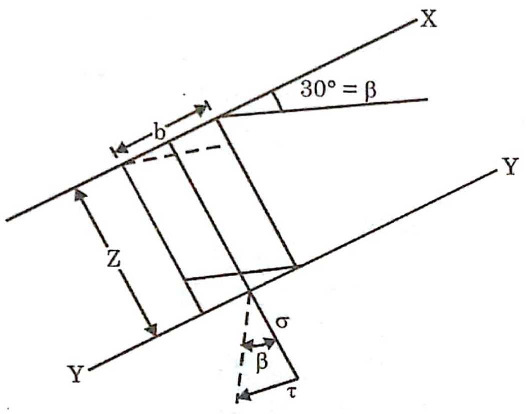
**Example 1**

A slope of infinite extent is made in a dense sand layer at angle of 30° to the horizontal. Determine the factor of safety of the slope against shear failure if the angle of internal friction of the soil be 36°.

**Sol.** With reference to figure (*a*) XX respresents the given slope and *YY* is a plane parallel to it at a depth *z*

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Vertical stress on YY $\sigma = \sigma_z \cos\beta$

$$= \left(\frac{\gamma Z b \cos\beta}{b}\right)\cos\beta \qquad [\because \quad W = (\gamma Z b \cos\beta)]$$

$$= \gamma Z \cos^2\beta \qquad \ldots (i)$$

Shear stress on $YY$

$$\tau = \sigma_Z \sin\beta = \left(\frac{\gamma Z\, b \cos\beta}{b}\right)\sin\beta$$

$$= (\gamma Z \sin\beta \cos\beta) \qquad \ldots (ii)$$

Shear strength of the soil on the $YY$-plane

$$\tau_f = (c + \sigma \tan\phi) \qquad [\because \quad c = 0 \quad \text{for dense sand}]$$

$$= (0 + \gamma Z \cos^2\beta \tan\phi)$$

$$= \gamma Z \cos^2\beta \tan\phi \qquad \ldots (iii)$$

Factor of safety against shear failure

$$F = \frac{\tau_f}{\tau} = \frac{\gamma Z \cos^2\beta \tan\phi}{(\gamma Z \sin\beta \cos\beta)}$$

$$F = \frac{\tan\phi}{\tan\beta} \qquad \ldots (iv)$$

**Data given**

$$\phi = 36°$$

$$\beta = 30°$$

$\therefore$

$$F = \left(\frac{\tan\phi}{\tan\beta}\right) = \left(\frac{\tan 36°}{\tan 30°}\right)$$

$$\boxed{F = 1.258}$$

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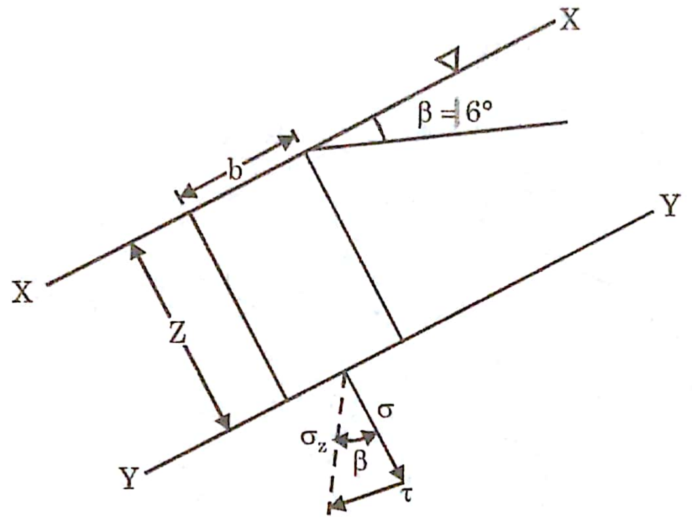
**Example 2**

A slope inclined at 16° to the horizontal is to made in a cohesionless deposit having the following properties $G = 2.70$, $e = 0.72$, $\phi = 35°$

Determine the factor of safety of the slope against shear failure if water percolates in a direction parallel to the surface of the slope.

**Sol.** From Figure

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Vertical stress on Y-Y $\sigma = \sigma_Z \cos\beta = \left(\frac{\gamma Z\, b \cos\beta}{b}\right)\cos\beta = \gamma Z \cos^2\beta$

Therefore Neutral stress at point $P = (\gamma_w\, Z \cos^2\beta)$

Total normal stress at point $P = (\gamma_{sat}\, Z \cos^2\beta)$

$\therefore$ Effective stress at $P = \gamma_{sat}\, Z \cos^2\beta - \gamma_w Z \cos^2\beta$

$$= (\gamma_{sat} - \gamma_w)\, Z \cos^2\beta$$

$$\bar{\sigma}_P = (\gamma_{sub}\, Z \cos^2\beta)$$

Shear stress at $P = \sigma_Z \sin\beta = \left(\frac{\gamma Z b \cos\beta}{b}\right)\sin\beta$

$$\tau = (\gamma_{sat}\, Z) \cos\beta\, \sin\beta \qquad \text{... (ii)}$$

Shear strength of the soil on Y–Y

$$\tau_f = (\bar{\sigma}_P \tan\phi + 0)$$

$$= (\gamma_{sub}\, Z \cos^2\beta\, \tan\phi) \qquad \text{... (iii)}$$

$\therefore$ Factor of safety $F = \left(\frac{\tau_f}{\tau}\right)$

$$= \frac{\gamma_{sub} Z \cos^2\beta \tan\phi}{\gamma_{sat} Z \cos\beta \sin\beta}$$

$$\boxed{F = \frac{(\gamma_{sub}\tan\phi)}{(\gamma_{sat}\tan\beta)}}$$

**Data Given:**

$\beta = 16°$

$G = 2.70$

$e = 0.72$

$\phi = 35°$

$\therefore \gamma_{sub} = (\gamma_{sat} - \gamma_w)$

$$= \frac{(G+e)\gamma_w}{(1+e)} - \gamma_w$$

$$= \frac{(G+e-1-e)\gamma_w}{(1+e)} = \frac{(G-1)\gamma_w}{(1+e)}$$

$\therefore$ $$\gamma_{sub} = \frac{(2.70-1)\times 1}{(1+0.72)} = 0.988 \text{ t/m}^3$$

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{(1+e)} = \frac{(2.70+0.72)\times 1}{(1+0.72)} = 1.99 \text{ t/m}^2$$

$\therefore$ $$F = \frac{(0.988\times\tan 35)}{(1.99\times\tan 16°)} = 1.21$$

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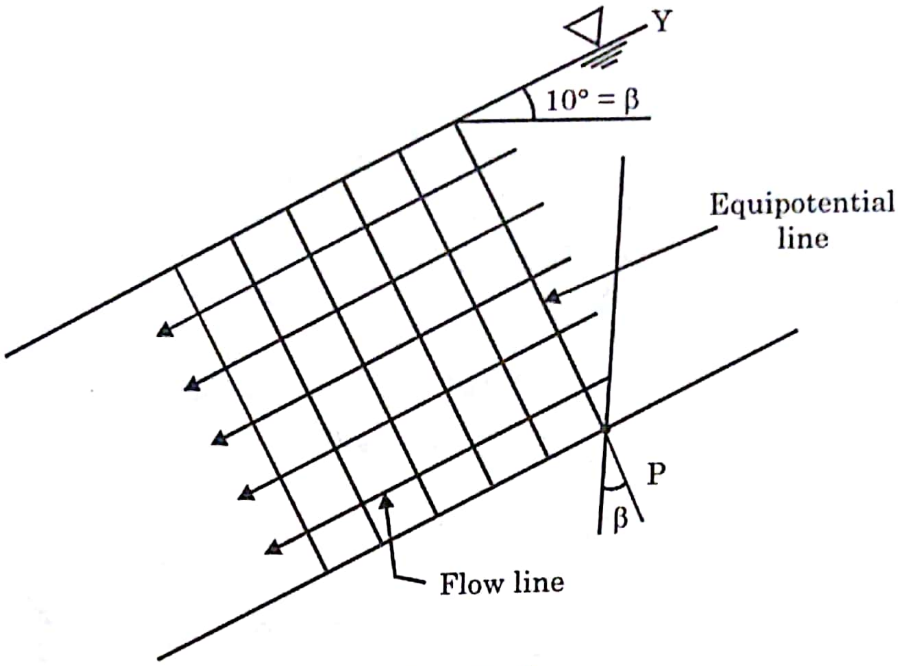
**Example 3**

A long natural slope in an overconsolidated clay ($c' = 10$ kN/m$^2$, $\phi' = 25°$, $\gamma_{sat} = 20$ kN/m$^3$) is inclined at 10° to the horizontal. The water table is at the surface and seepage is parallel to the slope. If a plane slip had developed at a depth of 5 m below the surface, determine the factor of saftey take $\gamma_w = 10$ kN/m$^3$.

**Sol. Data given**




$$c' = 10 \text{ kN/m}^2$$
$$\phi' = 25°$$
$$\gamma_{sat} = 20 \text{ kN/m}^3$$
$$\gamma_w = 10 \text{ kN/m}^3$$
$$\beta = 10°$$
$$\gamma_{sub} = (\gamma_{sat} - \gamma_w) = (20 - 10) = 10 \text{ kN/m}^3$$

We know the factor of safety of cohessive soil

$$F = \frac{c + \gamma_{sub} Z \cos^2 \beta \tan \phi}{(\gamma_{sat} Z \sin \beta \cos \beta)}$$

$$= \frac{(10 + 10 \times 5 \cos^2 10 \tan 25)}{(20 \times 5 \sin 10 \cos 10)}$$

$$= 1.90$$

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