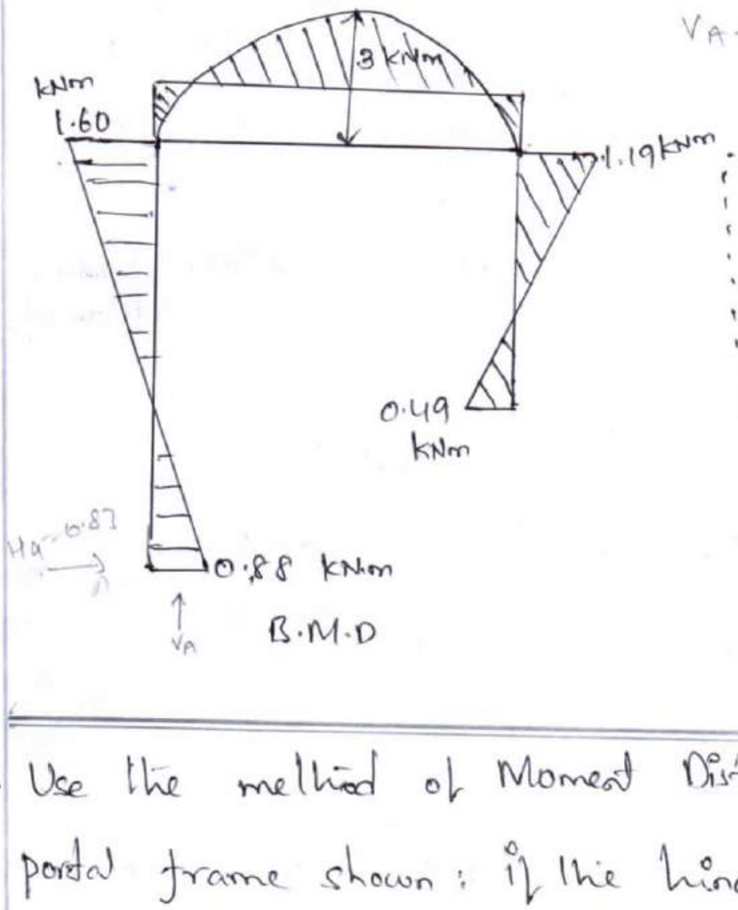
$H_A \times$

$2V_A - (0.83 \times 1) - (12 \times 1) + 0.88 - 0.49 = 0$

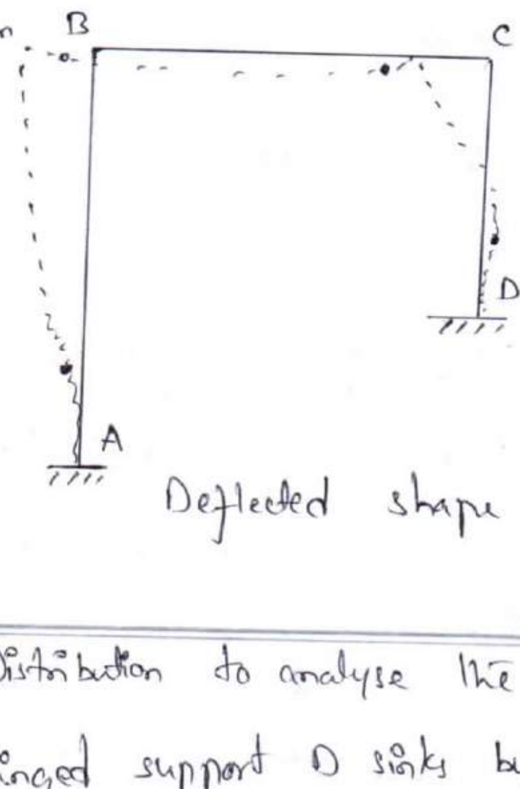
(or) $V_A = 6.22$ kN & $V_D = 12 - 6.22$

$V_D = 5.78$ kN. $\quad (6 \times 2) +$

$V_A + V_D = 12$ kN



B.M.D



Deflected shape

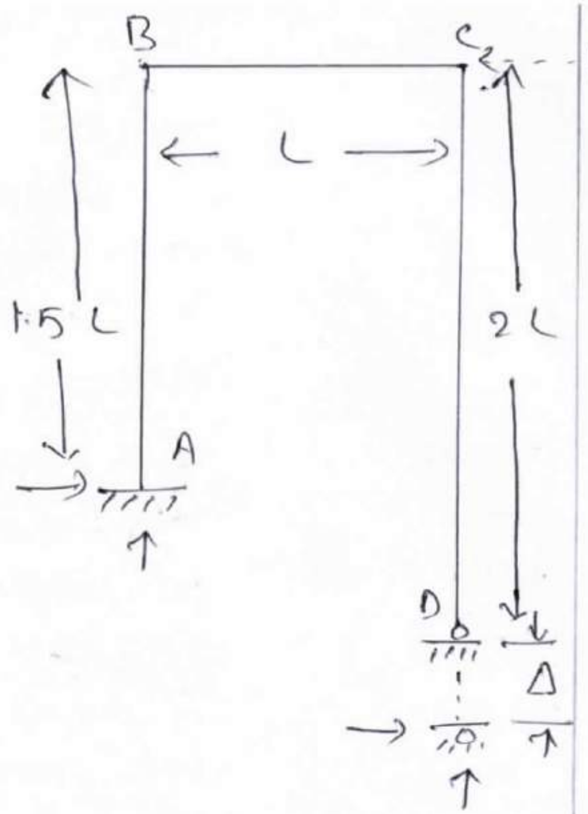
Prb: Use the method of Moment Distribution to analyse the portal frame shown: if the hinged support D sinks by an amount Δ. The members have the same uniform Cross-section.

Sol:- Let P be applied at C to prevent this sidesway.

When end D sinks, the end C will also settle.

Due to settlement, there will be side sway in the right side.

The settlement of end C will induce moments in BC in anticlockwise direction.



$$M_{FBC} = M_{FCB} = \frac{-6EI\Delta}{L^2}$$

$$= -6C \text{ [say]}$$

$$C = \frac{EI\Delta}{L^2}$$

b) Distribution factors :

Joint	Member	Relative Stiffness K	Sum ΣK	D.F $K/\Sigma K$
B	BA	$\frac{2I}{3L} = \frac{2I \cdot \cancel{1}}{3.0L}$	$\frac{5I}{3L}$	0.4
	BC	$I/L = 3I/3L$		0.6
C	CB	$I/L = \frac{8I}{8L}$	$\frac{11I}{8L}$	0.73
	CD	$\frac{3}{4} \times \frac{I}{L} = \frac{3I}{4 \times 2L} = \frac{3I}{8L}$		0.27

$\boxed{C\text{-}b}$ Horizontal Reaction @ A $= \frac{0.85c + 1.71c}{1.5L} = \frac{1.713c}{L}$

$$= \frac{1.713\, EI\Delta}{L^3} \ (\rightarrow)$$

Horizontal reaction @ D $= \frac{1.28c}{2L} = \frac{0.64\, EI\Delta}{L^3} \ (\rightarrow)$

$$P = (1.173 + 0.64)\frac{EI\Delta}{L^3}$$

$$= \frac{2.353\, EI\Delta}{L^3} \ (\leftarrow)$$

c) Moment Distribution Table : [with no side Sway]

A	B	B	C	C	D	
	0.4	0.6	0.73	0.27		
–	–	6.0 C	–6.0C	–	–	FEM
–	+2.4 C	+3.6 C	+4.36 C	+1.64 C	–	Bal
+1.2 C	–	+2.18C	+1.8 C	–	–	C.O
–	–0.87 C	–1.31 C	–1.31C	–0.49C	–	Bal
–0.44C	–	–0.66C	+0.66C	–	–	C.O
–	+0.26C	+0.40 C	+0.48C	+0.18C	–	Bal
+0.13C	–	+0.24 C	+0.20C	–	–	C.O
–	–0.10C	–0.14 C	–0.14C	–0.06C	–	Bal
–0.05C	–	+0.07C	–0.07C	–	–	C.O
–	+0.03C	+0.04C	+0.05C	+0.02C	–	Bal
+0.02 C	–	+0.03 C	+0.02C	–	–	C.O
–0.01 C ←	–0.01 C	–0.02 C	–0.01C	–0.01 C	–	B & C.O
+0.85 C	+1.71C	+1.71 C	–1.28C	+1.28C	0	–

c) Distribution of correcting moments :

A	B	B	C	C	D	
	0.4	0.6	0.73	0.27		
–8.0	–8.0	–	–	–2.25	–	FEM
–	+3.20	+4.80	+1.64	+0.61	–	Bal
+1.60	–	+0.82	+2.40	–	–	C.O
–	–0.33	–0.49	–1.75	–0.65	–	Bal
–0.16	–	–0.87	–0.24	–	–	C.O
–	+0.35	+0.52	+0.17	+0.07	–	Bal
+0.18	–	+0.09	+0.26	–	–	C.O
–	–0.04	–0.05	–0.19	–0.07	–	Bal
–0.02	–	–0.09	–0.03	–	–	C.O
+0.02 (←)	+0.04	+0.05	+0.02	+0.01	–	B & C.O
–6.38	–4.78	+4.78	+2.28	–2.28	–	–

d) Side Sway :

Apply a force $S = \frac{2.353\, EI\Delta}{l^3}$ at B in opposite direction to that of P. Side sway will ~~produce~~ induce anticlockwise moment A, B & C,

$$\frac{M_{BA} \text{ or } M_{AB}}{M_{CD}} = \frac{2I_1/L_1^2}{I_2/L_2^2} = \frac{8/9}{1/4} = \frac{32}{9}$$

$\therefore$ arbitary sway moments assumed are :

$$M_{BA} = M_{AB} = -8.0 \text{ kN.m}$$

$$M_{CD} = -2.25 \text{ kN.m}$$

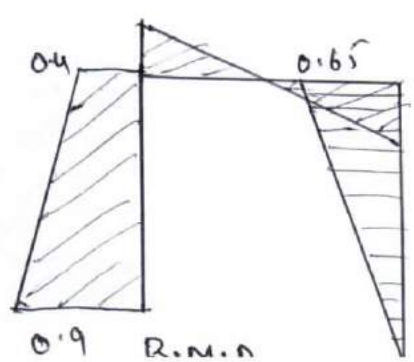
f) The horizontal reaction @ A $= -\frac{(6.38 + 4.78)}{1.5L} = \frac{7.44}{L}$ $(\leftarrow)$

The horizontal reaction @ D $= \frac{2.28}{2L} = \frac{1.14}{L}$ $(\leftarrow)$

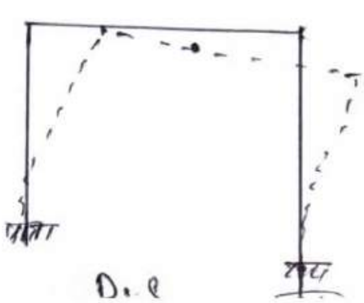
The sway force $= \frac{7.44}{L} + \frac{1.14}{L} = \frac{8.58}{L}$ $(\rightarrow)$

But the actual sway force is $\frac{2.353\, EI\Delta}{L^3}$ & hence the sway moments will have to be corrected according & added to the non-sway moments.

	A		B		C
1. Sway $= \frac{8.58}{L}$	-6.38	-4.78	$+4.78$	$+2.28$	-2.28
2. Sway $= \frac{2.535\, EI\Delta}{l^3}$	$-\frac{1.75\, EI\Delta}{l^3}$	$-\frac{1.31\, EI\Delta}{l^2}$	$+\frac{1.31\, EI\Delta}{l^2}$	$+\frac{0.63\, EI\Delta}{l^2}$	$-\frac{0.63\, EI\Delta}{l^2}$
3. Non sway moment	$+\frac{0.85\, EI\Delta}{l^2}$	$+\frac{1.71\, EI\Delta}{l^2}$	$-\frac{1.71\, EI\Delta}{l^2}$	$-\frac{1.28\, EI\Delta}{l^2}$	$+\frac{1.28\, EI\Delta}{l^2}$
4. Final moment	$-\frac{0.9\, EI\Delta}{l^2}$	$+\frac{0.4\, EI\Delta}{l^2}$	$-\frac{0.4\, EI\Delta}{l^2}$	$-\frac{0.65\, EI\Delta}{l^2}$	$\frac{0.65\, EI\Delta}{l^2}$



B.M.D


D.C

Portal frame with inclined leg:

Ans: Inclined members are used, though less frequently, in pitched roofs, in high trestles, & in framed girders for bridges.

→ Mostly inclined members are used in buildings for elegance in appearance.

→ Rigid jointed structures involving inclined members in their construction are of two types:

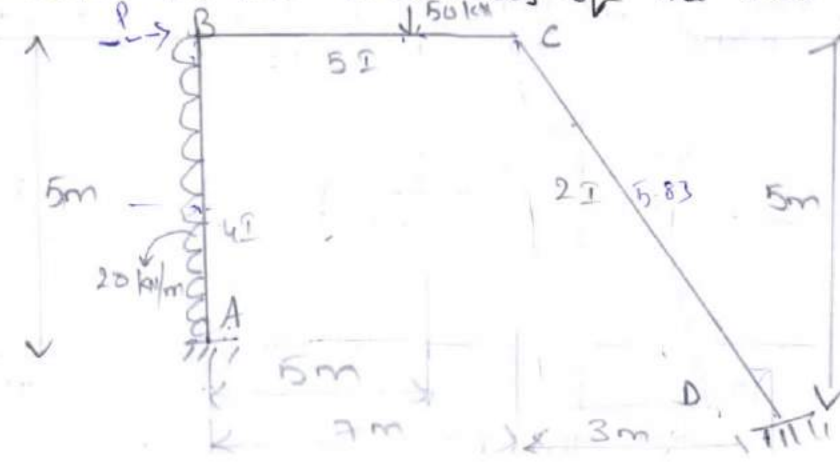
→ First one is the single or multi bay rigid portal frame. These types are used for construction of factories & it not only presents a clean & elegant appearance of but also provides unrestricted internal space by bracing member.

→ The second type is the open - frame cantilever or girder. This is used as trestles in vertical orientation or as a bridge in horizontal position.

→ In these types of application; the inclined members also help to resist lateral forces better in open frame. supports the roof in portal frame construction.

→ In such applications, the inclined member is both functional & pleasing

Prob:- Find the end moments in the members of the bent as shown below:



Sol: The length of the member CD = $\sqrt{5^2+3^2} = 5.83m$. (1)

The loading & geometrical configuration of the frame is unsymmetrical, it undergoes sway moment.

Moment Distribution under vertical loadings:

Relative stiffness:

Member	k	Σ k	D.F (k/Σk)
AB	4I/L	$\frac{4I}{5} = 0.8I$	$0.8I / 0.8I + 0.714I = 0.528 = D.F_{BA}$
BC	5I/L	$\frac{5I}{7} = 0.74I$	$0.74I / 0.714I + 0.343I = 0.675$ DF_{CB}
CD	2I/L	$\frac{2I}{5.83} = 0.343I$	$0.343I / 0.741I + 0.343I = 0.35$ DF_{CD}

$D.F_{AB} = 0$, $DF_{BA} = 0.528$, $D.F_{BC} = 0.472$, $DF_{CB} = 0.675$

$DF_{CD} = 0.325$

Fixed-End Moments:

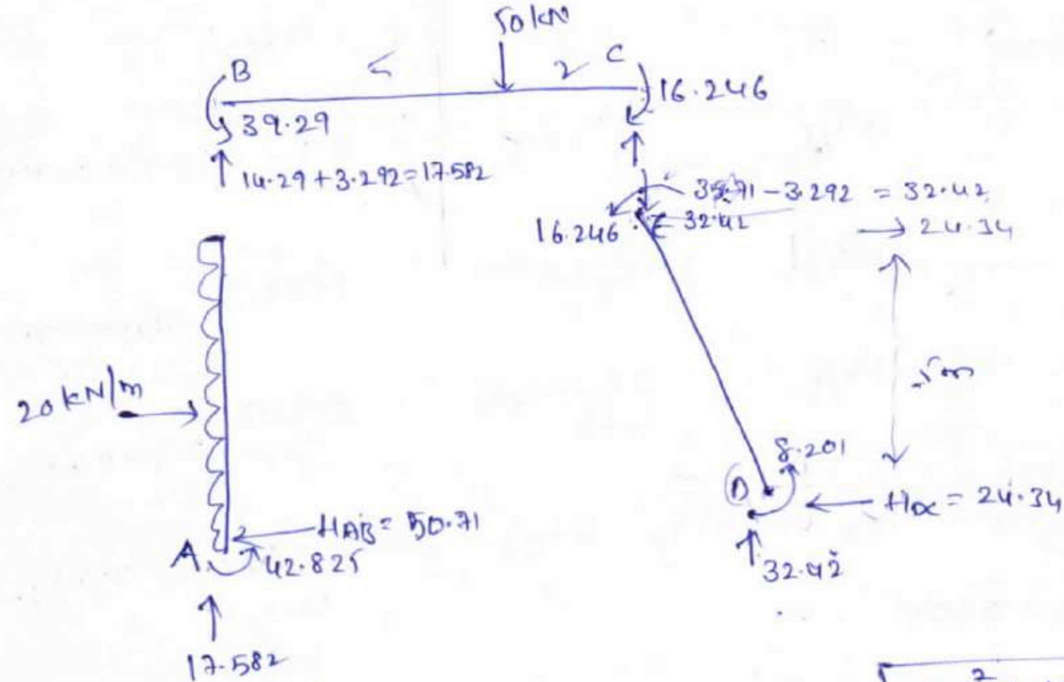
$MF_{BA} = \frac{20 \times 5^2}{12} = 41.67$ kN.m $\quad MF_{BC} = \frac{50 \times 5 \times 2^2}{7^2} = 20.41$ kN.m

$MF_{AB} = -41.67$ kN.m $\quad MF_{CB} = \frac{-50 \times 5^2 \times 2}{7^2} = -51.02$ kN.m

A	A	B		C		D	
0	0	0.528	0.472	0.675	0.325	0	D.F
~~41.67~~	+41.67	-41.67	20.41	-51.02	0.	0	FEM
	–	11.23	10.03	34.44	16.58	–	Bal
	5.615	–	17.22	5.015	–	8.29	C.O
	–	-9.09	-8.13	-3.385	-1.63	–	Bal
	-4.545	–	-1.693	-4.065	–	-0.815	C.O
	–	0.894	0.8	2.744	1.321	–	Bal
	0.447	0	1.372	0.4	0	0.661	CO
	–	-0.724	-0.648	-0.27	-0.13	–	Bal

(2)

A	B		C		D	
-0.362	-	-0.135	-0.324	-	0.65	C.O
-	0.0713	0.064	0.2187	0.1053	-	Bal
42.825	-39.29	39.29	-16.246	16.246	8.201	F.M



$\Sigma M_B = 0$ for member AB

$$= \frac{\sqrt{5.83^2 + 3^2}}{2} = 3.27.$$

$$-H_{AB} \times 5 + 20 \times 5 \times \frac{5}{2} + 42.825 - 39.29 = 0$$

$\therefore H_{AB} = 50.71$ kN.m

$\Sigma M_C = 0$ for member CD

$$-H_{DC} \times 5 + 8.201 + 32.42 \times 3 + 16.246 = 0$$

$H_{DC} = 24.34$ kN.m

The force P acting at 'C',

$$P = 20 \times 5 - (50.71 + 24.34)$$

$$P = 24.95 \text{ kN.m}$$

B 5 50 2 C

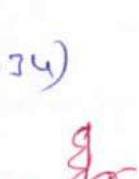
$R_B + R_C = 50$

$\Sigma M_B = 0$

$R_C \times 7 - 50 \times 5 = 0$

$R_C = 35.71$

$\therefore R_B = 14.29$



$R_A + R_D = \cancel{\frac{20 \times 20}{2}}$

$\cancel{R_B = 200 - R_A}$

$R_D \times 5 - \frac{20 \times 20}{2} = 0$

Moment Distribution for Horizontal loading:

	A	B		C		D	
	0	0.528	0.472	0.675	0.325	0	D.F
	96	96	-73.47	-73.47	41.17	41.17	FEM
	—	-11.90	-10.63	21.80	10.50	—	Bal
	-5.95	—	10.9	-5.315	—	5.25	C.O
	—	-5.755	-5.145	3.588	1.727	—	Bal
	-2.878	—	1.794	-2.573	—	0.864	C.O
	—	-0.947	-0.847	1.737	0.826	—	Bal
	-0.4735	-	0.8685	-0.4235	-	0.418	C.O
	—	-0.459	-0.411	0.286	0.138	-	Bal
	86.7	76.94	-76.99	-54.37	54.37	47.702	C.O F.M

The fixed End Moment due to translation is given by

$\frac{6EI\Delta}{l^2}$. Assume $EI\Delta = 100$

$$M_{FAB} = \frac{6E(4I)\Delta}{l^2} = \frac{6 \; 100\times4}{5^2} = 0.96\,\overset{100}{EI\Delta} = 96 \text{ kN.m}$$

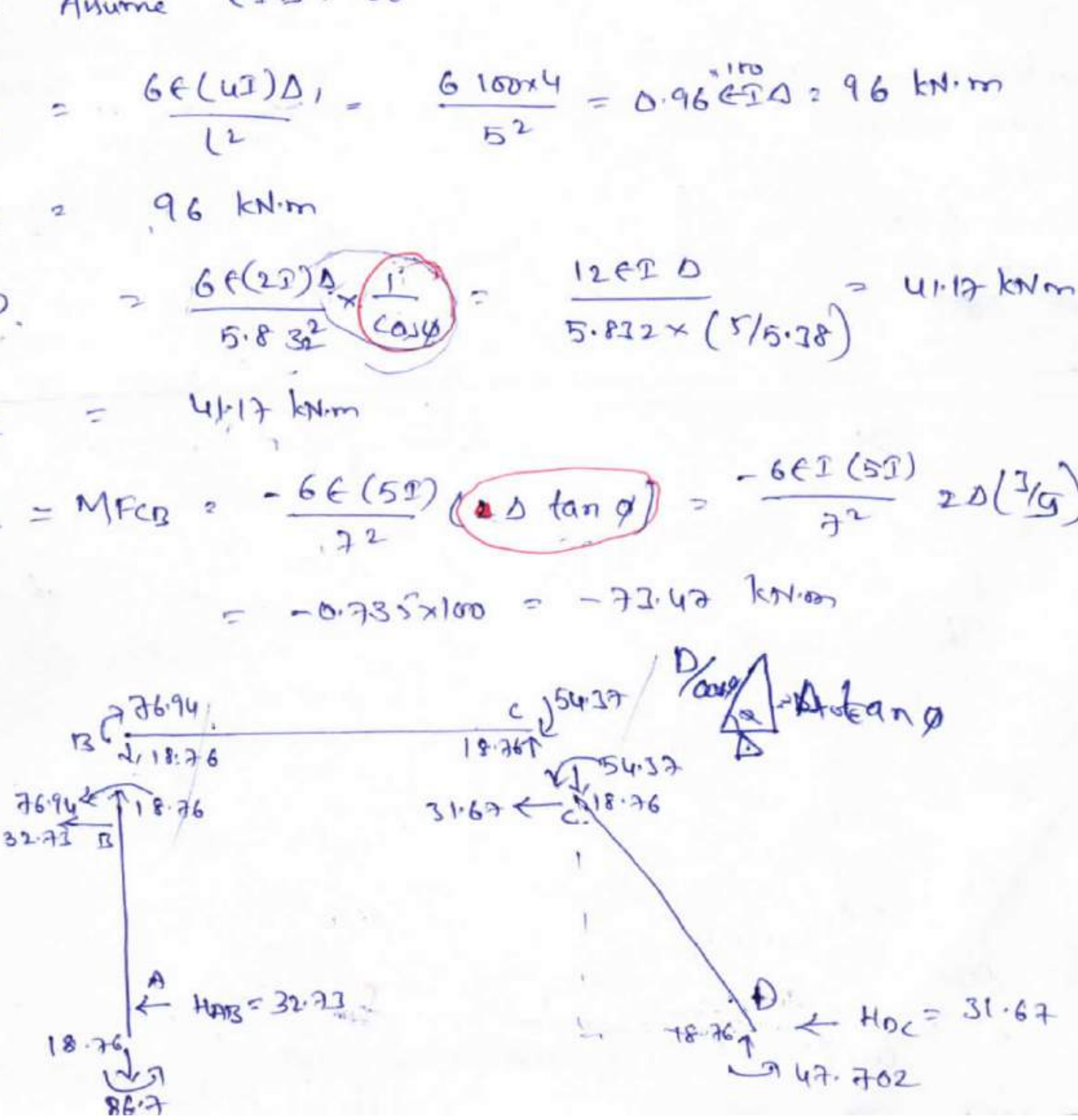
$$M_{FBA} = 96 \text{ kN.m}$$

$$M_{FCD} = \frac{6E(2I)\Delta}{5.832^2}\times\frac{1}{\cos\phi} = \frac{12EI\Delta}{5.832\times(5/5.38)} = 41.17 \text{ kNm}$$

$$M_{FDC} = 41.17 \text{ kN.m}$$

$$M_{FBC} = M_{FCB} = -\frac{6E(5I)}{7^2}(\Delta\tan\phi) = -\frac{6EI(5I)}{7^2}\,2\Delta(3/5)$$

$$= -0.735\times100 = -73.47 \text{ kN.m}$$



4

$\Sigma M_B = 0 \qquad -H_{AB} \times 5 + 86.7 + 76.94 = 0$

$\therefore H_{AB} = 32.73$ kN

$\Sigma M_C = 0$

$-H_{DC} \times 5 + 47.702 + 54.37 + 18.76 \times 3 = 0$

$\therefore H_{DC} = 31.67$ kN

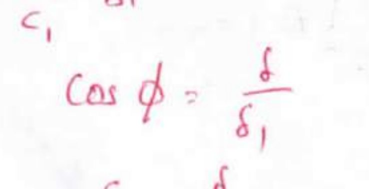
$\therefore$ The horizontal force $P' = 32.73 + 31.67$

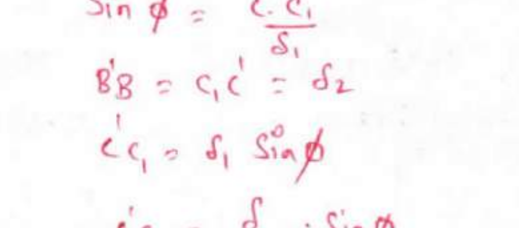
$= 64.4$ kN.

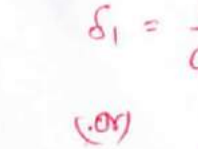
$\therefore$ The correction factor $k = \frac{24.95}{64.4} = 0.387$.

The End moments :

Member	AB	BA	BC	CB	CD	DC
Final moments under vertical loads	42.825	-39.29	39.29	-16.246	16.246	8.261
k x final moment under sway moment	33.55	29.78	-29.79	-21.04	21.04	18.46
final End moments kN-m	76.38	-9.51	9.5	-37.29	37.29	26.66







$\cos\phi = \frac{\delta}{\delta_1}$

$\delta_1 = \frac{\delta}{\cos\phi}$

(or)

$\delta_1 = \frac{\Delta}{\cos\phi}$ (or) $\boxed{\delta_1 = \Delta \times \frac{1}{\cos\phi}}$

$\delta_1 = \frac{\delta}{\cos\phi}$

$\sin\phi = \frac{C'C_1}{\delta_1}$

$B'B = C_1C' = \delta_2$

$C'C_1 = \delta_1 \sin\phi$

$C'C_1 = \frac{\delta}{\cos\phi} \cdot \sin\phi$

$\delta_2 = \delta \tan\phi$

(or)

$\boxed{\delta_2 = \Delta \tan\phi}$

Kani's Method:

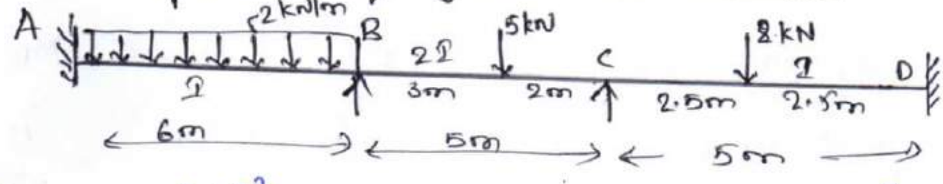
Analysis of continuous beams including settlement of supports.

Kani's method is given by Dr. Gasper Kani (1947). It is similar to moment distribution method.

The main difference b/w kani's method & MDM is that kani's method iterates the member end moments themselves rather than iterating their increments.

Prb:- A continuous beam ABCD consists of three spans, & is loaded as below. End A & D are fixed. Determine the ~~ABCD consists~~ of B.M at the supports, using kani's method. Also, plot the B.M.D & the deflected shape of the beam.

Sol:- Step-1. Computation of fixed End moments



$M_{FAB} = -\frac{2\times 6^2}{12} = -6.0$ kN·m $\qquad M_{FBA} = \frac{w\times l^2}{12} = \frac{2\times 6^2}{12} = +6.0$

$M_{FBC} = -\frac{5\times 3\times 2^2}{5^2} = -2.4$ kN·m $\qquad M_{FCB} = \frac{5\times 2\times 3^2}{5^2} = 3.6$ kN·m

$M_{FCD} = -\frac{8\times 5}{8} = -5.0$ kN·m $\qquad M_{FDC} = \frac{5\times 8}{8} = 5.0$ kN·m

2. Rotation Factors : Rotation factor is equal to -0.5 times the distribution factor used in moment distribution.

Joint	Member	Relative Stiffness	Sum	D.F	R.F $\doteq R = -0.5\times$ D.F
B	BA	$I/6$	$\frac{17I}{30}$	$5/17$	-0.147
	BC	$2I/5$		$12/17$	-0.353
C	CB	$2I/5$	$3I/5$	$2/3$	-0.333
	CD	$I/5$		$1/3$	-0.167

Note: The sum of rotation factors at a joint is equal to $\boxed{-0.5}$

$$R_{BA} + R_{BC} = -(0.147 + 0.353) = -0.5$$

3. Resultant Restraint moments :-

At each joint.

$$M_{Fi} = \sum_j M_{Fij}$$

$$M_{FB} = M_{FBA} + M_{FBC} = +6 - 2.4 = +3.6 \text{ kN·m}$$

$$M_{FC} = M_{FCB} + M_{FCD} = +3.6 - 5.0 = -1.4 \text{ kN·m}$$

A		B			C		D
-6.0	+6.0	-0.147 $\boxed{+3.6}$ -0.353	-2.4	+3.6	-0.333 $\boxed{-1.4}$ -0.167	-5.0	+5.0
M_{AB}		M_{BA}	M_{BC}		M_{CB}	M_{CD}	M_{DC}
0	i	-0.598	-1.435		+0.466	+0.234	0
	ii	-0.668	-1.604		+0.944	+0.473	
	iii	-0.676	-1.624		+1.000	+0.502	
	iv	-0.677	-1.626		+1.007	+0.505	
	v	-0.677	-1.627		+1.008	+0.505	

Step: 4 Kani's Iteration cycle :

$\theta_i = 0$; Note: M_{AB} & M_{DC} are zero, since ends A & D are fixed.

at joint C.

cycle - 1

$M_{BC} = 0$

$M_{CB} = R_{CB}(M_{FC}) = -0.333(-1.4) = +0.466$

$M_{CD} = R_{CD}(M_{FC}) = -0.167(-1.4) = +0.234$

@ joint B.

$m_{BC} = R_{BC}(M_{FB} + m_{CB}) = -0.353(+3.6 + 0.466)$

$= -1.435$

$m_{BA} = R_{BA}(M_{FB} + m_{CB}) = -0.147(+3.6 + 0.466)$

$= -0.598.$

Cycle-2 :

$m_{CB} = R_{CB}(M_{FC} + m_{BC}) = -0.333(-1.4 - 1.435) = +0.944$

$m_{CD} = R_{CD}(M_{FC} + m_{BC}) = -0.167(-1.4 - 1.435) = +0.473$

@ joint B

$m_{BC} = R_{BC}(M_{FB} + m_{CB}) = -0.353(+3.6 + 0.944) = -1.604$

$m_{BA} = R_{BA}(M_{FB} + m_{CB}) = -0.147(+3.6 + 0.944) = -0.668$

Cycle-3 :

$m_{CB} = -0.333(-1.4 - 1.604) = +1.000$

$m_{CD} = -0.167(-1.4 - 1.604) = +0.502$

@ joint B

$m_{BC} = -0.353(+3.6 + 1.000) = -1.624$

$m_{BA} = -0.147(+3.6 + 1.000) = -0.676$

Cycle-4 :

@ C, $M_{CB} = -0.333(-1.4 - 1.624) = +1.007$

$m_{CD} = -0.167(-1.4 - 1.624) = +0.505$

@ B $m_{BC} = -0.353(+3.6 + 1.007) = -1.626$

$m_{BA} = -0.147(+3.6 + 1.007) = -0.677$

Cycle-5 :

@ C, $m_{CB} = -0.333(-1.4 - 1.626) = +1.008$

$m_{CD} = -0.167(-1.4 - 1.626) = +0.505$

@ B, $m_{BC} = -0.353(+3.6 + 1.008) = -1.6277$

$m_{BA} = -0.147(+3.6 + 1.008) = -0.677$

The final values of the ~~rro~~ rotational components are:

$m_{BA} = -0.677$; $m_{BC} = -1.627$; $m_{CB} = +1.008$; $m_{CD} = +0.505$

displacement equations in iteration are:

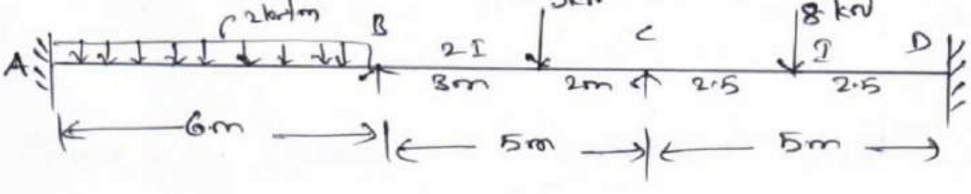
$$\theta_B = \frac{m_{BA}}{2E k_{BA}} = \frac{-0.677}{2E(I/6)} = -\frac{2.031}{EI}$$

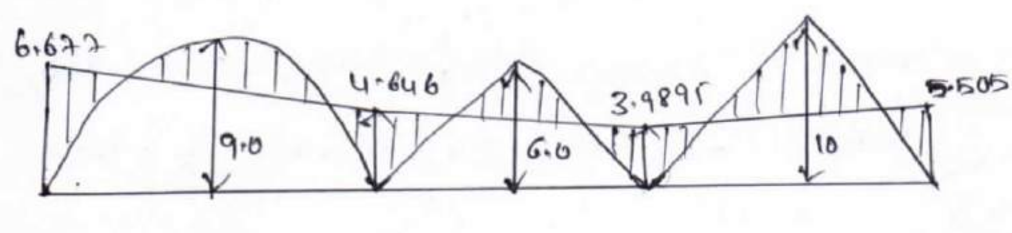
$$\theta_C = \frac{m_{CB}}{2E k_{CB}} = \frac{+1.008}{2E(2I/5)} = \frac{1.26}{EI}$$

5. Computation of final moments at joint.

$M_{AB} = M_{FAB} + 2m'_{AB} + m_{BA} \Rightarrow -6 + 2\times 0 + (-0.677)$
$= -6.677$ kN·m

M_{ij}	MF_{ij}	$2m_{ij}$	m_{ji}	Sum (kN·m)
M_{AB}	−6.0	0	−0.677	−6.677
M_{BA}	+6.0	−1.354	0	+4.646
M_{BC}	−2.4	−3.254	+1.008	−4.646
M_{CB}	+3.6	+2.016	−1.627	+3.989
M_{CD}	−5.0	+1.010	0	−3.990
M_{DC}	+5.0	0	+0.505	+5.505

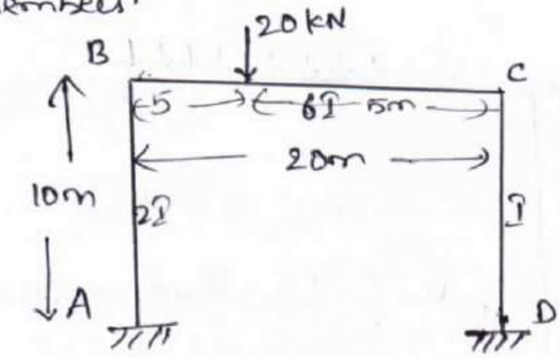




B.M.D & F.M.D

Analysis of Single bay Single storey & single bay two storey Frames by kani's Method Including Side sway.

Prb: Analyse the Portal frame shown below by kani's Method. Draw the B.M.D & sketch the deflected shape of the frame. Take EI constant for all the members.



Sol: The frame unsymmetrical.

Case-1

Step-1: Fixed End moments.

$$M_{FBC} = -\frac{20\times 5(15)^2}{20^2} = -56.25 \text{ kN.m}$$

$$M_{FCB} = +\frac{20\times 15\times 5^2}{20^2} = 18.75 \text{ kN.m}$$

Step-2: Rotation Factors:

$$k_{ij} = \frac{I_{ij}}{L_{ij}} \quad \& \quad R_{ij} = -0.5\frac{k_{ij}}{\Sigma k_{ij}}$$

$$K_{BA} = \frac{2I}{10}$$

$$K_{BC} = \frac{6I}{20} = \frac{3I}{10} = K_{CB}$$

$$K_{CD} = \frac{I}{10}$$

$$R_{BA} = -0.5\frac{2I/10}{\frac{2I}{10}+\frac{3I}{10}} = -0.5\times\frac{2}{5} = -0.2$$

$$R_{BC} = -0.5\frac{3I/10}{\frac{2I}{10}+\frac{3I}{10}} = -0.5\times\frac{3}{5} = -0.3$$

$$R_{CB} = -0.5\frac{3I/10}{\frac{3I}{10}+I/10} = -0.5\times\frac{3}{4} = -0.375$$

$$R_{CD} = -0.5\frac{I/10}{\frac{3I}{10}+\frac{I}{10}} = -0.5\times\frac{1}{4} = -0.125$$

Step: 3 : Displacement factors:

$$D_{ij} = -1.5 \frac{k_{ij}}{\sum k_{ij}} =$$

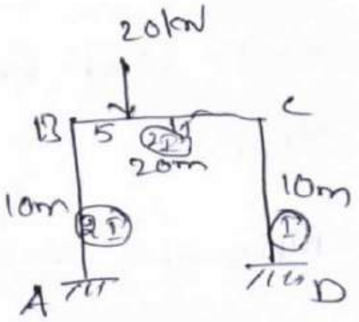
$$D_{BA} = -1.5 \left[\frac{k_{BA}}{k_{BA}+k_{CD}}\right] = -1.5\left[\frac{2I/10}{\frac{2I}{10}+\frac{I}{10}}\right] = -1.5\times\frac{2}{3}$$

$$D_{BA} = -1.0$$

$$D_{CD} = -1.5\left[\frac{k_{CD}}{k_{BA}+k_{CD}}\right] = -1.5\left[\frac{I/10}{\frac{2I}{10}+\frac{I}{10}}\right] = -1.5\times\frac{1}{3}$$

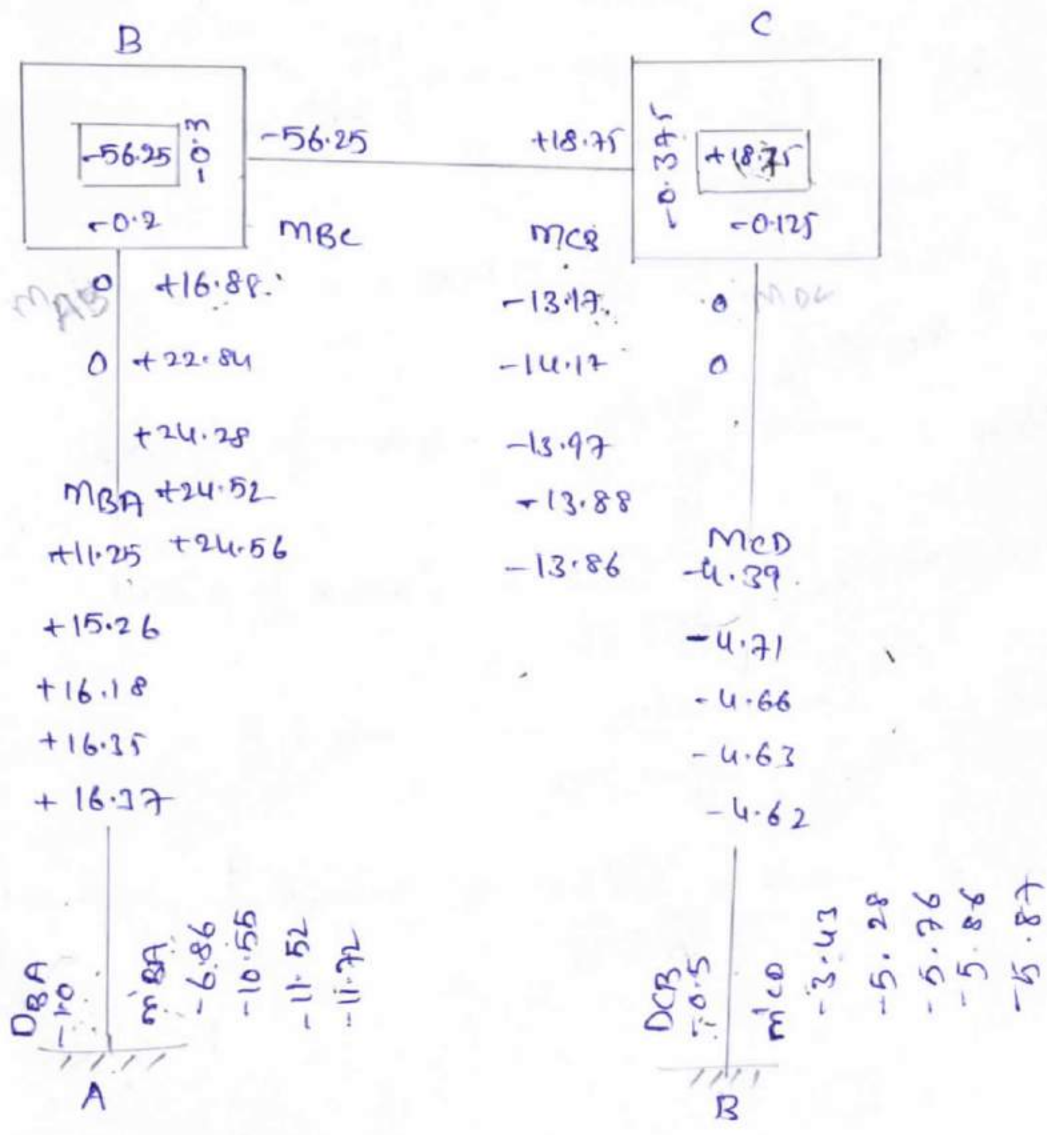
$$D_{CB} = -0.5$$

Step-4 : Resultant restraint moment:



$$M_{FB} = -56.25 \quad \& \quad M_{FC} = +18.75$$

Step-4 : Kani's Iteration Cycles :



Cycle 1 : Rotation Contribution m_{ij} @ joint

$$m_{ij} = R_{ij}\left[M_{Fi} + \sum_j (m_{ji} + m'_{ij})\right]$$

@ B $= m_{BC} = -0.3\left[-56.25 + 0 + 0\right] = +16.88.$

$m_{BA} = -0.2\left[-56.25 + 0 + 0\right] = +11.25$

@ C = $m_{CB} = R_{CB}\left[M_{FC} + m_{BC} + m_{DC} + m'_{CD}\right]$

$= -0.375\left[18.25 + 16.88 + 0 + 0\right] = -13.17$

$m_{CD} = R_{CD}\left[M_{FC} + m_{BC} + m_{DC} + m'_{CD}\right]$

$= -0.125\left[18.25 + 16.88 + 0 + 0\right] = 4.39$

Displacement contribution :

$m'_{BA} = D_{BA}\left[m_{BA} + m_{AB} + m_{CD} + m_{DC}\right] = -1.0\left[+11.25 + 0 - 4.39 + 0\right] = -6.86$

$m'_{CD} = D_{CD}\left[m_{BA} + m_{AB} + m_{CD} + m_{DC}\right] = -0.5\left[11.25 + 0 - 4.39 + 0\right]$

$= -3.43$

Cycle - 2 :

@ Rotation factors

@ joint B = $m_{BC} = R_{BC}\left[M_{FB} + m_{CB} + m_{AB} + m'_{BA}\right]$

$= -0.3\left[-56.25 - 13.17 + 0 - 6.86\right] = 22.84$

$m_{BA} = R_{BA}\left[M_{FB} + m_{CB} + m_{AB} + m'_{BA}\right]$

$= -0.2\left[-56.25 - 13.17 + 0 - 6.86\right] = 15.26$

@ joint C :

$m_{CB} = R_{CB}\left[M_{FC} + m_{BC} + m_{DC} + m'_{CD}\right]$

$= -0.375\left[+18.25 + 22.84 + 0 - 3.43\right]$

$= -14.12$

$m_{CD} = R_{CD}\left[M_{FC} + m_{BC} + m_{DC} + m'_{CD}\right]$

$= -0.125\left[+18.25 + 22.84 + 0 - 3.43\right] = -4.71$

b) Displacement factors :

$$m'BA = -1.0\left[+15.26 + 0 - 4.71 + 0\right] = -10.55$$

$$m'CD = -0.5\left[+15.26 + 0 - 4.71 + 0\right] = -5.28$$

Cycle 3:

(a) Rotation factors:

joint B: $mBC = -0.3\left[-56.25 - 14.12 + 0 - 10.55\right]$

$= +24.28$

$mBA = -0.2\left[-56.25 - 14.12 + 0 - 10.55\right]$

$= +16.18$

joint C : $mCB = -0.375\left[+18.25 + 24.28 + 0 - 5.28\right]$

$= -13.97$

$mCD = -0.125\left[18.25 + 24.28 + 0 - 5.28\right]$

$= -4.66$

(b) Displacement factors:

$$m'BA = -1.0\left[+16.18 + 0 - 4.66 + 0\right] = -11.52$$

$$m'CD = -0.5\left[+16.18 + 0 - 4.66 + 0\right] = -5.76$$

Cycle 4

(a) Rotation Factors

joint B : $mBC = -0.3\left[-56.25 - 13.97 + 0 - 11.52\right] = 24.52$

$mBA = -0.2\left[-56.25 - 13.97 + 0 - 11.52\right] = +16.35$

joint C : $mCB = -0.375\left[18.25 + 24.52 + 0 - 5.76\right] = -13.88$

$mCD = -0.125\left[18.25 + 24.52 + 0 - 5.76\right] = -4.63$

(b) Displacement factors :

$$m'BA = -1.0\left[+16.35 + 0 - 4.63 + 0\right] = -11.72$$

$$m'CD = -0.5\left[+16.35 + 0 - 4.63 + 0\right] = -5.86$$

Cycle - 5 :

(a) Rotation factors:

Joint B : $m_{BC} = -0.3\left[-56.25 - 13.88 + 0 - 11.72\right] = +24.56$

$m_{BA} = -0.2\left[-56.25 - 13.88 + 0 - 11.72\right] = +16.37$

Joint C : $m_{CB} = -0.375\left[+18.25 + 24.56 + 0 - 5.86\right] = -13.86$

$m_{CD} = -0.125\left[+18.25 + 24.56 + 0 - 5.86\right] = -4.62$

(b) Displacement factors :

$m'_{BA} = -1.0\left[+16.35 + 0 - 4.62 + 0\right] = -11.73$

$m'_{CD} = -0.5\left[+16.35 + 0 - 4.62 + 0\right] = -5.87.$

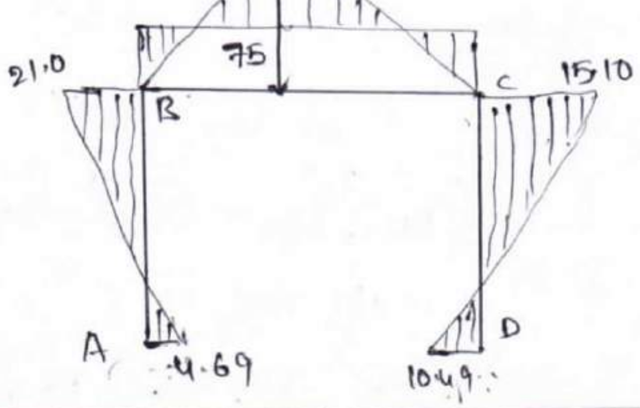
Step-6 : Final moments.

$M_{ij} = M_{Fij} + 2m_{ij} + m_{ji} + m'_{ij}$

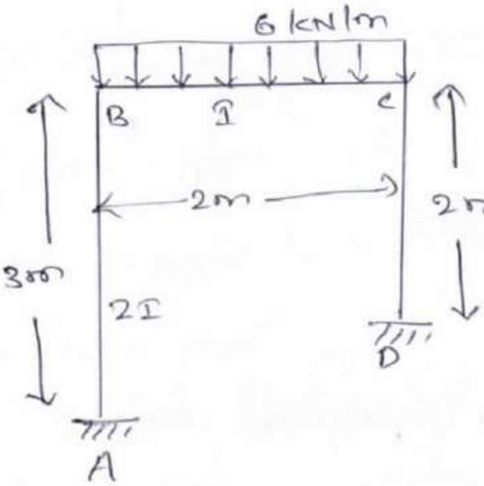
$M_{AB} = M_{FAB} + 2m_{ab} + m_{ba} + m'_{ab}$

$M_{BA} = M_{FBA} + 2m_{ba} + m_{ab} + m'_{ba}$

M_{ij}	M_{Fij}	$2m_{ij}$	m_{ji}	m'_{ij}	Total (kNm)
M_{BA}	0	+32.74	0	-11.73	+21.01
M_{BC}	-56.25	+49.12	-13.86	-	-20.99
M_{CB}	+18.25	-27.72	+24.56	-	+15.09
M_{CD}	0	-9.24	0	-5.87	-15.11
M_{AB}	0	0	+16.37	-11.73	4.64.
M_{DC}	0	0	-4.62	-5.87	-10.49



Prb: Draw the B.M.D & sketch the deflected shape of the frame:



Sol: 1) Fixed End Moments:

$$M_{FBC} = -\frac{6(2)^2}{12} = -2.0 \text{ kN.m}$$

$$M_{FCB} = +2.0 \text{ kN.m}$$

2) Rotation Factors:

$$k_{ij} = \frac{I_{ij}}{L_{ij}} \quad \& \quad R_{ij} = -0.5 \frac{k_{ij}}{\sum_j k_{ij}}$$

$$k_{BA} = \frac{2I}{3}; \quad k_{BC} = \frac{I}{2} = k_{CB}; \quad k_{CD} = I/2$$

$$\therefore R_{BA} = -0.5 \frac{2I/3}{\frac{2I}{3} + \frac{I}{2}} = -0.5 \times \frac{2}{3} \times \frac{6}{7} = -0.2857$$

$$R_{BC} = -0.5 \frac{I/2}{2/3 I + I/2} = -0.5 \times 1/2 \times 6/7 = -0.2143$$

$$R_{CB} = -0.5 \frac{I/2}{I/2 + I/2} = -0.5 + 1/2 \times 1 \times 1 = -0.25$$

$$R_{CD} = -0.5 \frac{I/2}{I/2 + I/2} = -0.5 \times 1/2 \times 1 = -0.25$$

Step: 3) Displacement factors:

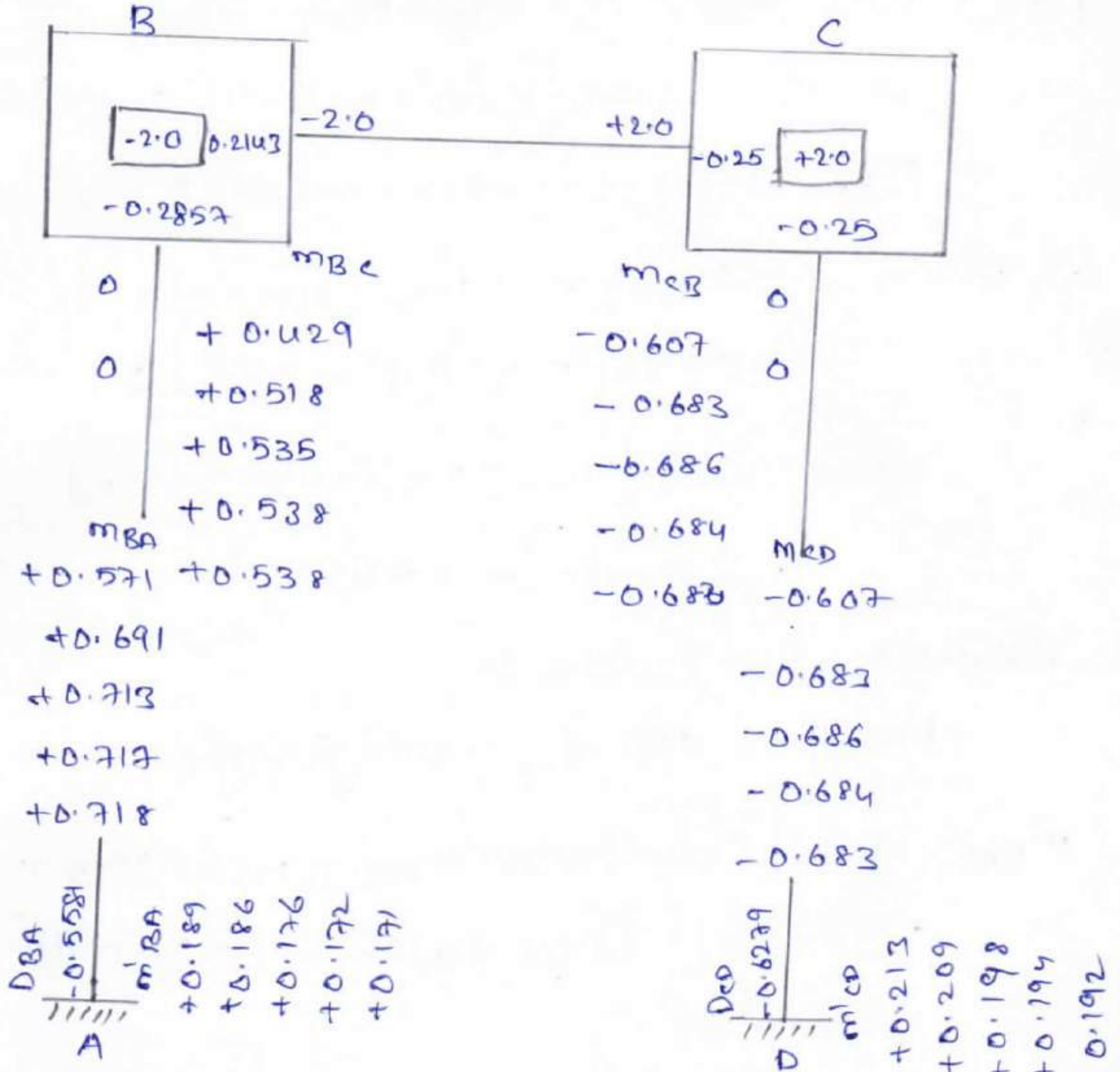
$$D_{ij} = -1.5 \frac{C_{ij} \cdot k_{ij}}{\sum_r C_{ij}^2 k_{ij}} \quad \text{[Unequal legs]}$$

Let reference height is 3m

$$\therefore C_{BA} = \frac{3}{3} = 1 \quad \& \quad C_{CD} = \frac{3}{2} = 1.5$$

$$\therefore D_{BA} = -1.5 \frac{C_{BA} \cdot k_{BA}}{(C_{BA})^2 \cdot k_{BA} + (C_{CD})^2 \cdot k_{CD}}$$

$$= -1.5 \frac{(1) \times 2I/3}{(1)^2 \times \frac{2I}{2} + (1.5)^2 \, I/2} = -0.5581$$



$$D_{CD} = -1.5 \frac{C_{CD} \cdot K_{CD}}{(C_{BA})^2 \cdot K_{BA} + (C_{CD})^2 \cdot K_{CD}} = -1.5 \frac{1.5 \times I/2}{(1)^2 \times \frac{2I}{3} + (1.5)^2 \cdot I/2}$$

$$D_{CD} = -0.6279$$

4) Resultant Restraint Moments:

$M_{FB} = -2.0$; $M_{FC} = +2.0$ kN-m

5) Kani's Iteration Cycle.

Cycle - 1 : Rotation contribution m_{ij} @ a joint i is,

$$m_{ij} = R_{ij}\left[M_{Fi} + \sum_j (m_{ji} + m'_{ij})\right]$$

m_{ji} & m'_{ji} are unknown (or) = 0

m_{ji} & $m'_{ji} = 0$

@ B = $M_{BC} = R_{BC}\left[M_{FB} + m_{CB} + m_{AB} + m'_{BA}\right]$

$= -0.2143\left[-2.0 + 0 + 0 + 0\right] = +0.429$

$m_{BA} = -0.2857\left[-2.0 + 0 + 0 + 0\right] = +0.571$

@ C = $m_{CB} = R_{CB}\left[M_{FC} + m_{BC} + m_{DC} + m'_{CD}\right]$

$= -0.25\left[2.0 + 0.429 + 0 + 0\right] = -0.607$

$m_{CD} = R_{CD}\left[M_{FC} + m_{BC} + m_{DC} + m'_{CD}\right]$

$= -0.25\left[+2.0 + 0.429 + 0 + 0\right] = -0.607$

Distribution Contributions :

$M_r = 0$; $m'_{ij} = D_{ij}\left[\sum_r C_{ij}\,(m_{ij} + m_{ji})\right]$

$m'_{BA} = D_{BA}\left[C_{BA}\,(m_{BA} + m_{AB}) + C_{CD}\,(m_{CD} + m_{DC})\right]$

$= -0.5581\left[1\,(0.571 + 0) + 1.5\,(-0.607 + 0)\right]$

$= +0.189$

$m'_{CD} = D_{CD}\left[1\,(m_{BA} + m_{AB}) + C_{CD}\,(m_{CD} + m_{DC})\right]$

$= -0.6279\left[1\,(0.571 + 0) + 1.5\,(-0.607 + 0)\right]$

$= +0.213.$

Cycle - 2 : @ Rotation factors.

@ B = $m_{BC} = -0.2143\left[-2.0 - 0.607 + 0 + 0.189\right] = +0.518$

$m_{BA} = -0.2857\left[-2.0 - 0.607 + 0 + 0.189\right] = +0.691$

@ C = $m_{CB} = -0.25\left[+2.0 + 0.518 + 0 + 0.213\right] = -0.683$

$m_{CD} = -0.25\left[+2.0 + 0.518 + 0 + 0.213\right] = -0.683$

ⓑ Displacement factors.

$m_{BA} = -0.5581\left[1 \times 0.691 - 1.5 \times 0.683\right] = +0.186$

$m'_{CD} = -0.6279\left[1 \times 0.691 - 1.5 \times 0.683\right] = +0.209$

Cycle-3: (a) Rotation factors

(a) B = $m_{BC} = -0.2143\left[-2.0 - 0.683 + 0 + 0.186\right] = +0.535$

$m_{BA} = -0.2857\left[-2.0 - 0.683 + 0 + 0.186\right] = +0.713$

(a) C = $m_{CB} = -0.25\left[+2.0 + 0.535 + 0 + 0.209\right] = -0.686$

$m_{CD} = -0.25\left[+2.0 + 0.535 + 0 + 0.209\right] = -0.686$

(b) Displacement factors!

$m'_{BA} = 0.5581\left[+1 \times 0.713 + 0 - 1.5 \times 0.686 + 0\right] = +0.176$

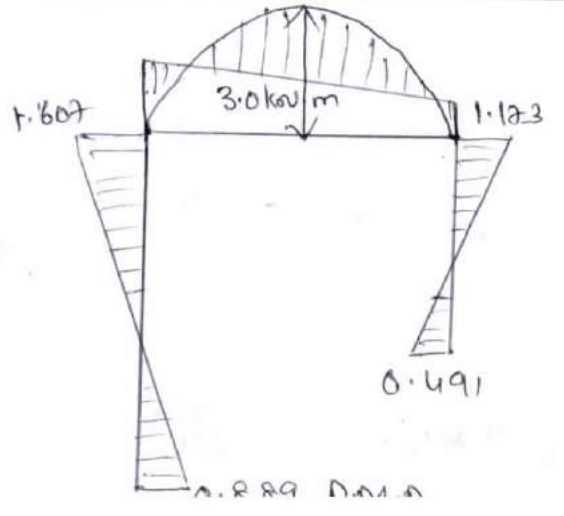
$m'_{CD} = -0.6279\left[+1 \times 0.713 + 0 - 1.5 \times 0.686 + 0\right] = +0.198.$

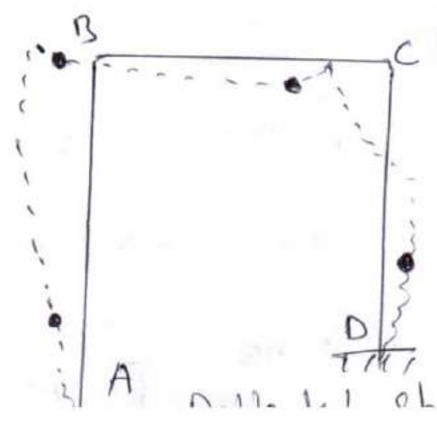
Similarly Cycle-4 & Cycle 5.

6) Final moments ;

$M_{ij} = M_{Fij} + 2m_{ij} + m_{ji} + m'_{ij} \Rightarrow M_{AB} = M_{FAB} + 2m_{ab} + m_{ba} + m'_{ab}$

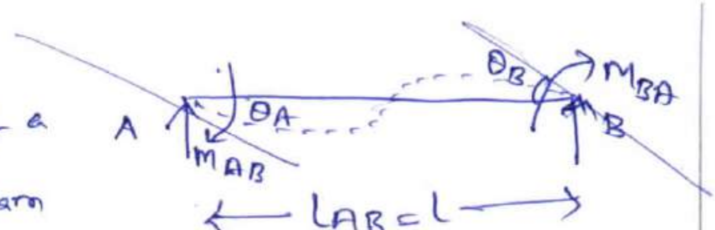
M_{ij}	M_{Fij}	$2m_{ij}$	m_{ji}	m'_{ij}	Total
M_{BA}	0	+1.436	0	+0.171	+1.607
M_{BC}	-2.0	+1.436	-0.683	–	-1.607
M_{CB}	+2.0	-1.366	0.538	–	+1.172
M_{CD}	+2.0	-1.366	+0.538	–	+1.174
M_{AB}	0	-1.366	0	+0.192	-1.174
M_{DC}	0	0	-0.683	+0.192	-0.491





Continuous beams :

The final deflected shape of a span AB of a continuous beam under imposed loads



let M_{AB} & M_{BA} are moments developed

θ_A & θ_B are rotations at A & B due to imposed loads

The slope deflection equation for span AB, at joint A is :

$$M_{AB} = M_{FAB} + 2EK_{AB}(2\theta_A + \theta_B) \longrightarrow (1)$$

where; $K_{AB} = \frac{I_{AB}}{L_{AB}}$

$$\therefore \quad M_{AB} = M_{FAB} + 4EK_{AB}\theta_A + 2EK_{AB} \cdot \theta_B$$

$$M_{AB} = M_{FAB} + 2m_{AB} + m_{BA} \longrightarrow (2)$$

where by definition $m_{AB} = 2EK_{AB} \cdot \theta_A$ & $m_{BA} = 2EK_{AB} \cdot \theta_B \longrightarrow (3)$

m_{AB} = rotational contribution of end A to M_{AB},

m_{BA} = rotational contribution of end. B to M_{AB}.

$$\therefore \quad \sum_B M_{AB} = 0$$

from eq-(2)

$$\sum_B M_{FAB} + \sum_B (2m_{AB} + m_{BA}) = 0 \longrightarrow (4)$$

let M_{FA} - resultant restraint moment at A.

$$M_{FA} = \sum_B M_{AB} \longrightarrow (5)$$

from eqn-(4)

$$\sum_B m_{AB} = -\frac{1}{2}\left(M_{FA} + \sum_B m_{BA}\right) \longrightarrow (6)$$

$\therefore$ For member AB,

$$m_{AB} = \frac{K_{AB}}{\sum_B K_{AB}} \cdot \sum_B m_{AB} \cdot \longrightarrow (7)$$

$$m_{AB} = \frac{K_{AB}}{\sum_B K_{AB}} \cdot \sum_B m_{AB} \quad \text{—— (7)}$$

(or)

$$m_{AB} = -\frac{1}{2}\left[\frac{K_{AB}}{\sum_B K_{AB}}\right]\left[M_{FA} + \sum_B m_{BA}\right]$$

$$m_{AB} = R_{AB}\left[M_{FA} + \sum_B m_{BA}\right] \quad \text{—— (8)}$$

where

R_{AB} = rotational factor for AB.

$$\therefore R_{AB} = -\frac{1}{2}\left[\frac{K_{AB}}{\sum_B K_{AB}}\right] \quad \text{—— (9)}$$

Note: Rotation factor $R = -\frac{1}{2}$ times D.F

i = Near end A & j = far near ends B.

$M_{ij} = M_{Fij} + 2Ek_{ij}(2\theta_i + \theta_j)$ where; $k_{ij} = \frac{I_{ij}}{L_{ij}}$

$$k_{ij} = \frac{I_{ij}}{L_{ij}} \quad \text{— (a)}$$

$$M_{ij} = M_{Fij} + 2m_{ij} + m_{ji} \quad \text{— (b)}$$

where $m_{ij} = 2Ek_{ij}\theta_i$ & $m_{ji} = 2Ek_{ij}\theta_j$ —— (c)

$$\sum_j M_{Fij} + \sum_j (2m_{ij} + m_{ji}) = 0 \quad \text{—— (d)}$$

$$M_{Fi} = \sum_j M_{Fij} \quad \text{— (e)}$$

$$\sum_j m_{ij} = -\frac{1}{2}\left[M_{Fi} + \sum_j m_{ji}\right] \quad \text{— (f)}$$

$$m_{ij} = R_{ij}\left(M_{Fi} + \sum_j m_{ji}\right) \quad \text{—— (g)}$$

where. $\boxed{R_{ij} = -\frac{1}{2}\left[\frac{k_{ij}}{\sum_j k_{ij}}\right]}$

Procedure for Kani's Method :

Step-1: Calculate the fixed end moments (M_{Fij}) in all the members of the structure & enter these outside the outer square.

Step-2 : Calculate the (k) values & rotation factors (R_{ij}) for all members meeting at each joint. These values are entered outside the first square but inside the second square towards each member.

Step-3 : Find the resultant restraint moment at each joint using eq-(5). Enter these values of resultant restraint moment within the inner square at each joint.

Step-4 : Compute rotational contribution (m_{ij}) of the two ends of all the members by Gauss-Seidel iteration from eq-(9), taking $m_{ij} = 0$ at all joints starting with the approximation that $\theta_i = 0$. Continue the iteration through several cycles till practically the same values of m_{ij} are obtained in two successive cycles. Each cycle gives improved approximation for the rotational contribution. All these values of m_{ij} are entered.

Step-5 : Using eq-(2) determine member end moments M_{ij}.

Kani's Method:

⇒ This method was developed by "Gasper kani" of Germany.

⇒ This method is an excellent method extension of the slope deflection method.

⇒ Kani's method is also known as "rotation contribution method.

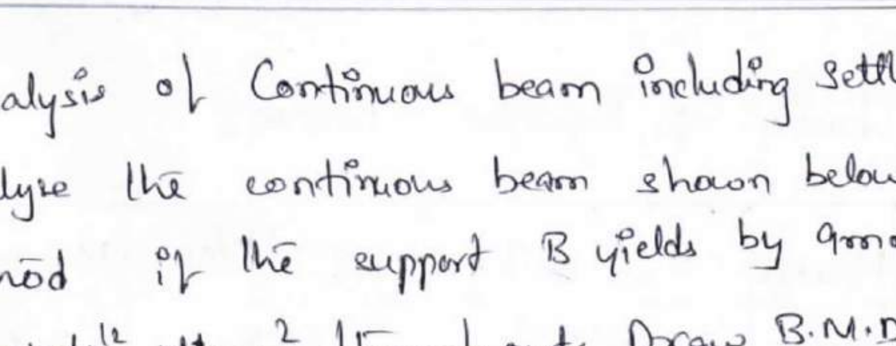
⇒ Rotation contribution method is used to analysis of statically indeterminate structures, like continuous beams, Portal frames etc.

Prb. Analysis of Continuous beam including settlement of Supports

1) Analyze the continuous beam shown below @ by kani's method if the support B yields by 9mm, Take $EI = 1\times10^{12}\ N\,mm^2$ through out. Draw B.M.D

$EI = 1\times10^{12}\ N\,mm^2$
$= 1\times10^{3}\ kNm^2$

$\Delta = 9mm$
$= 0.009m$



Sol:- Fixed End Moments:

$$M_{FAB} = -\frac{wab^2}{l^2} - \frac{6EI\delta}{l^2}$$

$$= -\frac{60\times2\times1^2}{3^2} - \frac{6\times1\times10^3\times0.009}{3^2} =$$

$$= -13.33 - 6 = -19.33\ kN.m$$

$M_{FAB} = -19.33$ kN.m

$$M_{FBA} = \left[\frac{Wa^2b}{L^2}\right] - \left[\frac{6.EI\delta}{L^2}\right] \Rightarrow \frac{60\times2^2\times1}{3^2} - \frac{6\times1000\times0.009}{3^2}$$

$= 20.67$ kN.m

$$M_{FBC} = -\frac{WL^2}{12} + \frac{6EI\delta}{L^2} = -\frac{20\times6^2}{\cancel{6}} + \frac{6\times1000\times0.009}{6^2} = -58.5 \text{ kN.m}$$

$$M_{FCB} = \frac{WL^2}{12} + \frac{6EI\delta}{L^2} = \frac{20\times6^2}{\cancel{6}\,12} + \frac{6\times1000\times0.009}{6^2}$$

$= 61.5$ kN.m.

M_{FCD} (cantilever moment) $= -W\times L \Rightarrow -20\times1$

$= -20$ kN.m.

$M_{FDC} = 0$

2) Distribution Factor & Rotation Factors:

S.NO	Joint	Member	k	Σk	$D.F = k/\Sigma k$	$R.F = -0.5\,D.F$
1.	B	BA	$I/L = I/3$	$0.5I$	0.667	-0.333
		BC	$I/L = I/6$		0.333	-0.166
~~2.~~	~~C~~	~~CB~~	~~$I/L = I/6$~~	~~$I/6 = 0.166$~~	~~1.0~~	~~-0.5~~
		~~CD~~	~~0~~		~~0~~	~~0~~

3) Rotation Contribution Moments:
Resultant Restraint

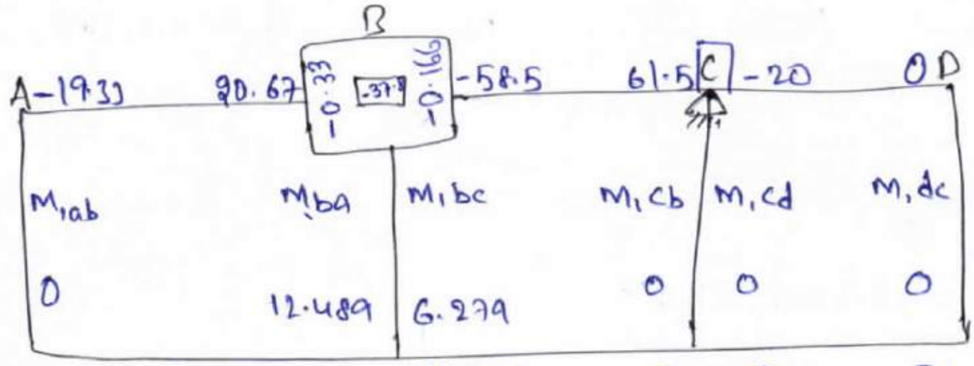
$M_{FB} = M_{FBA} + M_{FBC} = 20.67 + (-58.5) = -37.83$ kN.m

~~$M_{FC} = M_{FCB} + M_{FCD} = 61.5 - 20 = 41.5$ kN.m~~

3) Resultant Restraint Moments :

$M_{FB} = M_{FBA} + M_{FBC} = 20.67 + (-58.5) = -37.83$ kN·m

4) Rotation Contribution Table :



0 12.489 6.279 0 0 0

Rotation Contribution Moments :

$M'_{ab} = 0$ $\quad M'_{ba} = 12.489$ $\quad M'_{bc} = 6.279$ $\quad M'_{cb} = 0$ $\quad M'_{dc} = 0$

$M'_{cd} = 0$

5) Final Moments :

$M_{AB} = M_{FAB} + 2M'_{ab} + M'_{ba}$

$= -19.33 + 0 + 12.489 = -6.841$ kN·m

$M_{BA} = M_{FBA} + 2m'_{ba} + m'_{ab}$

$= 20.67 + 2 \times 12.489 + 0 = 45.648$ kN·m $= 45.47$

$M_{BC} = M_{FBC} + 2m'_{bc} + m'_{cb}$

$= -58.5 + 2 \times 6.279 + 0 = -45.942$ kN·m

$= -45.94$

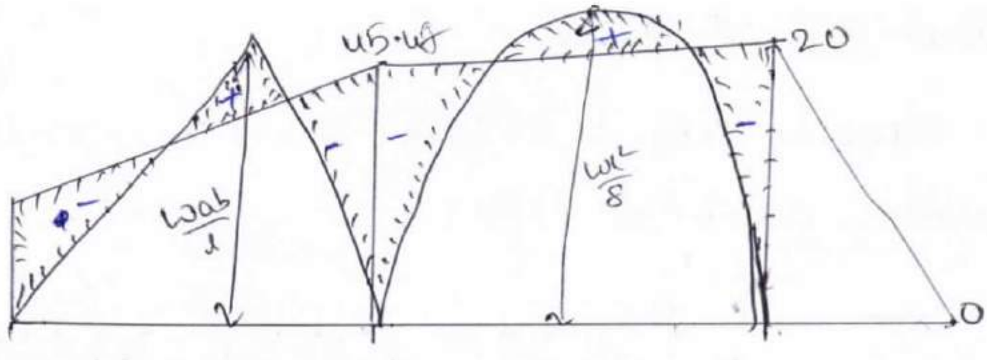
$M_{CB} = M_{FCB} + 2m'_{cb} + m'_{bc}$

$= 61.5 + (2 \times 0) + 6.279 = 67.779$ kN·m

$M_{CD} = M_{FCD} + 2m'_{CD} + M'_{dc}$

$= -20 + 0 + 0 = -20$ kN·m

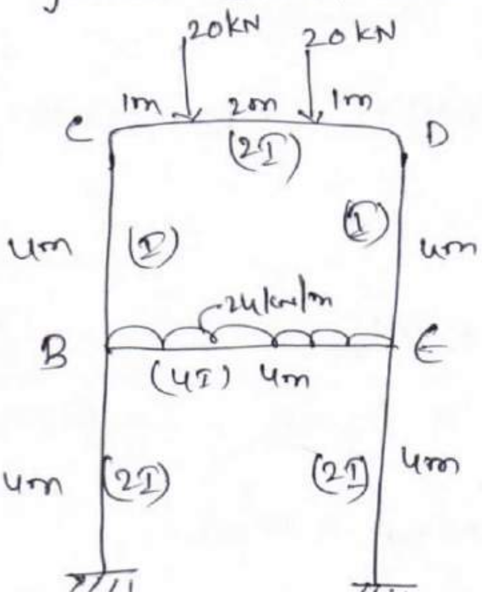
$M_{DC} = 0$



B.M.D

Analysis of Single bay Two-storey Portal Frame by kani's Method:

Prb: Analyse the frame shown below by Kani's method.



Sol:- Analysis will be carried out taking the advantage of symmetry.

a) Fixed End Moments:

$$M_{FCD} = -\left[\frac{Wab^2}{l^2} + \frac{Wab^2}{L^2}\right]$$

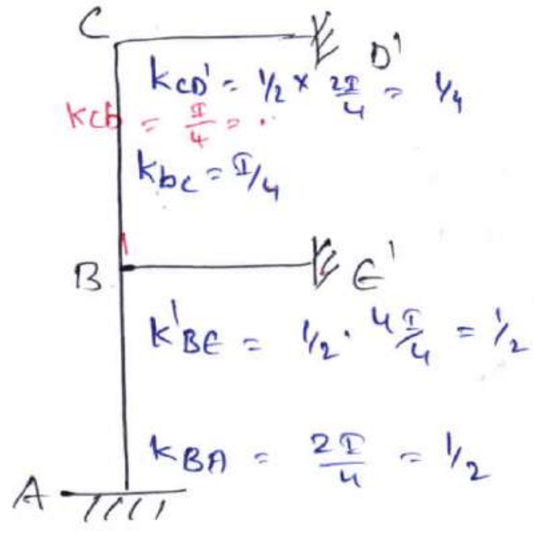
$$= -\left[\frac{20\times1\times3^2}{4^2} + \frac{20\times3\times1^2}{4^2}\right]$$

$$= -15\,kN.m$$

$$M_{FBC} = \frac{-WL^2}{12} = \frac{-24 \times 4^2}{12} = -32 \text{ kN.m}$$

b) Rotation Factors :

Joint	Member	k	Σk	D.F $k/\Sigma k$	R.F $= -0.5 \times D.F$
B	BA	$2I/4$		0.4	$-1/5$
	BE'	$1/2 \times \frac{4I}{4} = 1/2$	$5I/4$	0.4	$-1/5$
	BC	$I/4$		0.2	$-1/10$
C	CB	$I/4$	$2I/4$	0.5	$-1/4$
	CD'	$1/2 \times \frac{2I}{4} = 1/4$		0.5	$-1/4$

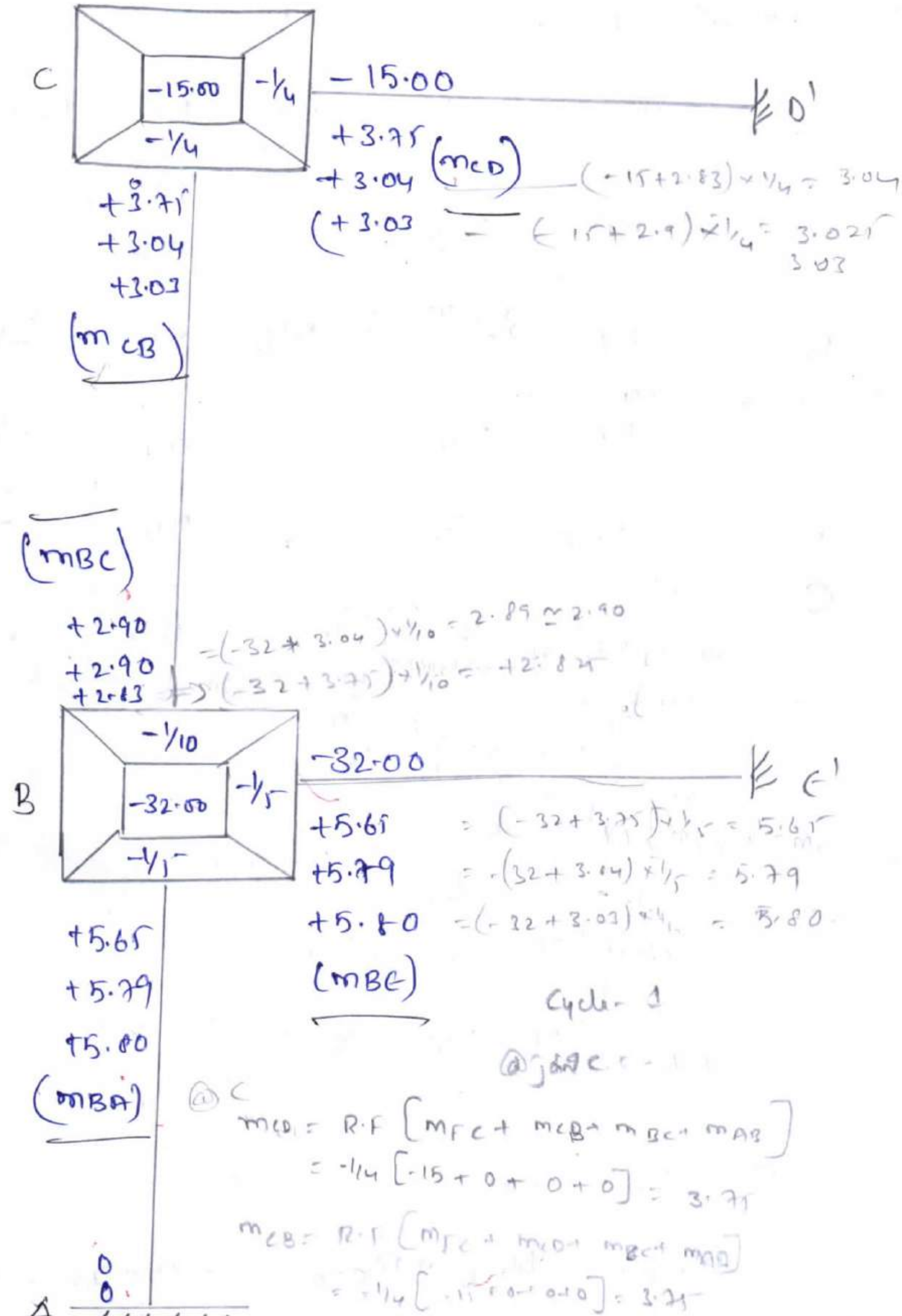


c) Resultant Restraint Moments :

$$M_{FC} = M_{FCD} + M_{FCB} = -15 + 0 = -15 \text{ kN.m}$$

$$M_{FB} = M_{FBE} + M_{FBA} = -32 + 0 = -32 \text{ kN.m}$$

d) Rotation Contributions calculations by Iteration cycles:



Cycle 1

@ joint C

@ C

$m_{CD} = R.F\left[M_{FC} + m_{CB} + m_{BC} + m_{AB}\right]$
$= -1/4\left[-15 + 0 + 0 + 0\right] = 3.75$

$m_{CB} = R.F\left[M_{FC} + m_{CD} + m_{BC} + m_{AB}\right]$
$= -1/4\left[-15 + 0 + 0 + 0\right] = 3.75$

@ B : $m_{BC} = R.F\left[M_{FB} + m_{CD} + m_{BA} + m_{AB}\right]$
$= -1/10\left[-32 + 3.75 + 0 + 0\right]$
$= +2.825$

$m_{BA} = R.F\left[M_{FB} + m_{BE} + m_{CD} + m_{AB}\right] \Rightarrow -1/5\left[-32 + 0 + 3.75 + 0\right] = +5.65$

Resultant ~~Resultant~~ Restraint Moments:

$m_{BE} = R.F\,[M_{FB} + \cancel{m_{BC}} + m_{DA} + m_{CB} + 0] \Rightarrow -\tfrac{1}{4}[-32 + 0 + 3.75 + 0]$

M_{AD} $= -\tfrac{1}{4}[-32 + 3.75] = 5.65$

Cy - Ⅱ

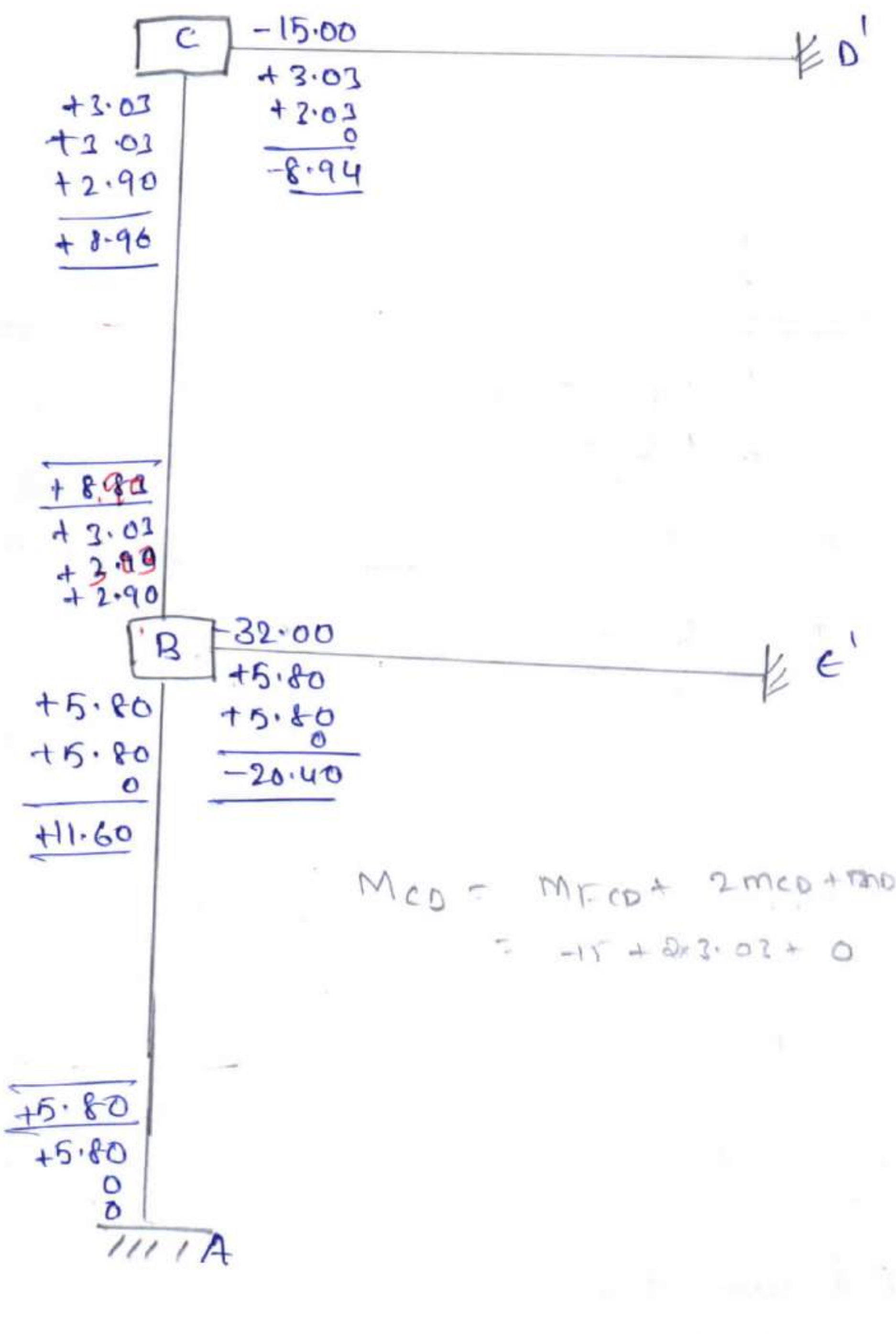
@ C $m_{CD} = R.F\,[M_{FC} + m_{BC} + m_{AB}] = -\tfrac{1}{4}[-15 + 2.83 + 0] = 3.0425$

$m_{CB} = R.F\,[M^{f}_{BC} + m_{BC} + m_{AB}] = -\tfrac{1}{4}[-15 + 2.83 + 0] = 3.04$

@ B $m_{BC} = R.F\,[M_{FB} + m_{CD} + m_{AB}] = -\tfrac{1}{10}[-32 + 3.04 + 0] = 2.896$

$m_{BE} = R.F\,[M_{FB} + 3.04 + 0] = -\tfrac{1}{5}[-32$

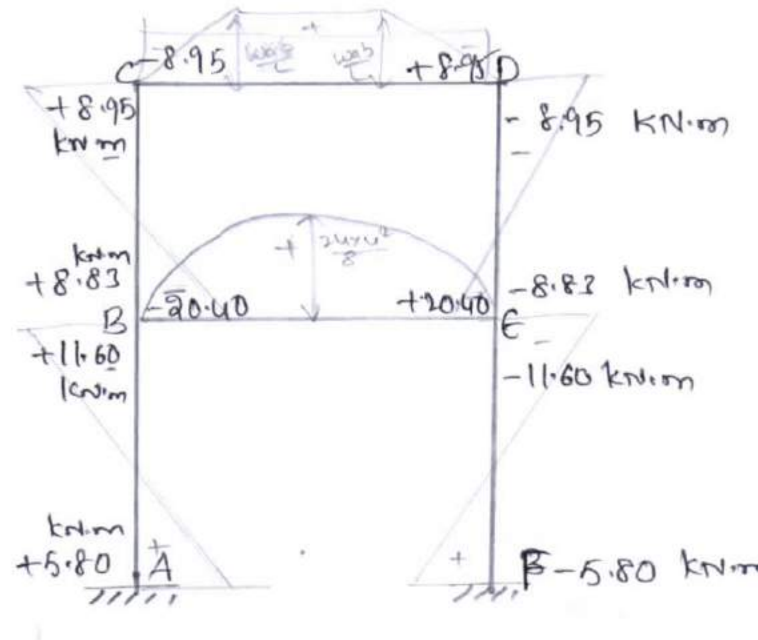
Calculation of final moments:



$M_{CD} = M_{FCD} + 2m_{CD} + m_{DC}$

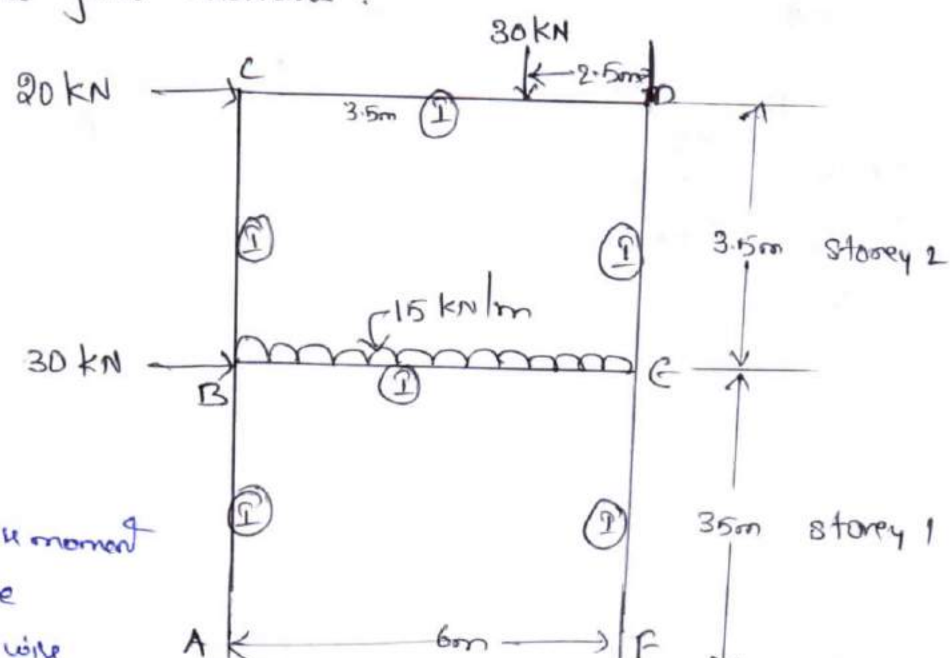
$= -15 + 2 \times 3.03 + 0$

Final End Moments



Two storey frames with lateral load: [Sway forces]

Pb: A two-storeyed frame along with lateral loading is shown below. The moments of Inertia of the different members are given. Find the joint moments:



Clock wise moment ↻ +ve
anticlock wise ↺ -ve

Sol:- Fixed-End Moments:

$$M_{FCD} = \frac{Wab^2}{L^2} = + \frac{30 \times 3.5 \times 2.5^2}{6^2} = 18.23 \text{ kN·m}$$

$$M_{FBC} = \frac{30 \times 3.5^2 \times 2.5}{6^2} = -25.52 \text{ kN·m}$$

$$M_{FBE} = +\frac{WL^2}{12} = \frac{+15 \times 6^2}{12} = +45 \text{ kN·m} \; ; \; M_{FEB} = -45 \text{ kN·m}$$

2. Rotation Factor Table :

Joint	Member	Relative Stiffness 'K'	Sum ΣK	D.F = $K/\Sigma K$	R.F = $-0.5 \times$ D.F
B	BA	$I/3.5$	$0.739\,I$	0.386	-0.193
	BE	$I/6$		0.225	-0.113
	BC	$I/3.5$		0.386	-0.194
C	CD	$I/6$	$0.453\,I$	0.363	-0.316
	DC	$I/6$		0.363	-0.184
D	DE	$I/3.5$	$0.453\,I$	0.631	-0.316
	DE	$I/3.5$		0.631	-0.194
E	EB	$I/6$	$0.739\,I$	0.225	-0.113
	ED	$I/3.5$		0.386	-0.194
	EF	$I/3.5$		0.386	-0.194

3. Displacement factors :

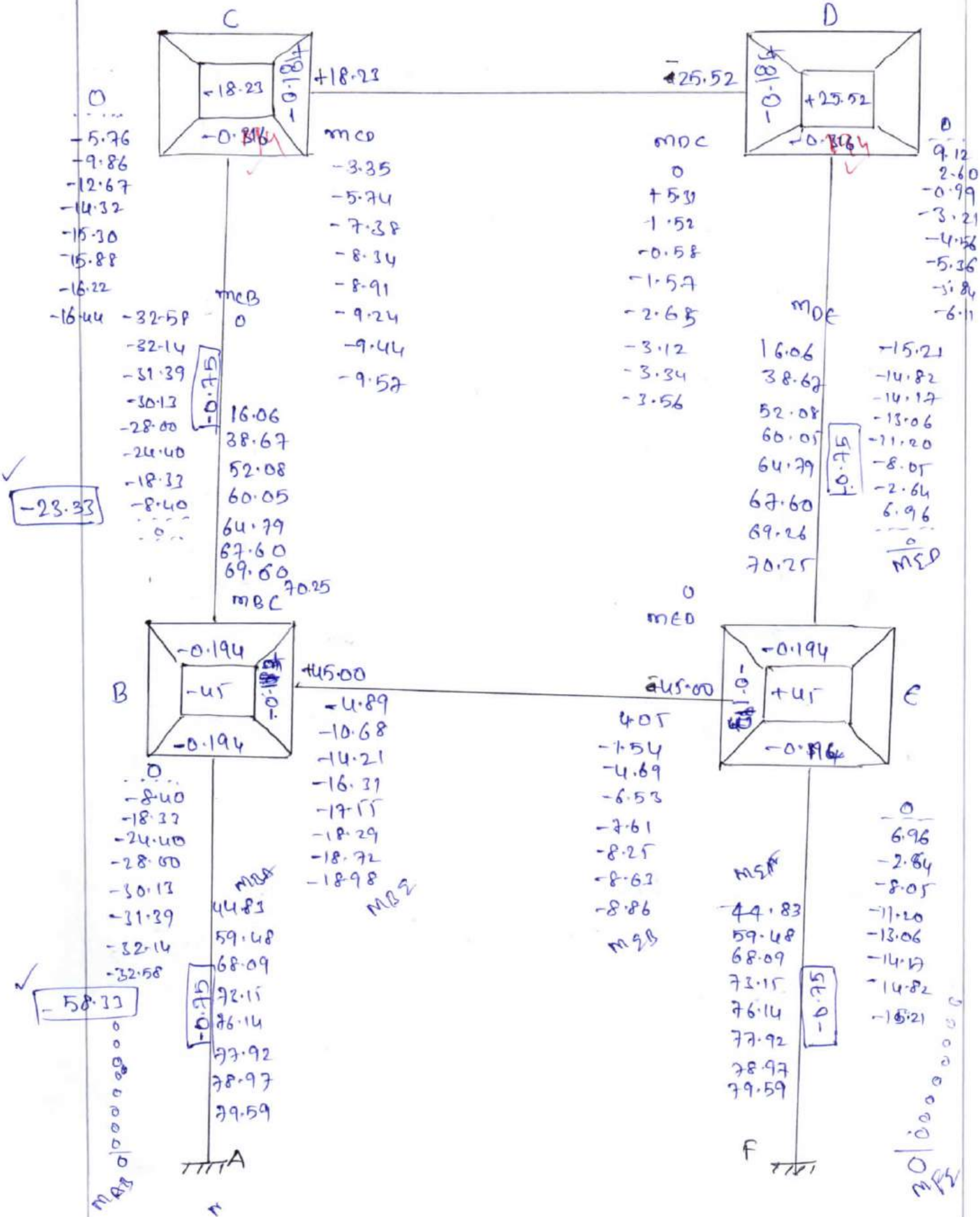
$$D_{BA} = -1.5 \left[\frac{K_{BA}}{K_{BA} + K_{CB} + K_{DE} + K_{EF}}\right] = -1.5 \times \left[\frac{1/3.5}{(1/3.5) + (1/3.5) + (1/3.5) + (1/3.5)}\right]$$

$$= -1.5 \times 0.25 = -0.375$$

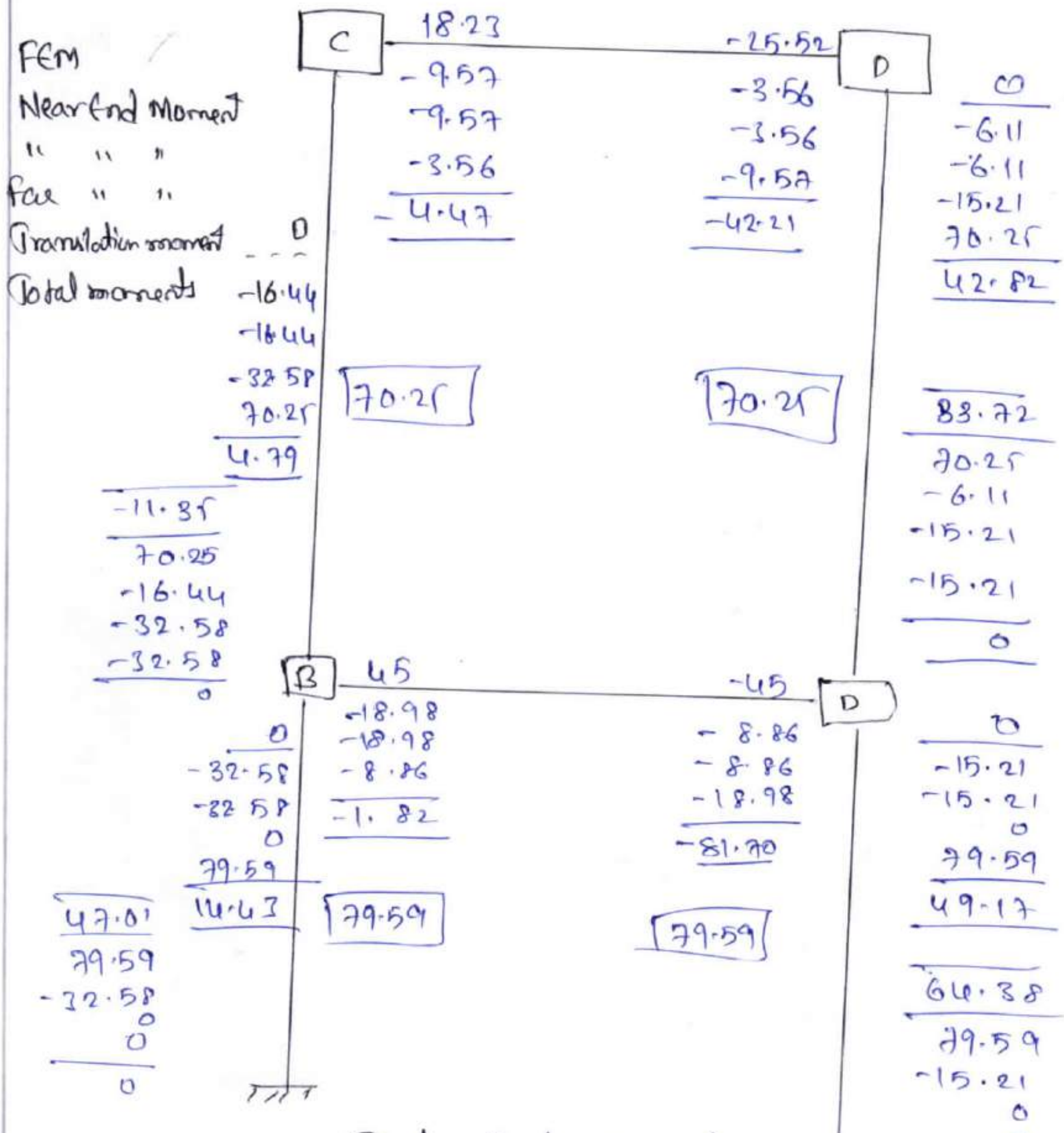
$D_{EF} = -\underline{\underline{0.375}}$ $\quad D_{BA} = D_{EF} = D_{EF} = D_{DE} = 0.375$

$$D.F = 0.75 \Rightarrow \left[-(0.375 + 0.375)\right]$$

4) Kani's Iteration Table:



Computation of rotation & Sway (translation moment)



Final End Moments

Storey Moments :

Storey - 2 : Due to lateral load of 20 kN @ C,

Storey shear $V_{r2} = -20$ kN

Storey moment $= M_{rF} = -20 \times \frac{3.5}{2} = -23.33$ kN·m

Storey - 1 : lateral loads @ B & C

Storey shear $V_{r1} = -20 - 30 = -50$ kN

Storey moments $M_{rF} = \frac{-50 \times 3.5}{3} = -58.33$ kN·m

Cycle-1 :

Joint C : The rotation moment & translation moments are assumed to be zero to start with.

Sum of joint moments = 18.23 kN·m

" of rotation " @ end D = 0 [assumed]

" " " " @ end B = 0 "

" " translation of column CB below = 0 "

Total = 18.23 kN·m

$M_{RCD} = (-0.184)(18.23) = -3.35$ kN·m

$M_{RCB} = (-0.316)(18.23) = -5.761$ kN·m

Joint D :

Sum of joint moments = -25.52 kN·m

" " rotation " @ C = -3.35 kN·m

" " " " @ E = 0 (assumed)

" " translation " column DE = 0 "

Total = -28.87 kN·m

$M_{RDC} = (-0.316) \times (-28.87) = 9.12$ kN·m

$M_{RDE} = (-0.184) \times (-28.87) = 5.31$ kN·m

Joint E :

Sum of joint moments = -45 kN·m

" " rotation " @ D = 5.31 "

" " " " @ B = 0 (assumed)

" " " " @ A = 0 "

Sum of translation " of column EF below = 0 "

" " " " " ED above = 0 "

Total = -39.69 kN·m

$\therefore$ $M_{RED} = (-0.194)(-39.69) = 7.70$ kN.m

$M_{REB} = (-0.113)(-39.69) = 4.48$ kN.m

$M_{REF} = (-0.194)(-39.69) = 7.70$ kN.m

Joint B :

Sum of joint moments = 45.0 kN.m

" " rotation " @ end E = 4.48 "

" " " " @ end C = -5.76 "

" " " " @ end A = 0 kN.m (assumed)

" " translation " column BC below = 0 " "

" " " " " BA " = 0 " "

Total = 43.72 kN.m

$M_{RBE} = (-0.113)(43.72) = -4.94$ kN.m

$M_{RBC} = M_{RBA} = (-0.194)(43.72) = -8.48$ kN.m

MBC (-0.194)(43.72) = -8.4 = BA

Calculation of Storey Moments:

storey - 2:

Storey moments: = -23.33 kN.m

Rotation moments of columns @ end C of column BC = -5.76 kN.m

" " " " " " D " " DE = 5.31 "

" " " " " " B " " BC = -8.48 "

" " " " " " E " " DE = 7.70 "

Total = -24.56 kN.m

$M_{TBC} = M_{TED} = (-0.75) \times (-24.56) = 18.42$ kN.m

Storey - 1 :

Storey moments = -58.33 kN.m

" " " " " " B " " AB = -8.48 kN.m

" " " " " " E " " FE = 7.70 "

" " " " " " A " " AB = 0 " (assumed)

" " " " " " F " " FG = 0 " "

Total = -59.11 kN.m

$M_{TAB} = M_{TFE} = (-0.75) \times (-59.11) = 44.33$ kN.m

UNIT-1:Objective Questions:

1.In moment distribution method ,the sum of distribution factors of all the members meeting at any joint

a) zero

b) <1

c)1

d) >1

2. In the slope deflection equations, the deformations are caused by

a) shear force

b) Bending moment

c) axial force

d) none

3. Which of the following methods of structural analysis is a force method ?

a) slope deflection method

b) column analogy method

c) moment distribution method

d) none of the above

4. Which of the following methods of structural analysis is a displacement method ?

a) moment distribution method

b) column analogy method

c) three moment equation

d) none of the above

5. In the displacement method of structural analysis, the basic unknowns are

a) displacements

b) force

c) displacements and forces

d) none of the above

6. Which of the following is not the displacement method?

a) Equilibrium method

b) Column analogy method

c) Moment distribution method

d) Kani's method

7. Select the correct statement

a) Flexibility matrix is a square symmetrical matrix

b) Stiffness matrix is a square symmetrical matrix

c) both (a) and (b)

d) none of the above

8. When a load crosses a through type Pratt truss in the direction left to right, the nature of force in any diagonal member in the left half of the span would

a) change from compression to tension

b) change from tension to compression
c) always be compression
d) always be tension

9.The ratio of a stiffness of a beam at a joints with one end hinged and other end fixed is

a) 1/2
b) 3/4
c) 1
d) 4/3

10Moment distribution method is developed by

a) Maney

b) Hardy cross

c) Muller

d) Gumbel

Fill in the blanks:

1. If the free end of acantilever of span l and flexural rigidity EI undergoes a unit displacement , the bending moment induced at the fixed end --------------[IES-08]
2.The factor by which the applied moment is multiplied to obtain the end moment of any member is known ---
3.In simply suppoted beam carrying point load at centre ,slope is maximum at -------------
4.In simply suppoted beam carrying point load at centre deflection is maximum at -------------
5. In cantilver deflection is maximum at ----------------------
6. Moment distribution method is developed by -------------------------
7.The relative stiifness of member with far end fixed is---------------------
8.The relative stiifness of member with far end hinged is---------------------
9.. A single bay single storey portal fgrames has a hinged left support and a fixed right support .it is loaded with udl on th e beam .The deformation of the frame is------------------[GATE-95]
10.the factor by which the moment at simply supported end is multiplied to get moment carried over to the other end is-----------------

Key :

1.c 2.b, 3.b,4.a,5.a ,6.b,7.c,8.a 9.b, 10.b
Fill in the blanks: 1.6EI/L^2 , 2. Distribution factor 3.ends 4. Centre 5. Free end 6.Hardy cross 7.I/L 8.3I/4L 9- it would sway to the left side, 10.Carry over factor.
UNIT-2:Objective Questions:

SLOPE DEFLECTION METHOD

Analysis of Single Bay - single storey Portal Frames by Slope Deflection Method Including Side Sway. Shear force & B.M.D Elastic Curve.

When portal frames sway, the joint translations become additional unknown quantities. Some additional conditions will, therefore, be required for analyzing the frame.

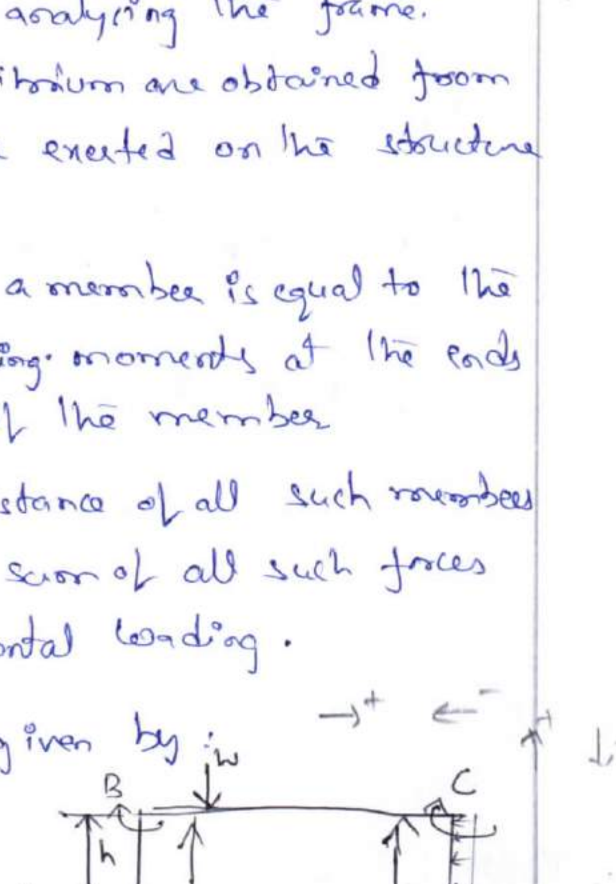
→ The additional conditions of equilibrium are obtained from the consideration of the shear force exerted on the structure by the external loading.

→ The horizontal shear exerted by a member is equal to the algebraic sum of the external loading moments at the ends

→ divided by the length of the member

→ Thus the horizontal shear resistance of all such members can be found & the algebraic sum of all such forces must balance the external horizontal loading.

The horizontal reactions were given by :



$$H_A = \frac{M_{AB} + M_{BA} - Ph}{L_1} \rightarrow (1)$$

$$H_D = \frac{M_{CD} + M_{DC} + \frac{1}{2} w l_2^2}{L_2} \rightarrow (2)$$

ΣM (end moments) are clockwise [assume]

$\Sigma H = 0$ — (3)

$\therefore H_A + H_D + P - wl_2 = 0$

The above equation is also known as shear equation.

By substituting values we get.

$$\frac{M_{AB}+M_{BA}-Ph}{L_1} + \frac{M_{CD}+M_{DC}+\frac{1}{2}wl_2^2}{l_2} + P - wl_2 = 0 \quad \text{--- (4)}$$

eqn-(4) gives the general expression of shear eqn.

If; $P = 0$

$$\frac{M_{AB}+M_{BA}}{L_1} + \frac{M_{CD}+M_{DC}+\frac{1}{2}wl_2^2}{l_2} - wl_2 = 0$$

$$\& \quad \frac{M_{AB}+M_{BA}}{L_1} + \frac{M_{CD}+M_{DC}}{l_2} - \frac{wl_2}{2} = 0 \quad \text{--- 4(a)}$$

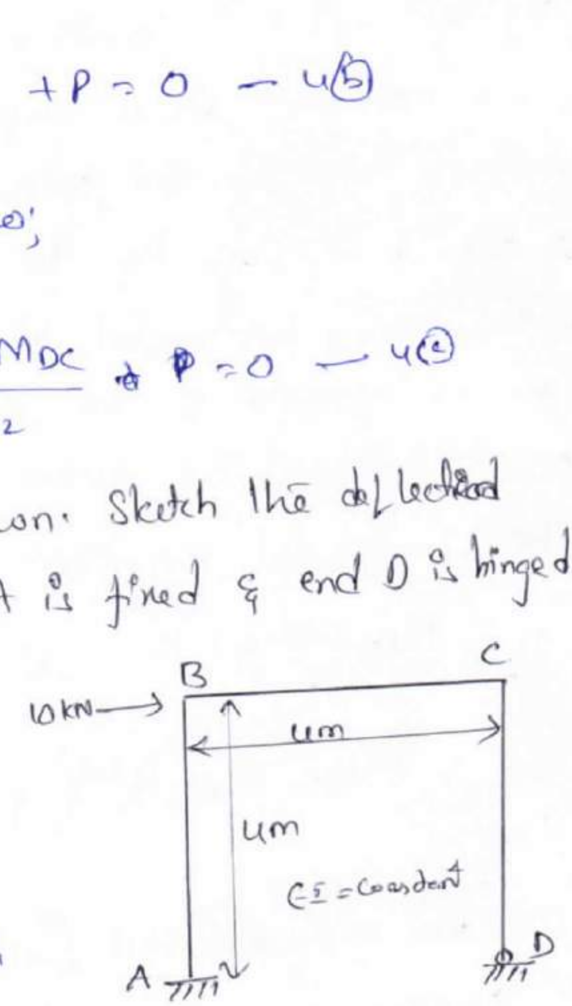
If; $w = 0$; from eq.(4)

$$\frac{M_{AB}+M_{BA}-Ph}{L_1} + \frac{M_{CD}+M_{DC}}{L_2} + P = 0 \quad \text{--- 4(b)}$$

If both P & w are zero;

$$\frac{M_{AB}+M_{BA}}{L_1} + \frac{M_{CD}+M_{DC}}{l_2} + \cancel{P} = 0 \quad \text{--- 4(c)}$$

Pbs:- Analyse the Portal frame shown. Sketch the deflected shape of the frame. The end A is fixed & end D is hinged.



Sol:- a) There is no fixed End moments since the load acting at joint.

b) Slope deflection equations:

The unknowns are θ_B, θ_C, θ_D & the joint translation δ.

The horizontal moment of B & C will equal to δ.

$$R = \delta/4$$

$$M_{AB} = M_{FAB}^{0} + \frac{2EI}{L}(2\theta_A^{0}+\theta_B-3R)$$

$$M_{AB} = 2EK(\theta_B - 3R) \quad \text{--- (1)}$$

$$M_{BA} = 2EK(2\theta_B - 3R) \quad \text{--- (2)}$$

$$M_{BC} = 2EK(2\theta_B + \theta_C) \quad \text{--- (3)}$$

$$M_{CB} = 2EK(2\theta_C + \theta_B) \quad \text{--- (4)}$$

$$M_{CD} = 2EK(2\theta_C + \theta_D - 3R) \quad \text{--- (5)}$$

$$M_{DC} = 2EK(2\theta_D + \theta_C - 3R) \quad \text{--- (6)}$$

$\therefore M_{DC} = 0$, θ_D can be expressed in terms of θ_C.

$$M_{DC} = 0 = 2\theta_D + \theta_C - 3R$$

(or)

$$\theta_D = \frac{3R - \theta_C}{2}$$

(c) Equilibrium Equations:

(a) joint B : $M_{BA} + M_{BC} = 0$

$$2EK(2\theta_B - 3R) + 2EK(2\theta_B + \theta_C) = 0$$

$$8EK\theta_B - 6EKR + 2EK\theta_C = 0$$

(or) $4\theta_B - 3R + \theta_C = 0$ —— (7)

(or) $M_{CB} + M_{CD} = 0$.

(a) joint C,

$$2EK(2\theta_C + \theta_B) + 2EK(2\theta_C + \theta_D - 3R) = 0$$

$$4\theta_C + \theta_B + \theta_D - 3R = 0$$

Substituting the values of $\theta_D = \frac{3R - \theta_C}{2}$,

$$\therefore 4\theta_C + \theta_B + \frac{3R - \theta_C}{2} - 3R = 0$$

$$\therefore 7\theta_C + 2\theta_B - 3R = 0 \quad \text{—— (8)}$$

(d) Shear Equations:

$$\frac{M_{BA} + M_{AB}}{4} + \frac{M_{CD}}{4} + P = 0$$

$$\frac{2EK(2\theta_B - 3R) + 2EK(\theta_B - 3R)}{4} + \frac{2EK(2\theta_C + \theta_D - 3R)}{4} + 10 = 0$$

$$\frac{4EK}{4}\left[2\theta_B - 3D + \cancel{2}\theta_B - 3D\right] + \frac{\cancel{2}EK}{4}\left[2\theta_C + \theta_D - 3D\right] + 10 = 0$$

$\cancel{+10} = \cancel{0}$

$3\theta_B - 6D + 2\times$

$$3\theta_B - 6R + \frac{20}{EK} + 2\theta_C + \left(\frac{3R - \theta_C}{2}\right)_{\theta_D} - 3R = 0$$

$$6\theta_B - 15R + \frac{40}{ER} + 3\theta_C = 0$$

From equation 7, $\theta_C = 3R - 4\theta_B$

substituting in eq-(8)

$21R - 28\theta_B + 2\theta_B - 3R = 0 \Rightarrow 18R - 26\theta_B = 0$

$18R = 26\theta_B$

$R = \frac{26}{18}\theta_B$ $R = \frac{13}{9}\theta_B$

$R = 13/9\ \theta_B$ ——— (i)

$\therefore \theta_C = \frac{3\times 13}{9}\theta_B - 4\theta_B = \frac{\theta_B}{3}$ ——— (ii)

$\theta_D = \frac{3\times\frac{13}{9}\times\theta_B - \frac{\theta_B}{3}}{2}$ ——— (iii)

substituting in eqn -(9) we get

$6\theta_B - 15\times\frac{13}{9}\cdot\theta_B + \frac{40}{\Sigma k} + \theta_B = 0$

or $\frac{44}{3}\theta_B = \frac{40}{\Sigma k}$

or $\theta_B = \frac{30}{11Ek} = \frac{2.727}{Ek}$

$\therefore R = \frac{130}{33Ek} = \frac{3.939}{\Sigma k}$

$\therefore \theta_C = \frac{10}{11Ek}$ & $\theta_D = \frac{60}{11Ek}$

e) Final moments :

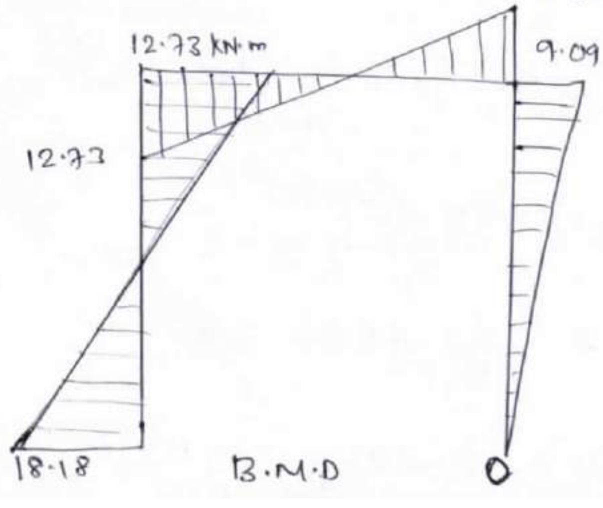
$M_{AB} = 2Ek\left(\frac{30}{11Ek} - \frac{3\times 130}{33Ek}\right) = -18.18$ kN·m

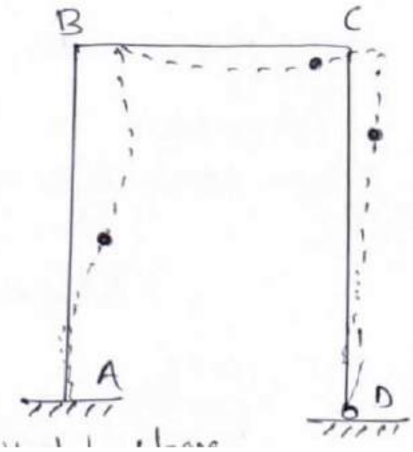
$M_{BA} = 2Ek\left(\frac{2\times 30}{11Ek} - \frac{3\times 130}{33Ek}\right) = -12.73$ kN·m

$M_{BC} = 2Ek\left(\frac{2\times 30}{11Ek} + \frac{10}{11Ek}\right) = 12.73$ kN·m

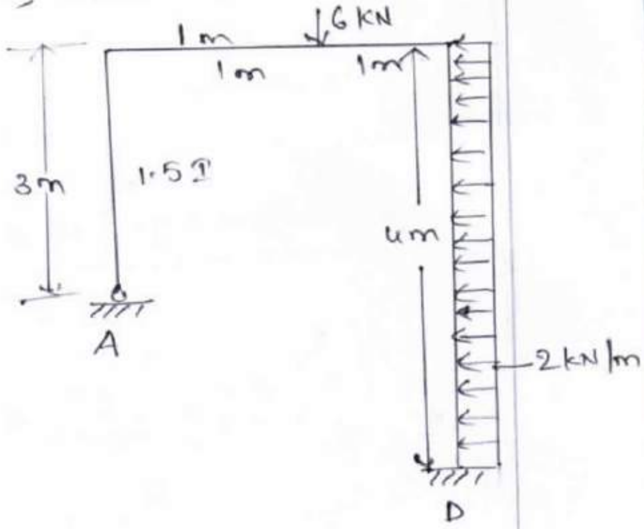
$M_{CB} = 2Ek\left(\frac{20}{11Ek} + \frac{30}{11Ek}\right) = +9.09$ kN·m

$M_{CD} = 2Ek\left(\frac{20}{11Ek} + \frac{60}{11Ek} - \frac{3\times 130}{33Ek}\right) = -9.09$ kN·m





Prb: A portal frame ABCD is hinged at A & fixed at D & has stiff joints at B & C. The loading is shown; Draw the B.M.D & deflected shape of the frame.



a) Fixed End Moments :

$$M_{FBC} = -\frac{6\times 2}{8} = -1.5 \text{ kN.m}$$

$$M_{FCB} = +1.5 \text{ kN.m}$$

$$M_{FCD} = \frac{-2\times 4^2}{12} = -\frac{8}{3} \text{ kN.m}$$

$$M_{FDC} = +8/3 \text{ kN.m}$$

b) Slope deflection equations :

joints B & C move horizontally by δ. There are four unknowns : $\theta_A, \theta_B, \theta_C$ & δ.

$$M_{AB} = \frac{2E\times 3I}{2\times 3}\left(2\theta_A+\theta_B-\frac{3\delta}{3}\right) = EI(2\theta_A+\theta_B-\delta) \quad (1)$$

$$M_{BA} = \frac{2E\times 3I}{2\times 3}\left(2\theta_B+\theta_A-\frac{3\delta}{3}\right) = EI(2\theta_B+\theta_A-\delta) \quad (2)$$

$$\frac{2EI}{L}(2\theta_B+\theta_C) + M_{FBC}$$

$$M_{BC} = \frac{2EI}{2}(2\theta_B+\theta_C) - 1.5 = EI(2\theta_B+\theta_C) - 1.5 \quad (3)$$

$$M_{CB} = \frac{2EI}{2}(2\theta_C+\theta_B) + 1.5 = EI(2\theta_C+\theta_B) + 1.5 \quad (4)$$

$$M_{CD} = \frac{2EI}{4}\left(2\theta_C-\frac{3\delta}{4}\right) - 8/3 = \frac{EI}{2}\left(2\theta_C-\frac{3\delta}{4}\right) - 8/3 \quad (5)$$

$$M_{DC} = \frac{2EI}{4}\left(\theta_C-\frac{3\delta}{4}\right) + 8/3 = \frac{EI}{2}\left(\theta_C-3\delta/2\right) + 8/3 \quad (6)$$

c) Equilibrium Equations :

@ joint B, $M_{BA}+M_{BC}=0$

or $EI(2\theta_B+\theta_A-\delta) + EI(2\theta_B+\theta_C) - \frac{3}{2} = 0$

$$4\theta_B+\theta_A+\theta_C-\delta-\frac{3}{2EI} = 0 \quad (7)$$

@ joint C, $M_{CB}+M_{CD}=0$

$\therefore EI(2\theta_C+\theta_B) + \frac{3}{2} + \frac{EI}{2}\left(2\theta_C-\frac{3\delta}{4}\right) - \frac{8}{3} = 0$

$$3\theta_C + \theta_B - \frac{3}{8}\delta - \frac{7}{6EI} = 0 \quad \text{—} (8)$$

d) Shear equation :

$$\frac{M_{AB} + M_{BA}}{L_1} + \frac{M_{CD} + M_{DC}}{L_2} = \frac{w \cdot L_2}{2}$$

or $$\frac{EI(2\theta_A + \theta_B - \delta) + EI(2\theta_B + \theta_A - \delta)}{3} + \frac{\frac{EI}{2}\left(2\theta_C - \frac{3\delta}{4}\right) - \frac{8}{3} +}{}$$

$$+ \frac{\frac{EI}{2}\left(\theta_C - \frac{3\delta}{4}\right) + \frac{8}{3}}{4} = \frac{2\times 4}{2}$$

or $8\theta_A + 4\theta_B - 4\delta + 8\theta_B + 4\theta_A - 4\delta + 3\theta_c - \frac{9}{8}\delta + \frac{3\theta_c}{2}$

$- \frac{9}{8}\delta = \frac{48}{EI}$

or $12\theta_A + 12\theta_B + \frac{9}{2}\theta_C - \frac{41}{4}\delta = \frac{48}{EI}$

or $12\theta_A + 12\theta_B + \frac{9}{2}\theta_C - \frac{41}{4}\delta = \frac{48}{EI}$ — (9)

The end A is hinged so, $M_{AB} = 0$

i.e; $EI(2\theta_A + \theta_B - \delta) = 0$

or $\theta_B = \delta - 2\theta_A$ —— (10)

Substituting the value of θ_B in eqn - 7.

$4\delta - 8\theta_A + \theta_A + \theta_C - \delta - \frac{3}{2EI} = 0$

or $\theta_C - 7\theta_A + 3\delta - \frac{3}{2EI} = 0$

or $\theta_C = \frac{3}{2EI} + 7\theta_A - 3\delta$ —— (11)

Substituting the value of θ_B in eqn - 7.

$4\delta - 8\theta_A + \theta_C - \delta - \frac{3}{2EI} = 0$

or $\theta_C - 7\theta_A + 3\delta - \frac{3}{2EI} = 0$

or $\theta_C = \frac{3}{2EI} + 7\theta_A - 3\delta$ —— (11)

Substituting the value of θ_B in eqn - 8,

$$3\theta_C + \delta - 2\theta_A - \frac{3}{8}\delta - \frac{7}{6EI} = 0 \quad \text{—(12)}$$

$$\text{or } 3\theta_C - 2\theta_A + \frac{5}{8}\delta - \frac{7}{6EI} = 0$$

Substituting the values of θ_B & θ_C in eqn - 9, we get

$$12\theta_A + 12\delta - 24\theta_A + \frac{27}{4EI} + \frac{63}{2}\theta_A - \frac{27}{2}\delta - \frac{41}{4}\delta = \frac{48}{EI}$$

$$\text{or } \theta_A = \frac{330}{156\,EI} + \frac{47}{78}\delta.$$

Substituting the values of θ_C & θ_A in eq - 12, we get

$$\frac{9}{2EI} + \frac{19 \times 330}{156\,EI} + \frac{47 \times 19}{78}\delta - 9\delta + \frac{5}{8}\delta - \frac{7}{6EI} = 0$$

$$\frac{959}{312}\delta = -\frac{3395}{78EI} \quad (\text{or}) \quad \delta = -\frac{3395}{78EI} \times \frac{312}{959} = -\frac{13580}{959\,EI} \quad \text{—(i)}$$

$$\text{Hence, } \theta_A = \frac{330}{156\,EI} - \frac{47}{78} \times \frac{13580}{959\,EI} = -\frac{6.42}{EI} \quad \text{—(ii)}$$

$$\theta_C = \frac{3}{2EI} - \frac{7 \times 6.42}{EI} + \frac{3 \times 13580}{959\,EI} = -\frac{0.94}{EI} \quad \text{—(iii)}$$

$$\text{\& } \theta_B = \frac{-13580}{959\,EI} + \frac{2 \times 6.42}{EI} = -\frac{1.33}{EI} \quad \text{—(iv)}$$

c) Final moments:

Substituting the values of $\theta_A, \theta_B, \theta_C$ & δ in eqn 2 to 6;

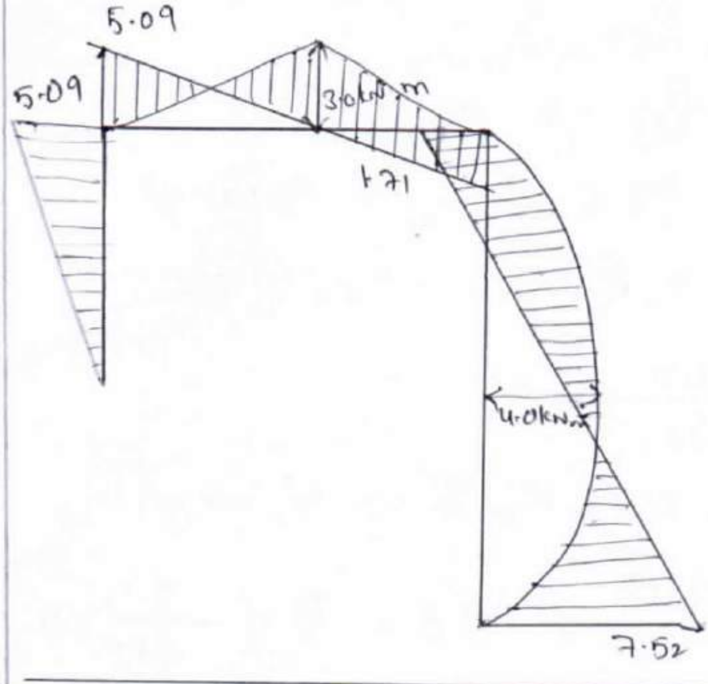
$$M_{BA} = EI\left(\frac{-2 \times 1.33}{EI} - \frac{6.42}{EI} + \frac{13580}{959\,EI}\right) = +5.09 \text{ kN.m}$$

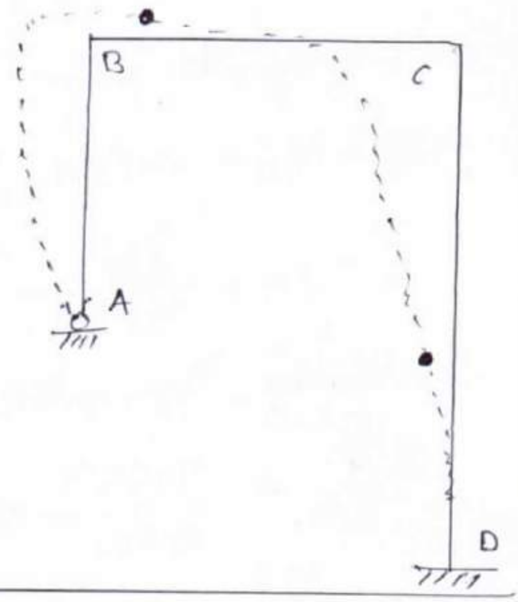
$$M_{BC} = EI\left(\frac{-2 \times 1.33}{EI} - \frac{0.94}{EI}\right) - 1.5 = -5.09 \text{ kN.m}$$

$$M_{CB} = EI\left(\frac{-2 \times 0.94}{EI} + \frac{1.33}{EI}\right) + 1.5 = -1.71 \text{ kN.m}$$

$$M_{CD} = \frac{EI}{2}\left(\frac{-2 \times 0.94}{EI} + \frac{3}{4} \times \frac{13580}{959\,EI}\right) - \frac{8}{3} = +1.71 \text{ kN.m}$$

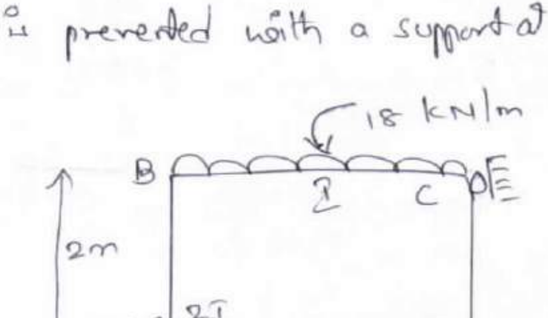
$$M_{DC} = \frac{EI}{2}\left(\frac{-0.94}{EI} + \frac{3 \times 13580}{4 \times 959\,EI}\right) + 2.67 = +7.52 \text{ kN.m}$$





Laterally Restrained Portal frame:

Prb: Analyse the frame shown below by slope-deflection method. The lateral movement of the frame is prevented with a support at joint C. Draw S.F.D & B.M.D



Sol:- 1) Fixed - End Moments:

$M_{FAB} = + \frac{WL}{8} = \frac{-25 \times 4}{8}$

$\rightarrow^{+} \overleftarrow{\ }$

$= ^{+}12.5$ kN.m

$M_{FBA} = -12.5$ kN.m

$M_{FBC} = \frac{WL^2}{12} = \frac{18 \times 4^2}{12}$

$= 24$ kN.m

$M_{FCB} = -24$ kN.m

2) Slope-deflection Equations:

Support A & D are fixed $\therefore$ θ_A & $\theta_D = 0$

Horizontal sway or movement is prevented

$\psi = 0$ $[\delta = 0]$

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= 12.5 + \frac{2E(2I)}{4}(2\theta_A + \theta_B)$$

$$M_{AB} = +12.5 + EI\theta_B \quad \text{—— (1)}$$

$$M_{BA} = -12.5 + \frac{2E(2I)}{4}(2\theta_B + \theta_A)$$

$$= -12.5 + 2EI\theta_B \quad \text{—— (2)}$$

$$M_{BC} = 24 + \frac{2E(I)}{4}(2\theta_B + \theta_C)$$

$$= 24 + EI\theta_B + 0.5EI\theta_C \quad \text{—— (3)}$$

$$M_{CB} = -24 + \frac{2EI}{4}(2\theta_C + \theta_B)$$

$$= -24 + EI\theta_C + 0.5EI\theta_B \quad \text{—— (4)}$$

$$M_{CD} = 0 + \frac{2EI}{4}(2\theta_C + \theta_D)$$

$$= EI\theta_C \quad \text{—— (5)}$$

$$M_{DC} = 0 + \frac{2EI}{4}(2\theta_D + \theta_C)$$

$$= 0.5EI\theta_C \quad \text{—— (6)}$$

3) Joint Conditions / Equilibrium Equations:

Joint B: $M_{BA} + M_{BC} = 0$

Joint C: $M_{CB} + M_{CD} = 0$

$$-12.5 + 2EI\theta_B + 24 + EI\theta_B + 0.5EI\theta_C \quad \text{—— (7)}$$

$$-24 + EI\theta_C + 0.5EI\theta_B + EI\theta_C \quad \text{—— (8)}$$

$2EI\theta_B + 0.5EI\theta_B = 24$ (or) $3EI\theta_B + 0.5EI\theta_C = -11.5$ — (7)

$0.5EI\theta_B + 2EI\theta_C = 24$ — (8)

Multiply 4 by eq-(7)

$4[-12.5 + 2EI\theta_B + 24 + EI\theta_B + 0.5EI\theta_C] = 0$

(or)

$4[3EI\theta_B + 0.5EI\theta_C - 11.5] = 0$

$12EI\theta_B + 2EI\theta_C - 46 = 0$

$12EI\theta_B + 2EI\theta_C = 46$ — (9)

Eqn-(8) & Eqn-(9)

$12EI\theta_B + \cancel{2EI\theta_C} = -46$

$0.5EI\theta_B + \cancel{2EI\theta_C} = 24$

$11.95\,\theta_B EI = -70$

$\theta_B =$

$\theta_B EI = -70/11.95 = -6.087$

$\boxed{\theta_B = \frac{6.087}{EI}}$ ——— (10)

Substitute θ_B in (8) eq

$0.5EI\theta_B + 2EI\theta_C = 24$

$0.5\cancel{EI} \times \frac{6.087}{\cancel{EI}} + 2EI\theta_C = 24$

$-1.043\,\theta_C EI = 24$

$-\theta_C EI = 24/1.0431$

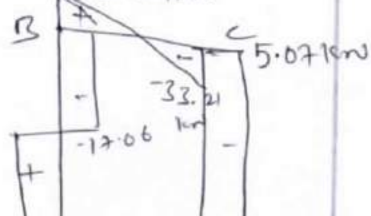
$$0.5\,EI\left[\frac{-6.087}{EI}\right] + 2EI\,\theta_C = 24$$

$$2EI\,\theta_C = 24 + 3.044$$

$$\theta_C = \frac{27.044}{2EI}$$

$$\boxed{\theta_C = \frac{13.52}{EI}} \quad \text{—— (11)}$$

Substituting (10) & (11) in slope deflection equations to get final moments.



S.F.D

$$M_{AB} = 12.5 + EI\left[\frac{-6.087}{EI}\right] = +6.413 \text{ kN·m}$$

$$M_{BA} = -12.5 + 2EI\left[\frac{-6.087}{EI}\right] = -24.67 \text{ kN·m}$$

$$M_{BC} = 24 + EI\left[\frac{-6.087}{EI}\right] + 0.5EI\left[\frac{13.52}{EI}\right] = 24.67 \text{ kN·m}$$

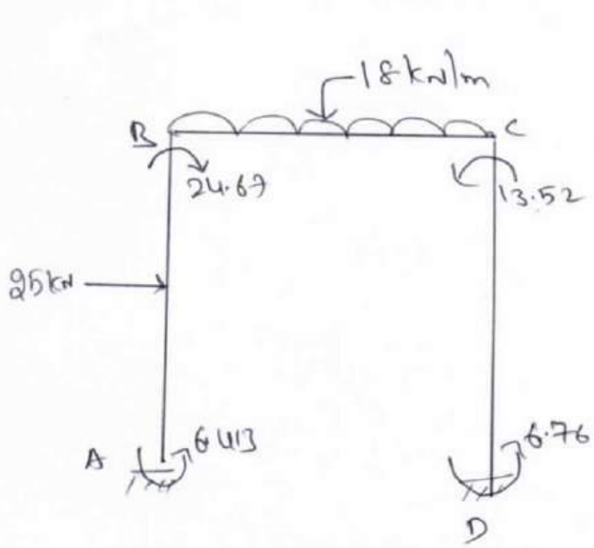
$$M_{CB} = -24 + EI\left[\frac{13.52}{EI}\right] + 0.5EI\left[\frac{-6.087}{EI}\right] = -13.52 \text{ kN·m}$$

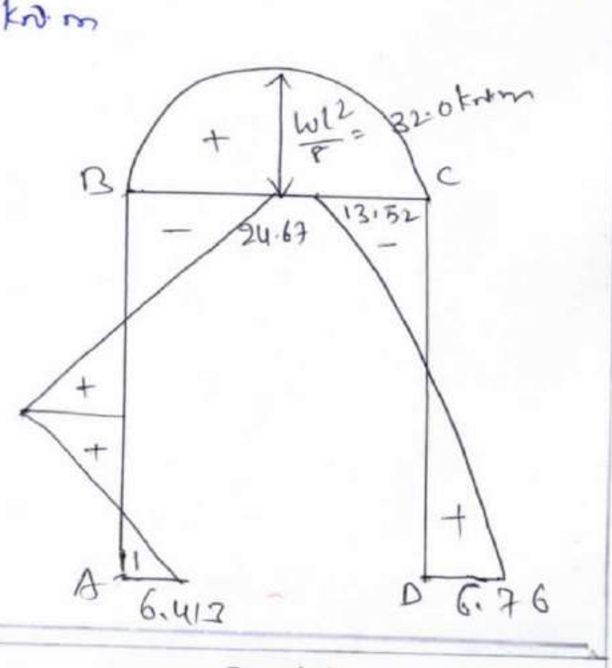
$$M_{CD} = 0 + EI\left[\frac{13.52}{EI}\right] = 13.52 \text{ kN·m}$$

$$M_{DC} = 0.5EI\left[\frac{13.52}{EI}\right] = 6.76 \text{ kN·m}$$

$R_B \cdot 4 - 72 = 0$

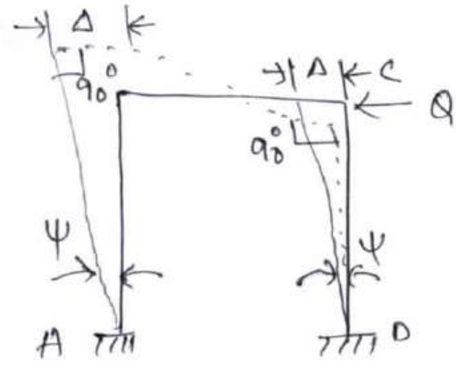
$R_B = 72/4$



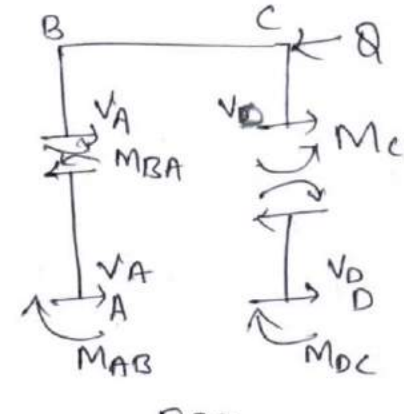


B.M.D

Analysis of frames with sway:



Deflected shape of frame with sideway



FBD

$\rightarrow^{+}$ $\leftarrow^{-}$ $\uparrow^{+}$ $\downarrow^{-}$

The horizontal displacement of the top of both the columns are the same as Δ. This displacement induces a counter-clockwise chord rotation in both the columns of the frame & is given as $\psi = \frac{\Delta}{h}$ (where 'h' is height of the column.

There are three independent displacements in the frame i.e, rotation at joint B (θ_B) & joint C (θ_C) & the chord rotation ψ. ~~to~~ Three equilibrium equations are need to ~~ana~~ analyse the frame.

The two equations are due to moments acting @ joints B & C. The third equation which is based on equilibrium of forces need to develop. This eqn is called shear equation.

The shear equation is obtained by summing the forces acting on the free body of the girder in the horizontal direction.

for the girder BC,

V_A - shear in the column AB

V_D - shear in the column in CD.

$\leftarrow + \Sigma f_x = 0,$

$$Q - V_A - V_D = 0 \quad — (1)$$

$\circlearrowright + \Sigma M_A = 0$

$$V_A h - M_{AB} - M_{BA} = 0$$

$$\therefore \boxed{V_A = \frac{M_{AB} + M_{BA}}{h}} \quad — (2)$$

Ily; V_D ~~MCD~~ in column CD by summing moment about point D. $\circlearrowright + \Sigma M_D = 0$

$$V_D h - M_{CD} - M_{DC} = 0$$

$$\boxed{V_D = \frac{M_{CD} + M_{DC}}{h}} \quad — (3)$$

The third equilibrium equation is

$$V_A + V_D = 0$$

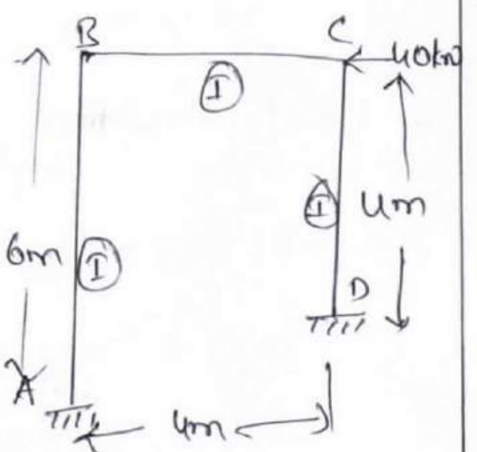
$$Q - V_A - V_D = 0$$

$$\boxed{Q - \frac{(M_{AB} + M_{BA})}{h} - \frac{(M_{CD} + M_{DC})}{h} = 0}$$ is

the shear equation.

Analyse the frame shown, by slope deflection method. EI is constant for all members. Draw SFD & BMD.

Sol:- The frame is fixed at A & D.



$\theta_A = \theta_D = 0.$

The frame sways to left

The chord rotation of AB is

$\psi_{AB} = \frac{\Delta}{6}$; $\psi_{CD} = \frac{\Delta}{4}$

$\psi_{AB} = \frac{\Delta}{6}$

$\psi_{CD} = \frac{\Delta}{4}$

$\frac{\Delta}{4} \times \psi_{AB} = \frac{\Delta}{6}\,\psi_{CD}$

$\psi_{CD} = \frac{6}{\Delta} \times \frac{\Delta}{4} \times \psi_{AB}$

$= \frac{6}{4}\psi_{AB}$

$\psi_{CD} = 1.5\,\psi_{AB}$

$\psi_{CD} = 1.5\,\psi_{AB}.$

There are no fixed-end moments since no loads acting on any member.

2. Slope deflection equations:

$M_{AB} = 0 + \frac{2EI}{L}(2\theta_A + \theta_B - 3\psi_{AB})$

$= 0 + \frac{2EI}{6}(2\theta_A + \theta_B - 3\psi_{AB})$

$= 0.33\,EI\,\theta_B - EI\,\psi_{AB}$ —— (1)

$M_{BA} = 0 + \frac{2EI}{6}(2\theta_B + \theta_A - 3\psi_{AB})$

$= 0.67\,EI\,\theta_B - EI\,\psi_{AB}$ —— (2)

$M_{BC} = 0 + \frac{2EI}{4}(2\theta_B + \theta_C)$

$= EI\,\theta_B + 0.5\,EI\,\theta_C$ —— (3)

$M_{CB} = 0 + \frac{2EI}{4}(2\theta_C + \theta_B)$

$= EI\,\theta_C + 0.5\,EI\,\theta_B$ —— (4)

$M_{CD} = 0 + \frac{2EI}{4}(2\theta_C + \theta_D - 3\psi_{CD})$

$\therefore \psi_{CD} = 1.5 \times \psi_{AB}$

$= EI\,\theta_C - 1.5\,EI\,\psi_{CD}$

$EI\,\theta_C - 2.25\,EI\,\psi_{AB}$ —— (5)

$$M_{DC} = \frac{2EI}{4}(2\theta_D + \theta_C - 3\psi_{CD}) \quad \text{——} (6)$$

$$= 0.5 EI\theta_C - 1.50 EI\,\psi_{CD}$$

$\psi_{CD} = 1.5\,\psi_{AB}$

$$= 0.5 EI\theta_C - 2.25 EI\,\psi_{AB} \quad \text{——} (7)$$

3. Joint Conditions | Equilibrium Equations :

Joint B : $M_{BA} + M_{BC} = 0$

$$0.67 EI\,\theta_B - EI\,\psi_{AB} + EI\theta_B + 0.5 EI\theta_C = 0$$

$$1.67\,\theta_B + 0.5\theta_C - \psi_{AB} = 0 \quad \text{——} (8)$$

Joint C : $M_{CB} + M_{CD} = 0$

$$EI\,\theta_C + 0.5 EI\theta_B + EI\theta_C - 2.25 EI\,\psi_{AB} = 0$$

$$0.5\,\theta_B + 2\theta_C - 2.25\,\psi_{AB} = 0 \quad \text{——} (9)$$

Shear equation

$$Q - \frac{(M_{AB}+M_{BA})}{L} - \frac{(M_{CD}+M_{DC})}{L} = 0$$

$$40 - \frac{[(0.33 EI\theta_B - EI\psi_{AB}) + (0.67 EI\theta_B - EI\psi_{AB})]}{6} - \frac{[(EI\theta_C - 2.25 EI\,\psi_{AB}) + (0.5 EI\theta_B - 2.25 EI\,\psi_{AB})]}{4} = 0$$

$$0.167 EI\,\theta_B + 0.375 EI\,\theta_C - 1.458\,EI\,\psi_{AB} = 40 \quad \text{——} (10)$$

from eq (8)

$$\psi_{AB} = 1.67\,\theta_B + 0.5\,\theta_C \quad \text{——} (11)$$

put eq (11) in (9)

$$0.5\theta_B + 2\theta_C - 2.25[1.67\,\theta_B + 0.5\theta_C] = 0$$

$-3.2575\,\theta_B + 0.875\,\theta_C = 0$ — (12)

put eq-(11) in (10)

$0.167\,EI\,\theta_B + 0.375\,EI\,\theta_C - 1.658\,EI\left[1.67\theta_B + 0.5\theta_C\right] = 40$

$-2.268\,\theta_B - 0.354\,\theta_C = \frac{40}{EI}$ — (13)

Multiplying eq-(12) × 0.354,

$-1.153\,\theta_B + 0.30975\,\theta_C = 0$ — (14)

multiplying eq-13 by 0.875,

$-1.9845\,\theta_B - 0.30975\,\theta_C = \frac{35}{EI}$ — (15)

adding eq-(15) & (14)

$-3.1375\,\theta_B = 35\,EI$

$$\boxed{\theta_B = -\frac{11.16}{EI}} \quad \text{— (16)}$$

Substituting (16) in (12)

$-3.2575 \times \left(+\frac{11.16}{EI}\right) + 0.875\,\theta_C = 0$

$$\boxed{\theta_C = -\frac{41.55}{EI}} \quad \text{— (17)}$$

Substituting θ_B & θ_C in eq-(11)

$\psi_{AB} = -1.67\,EI\left[-\frac{11.16}{EI}\right] + 0.5\left[-\frac{41.55}{EI}\right]$

$\psi_{AB} = \left(-\frac{18.6372}{EI}\right) - \left(\frac{20.775}{EI}\right)$

$\psi_{AB} = -\frac{39.4122}{EI}$

$$\psi_{CD} = 1.5\ \psi_{AB}$$

$$= 1.5 \times \left[-\frac{39.4122}{EI} \right]$$

$$\psi_{CD} = -\frac{59.1183}{EI}$$

Check: substituting the values of $\cancel{\psi_{CD}}$, ψ_{AB} & θ_B, θ_C in (9)

$$L.H.S = 0.5\left[-\frac{11.16}{EI}\right] + 2\left[-\frac{41.55}{EI}\right] - 2.25\left[-\frac{39.4122}{EI}\right]$$

$$= \underline{\underline{0}}$$

L.H.S = R.H.S

Hence ok

End Moments / Final Moments:

$$M_{AB} = 0.33\,EI\left[\frac{-11.16}{EI}\right] - EI\left[-\frac{39.4122}{EI}\right] = 35.73\ \text{kN·m}$$

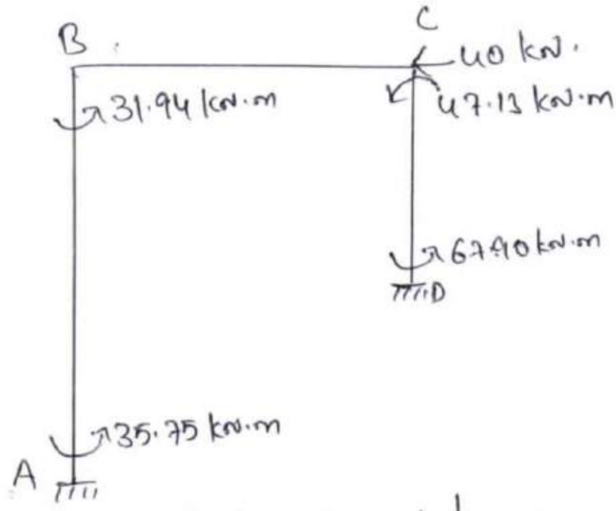
$$M_{BA} = 0.67\,EI\left[\frac{-11.16}{EI}\right] - EI\left[-\frac{39.4122}{EI}\right] = 31.94\ \text{kN·m}$$

$$M_{BC} = EI\left[\frac{-11.16}{EI}\right] + 0.5\,EI\left[\frac{-41.55}{EI}\right] = -31.94\ \text{kN·m}$$

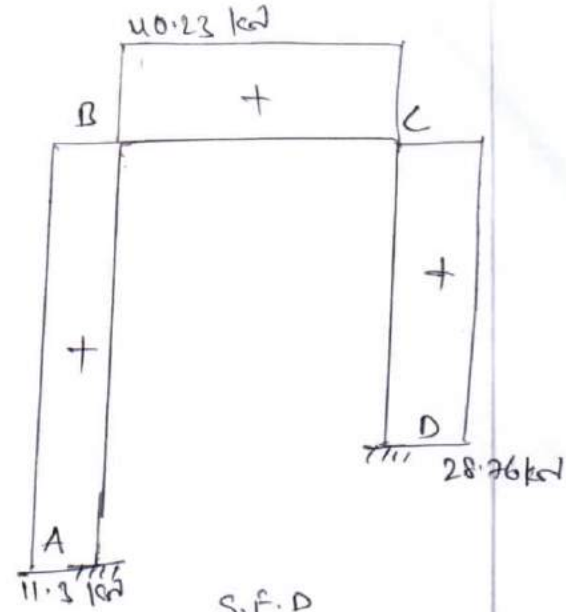
$$M_{CB} = EI\left[\frac{-41.55}{EI}\right] + 0.5\,EI\left[\frac{-11.6}{EI}\right] = -47.13\ \text{kN·m}$$

$$M_{CD} = EI\left[\frac{-41.55}{EI}\right] - 2.25\,EI\left[\frac{-39.4122}{EI}\right] = 47.13\ \text{kN·m}$$

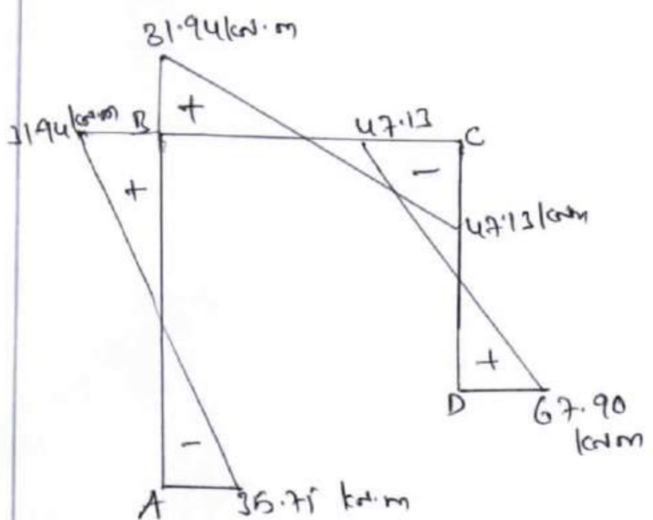
$$M_{DC} = 0.5\,EI\left[\frac{-41.55}{EI}\right] - 2.25\,EI\left[\frac{-39.4122}{EI}\right] = 67.90\ \text{kN·m}$$



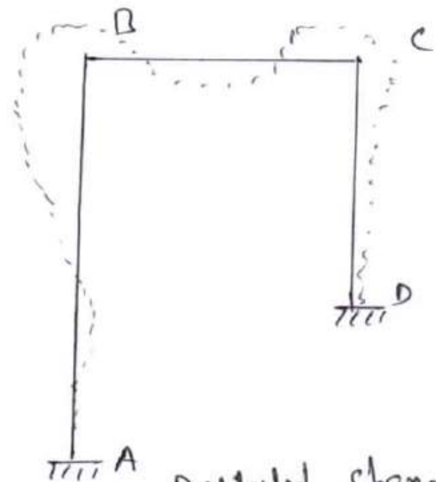
End Moments / Final Moments



S.F.D



B.M.D



Deflected shape

16.975
11.82

S.F.D

$\frac{m_{AB}}{L} = \frac{35.75}{6} = 5.95$

$\frac{m_{BA}}{L} = \frac{31.94}{6} = 5.323$

$m_{AB} + m_{BA} = 10.646$

$\frac{46.9}{4} = 11.73$

$\frac{47.13}{4} = 11.78$

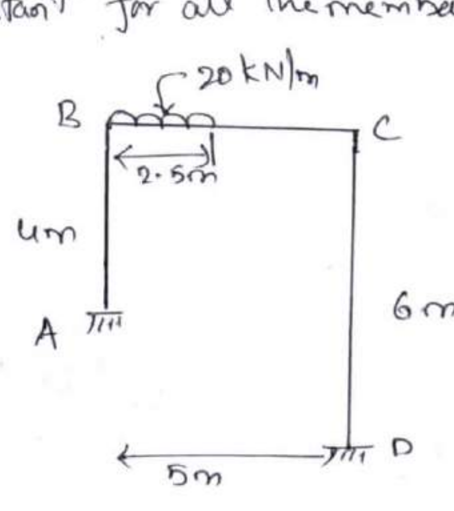
11.73 + 11.78

=>

Slope - Deflection - Method for Sway frames:

Prob: Analyse the frame by SDM. EI is constant for all the members.

Sol: A & D are fixed: $\theta_A = \theta_D = 0$



$\psi_{AB} = \frac{\delta}{4}$, $\psi_{CD} = \frac{\Delta}{6}$.

$\psi_{CD} = 0.67\ \psi_{AB}$

1. Fixed End Moments:

$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$

$M_{FBC} = \frac{11 \times 20 \times 5^2}{192} = 28.65$ kN·m

$M_{FCB} = \frac{-5 \times 20 \times 5^2}{192} = -13.02$ kN·m

$\psi_{AB} = \frac{\Delta}{4}$, $\psi_{CD} = \frac{\Delta}{6}$

$\psi_{AB} = \frac{\Delta}{4}$

$\psi_{CD} = \frac{\Delta}{6}$

$\psi_{AB} \times \frac{\Delta}{6} = \frac{\Delta}{4}\psi_{CD}$

$\psi_{CD} = \frac{4}{\Delta} \times \frac{\Delta}{6}\psi_{AB} = \frac{4}{6}\psi_{AB}$

2. Slope - Deflection Equations:

$M_{AB} = \frac{2EI}{4}\left(2\theta_A + \theta_B - 3\psi_{AB}\right) = 0.5EI\theta_B - 1.5EI\psi_{AB}$ — (1)

$M_{BA} = \frac{2EI}{4}\left[2\theta_A + \theta_B - 3\psi_{AB}\right) = EI\theta_B - 1.5EI\psi_{AB}$ — (2)

$M_{BC} = 28.65 + \frac{2EI}{5}(2\theta_B + \theta_C) = 28.65 + 0.8EI\theta_B + 0.4EI\theta_C$ — (3)

$M_{CB} = -13.02 + \frac{2EI}{5}(2\theta_C + \theta_B) = -13.02 + 0.8EI\theta_C + 0.4EI\theta_B$ — (4)

$M_{CD} = \frac{2EI}{6}(2\theta_C + \theta_D - 3\psi_{CD})$

$= 0.67EI\theta_C - EI\psi_{CD} = 0.67EI\theta_C - 0.67EI\psi_{AB}$ — (5)

$M_{DC} = \frac{2EI}{6}(2\theta_D + \theta_C - 3\psi_{CD}) = 0.33EI\theta_C - EI\psi_{CD}$

$= +0.33EI\theta_C - 0.67EI\psi_{AB}$ — (6)

3) Joint Conditions | Equilibrium equations:

Joint B $\therefore$ $M_{BA} + M_{BC} = 0$

Joint C $\therefore$ $M_{CB} + M_{CD} = 0$

Joint B: $EI\theta_B - 1.5EI\psi_{AB} + 28.65 + 0.8EI\theta_B + 0.4EI\theta_C = 0$

$$1.8\theta_B + 0.4\theta_C - 1.5\psi_{AB} = \frac{-28.65}{EI} \quad \text{—} (7)$$

Joint C: $-13.02 + 0.8EI\theta_C + 0.4EI\theta_B + 0.67EI\theta_C - 0.67EI\psi_{AB} = 0$

$$0.4\theta_B + 1.47\theta_C - 0.67\psi_{AB} = \frac{13.02}{EI} \quad \text{—} (8)$$

Shear equation: $0 - \left[\frac{M_{AB} + M_{BA}}{L}\right] - \left[\frac{M_{CD} + M_{DC}}{L}\right] = 0$

$$0 - \frac{(0.5EI\theta_B - 1.5EI\psi_{AB} + EI\theta_B - 1.5EI\psi_{AB})}{6} -$$

$$\frac{(0.6EI\theta_C - 0.67EI\psi_{AB} + 0.33EI\theta_C - 0.67EI\psi_{AB})}{6} = 0 \quad \text{—} (9)$$

$$-0.375EI\theta_B - 0.167EI\theta_B + 0.172\theta_C = 0$$

from eq- 9, $\psi_{AB} = 0.385\theta_B + 0.172\theta_C$ — (10)

put eq-(10) in eq-(7)

$$1.8EI\theta_B + 0.4\theta_C - 1.5EI(0.385\theta_B + 0.172\theta_C) = -\frac{28.65}{EI}$$

$$1.2225\theta_B + 0.142\theta_C = -\frac{28.65}{EI} \quad \text{—} (11)$$

put eq-(10) in eq-(8)

$$0.4\theta_B + 1.47\theta_C - 0.67(0.385\theta_B + 0.172\theta_C) = \frac{13.02}{EI}$$

$$0.14205\theta_B + 1.3548\theta_C = \frac{13.02}{EI} \quad \text{—} (12)$$

Multiplying eq-(11) by 0.14205, $0.1737\theta_B + 0.020\theta_C = \frac{-4.07}{EI}$ (13)

Multiplying eq-(12) by 1.2225, $0.1737\theta_B + 1.656\theta_C = \frac{15.917}{EI}$ (14)

subtracting eq-(14) from eq-(13),

$$-1.636\theta_C = -\frac{19.987}{EI}; \quad \boxed{\theta_C = \frac{12.217}{EI}} \quad \text{—} (15)$$

Substituting eq (15) into eq-(11),

$$1.2225\theta_B + 0.142\left(\frac{12.217}{EI}\right) = \frac{-28.65}{EI}$$

$$\theta_B = -\frac{24.855}{EI} \quad \text{--- (16)}$$

Substituting Eqs. (15) & (16) into Eq. (10)

$$\psi_{AB} = 0.385\left(\frac{-24.855}{EI}\right) + 0.172\left(\frac{12.217}{EI}\right) = \frac{-7.468}{EI} \quad \text{--- (17)}$$

$$\psi_{CD} = 0.67\left(-\frac{7.468}{EI}\right) = -\frac{5}{EI}.$$

$$\psi_{CD} = -\frac{5}{EI}.$$

Check: Substituting Eqns (15) - (17) into Eq. (8)

$$L.H.S = 0.4\left[\frac{-24.855}{EI}\right] + 1.47\left[\frac{12.217}{EI}\right] - 0.67\left[\frac{-7.468}{EI}\right]$$

$$= -\frac{9.942}{EI} + \frac{17.959}{EI} + \frac{5}{EI} = \frac{13.02}{EI} = R.H.S = ok.$$

End moments:

$$M_{AB} = 0.5EI\left(\frac{-24.855}{EI}\right) - 1.5EI\left(\frac{-7.468}{EI}\right) = -1.2255 \text{ kN.m}$$

$$M_{BA} = EI\left[\frac{-24.855}{EI}\right] - 1.5EI\left[\frac{-7.468}{EI}\right] = -13.653 \text{ kN.m}$$

$$M_{BC} = 28.65 + 0.80EI\left[\frac{-24.855}{EI}\right] + 0.4EI\left(\frac{12.217}{EI}\right) = 13.65 \text{ kN.m}$$

$$M_{CB} = -13.02 + 0.80EI\left[\frac{12.217}{EI}\right] + 0.4EI\left[\frac{-24.855}{EI}\right] = -13.188 \text{ kN.m}$$

$$M_{CD} = 0.67EI\left[\frac{12.217}{EI}\right] - EI\left[\frac{-5}{EI}\right] = 13.185 \text{ kN.m}$$

$$M_{DC} = 0.33EI\left[\frac{12.217}{EI}\right] - EI\left[\frac{-5}{EI}\right] = 9.032 \text{ kN.m}.$$

TWO HINGED ARCHES

Introduction :

Arch is subjected to three restraining forces, i) thrust, ii) shear force iii) bending moment.

Arches carries the transverse loading which are frequently vertical. Since the transverse loading at any section normal to the axis of the girder is at an angle to the normal face.

Arches are of two types :

1) Two hinged arches 2) Three hinged arches.

Two-hinged arches have degree of indeterminacy one. the c/s of the arches may be of different shapes of which the commonly used ones are Olar & parabolic.

The analysis is restricted to circular & parabolic arches only.

Arch is a curved member subjected to external loads & supports at the ends.

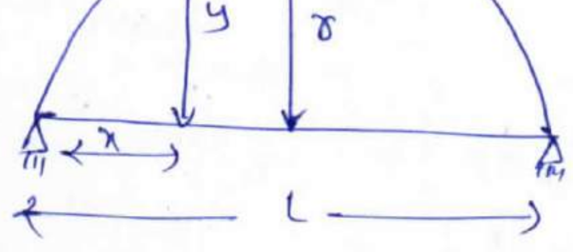
Types of arches : i) 3-hinged arches

ii) Two-hinged arches

Classification of arches based on shaped :

i) Parabolic arch ii) Circular arch iii) Segmental arch

iv) Semi circular arch



L - span

r - central rise

y - arched end at any s/n

x - horizontal distance from support to section.

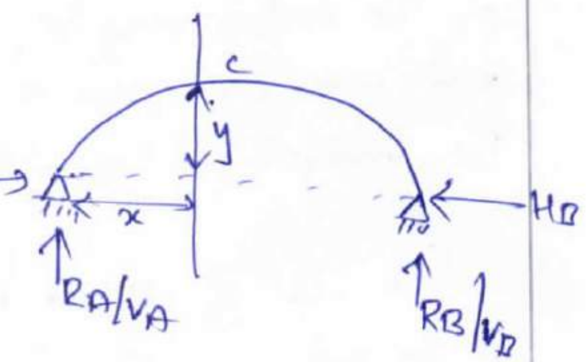
A Two hinged arch is statically indeterminate to single degree, since there are four reaction components to be determined while the no. of equations from statical equilibrium is only three. Considering 'H' to be redundant reaction, it can be found out by the use of Castigliano's theorem..

assuming the horizontal span; unchanged,

$$\frac{\partial U}{\partial H} = 0$$

where U – is the total strain energy stored in the arch. Strain-energy stored due to thrust & shear will be considered negligible in comparision to that due to bending.

$$U = \int \frac{M^2 ds}{2EI}$$



It is statically Indeterminate structure.

$$M_x = V_A \cdot x - H \cdot y$$

$$\boxed{M = V_A \cdot x}$$

$$M_x = M - H \cdot y.$$

$D = R - 3$

$= 4 - 3$

$\underline{\underline{D = 1}}$

$$\frac{\partial U}{\partial H} = 0 \; ; \quad U = \int \frac{m^2 ds}{2EI}$$

ds = length of arch

M = B.M of beam

$$= \int \frac{(M - Hy)^2 ds}{2EI}$$

$$\frac{\partial U}{\partial H} = \int \frac{2(m - H \cdot y)(-y)\, ds}{2EI} = 0$$

$$= \int (-my + Hy^2)\, ds = 0$$

$$\int My \cdot ds = H \int y^2 ds$$

$\int (m - Hy)(-y)\, ds = 0$

$= \int (-(my \cdot ds) + Hy^2 ds) = 0$

$\Rightarrow \int (-my)\, ds + \int Hy^2 ds = 0$

$= \int Hy^2 ds = \int my\, ds$

$\int Hy^2 ds = \int my \cdot ds$

$H \int y^2 ds = \int my\, ds$

$H = \dfrac{\int my\, ds}{\int y^2 ds}$

$$\boxed{H = \frac{\int M y \cdot ds}{\int y^2 ds}}$$

Two higned parabolic arch ⇒ Horizontal Thrust H.

$$y = \frac{4r}{l^2} x (l - x)$$

Normal thrust $= N = H\cos\theta + V\sin\theta$ $(V\sin\theta + H\cos\theta)$

Radial shear $= +(H\sin\theta \mp V\cos\theta) = H\sin\theta - V\cos\theta$

$(V\cos\theta - H\sin\theta)$

Two Hinged Arches:

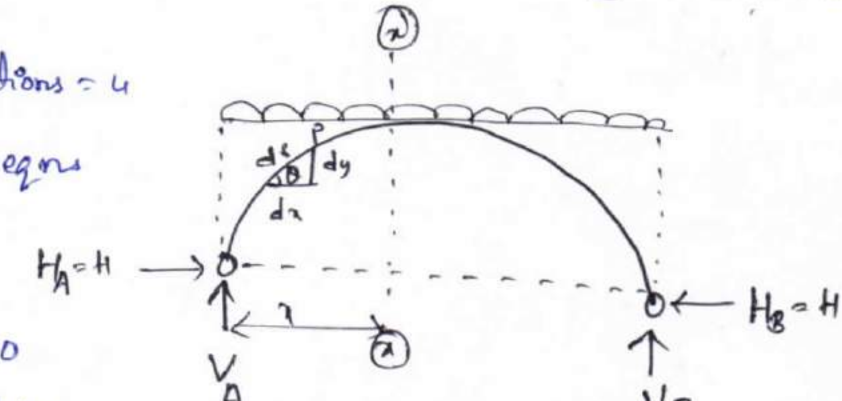
No. of unknown reactions = 4

No. of equilibrium eqns

= 3

i.e; $\Sigma F_x = 0$; $\Sigma F_y = 0$

$\Sigma M = 0$ available



∴ Two hinged arch is a statically indeterminate structure

Degree of static indeterminancy $D_s = r - 1$

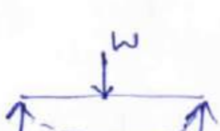
(or) $= 4 - 3$

Degree of redundancy $\boxed{D_s = 1}$

Select 'H' as redundant reaction.

$$\frac{\partial U}{\partial H} = 0$$

H → Horizontal reaction.



$\cos\theta = \frac{dx}{ds}$

$\sec\theta = \frac{ds}{dx}$

$ds = dx \cdot \sec\theta$

strain energy stored in the beam due to bending.

$$U = \int \frac{M^2 dx}{2EI}$$

$$U = \int \frac{M^2}{2EI} ds$$

$$\frac{\partial U}{\partial H} = \int \frac{\cancel{2}M}{\cancel{2}EI} \cdot \frac{\partial M}{\partial H} \cdot ds$$

$$= \int \frac{M}{EI} \cdot \frac{\partial m}{\partial H} \cdot ds$$

$$\frac{\partial U}{\partial H} = \int \frac{M}{EI} \cdot \frac{\partial M}{\partial H} (dx \cdot \sec\theta) = 0$$

Consider x–x @ 'a' distance of x from 'A'.

$$\frac{\partial U}{\partial H} = \int \frac{M_{x-x}}{EI} \cdot \frac{\partial M_{x-x}}{\partial H} \cdot ds$$

$$\boxed{M_{x-x} = M - Hy}$$

$$\frac{M_{x-x}}{\partial H} = 0 - y$$

$$\frac{\partial M_{xx}}{\partial y} = -y$$

$$\frac{\partial U}{\partial H} = \int \frac{M - Hy}{EI} (-y)\, ds$$

$$0 = \int \frac{M - Hy}{EI} (-y)\, ds$$

$$\int \frac{M - Hy}{EI} (-y) (dx \cdot \sec\theta) = 0$$

$$\int \frac{-My + Hy^2}{EI} \sec\theta \cdot dx = 0$$

$$-\frac{My + Hy^2}{EI} \sec\theta \int dx = 0$$

$$\left(\frac{-My + Hy^2}{EI}\right) \sec\theta\, x = 0$$

$$\frac{Hy^2}{} \cancel{\sec}\theta = \frac{My}{EI} \cancel{\sec}\theta\, x$$

$$Hy = M$$

$$\boxed{H = \frac{M}{y}}$$

(or) $- \int \frac{My}{EI} . ds + \int \frac{Hy^2}{EI} . ds = 0$

$$H \int \frac{y^2}{EI} ds = \int \frac{My}{EI} ds$$

$$\boxed{H = \frac{\int \frac{My}{EI} . ds}{\int \frac{y^2}{EI} ds}} \quad \text{(or)} \quad \boxed{H = \frac{\int My \, dx}{\int y^2 dx}}$$

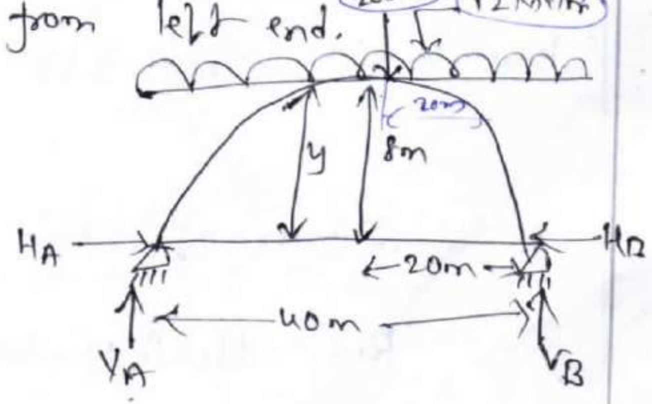
let $I \rightarrow$ M.I of curved axis

$I_0 \rightarrow$ Moment of Inertia of the crown.

$$\boxed{I = I_0 \sec\theta}$$

$$\boxed{ds = dx . \sec\theta}$$

Prob:- A 2-hinged parabolic arch of span 40 m & central rise 8 m carries a U.D.L 12 kN/m over the entire of the span & 200 kN load at a distance 20 m from right end. Calculate the Horizontal thrust & draw B.M.D. Also calculate R.S & N.T at section 20 m from left end.



Sol:- $\Sigma y = 0$

$V_A + V_B = 12 \times 40 + 200$

$= 680$ kN

$\Sigma M_A = 0$

$12 \times 40 \times \frac{40}{2} + 200 \times 20 - 40 V_B = 0$

$V_B = 340$ kN

$V_A = 340$ kN

$H = H_1 + H_2$

$= \frac{wl^2}{8r} + \frac{25}{128} \cdot \frac{Wl}{r}$

$H = \frac{12 \times 40^2}{8 \times 8} + \frac{25 \times 200 \times 40}{128 \times 8}$

$\boxed{H = 495.3 \text{ kN}}$

B.M @ A = 0

B.M @ B = 0

B.M @ C = $V_B \times 20 - H \times 8$

$= 340 \times 20 - 495.3 \times 8$

$= 2837.6$

$M_x = V_A \cdot x - H \cdot y - 12 \cdot x \cdot \frac{x}{2}$

$= 340 \cdot x - 495.3 \times \frac{4 \times 8}{40^2}\left[40x - x^2\right] - 6x^2$

$\frac{dM_x}{dx} = 340 - 9.9\left[40 - 2x\right] - 12x = 0$

$= 340 - 396 + 19.8x + 12x = 0$

$7.8x = 56$

$x = \frac{56}{7.8} = 7.17 m$

$x = 7.17 m$

Max B.M $= 340 \times 7.17 - 9.9\left[40 \times 7.17 - 7.17^2\right] - 6 \times 7.17^2$

$= 415.8$ kNm

At a distance of 20m section from left end:

$R.S = H \cdot \sin\theta - V_x \cdot \cos\theta$

$N.T = H \cos\theta + V_x \cdot \sin\theta \Rightarrow \cancel{495.3 \times \cos \times \theta + V_x \times \sin\theta}$

$\Rightarrow$

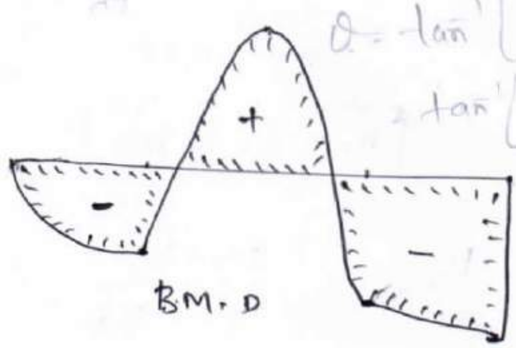
$V_x = 495.3$ $\qquad V_x =$

$y = \frac{4 \times 8}{40^2}\left[40x - x^2\right]$

$\tan\theta = 0$

$\theta = \tan^{-1} 0$

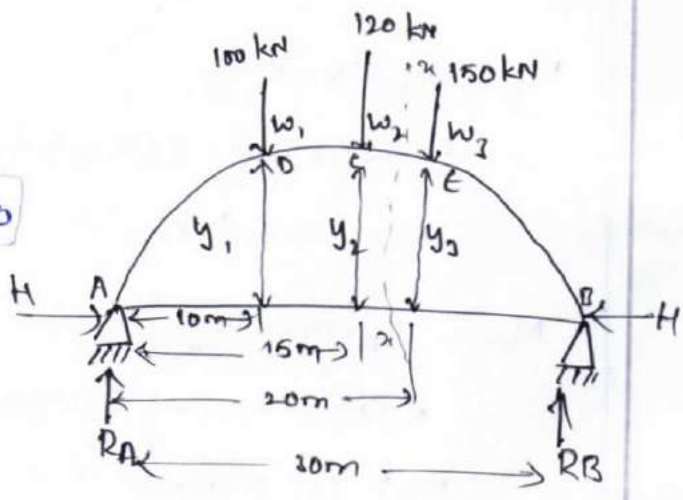
$\underline{\theta = 0}$



Prb: A 2-hinged parabolic arch of span 30m & Central rise of 7.5m carries 3 point loads 100 kN, 120 kN, 150 kN at a distance of 10 m, 15m, & 20 m respectively from left end. Calculate radial shear & Normal thrust at a section of 18m from left end. Also calculate max B.M?

Sol:- $R_A + R_B = 100 + 120 + 150$

$= 370$ kN



$\Sigma_A = 0$

$R_A \times 0 - [30 \times R_B + 10 \times 100 + 15 \times 120 + 20 \times 150] = 0$

$30 R_B = 1000 + 1800 + 3000$

$R_B = \frac{5800}{30}$

$\boxed{R_B = 193.3 \text{ kN}}$

$R_A + R_B = 370$

$R_A = 370 - 193.3 = 176.67$ kN

$\boxed{R_A = 176.67 \text{ kN}}$

$H_1 = \frac{5}{8} \times \frac{Wa}{rl^3} (l - a)(l^2 + al - a^2)$

$H_1 = \frac{5}{8} \times \frac{Wa}{rl^2}\left[l-a\right]\left[l^2+al-a^2\right]$

$= \frac{5}{8}\times \frac{100\times 10}{7.5\times 30^3}\left[30-10\right]\left[30^2+10\times 30-10^2\right]$

$= 67.90218$

$\boxed{H_1 = 67.90\ kN}$

$H_2 = \frac{25\times wL}{128\times 7.5} = \frac{25\times 120\times 30}{128\times 7.5} = 93.75$

$\boxed{H_2 = 93.75\ kN}$

$H_3 = \frac{5}{8}\times \frac{150\times 20}{7.5\times 30^3}(30-20)(30^2+20\times 30-20^2)$

$\boxed{H_3 = 101.85\ kN}$

$H = H_1+H_2+H_3 = 67.9+93.75+101.85$

$\boxed{H = 263.50\ kN}$

$M_D = V_A\times 10 - H\cdot y_D$

$= 176.67\times 10 - 263.5\times 6.67 = 91.55\ kN\cdot m$

$M_D = 9.15\ kN\ m$

$M_C = V_A\times 15 - H\cdot y_C - 100\times 5$

$= 176.67\times 15 - 263.5\times 7.5 - 100\times 5$

$M_C = 173.8\ kN\cdot m$

$H = H_1+H_2+H_3$
$\Rightarrow 263.5\ kN$

$M_E = V_B\times 10 - H\cdot y_E$

$= 193.3\times 10 - 263.5\times 6.67$

$M_E = 175.45\ kN\cdot m$

Max B.M acts at under any of the loads:

Max -ve B.M - 'x' from 'B'

$V_B\times x - H\times y - 150\times \overset{(x-10)}{10} = 0$

$193.3\times 263.5\times 6.67 - 150\times 10 = 0$

$-3064.245\ kN\cdot m$

$y_D = \frac{4r}{l^2}\left[lx-x^2\right]$

$= \frac{4\times 7.5}{30^2}\left(30\times 10-10^2\right)$

$= \frac{4\times7.5}{30^2}\left[30\times10 - 10^2\right]$ $\quad y_E = \frac{4\times7.5}{30^2}\left[30\times10-10^2\right]$

$y_D = 6.667\,m.$ $\quad y_E = 6.67\,m.$

$y_C = 7.5m$

$\frac{dy}{dx} = \tan\theta = \frac{4r}{l^2}(l-2x) \qquad \therefore x = 18m$

$\theta = \tan^{-1}\left[\frac{4r}{l^2}(l-2x)\right]$

$= \tan^{-1}\left[\frac{4\times7.5}{30^2}(30-2\times18)\right]$

$\theta = -11.30^\circ$

Radial shear $= H\sin\theta - V_x\cdot\cos\theta$

$= 263.3\times\sin 11.30^\circ - V_x\cdot\cos 11.30^\circ$

$R.S = -285.05\,kN$

$V_x = -V_B + 150$

$= 193.3 + 150$

$V_x = -43.3\,kN$

$R_S = 263.3\times\sin 11.30^\circ - (-43.3)\cos 11.30^\circ$

$\boxed{R.S = -285.05\,kN}$

Normal thrust : $H\cos\theta + V_x\sin\theta$

$= 263.3\times\cos 11.30^\circ - (-43.3)\times\sin 11.30^\circ$

$\boxed{N.T = 190.92\,kN}$

$193.3x - 263.5\times\left[\frac{4\times7.5}{30^2}(30x - x^2)\right] - 150(x-10) = 0$

$\frac{dm_x}{dx} \Rightarrow 193.3 - 263.5\left[0.033(30-2x)\right]$

$193.3 - 263.5(0.99 - 0.066x) - 1500 = 0$

$193.3 - 260.865 + 17.39x - 1500 = 0$

$x = \frac{1567.5}{17.39}$

$x = 90.14\,m$

$V_B\times x - 150(x-10) - H\times y = 0$

$= 193.3x - 150x - 1500 - 263.5\times\left[\frac{4\times7.5}{30^2}(30x - x^2)\right]$

$= 43.3x - 1500 - 263.5\left[0.033(30x - x^2)\right]$

$= 43.3x - 1500 - 263.5\left[1x - 0.033x^2\right]$

$= 43.3x - 1500 - 263.5x - 8.6955x^2$

$\frac{dm_x}{dx} = 0: 43.3 - 1500 - 263.5 - 8.6955\times2x$

$17.39x = \frac{1720.2}{17.39}$

$x =$

Max -ve B.M $= 193.3\times3.91 - 263.5\times\left[\frac{4\times7.5}{30^2}\times(30\times3.91 - 3.91^2)\right]$

$= 140.20\,kN\cdot m$

$x = 3.91m$

$193.3x - 263.5\times\left[\frac{4\times7.5}{30^2}(30x - x^2)\right] = 0$

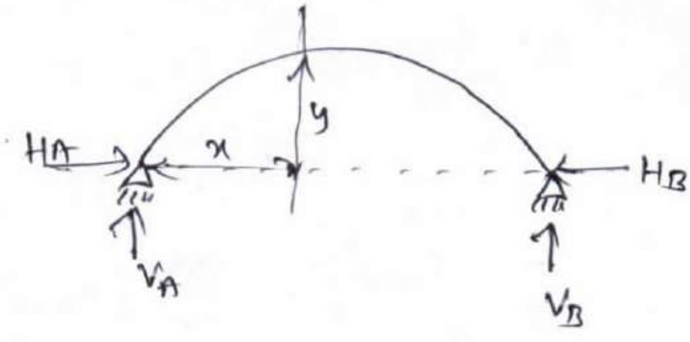
$193.3 - 263.5\left(0.033(30-2x)\right)$

$= 193.3 - \cancel{8.698} - 260.8 - 17.39x = 0$

$x = 67.5/...$

Two hinged arches:

It is a statically Indeterminate structure.



$M_x = V_A \cdot x - H \cdot y$

$\rightarrow M = V_A \cdot x$

$M_x = M - Hy$

$\frac{\partial U}{\partial H} = 0$

$U = \int \frac{M^2 ds}{2EI} = \int \frac{(M-Hy)^2 ds}{2EI}$

ds = length of arch

M = B.M of beam

$\frac{\partial U}{\partial H} = \int \frac{\cancel{2}(M - Hy)(-y)\,ds}{\cancel{2}EI} = 0$

$= \int (M-Hy)(-y) ds = 0$

$-\int My\,ds + \int Hy^2 ds = 0$

$\int Hy^2 ds = \int My\,ds$

$H \int y^2 ds = \int My\,ds$

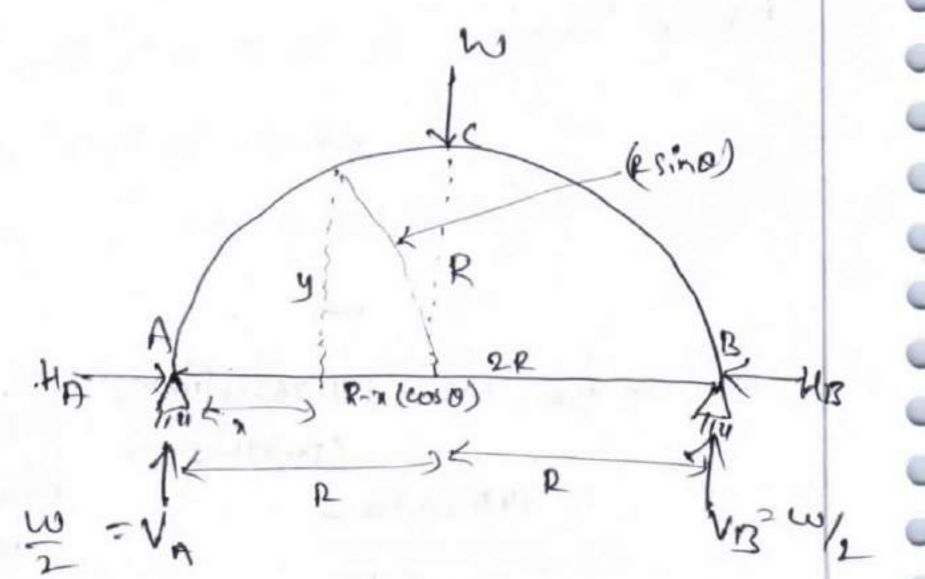
$H = \frac{\int My\,ds}{\int y^2 ds}$

$\Rightarrow \int (My + Hy^2)\,ds = 0$

$\int My \cdot ds = H \int y^2 ds$

$$\boxed{H = \frac{\int My\,ds}{\int y^2 ds}}$$

Show that the horizontal thrust in a two hinged semicircular arch carries a point load 'W' at centre is '$\frac{W}{\pi}$'?



Sol: $V_A + V_B = W$

$\Sigma M_A = 0$

$V_A \times 2R - W \times R = 0$

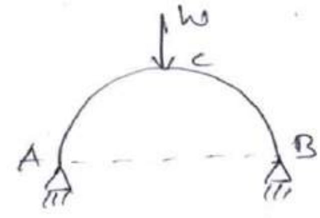
$V_A \times 2R = WR$

$V_A = \frac{W\cancel{R}}{2R}$

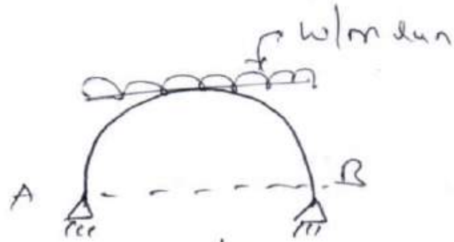
$\boxed{V_A = W/2}$

Parabolic arches:

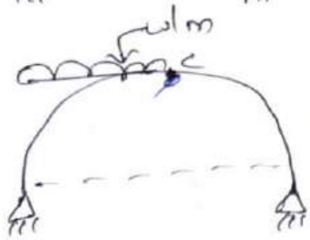
$$H = \frac{25}{128} \cdot \frac{Wl}{h}$$



$$H = \frac{wl^2}{8r \text{ (or) } h}$$

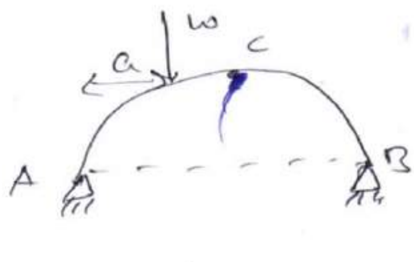


$$H = \frac{wl^2}{16h}$$



Two hinged

$$H = \frac{5}{8} \cdot \frac{Wa}{hl^3} (l-a)(l^2 + la - a^2)$$

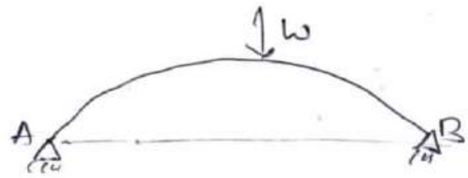


$$H = \frac{wa^2}{16hl^3}\left[5l^3 - 5la^2 + 2a^3\right]$$

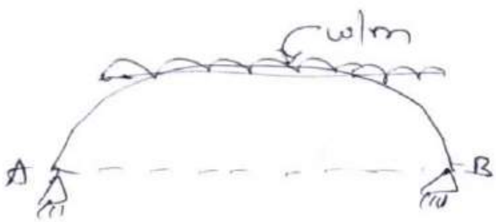
3-hinged

Semicircular arches:

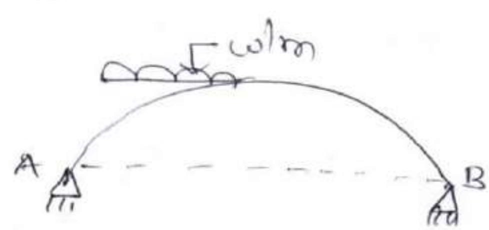
$$H = W/\pi$$



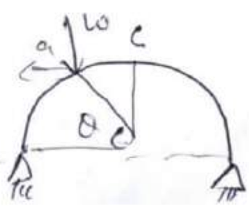
$$H = \frac{4wR}{3\pi}$$



$$H = \frac{4wR}{6\pi}$$



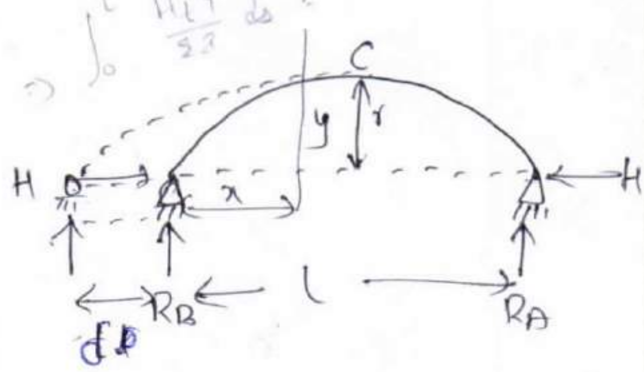
$$H = \frac{W}{\pi} \cdot \sin^2\theta.$$



Effect of Temperature in 2-hinged arches.

$M_x = H \cdot y$ $\quad M = H_t \cdot y$ $\quad \int_0^L \frac{My}{EI} ds \Rightarrow \int_0^L \frac{H_t y^2}{EI} ds =$

$U = \int \frac{M_x^2 \cdot dx}{2EI}$



$= \int \frac{(H(y))^2 \cdot dx}{2EI}$

$$\frac{dU}{dH} = \frac{\cancel{2}H \int y^2 \cdot dx}{\cancel{2}EI} = \left[\delta\ell = \alpha \cdot l \cdot t\right]$$

$$\boxed{H = \frac{\alpha \cdot l \cdot t \cdot E \cdot I}{\int y^2 \cdot dx}}$$

$M = H \cdot r$

$f = \frac{H \cdot r}{I} \cdot y$

$= \frac{\alpha \cdot l \cdot t \cdot E\cancel{I}\, r}{\int y^2 \cdot dx} \times \left[\frac{y}{\cancel{I}}\right] = \frac{\alpha \cdot t \cdot E \cdot l \cdot y \cdot r}{\int y^2 \cdot dy}$

$$\boxed{f = \frac{\alpha \cdot t \cdot E \cdot l \cdot y \cdot r}{\int y^2 \cdot dy}}$$

$\therefore y = D/2$

$I = bD^3/12$

Horizontal thrust due to temperature effect on two-hinged arch:

$$H_t = \frac{EI \alpha \cdot T \cdot l}{\int_0 y^2 \cdot ds}$$

$$y = \frac{4h}{l^2}(lx - x^2) \Rightarrow \frac{8}{15} h^2 l \text{ (or) } \frac{8}{15} r^2 l.$$

Prb: A two-hinged parabolic arch of span 30m & Central rise 6m, subjected to change in temperature of 30°c. Cal. the bending stress developed in a depth of 0.70 m from given data. Take $\alpha = 12 \cdot 10^{-6}$ per degree centigrade, $E = 2\times10^5 N/mm^2$ $b = 0.3m$.

Sol:- $r = h = 6m$, span $l = 30m$, $t = 30°c$, $\alpha = 12.0\times10^{-6}/°c$

$D = 0.7m$, $E = 2\times10^5 N/mm^2$, $b = 0.3m = 300mm$

$f = ?$

$$H = \frac{\alpha . l . t . E . I}{\int y^2 . dx}$$

$$I = \frac{b D^3}{12} = \frac{0.3\times0.7^3}{12} = \frac{300\times700^3}{12}$$

$$I = 8.575\times10^9 mm^4$$

$$Y = D/2 = \frac{700}{2}$$

$$Y = 350mm$$

$$y = \frac{4r}{l^2}\left[lx - x^2\right]$$

$$y = \frac{4\times6}{30^2}\left[30x - x^2\right] \quad y = \frac{8}{15} r^2 l$$

$$\int y^2 dx = \int \left(\frac{24}{900}\right)^2 \times \left(30x - x^2\right)^2 . dx$$

$$= \left(\frac{24}{900}\right)^2 \int_0^{30} (900x^2 + x^4 - 60x^3)\, dx$$

$$= 7.1\times10^{-04}\left[\frac{900x^3}{3} + \frac{x^5}{5} - \frac{60x^4}{4}\right]_0^{30}$$

$$= 7.1\times10^{-4}\left[\frac{900\times30^3}{3} + \frac{30^5}{5} - \frac{60\times30^4}{6}\right]$$

$$\int y^2 dx = 575.1$$

$$\therefore f = \frac{M}{I} . y = H . r\left[\frac{y}{I}\right] \Rightarrow \frac{\alpha . t . E . l . \cancel{I} . r}{\int y^2 . dx} \times \frac{y}{\cancel{I}}$$

$$f = \frac{\alpha . t . E . l . r}{\int y^2 dx}$$

$$f = \frac{12\times10^{-6}\times 30^{0}\times 2\times10^{5}\times 30\times10^{-3}\times 6\times10^{-3}}{576}\times 370$$

$$f = 7.871\times10^{-3}\ N/mm^2$$

$$\boxed{f = 7871\ N/m^2}$$ ⇒ Bending stress.

Horizontal thrust in Parabolic arch due to change in temperature:

$$y = \frac{4r}{l^2}\left[lx - x^2\right]$$

$$\int y^2\,dx = \int_0^l \left(\frac{4r}{l^2}\right)^2 (lx - x^2)^2\,dx$$

$$= \left(\frac{4r}{l^2}\right)^2 \int_0^l (lx - x^2)^2\,dx$$

$$\Rightarrow \frac{4^2 r^2}{l^4}\left[\int_0^l l^2x^2 + x^4 - 2lx\cdot x^2\,dx\right]$$

$$\Rightarrow \frac{16r^2}{l^4}\left[l^2\cdot\frac{x^3}{3} + \frac{x^5}{5} - 2l\cdot\frac{x^4}{4}\right]_0^l$$

$$\Rightarrow \frac{16r^2}{l^4}\left[l^2\frac{l^3}{3} + \frac{l^5}{5} - 2l\cdot\frac{l^4}{4}\right]$$

$$\Rightarrow \frac{16r^2}{l^4}\left[\frac{l^5}{3} + \frac{l^5}{5} - \frac{\cancel{2}\cdot l^5}{\cancel{4}_2}\right]$$

$$\Rightarrow \frac{16r^2}{l^4}\left[\frac{10l^5 + 6l^5 - 15l^5}{30}\right]$$

$$\Rightarrow \frac{16r^2}{l^4}\left[\frac{16l^5 - 15l^5}{30}\right] \Rightarrow \frac{16r^2}{l^4}\left[\frac{l^5}{30}\right]$$

$$\Rightarrow \frac{16r^2}{l^4}\left[\frac{l^5}{30}\right] \Rightarrow \frac{\overset{8}{\cancel{16}}\cdot r^2\cdot \cancel{l^5}}{\underset{15}{\cancel{30}}\,\cancel{l^4}}$$

$$\Rightarrow \frac{8r^2 l}{15}$$

Horizontal thrust due to yielding of supports :

$$\boxed{dH = \frac{EI\delta}{\int y^2 ds}}$$

δ - yielding in supports

Net horizontal thrust :

Horizontal thrust due to loads — Horizontal thrust due to yielding :

$$\boxed{N.H.T = H - dH}$$

Horizontal thrust due to rib shortening :

$$H = \frac{\int My\,ds/EI}{\int \frac{y^2 ds}{EI} + \frac{l}{AmE}}$$

Prb: A 2-hinged parabolic arch of span 20 m of rise 8m carries a UDL of 20 kN/m. Calculate the horizontal thrust if a supports yields laterally with respect to other support by 0.02m. What will be the horizontal thrust?

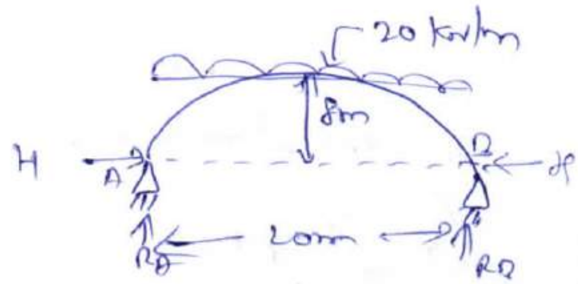
Sol: $I = 1.2\times10^7\,mm^4$, $E = 2\times10^5\,N/mm^2$ [Assume]

$w = 20\,kN/m$

$h = 8m$

$l = 20m$

$\delta = 0.02m = 0.002\times10^3\,mm$



$\Sigma M_B = 0$ $\quad R_A + R_B = 20\times20$

$= 400\,kN$

$R_A\times0 + R_B\times20 - 20\times\frac{20}{2}\times20 = 0$

$R_B = 200\,kN$

$R_A = 200\,kN$

$$H = \frac{Wl^2}{8h}$$

$$= \frac{20 \times 20^2}{8 \times 8} = 125 \text{ kN}$$

$$\underline{H = 125 \text{ kN}}$$

dH (or) decrease in horizontal thrust due to yielding

$$dH = \frac{EI\delta}{\int_0^l y^2 ds}$$

$$dH = \frac{2\times10^5 \times 1.7\times10^7 \times 0.02\times10^3}{\int_0^l y^2 ds}$$

$$\therefore \int_0^l y^2 ds = \frac{8}{15} \cdot h^2 \cdot l$$

$$= \frac{2\times10^5 \times 1.7\times10^7 \times 0.02\times10^3}{6.826\times10^4}$$

$$= \frac{8}{15} \times (8\times10^3)^2 \times 20\times10^3$$

$$= 6.826\times10^4 \text{ mm}$$

$$dH = 99.60 \text{ kN}$$

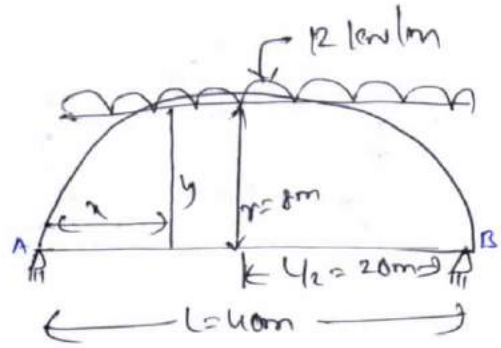
$$= 0.099 \text{ kN}.$$

$\therefore$ Horizontal thrust $= H - dH \Rightarrow 125\times10^3 - 99.60$

$\Rightarrow 124900.4$ ~~kN~~ kN

$\Rightarrow 124.900$ ~~kN~~ kN

Prb:



$V_A + V_B = W \times l = 12 \times 40 = 480$ kN

$\Sigma M_A = 0 \Rightarrow V_B \times l - W \times l \times l/2 = 0$

$V_B \times 40 - 12 \times 40 \times 40/2 = \frac{9600}{40}$ kN $= V_B$

$V_B = 240$ kN

$V_A = 480 - 240$

$V_A = 240$ kN.

Horizontal thrust $H = \frac{wL^2}{8r} = \frac{12 \times 40^2}{8 \times 8} = 300$ kN

Cal. of B.M:

B.M @ A = 0

B.M @ B = 0

B.M @ C = $V_B \times 20 - H \times r$

$= 240 \times 20 - 300 \times 8$

$= 2400$ kN·m

B.M @ x-x =

$M_x = V_A \times x - w \times x \cdot \frac{x}{2} - H \times y$

$= 240 \times x - 12 \times \frac{x^2}{2} - 300 \times y$

$y = \frac{4r}{L^2}\left[Lx - x^2\right]$

$M_x = 240x - \frac{12x^2}{2} - 300\left[\frac{4r}{40^2}\left(40 \times x - x^2\right)\right]$

$= 240x - 6x^2 - 300\left[\frac{32}{1600}\left[40x - x^2\right]\right]$

$= 240x - 6x^2 - 300\left[0.8x - 0.02x^2\right]$

$= 240x - 6x^2 - \left(240x - 6x^2\right)$

$\frac{dM_x}{dx} = \cancel{240 - 12x - 240 - 12x} = 0$

$M_x = \cancel{240x - 240x - 12x^2} =$

$= 240x + \underline{1440x^3 + 36x^2} = 0$ $\quad 240x + 1440x^3 + 36x^4 = \frac{dM_x}{dx}$

$\frac{dM_x}{dx} = \cancel{240 - 240 - 12 \times 2x = 0}$

$0 = 240 + 4320x + 144x = 0$ $\quad 240 + (3 \times 1440x) + 144x = 0$

$x = 18.6$ m $\quad x = \frac{4464}{240} = 18.6$

$x = \cancel{18.6}0$ m $= 0$

$\therefore$ Max B.M = $V_A \times x - w \times x \times \frac{x}{2} - H \times y$

$= 240 \times 18.6 - 12 \times 18.6 \times 18.6/2 - 300 \times \left[\frac{4 \times 8}{?}\left(40 \times 18.6 - 18.6^2\right)\right]$

$0 = 4464 - 2075.76 - 7.96$

$B.M = M_x = \underline{2380.28}$ kN.m

<u>At a distance of 20m from the left end</u>

$R.S = H \sin\theta - V_x \cos\theta.$

$$\tan\theta = \frac{dy}{dx}$$

$$\tan\theta = \frac{d}{dx}\left[\frac{4r}{l^2}(lx - x^2)\right]$$

$$\theta = \tan^{-1}\left[\frac{4r}{l^2}(l - 2x)\right]$$

$$= \tan^{-1}\left[\frac{4\times 8}{40^2}\left[40 - 2\times \cancel{18.96}\ 20\right]\right]$$

$$= \tan^{-1}\left[\cancel{+.684}\ 0\right]$$

$$\theta = \cancel{58^\circ 59'} / \cancel{58.995}\ 0$$

$V_x = V_A$

$R.S = H \sin\theta - V_A \cos\theta$

$= 300 \times \sin(0) - 240 (\cos 0)$

$= -240$ kN

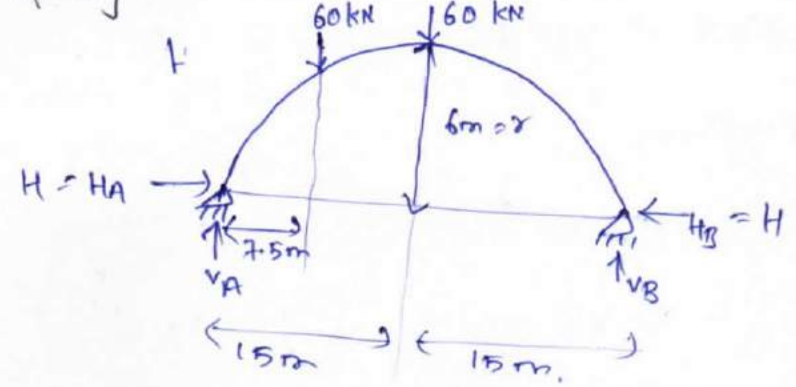
$N.T = H \cos\theta + V \sin\theta$

$= 300 \times \cos(0) + 240 (\sin 0)$

$= \underline{300 \text{ kN}}$

Ex:- A Two-hinged parabolic arch of span 30m & rise 6m carries two point loads, each 60 kN acting at 7.5m & 15m from the left end, resp. The moment of inertia varies, as the secant of slope. Determine the horizontal thrust & max. positive & negative moments in the arch rib.

Sol:-



$V_A + V_B = 60 + 60 = 120\ kN$

$\Sigma m_A = 0$

$V_B \times 30 - 60 \times 15 - 60 \times 7.5 = 0$

$V_B = \frac{1350}{30} = 45\ kN$

$V_B = 45$

$V_A = 120 - 45 = 75\ kN$

$H_1 = \frac{5}{8} \times \frac{w}{r l^3}\ a \cdot [l - a][l^2 + la - a^2]$

$= \frac{5}{8} \times \left(\frac{60}{6 \times 30^3}\right) \times 7.5 [30 - 7.5] [30^2 + 30 \times 7.5 - 7.5^2]$

$= \cancel{1265} \qquad [\cancel{1068.75}]\ 198.75$

$\Rightarrow 48.71\ kN$

$H_2 = \frac{25}{128} \frac{wl}{r} = \frac{25}{128} \times \frac{60 \times 30}{6}$

$= 58.59\ kN$

$H = H_1 + H_2 \Rightarrow 48.75 + 58.59$

$= 100.34\ kN$

Max B.M (+ve) $= M_x = \cancel{V_B \times x} - 60 \times 15 =$

$= V_B \times x - H \times y$ $\qquad x = 15$

$y = \frac{4r}{l^2}\left[lx - x^2\right] = \frac{4 \times 6}{30^2}\left[30 \times 15 - 15^2\right]$

$= 6\,m$

$= 45 \times 15 - H \times 6 = 45 \times 15 - 100.34 \times 6 = \cancel{332}.04$

$= 72$ kN.m

$m_x = 72.96$ kN.m.

Max (-ve) B.M $= M_x = V_A \times x - H \times y$

$= 75x - 100.34 \times \left[\frac{4r}{l^2}(lx - x^2)\right]$

$\frac{dm_x}{dx} = 0$

$= 75x - 100.34\left[\frac{4 \times 6}{30^2}(30x - x^2)\right]$

$\frac{dm_x}{dx} = 0 \Rightarrow 75 - 100.34\left[0.0266\right]\left[30 - 2x\right]$

$0 = 75 - 2.667 \times (30 - 2x)$

$0 = 75 - \cancel{7}9.81 - 5.334x$

$x = \frac{5.334}{5.01}$

$x = 1.06\,m$

$M_x = V_B \times x - H \times y$

$= 45 \times x - 100.34 \times \left[\frac{4r}{l^2}(lx - x^2)\right]$

$= 45x - 100.34\left[\frac{4 \times 6}{30^2}(30x - x^2)\right]$

$= 45x - 2.676(30x - x^2)$

$\frac{dm_x}{dx} = 0$

$= 45 - 2.676(30 - 2x)$

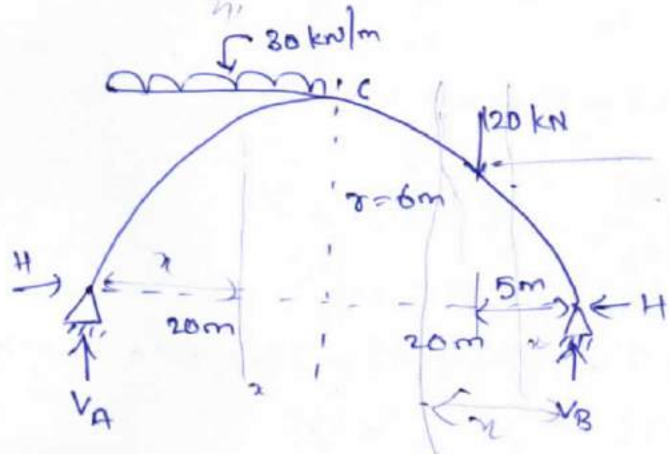
$0 = 45 - 80.28 + 5.35x$

$x = \frac{35.3}{5.35}$ $\qquad x = 6.59\,m$

Max -ve B.M $= M_x = 45(6.59) - 2.676.(30\times 6.59 - 6.59^2)$

$= -116.28$ kN.m

Prob:- A two hinged parabolic arch is given below. Determine the
1) horizontal thrust 2) Max (+ve) & (-ve) moments
3) Shear force & Normal thrust at 10 m from the left support.



$V_A + V_B = 120 + 30\times 20 = 720$ kN

$\Sigma M_A = 0$

$V_B \times L - 120\times 35 - 30\times 20\times \frac{20}{2} = 0$

$V_B = 10200/40 = 255$ kN $= \frac{10200}{40} = 255$ kN

$V_B = 720 - 255 = 465$ kN $= 720 - 255 = 465$ kN

$H_1 = \frac{wl^2}{16r} = \frac{30\times 40^2}{16\times 6} = 500$ kN

$H_2 = 5/8 \left[\frac{wa}{rl^3}\right]\left[l-a\right]\left[l^2 + la - a^2\right]$

$= \frac{5}{8}\left[\frac{120\times 5}{6\times 40^3}\right]\left[40-5\right]\left[40^2 + 40\times 5 - 5^2\right]$

$= 60.699$ kN

$H = H_1 + H_2$

$= (500 + 60.669)$ kN

$H = 560.669$ kN

B.M @ A = 0

B = 0

$$x = M_x = V_A \times x - w \times x \times \frac{x}{2} - H \times y$$

$$= 465 \times x - \frac{30x^2}{2} - 560.669 \times \left[\frac{4 \times 6}{40^2} \times (40x - x^2)\right]$$

$$= 465x - 15x^2 - 8.41(40x - x^2)$$

$$= 465x - 15x^2 - 336.4x + 8.41x^2$$

$$= 128.6x - 6.59x^2$$

$$\frac{dm_x}{dx} = 0$$

$$0 = 465 - 30x - 336.4 + 16.82x$$

$$x = \frac{128.6}{13.18} = 9.757 \text{ m}$$

$$M_{max} = 465 \times 9.757 - 30 \times 9.757 - 336.4 + 16.82 \times 9.757$$

$$= 4537.01 - 292.71 + 164.1127$$

(or)

$$= 128.6(x) - 6.59x^2 = 0$$

$$128.6(9.757) - 6.59 \times 9.757^2 = 0$$

$$\cancel{1254.7} - 627.36$$

$$1247.42 - 627.36 = 0$$

$$M_{x max} = 627.388 \text{ kN.m}$$

Max -ve B.M =

$$M_x = V_B \times x - 120(x-5) - H \times y$$

$$= 255x - 120(x-5) - 560.669 \times y$$

$$= 255x - 120x - 600 - 560.669\left[\frac{4 \times 6}{40^2} \times 40x - x^2\right]$$

$$= 255x - 120x - 600 - 8.41[40x - x^2]$$

$$= 600 - 201.4x + 8.41x^2$$

$$\frac{dm_x}{dx} = 0$$

$$= -201.4 + 2x(8.41)$$

$$0 = -201.4x + 8.41x^2$$

$$x = \frac{201.4}{16.82} = 11.974\,m$$

$$M_{min} = 600 - 201.4x + 8.41x^2$$

$$= 600 - 201.4(11.974) + 8.41(11.974)^2$$

$$= -605.766\ kN.m$$

b/w $\underline{\underline{BD}}$

$$M_x = 255x - 560.669\,y$$

$$= 255x - 8.41(40x - x^2)$$

$$= -81.4x + 8.41x^2$$

$$x = \frac{81.4}{16.82} = 4.839m$$

$$M = -81.4(4.839) + 8.41(4.839)^2$$

$$= \underline{-196.967\ kN.m}$$

+ve max = 627.388 kN.m ; at 9.757 m

max -ve = -605.766 kN.m at 11.974m.

At 10 m from the right support:

$$\tan\theta = \frac{dy}{dx} = \frac{d}{dx}\left[\frac{4r}{L^2}\left(Lx - x^2\right)\right]$$

$$= \frac{4r}{L^2}(L - 2x)$$

$$= \frac{4\times(6)}{40^2}(40 - 2\times 9.757)$$

$$\tan\theta = 0.3$$

$$\theta = \tan^{-1} 0.3$$

$$\underline{\theta = 16.694^\circ}$$

$$N = V\sin\theta + H\cos\theta \Rightarrow (225 * 120)\sin\times 16.699^\circ +$$

$$560.669\times\cos\times 16.699^\circ$$

N.T $\Rightarrow$ 575.815 kN

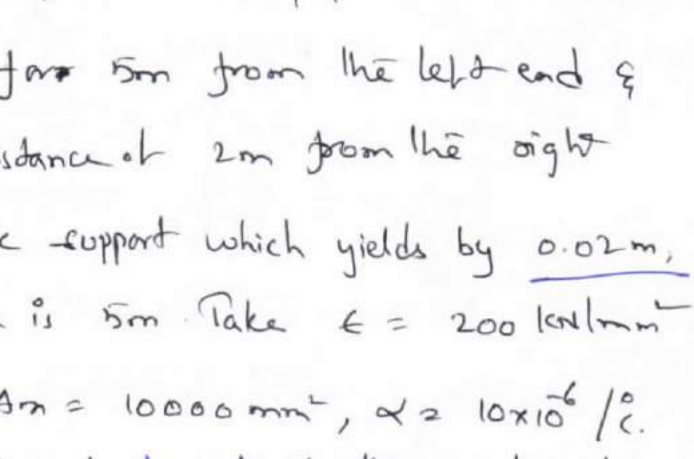
R.S $\Rightarrow V\cos\theta - H\sin\theta$

$\Rightarrow (255-120)\times\cos 16.699^\circ - 560.679\times\sin 16.699^\circ$

$\Rightarrow -31.800$ kN

Rib Shortening & yielding of supports:

A two hinged parabolic of arch of span 10 m carries a udl of 12 kN/m for 5m from the left end & a point of 20 kN at a distance of 2m from the right support. It has an elastic support which yields by 0.02 m, The central rise of arch is 5m Take $E = 200$ kN/mm^2 $I = 5\times10^9$ mm^4 & area $A_m = 10000$ mm^2, $\alpha = 10\times10^{-6}$ /°C. Calculate the horizontal thrust also calculate the H due to rib shortening.



Sol:-
$r = 5m$ $\quad \delta = 0.02m$

$E = 200$ kN/mm^2

$I = 5\times10^9$ mm^4

$A_m = 10000$ mm^2

$\alpha = 10\times10^{-6}$ °/c

$H = ?$

$R_A + R_B = (12\times5)+20 = 80$ kN.

$\Sigma M_A = 0$

$V_B\times l - 20\times8 - 12\times5\times\frac{5}{2} = 0$

$V_B = \frac{310}{10}$

$V_B = 31$ kN

$V_A = 80-31 = 49$ kN

$$H = \frac{wl^2}{16r} + \frac{5}{8} \cdot \frac{Wa}{rL^3}\left[L-a\right]\left[L^2+La-a^2\right]$$

$$= \frac{12\times10^2}{16\times5} + \frac{5}{8}\times\frac{20\times8^2}{5\times10^3}(10-8)\left(10^2+10\times8-8^2\right)$$

$$= 15 + (0.02)\times116$$

$$H = 17.32\ kN \Rightarrow 17.32\times10^3\ N$$

Horizontal thrust due to loading $= H = 17.32$

Horizontal thrust due to yielding $= dH = \oint \frac{EI\delta}{\int y^2 dx} = \frac{EI\delta}{\int y^2 dx}$

$$\int_0^L y^2 dx = \frac{8}{15} r^2 l = \frac{8}{15}\times[5]^2\times[10]$$

$$= 133.33\ m$$

$1m = 1000mm$

$0.02 = x.$

$$d_H = \frac{EI\delta}{\int y^2 dx} = \frac{200\ kN/mm^2\times5\times10^9\times0.02\times10^3}{133.33\times10^3} = 150.0\times10^6$$

$$\frac{\frac{1}{mm^2}\times mm\times mm^{4}}{mm\ \frac{m^2}{mm}}$$

$$= 150.003\times10^6\ kN.mm^2$$

$$= 150.003\times10^3\ N.mm$$

$$= 150.003\ N$$

Net horizontal thrust $\Rightarrow H - dH \Rightarrow 17.32\times10^3 - 150$

$$\Rightarrow 17170\ N$$

$$\Rightarrow 17.17\ kN$$

Horizontal thrust due to rib shortening $\Rightarrow H = \dfrac{\int \frac{My\,ds}{EI}}{\int \frac{y^2 dx}{EI} + \frac{L}{AmE}}$

$$M = H\times y.$$

$$y = \frac{4r}{l^2}\left(lx - x^2\right)$$

B.M @ x-$x \Rightarrow R_A\times x - w\times x\times\frac{x}{2} - H\times y$

$$M_x \Rightarrow 49\times x - \frac{12\times x^2}{2} - 17.32\times\left[\frac{4\times5}{10^2}\left[10x - x^2\right]\right]$$

$$\frac{dM_x}{dx} = 0 \quad 0 = 49 - 12x - 17.32\times\left[0.2\left(10-2x\right)\right]$$

$0 = 49 - 12x - 17.32\,(2 - 0.4x)$

$0 \Rightarrow 49 - 12x - 34.64 + 6.928x$

$0 = 14.36 - 5.072x$

$x = 2.86\ m$

$y = \frac{4 \times 8}{10^2}\left[10 \times 2.86 - 2.86^2\right]$

$y = 6.535\ m$

$M_x = 49x - \frac{12x^2}{2} - 17.32\left(\frac{32}{10^2}\right)[\ \times 6.535$

$\Rightarrow 49 \times 2.86 - 6 \times 2.86^2 - 17.32 \times 6.535$

$M_x \Rightarrow 22.12\ kN.m$ (or) $H \times y \Rightarrow 17.32 \times 6.535 = 113.2$

$$\boxed{H = \frac{\int M y\, ds / EI}{\int \frac{y^2 ds}{EI} + \frac{L}{AmE}}} \Rightarrow$$

Effect of Temperature on Two-hinged Arches

Let 'H_t' be the horizontal thrust induced due to temperature rise by $t°$.

The increase in horizontal span of arch $= L.\alpha.t$

where α = coefficient of thermal expansion.

The B.M on any element at a height y is

$$M = H_t \cdot y.$$

Total increase in span due to bending of curved bar $= \int \frac{M_x^2\, y}{2EI}\,dx \quad \int \frac{M_x^2\, dh}{2EI}$

$$H = \int_0^L \frac{My}{EI}\,ds \quad \text{or} \quad \int_0^L \frac{M_x^2\, dh}{2EI}$$

$\int H_t^2 y^2 dh = \frac{2H_t \int y^2 dh}{2EI}$

$H = L\alpha t$

$$L.\alpha.t = \int_0^L \frac{My}{EI}\,ds \quad \text{or} \quad \int_0^L \frac{(H_t\, y)^2}{2EI}\,dh = L\alpha t$$

$$\frac{2H_t \int_0^L y^2\, dy}{2EI} = L\alpha t$$

$$H_t \int_0^l y^2 \, ds = EI \, l \cdot \alpha \cdot t$$

$$\boxed{H_t = \frac{EI \cdot l \cdot \alpha \cdot t}{\int_0^l y^2 \, ds}}$$

Pb: A two hinged parabolic arch of span 40 m & rise 8 m is subjected to a temperature rise of 22 k. Calculate the maximum bending stress at the crown due to the temperature rise if $\alpha = 11 \times 10^{-6}$ per 1°k & $E = 2.1 \times 10^5$ N/mm². The rib section is symmetrical & 1 m deep.

Sol: The eqn of parabola = $y = \frac{4r}{l^2}[lx - x^2]$

$$y = \frac{4 \times 8}{40^2}\left[40 \times x - x^2\right] = \frac{x}{50}(40 - x)$$

$$\int_0^l y^2 \, ds = \int_0^{40} \left[\frac{x}{50}(40-x)\right]^2 dx$$

$$= \int_0^{40} \frac{x^2}{2500}[40 - x]^2 \, dx$$

$\int y^2 ds = \frac{8}{15} r^2 L$

$$\Rightarrow \int_0^{40} \frac{x^2}{2500}\left[1600 + x^2 - 80x\right] dx$$

$\frac{8}{15} \times 8^2 \times 40$

$\Rightarrow 1365.3 \text{ m}$

$$= \frac{1}{2500}\left[1600\frac{x^3}{3} + \frac{x^5}{5} - \frac{80x^4}{4}\right]_0^{40}$$

$$= \frac{(40)^2}{2500}\left[\frac{1600}{3} + \frac{1600}{5} - \frac{80 \times 40}{4}\right] m^3$$

$$= 1360 \text{ m}^3$$

$E = 2.1\times10^{5}\ N/mm^2 = 2.1\times10^{8}\ kN/m^2$

$EI.L.\alpha.t = (2.1\times10^{8})\ I\ [40\times11\times10^{-6}\times22]$

$= 2.0328\times10^{6}\ I\ kN\text{-}m^2$ $\quad 2.1\times10^{5}$

where - $\underline{I = m^4 \text{ units}}$ $\quad 2.1\times10^{2}+10^{3}$

$H_t = \dfrac{EIL\alpha t}{\int_0^L y^2 ds} = \dfrac{2.0328\times10^{6}\ I}{1360} = 1495\ I\ kN$ $\quad 2.1\times10^{2}\ kN/mm^2$

$\underline{H_t = 1495\ I\ kN}$ $\quad \dfrac{2.1\times10^{2}}{(10^{-3})^2}\ kN/m^2$

Max B.M @ crown $= H_t\times y_r$ $\quad \dfrac{2.1\times10^{2}}{10^{-6}}$

$= 1495I\times8$ $\quad 2.1\times10^{2}\times10^{6}$

$= 11960\ I\ kN.m$ $\quad kN/m^2$

$2.1\times10^{8}\ kN/m^2$

Max Bending stress @ crown

$\dfrac{f}{I} = \dfrac{M}{Z}$ $\quad Z = \dfrac{I}{y} = \dfrac{I}{0.5}$

$D = 1m$

$y = D/2 = 1/2 = 0.5$

$f = \dfrac{M}{Z} = \dfrac{11960\ I}{\frac{I}{0.5}}$

$= \dfrac{11960\ \cancel{I}}{\cancel{I}}\times0.5 = 5980\ kN/m^2$

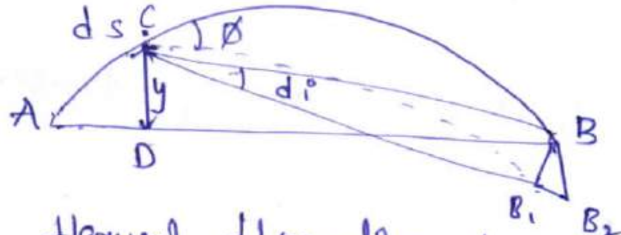
$\underline{f = 5.98\ N/mm^2}$

(or)

$\dfrac{f}{y} = \dfrac{M}{I} \Rightarrow f = \dfrac{M}{I}\times y \Rightarrow \dfrac{11960\cancel{I}}{I}\times0.5$

$\Rightarrow 5980\ kN/m^2$

Derivation of Horizontal Thrust of a Two hinged Arch.



Consider the flexural deformation of a curved rib.

Let ACB represents the centre line of a curved rib subjected to variable B.M.

Let us find the horizontal & vertical displacements of of end B with reference to A.

Consider the effect of B.M on an element of length "ds".

The element turn through an angle di, the part AC of the rib being unchanged & the chord CB will therefore turn to a position CB_1 through an angle "di".

Thus B_1B_2 gives horizontal displacement, & BB_2 gives vertical displacement of B.

$\therefore$ ~~B_1B_2~~ =

$\cos[BB_1B_2] \Rightarrow \frac{B_1B_2}{BB_1}$

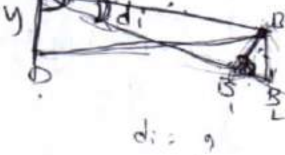
$\therefore BB_1(\cos BB_1B_2) = B_1B_2 \longrightarrow$ (a)

$BB_1 = CB \times di \longrightarrow$ (b)

put (b) in (a)

$\therefore B_1B_2 = (CB \times di)\cos BB_1B_2$

$\Rightarrow CB \cdot di \times \cos BCD$



(Sin) di = $\frac{BB_1}{CB}$

$\Rightarrow di \cdot (CB)\cos BCD$

(Sin) di

$di \times CB = BB_1 \Rightarrow di\,(CD)$

~~cos~~ $di \Rightarrow$

$\theta = \frac{BB_1}{CB}$

$B_1B_2 \Rightarrow di \cdot y \longrightarrow$ (1)

from Bending eqn

$$\frac{M}{I} = \frac{E}{R}$$

$\therefore R =$ radius of curvature $= \frac{ds}{di}$

$$\frac{M}{I} = \frac{E}{\frac{ds}{di}}$$

$$\frac{M}{I} = E \times \frac{di}{ds} \Rightarrow di = \frac{M}{I} \times \frac{ds}{E}$$

$$\therefore \frac{M\,ds}{I.E} = di \quad \text{--- (2)}$$

$$\int \frac{My\,ds}{EI} = 0$$

$$M = \mu - Hy$$

Substituting (2) in (1)

$B_1B_2 = di$ ED (or) $y.di$

$$\int \frac{(\mu - Hy)y\,ds}{EI} = 0$$

$$\Rightarrow y \times \frac{M\,ds}{EI}$$

$$\frac{\int(\mu y\,ds - Hy^2\,ds)}{\int EI} = 0$$

$$B_1B_2 \Rightarrow \frac{My\,ds}{EI}$$

$$\int \mu y\,ds - \int Hy^2\,ds = 0$$

$\therefore$ The total horizontal displacement of B

$$\int \mu y\,ds = H \int y^2\,ds$$

$$B \Rightarrow \int \frac{My\,ds}{EI} \longrightarrow \text{(3)}$$

$$\frac{\int \mu y\,ds}{\int y^2\,ds} = H.$$

lly; The total vertical displacement of B

$$B = \int \frac{Mx\,ds}{EI}$$

Two hinged arch having no yielding of supports $\Rightarrow$

$$\int \frac{My\,ds}{EI} = 0 \text{ (or) } \int \frac{(\mu - Hy)y\,ds}{EI} = 0$$

$$0 = \int \frac{\mu y - Hy^2\,ds}{EI} \Rightarrow H \int \frac{\mu y\,ds - y^2\,ds}{EI}$$

$$\cancel{H \int \frac{\mu y\,ds}{EI}} - \cancel{\int y^2 ds} = 0$$

$$\cancel{H \int \frac{\mu y\,ds}{EI} = \int \frac{y^2\,ds}{EI}}$$

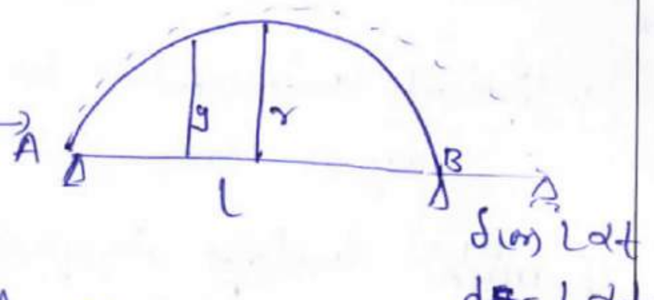
$$\cancel{H = \frac{\int \mu y\,ds}{\int y^2\,ds}}$$

$$\boxed{H = \frac{\int \mu y\,ds}{\int y^2\,ds}}$$

$$H = \frac{\int \mu y\,ds}{\int y^2\,ds.}$$

$$M = (\mu - Hy)$$

let H_t be horizontal thrust due to rise in temperature by t°.



The increase in horizontal span

$\delta \Rightarrow L\alpha t$

where, t = temperature
α = co-efficient of thermal expansion.

The B.M on element at 'y' height 'y' $\Rightarrow M = H_t \cdot y$

$$M = H_t \cdot y$$

∴ Total increase in span due to bending of curved bar $= \int_0^L \frac{My}{EI} \cdot ds$.

$$\therefore \int_0^L \frac{My}{EI} ds = L\alpha t$$

$$\int_0^L \frac{H_t \cdot y \cdot y \; ds}{EI} = L\alpha t$$

$$\int_0^L \frac{H_t y^2 ds}{EI} = L\alpha t$$

$\frac{H_t}{EI} \int y^2 ds$

$$\frac{H_t \int_0^L y^2 ds}{EI} = L\alpha t$$

$H \int y^2 ds = EI$

$$\boxed{H_t = \frac{EI \cdot L\alpha t}{\int_0^L y^2 ds}}$$

$H_t \int y^2 ds = EI L\alpha t$

$H_t = \frac{EI L\alpha t}{\int y^2 ds}$

Two hinged arches	Three hinged arches
1. Statically Indeterminate to degree one.	Statically determinate
2. Might develope temperature stresses	Increase in temp. causes increase in central rise. but no stresses
3. ~~Easy to analyse~~ Structurally more efficient	Easy to analyse, But in construction, the central hinge, may involve additional expenditure
4. Will develope stresses due to sinking or yielding of supports	Since this is determinate, no stresses due to support sinking

For Parabolic arch

$$\text{Hori: thrust @ support} = H = \frac{15 \alpha T E I}{8 r^2}$$

$$\text{Max. B.M} = H \times r$$

$$M = \frac{15 \alpha T E I \cancel{r}}{8 r^{\cancel{2}}} = \frac{15 \alpha T E I}{8 r}$$

Objective Questions:

1. In the slope deflection equations, the deformations are considered to be caused by

i) bending moment ii) shear force iii) axial force

The correct answer is

a) only (i)

b) (i)and(ii)

c) (ii) and (iii)

d) (i), (ii) and (iii)

2. For a symmetrical two hinged parabolic arch, if one of the supports settles horizontally, then the horizontal thrust

a) is increased

b) is decreased

c) remains unchanged

d) becomes zero

3..Slope-deflection is developed by

a) Maney

b) Hardy cross

c) Muller

d) Gumbel

4.The slope deflection equations give the relationship between

a)slope and deflection only

b)BM and rotation only

c)BM and vertical deflection only

d)BM ,rotation and deflections

5.In slope deflection method the displacements considered are due to

a)SF

b)BM

c)Axial force and BM

D)SF and BM

6.The number of simultaneous equations to be solved in the slope deflection method is equal to

a)static indeterminacy

b)kinematic indeterminacy

c)no of joint displacements in the structure

d)none of the above

7.The slope deflection method formulates

a)equilibrium conditions only

b)compatibility conditions only

c)both equilibrium and compatibility conditions

d)either equilibrium or compatibility conditions

8.The slope deflection method in structural analysis falls in the category of

a)force method

b)flexibility method

c)consistent deformation method

d)stiffness method

9.In slope deflection method the no of unknown rotations at various joints are determined

by considering

a)the equilibrium of joints

b)the rigidity of joint

c)the equilibrium of structure

d)none of the above

10.In slope deflection method , the end moments for any member are expressed

a)as zero

b)in terms of unknown end rotations

c)as equal

d)none of the above

Fill in the blanks:

11.Slope-deflection method primarily gives ------------------------------

12. The horizontal thrust for two hinged parabolic arch is------------

13. Degree of static indeterminacy of a two hinged arch is ---------------

14. Degree of static indeterminacy of a three hinged arch is ---------------

15. The strain energy due to volumetric strain---------------------- to volume

16. The strain energy due to volumetric strain---------------------- to bulk modulus

17. The effect of normal thrust in the arch is to shorten the ---------- of the arch.

18. The radial shear of two hinged parabolic arch is-----

19. The normal thrust of any arch is ------

20. The net horizontal thrust due to temperature effect is------

KEY 1- a, 2-b,3-d, 4-b,5-b,6-c, 7-a,8-d,9-c,10-b

11-displacements, 12-H=$\int My\, ds / \int y^2 ds$, 13-1 , 14-zero, 15-4, 16-2

17-rib, 18-R.S=$H_a \sin \theta - V_a \cos \theta$, 19 –N.T-=$H_a \sin \theta - V_a \cos \theta$, 20-H.T=[Dh/H=-dh/h]