STABILITY OF INFINITE SLOPES

- In Fig. (a), X-X represents an infinite slope which is inclined to the horizontal at an angle β. On any plane YY (YY||XX) at a depth z below the ground level the soil properties and the overburden pressure are constant, Hence, failure may occur along a plane parallel to the slope at some depth. The conditions for such a failure may be analysed by considering the equilibrium of the soil prism ABCD of width b.

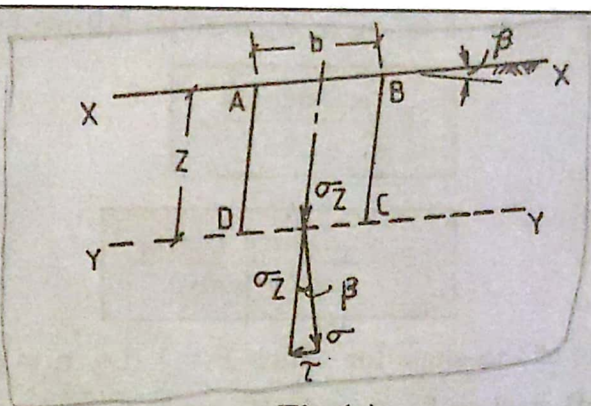

Fig (a)

remember this figure

Considering unit thickness, volume of the prism $\boxed{V = z\ b\ \cos\beta}$

and, weight of the prism, $\boxed{W = \gamma z\ b\ \cos\beta}$

Remember this

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Vertical stress on YY due to self-weight,

$$\boxed{\sigma_z = \frac{W}{b} = \gamma z \cos\beta} \quad \text{...}(i)$$

This vertical stress can be resolved into the following two components:

$$\boxed{\sigma = \sigma_z \cos\beta = \gamma z \cancel{b} \cos^2\beta} \quad \text{...}(ii)$$

and,

$$\boxed{\tau = \sigma_z \sin\beta = \gamma z \cos\beta \sin\beta} \quad \text{...}(iii)$$

Failure will occur if the shear stress τ exceeds the shear strength τ_f of the soil. The factor of safety against such failure is given by.

$\tau \rightarrow$ shear stress

$\tau_f \rightarrow$ shear strength

$$\boxed{F = \frac{\tau_f}{\tau}} \quad \text{...(iv)}$$

$F \rightarrow$ Factor of safety

(i) **Cohesionless soils :** from Coulomb's equation, we have

$$\tau_f = c + \sigma \tan\phi$$

For a cohesionless soil, $c = 0$,

$$\tau_f = \sigma \tan\phi$$

Substituting in eqn. (iv)

$$F = \frac{\sigma \tan\phi}{\tau}$$

Again, substituting the expressions for σ and τ.

$$\boxed{F = \frac{\gamma z \cos^2\beta \tan\phi}{\gamma z \cos\beta \sin\beta} = \frac{\tan\phi}{\tan\beta}} \quad \text{...(v)}$$

When $\phi = \beta$, $F = 1$. Thus a slope in a cohesionless soil is stable till $\beta \le \phi$, provided that no external force is present.

(ii) **c − ϕ soils:** In this case, the factor of safety against slope failure is given by,

$$\boxed{F = \frac{c + \sigma \tan\phi}{\tau}}$$

or

$$\boxed{F = \frac{c + \gamma z \cos^2\beta \tan\phi}{\gamma z \cos\beta \sin\beta}} \quad \text{...(vi)}$$

Let H_c be the critical height of the slope for which $F = 1$ (i.e. $\tau_f = \tau$)

$$\therefore \quad \gamma H_c \cos\beta \sin\beta = c + \gamma H_c \cos^2\beta \tan\phi$$

or,

$$H_c = \frac{c}{\gamma \cos\beta(\sin\beta - \cos\beta \tan\phi)}$$

or,

$$H_c = \frac{c}{\gamma \cos^2 \beta (\tan\beta - \sin\phi)} \quad \text{...(vii)}$$

Eqn. (vii) may also be written as:

$$\frac{c}{\gamma H_c} = \cos^2 \beta(\tan\beta - \tan\phi) \quad \text{...(viii)}$$

or,

$$S_n = \cos^2\beta \ (\tan\beta - \tan\phi) \quad \text{...(ix)}$$

where,

S_n is a dimensionless quantity known as the stability number and is given by:

$$S_n = \frac{c}{\gamma H_c} \quad \text{...(x)}$$

If a factor of safety F_c is applied to the cohesion such that the mobilised cohesion at a depth H is,

$$c_m = \frac{c}{F_c} \quad \text{...(xi)}$$

γ may be γ_{sat}, γ_{sub} or γ_{dry} depending upon situation

Then,

$$S_n = \frac{c_m}{\gamma H} = \frac{c}{F_c \gamma H} \quad \text{...(xii)}$$

From eqns. (x) and (xii), we get,

$$\frac{c}{\gamma H_c} = \frac{c}{F_c \gamma H}$$

or,

$$F_c = \frac{H_c}{H} = F_H$$

Hence, the factor of safety against cohesion, F_c, is the same as the factor of safety with respect to height, F_H.