Concept of fixed end moments

Obtained using unit load method

Derivation of the Slope-Deflection Equation

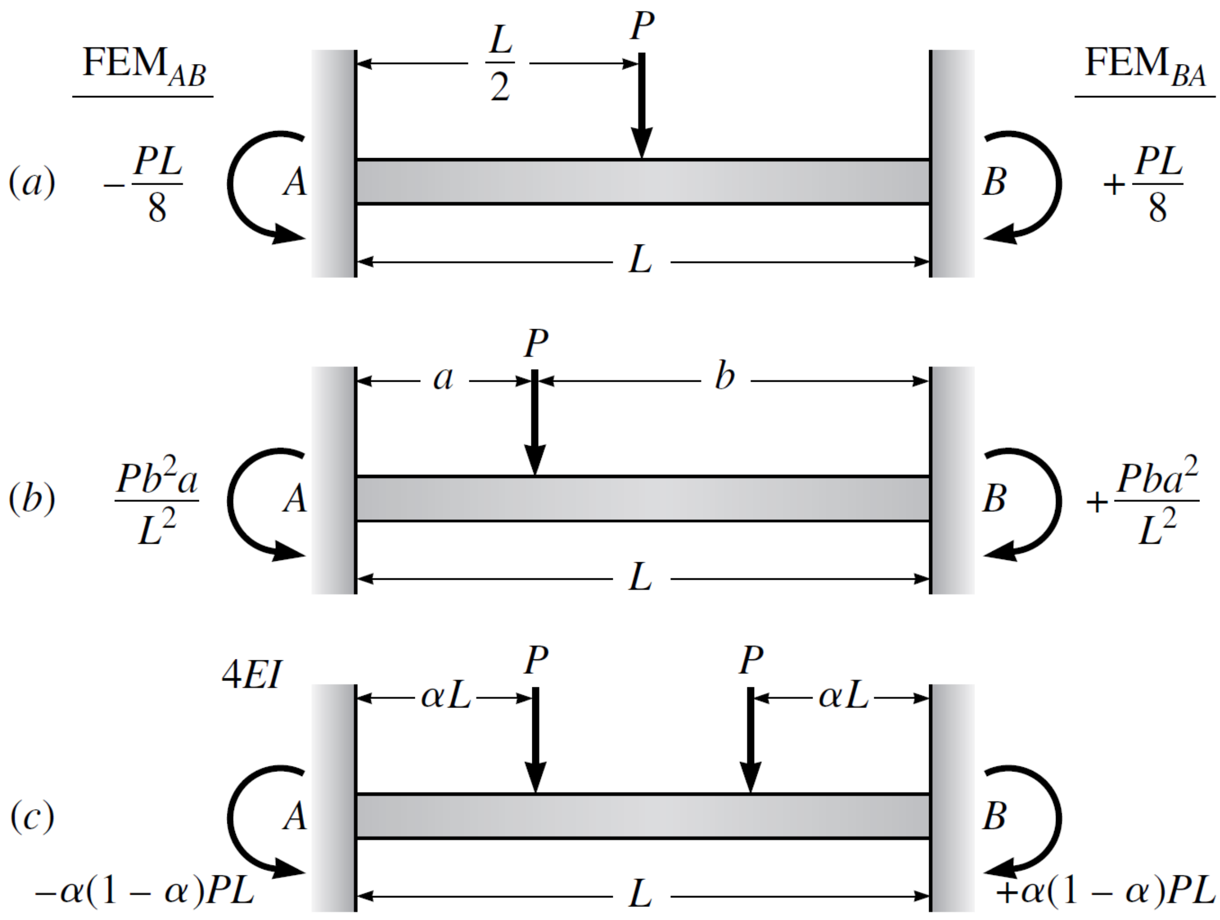


Figure 12.5 Fixed-end moments

Derivation of the Slope-Deflection Equation

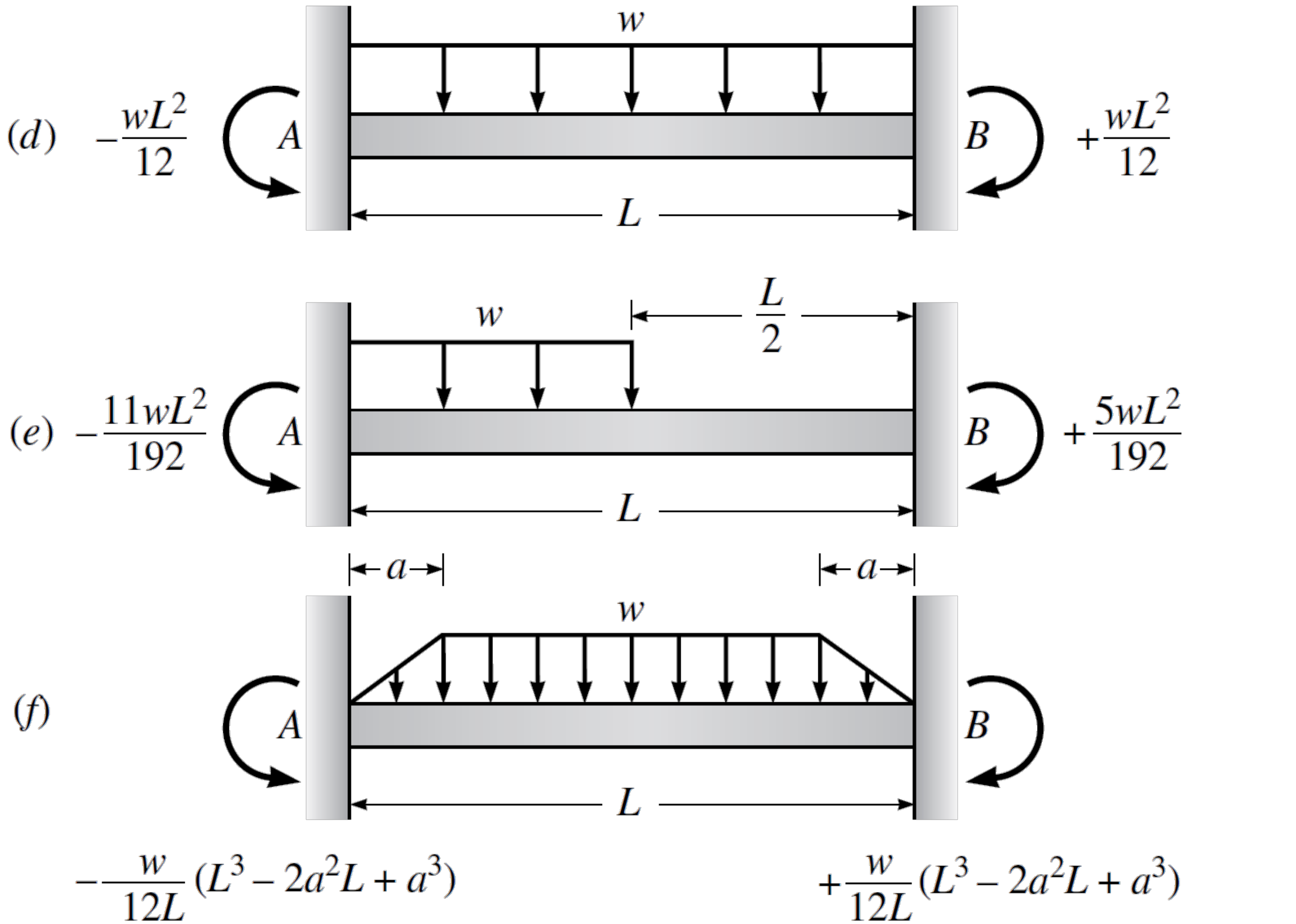


Figure 12.5 Fixed-end moments (continued)

Derivation of the Slope-Deflection Equation

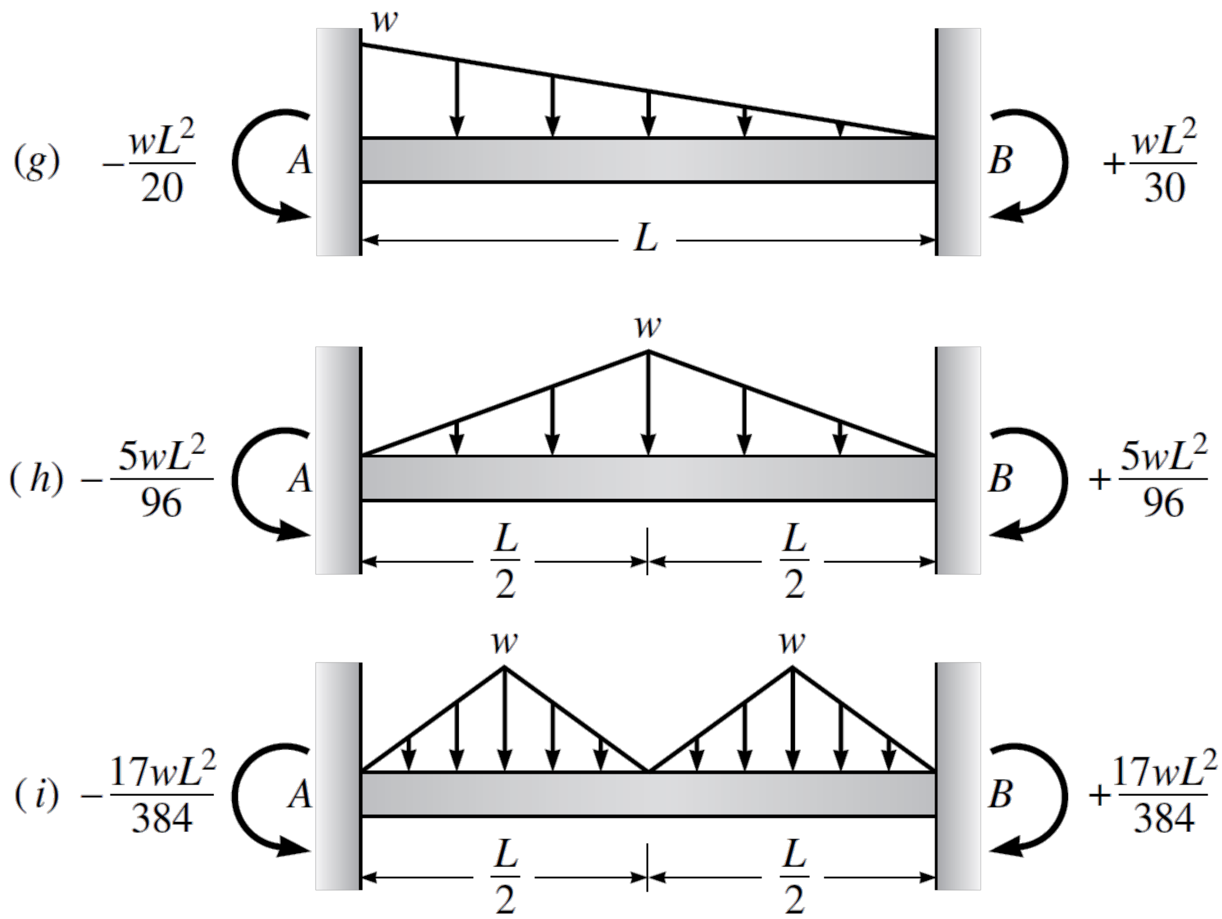


Figure 12.5 Fixed-end moments (continued)

Derivation of the Slope-Deflection Equation

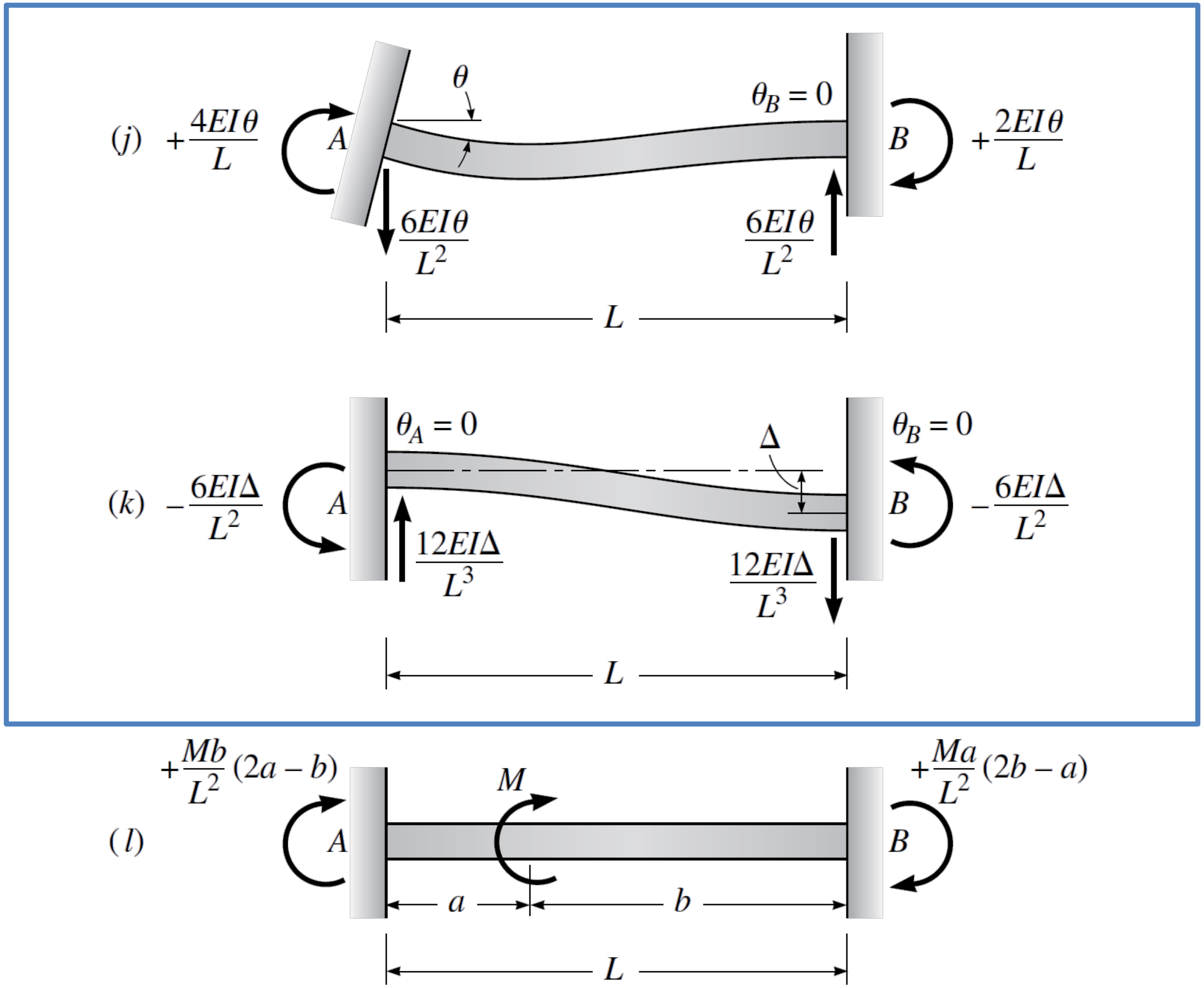
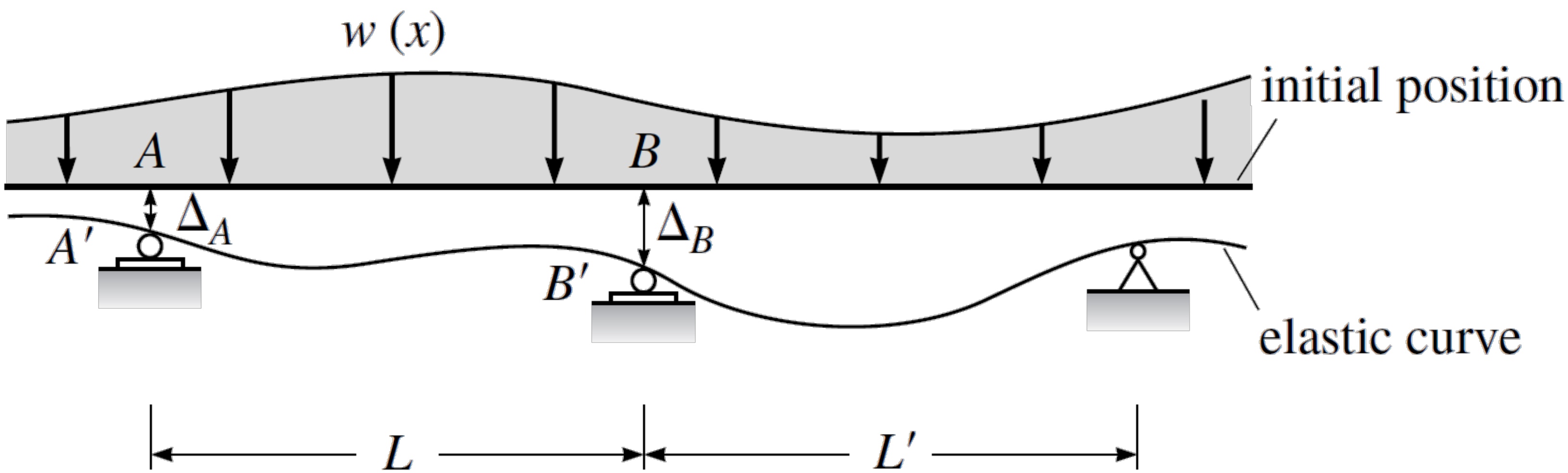


Figure 12.5 Fixed-end moments (continued)

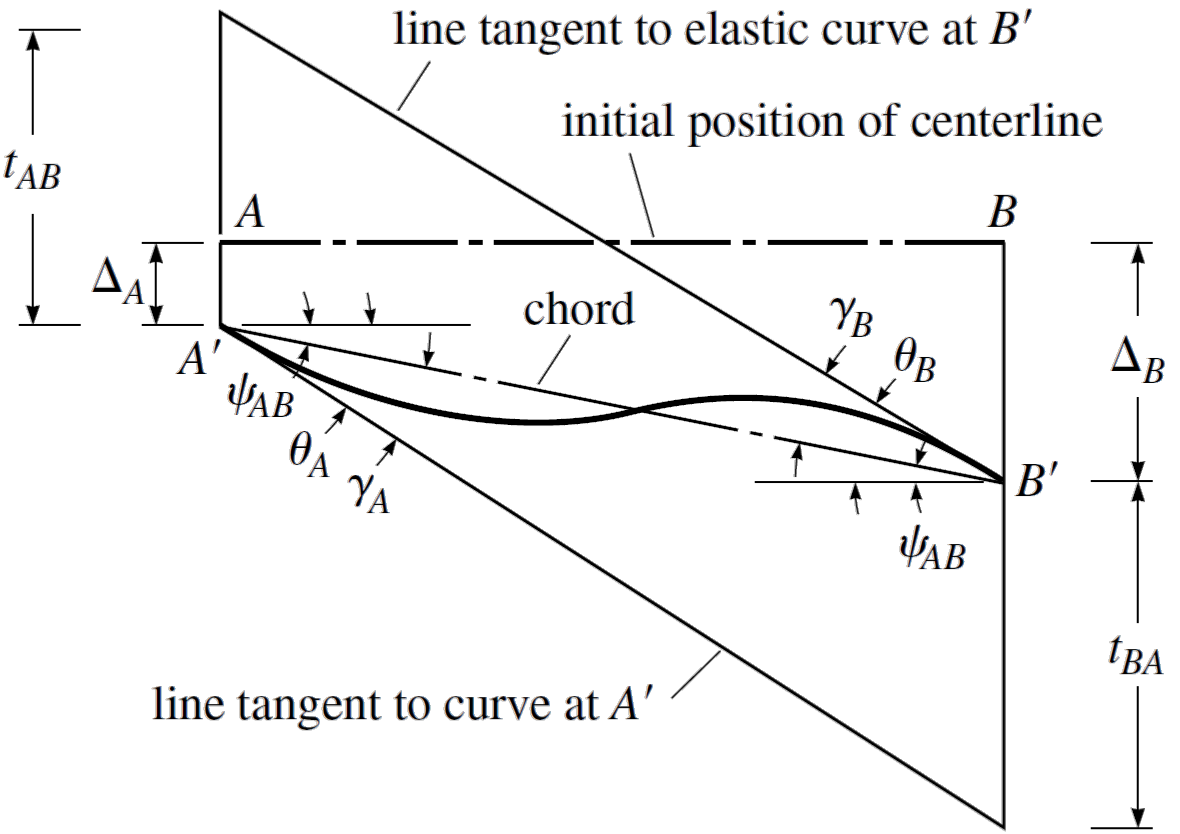
Derivation of the Slope-Deflection Equation



Continuous beam whose supports settle under load

Figure 12.2

§12.3 Derivation of the Slope-Deflection Equation



$$\psi_{AB} = \frac{\Delta_B - \Delta_A}{L}$$

Deformations of member *AB* plotted to an exaggerated vertical scale

Derivation of the Slope-Deflection Equation

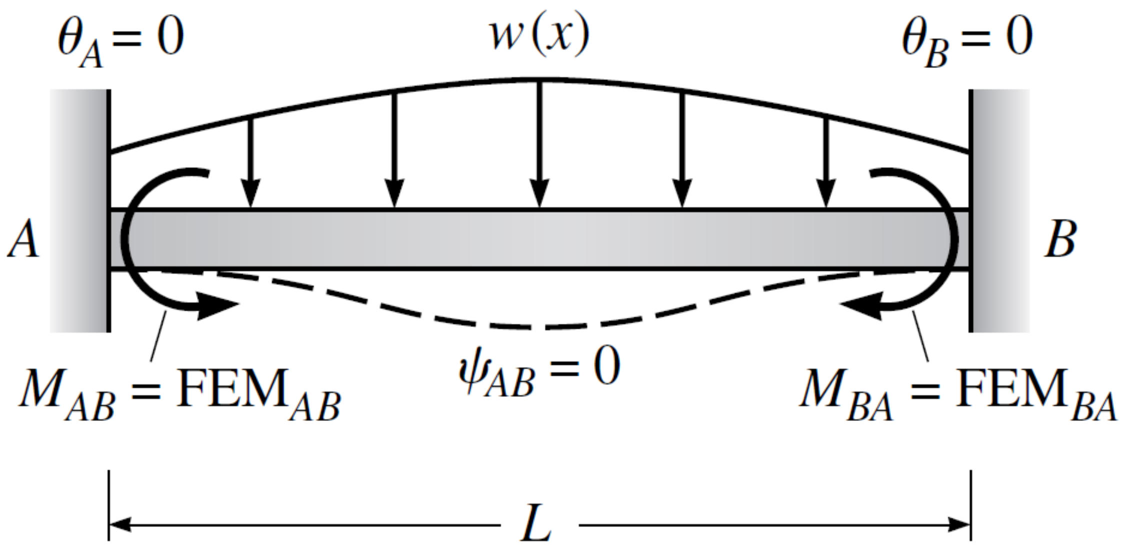
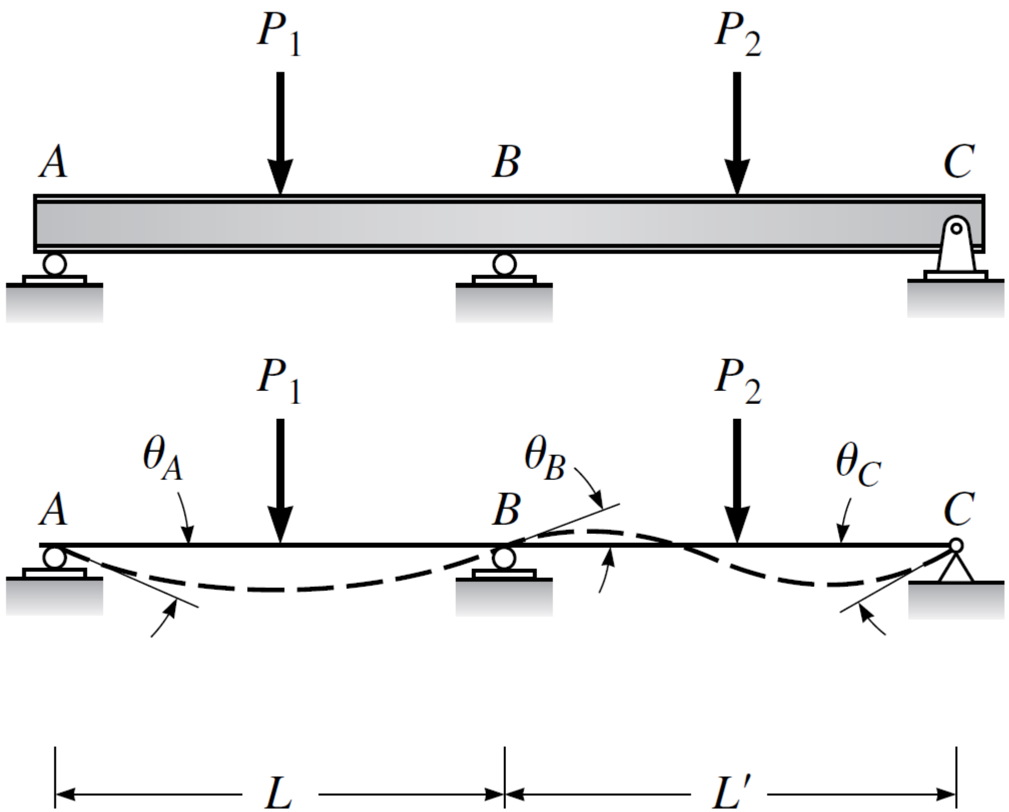


Figure 12.4

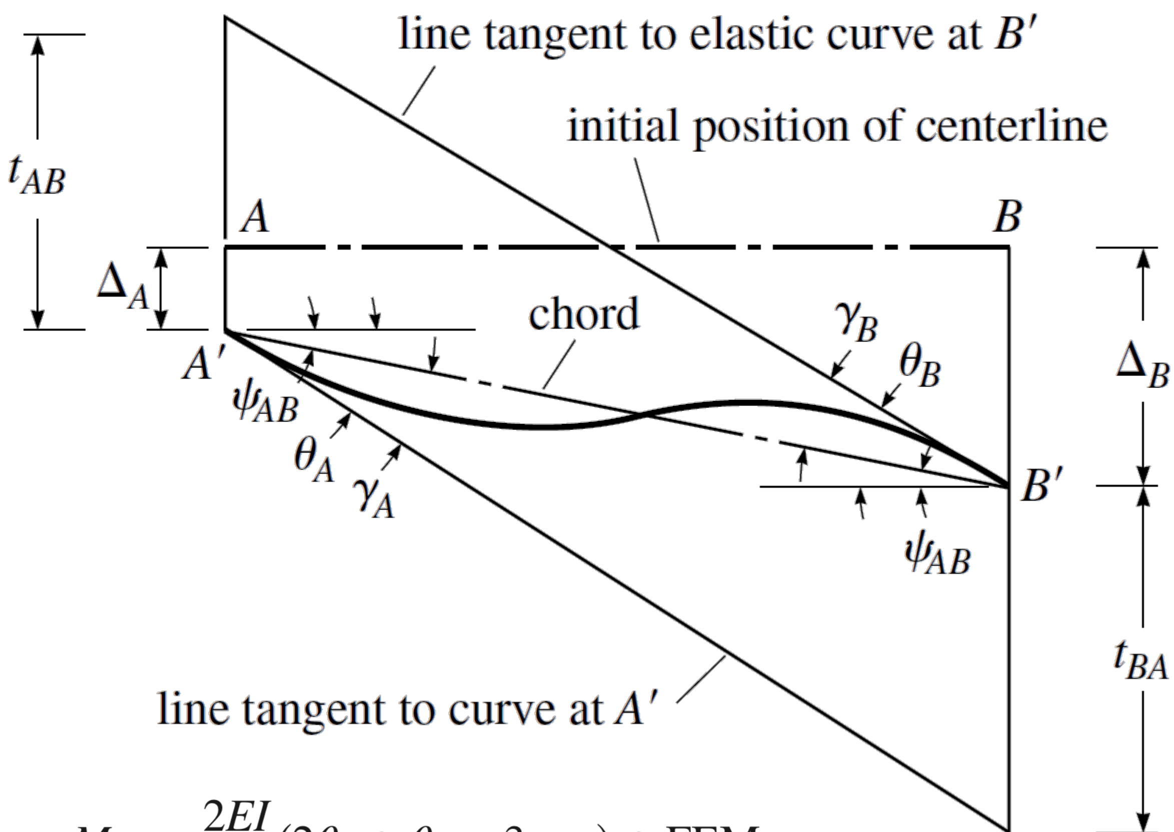
Illustration of the Slope-Deflection Method



Continuous beam with applied loads
(deflected shape shown by dashed line)

Figure 12.1

§12.3 Derivation of the Slope-Deflection Equation



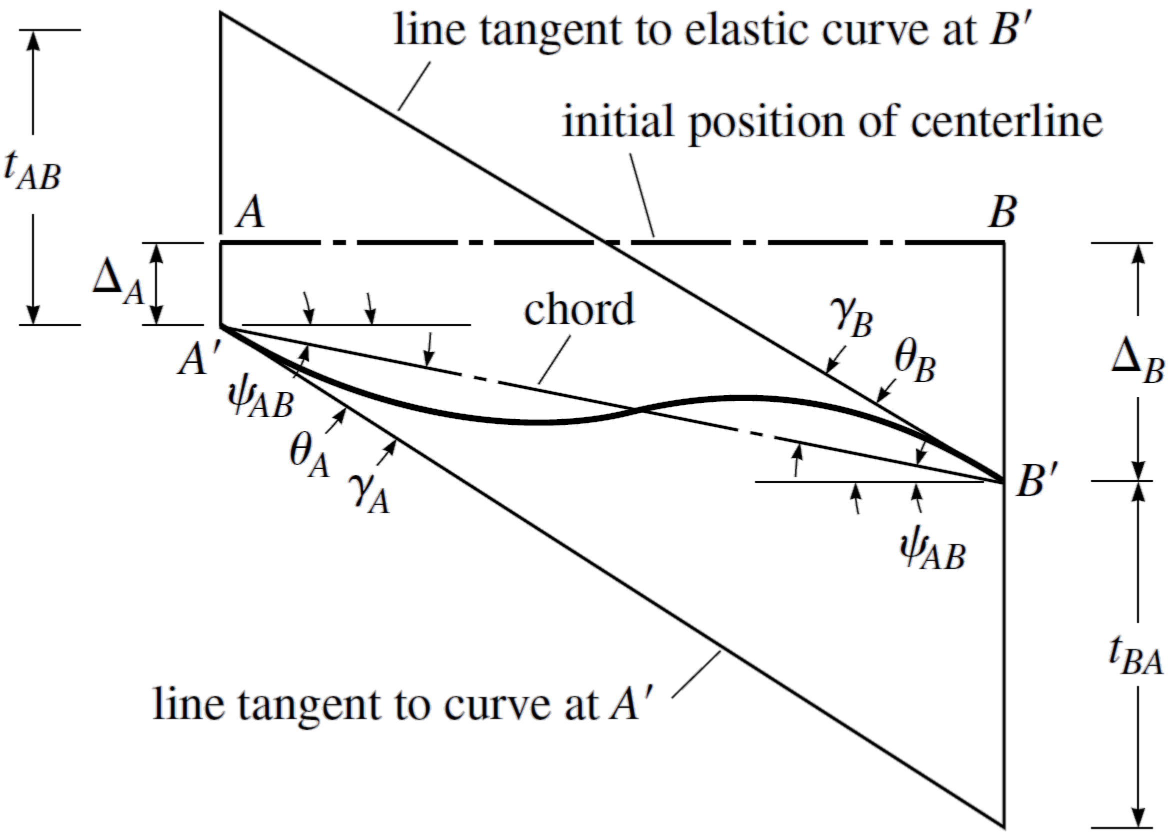
$$\psi_{AB} = \frac{\Delta_B - \Delta_A}{L}$$

Deformations of member *AB* plotted to an exaggerated vertical scale

$$M_{AB} = \frac{2EI}{L}(2\theta_A + \theta_B - 3\psi_{AB}) + \text{FEM}_{AB}$$

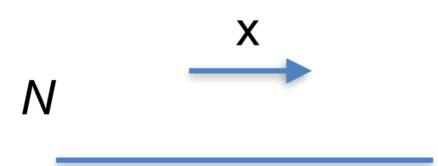
$$M_{BA} = \frac{2EI}{L}(2\theta_B + \theta_A - 3\psi_{AB}) + \text{FEM}_{BA}$$

§12.3 Derivation of the Slope-Deflection Equation



$$\psi_{AB} = \frac{\Delta_B - \Delta_A}{L}$$

Deformations of member *AB* plotted to an exaggerated vertical scale



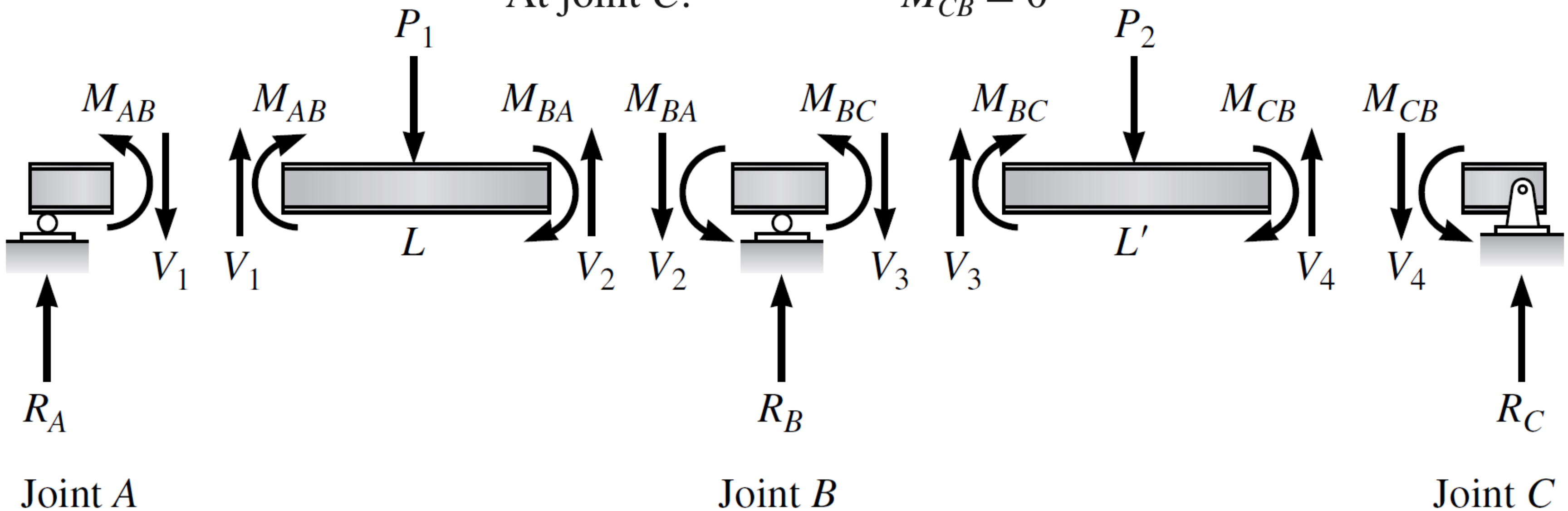
$$M_{NF} = \frac{2EI}{L}(2\theta_N + \theta_F - 3\psi_{NF}) + \mathrm{FEM}_{NF}$$

Illustration of the Slope-Deflection Method

At joint A: $\quad M_{AB} = 0$

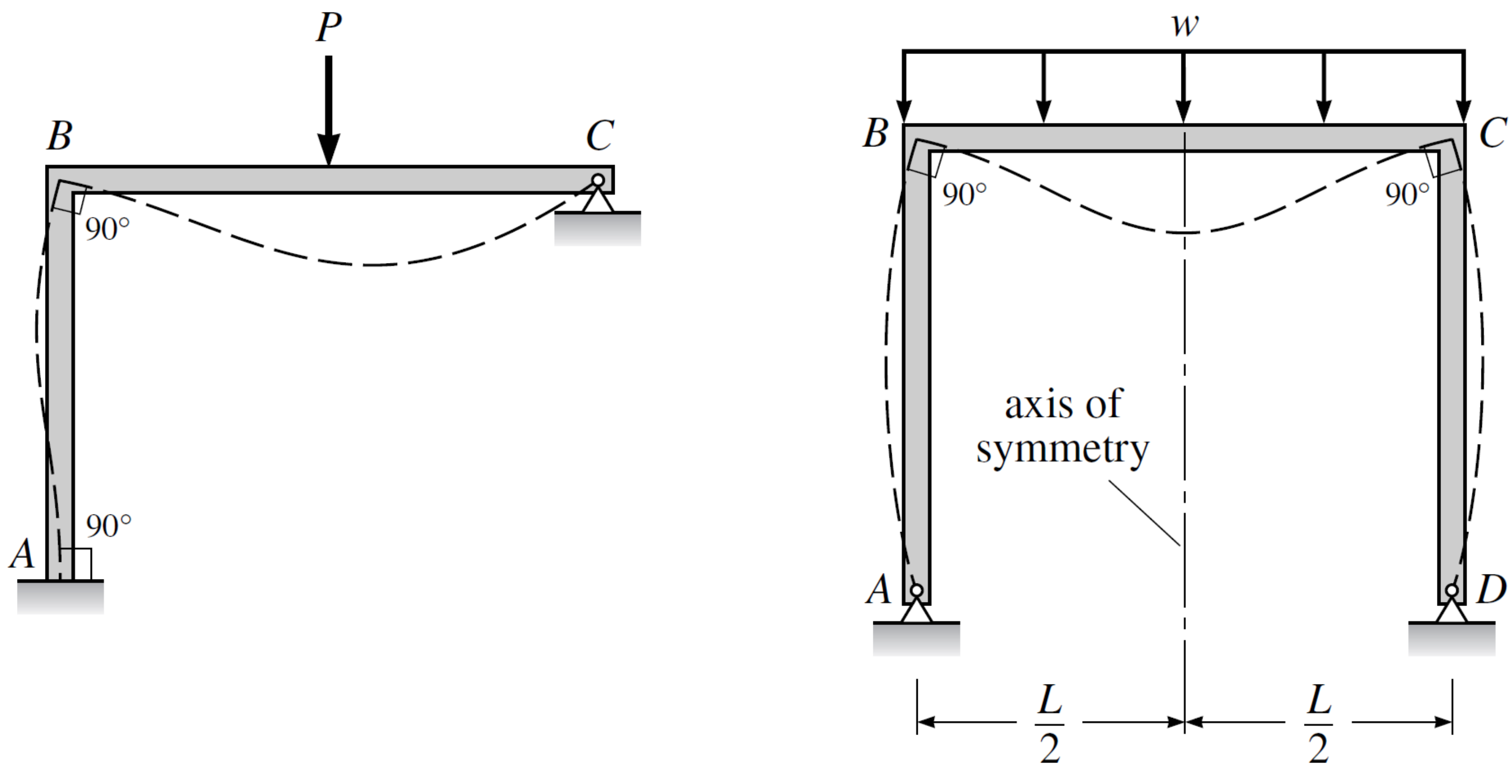
At joint B: $\quad M_{BA} + M_{BC} = 0$

At joint C: $\quad M_{CB} = 0$



Free bodies of joints and beams (sign convention: **Clockwise moment on the end of a member is positive**)

Analysis of Structures by the Slope-Deflection Method

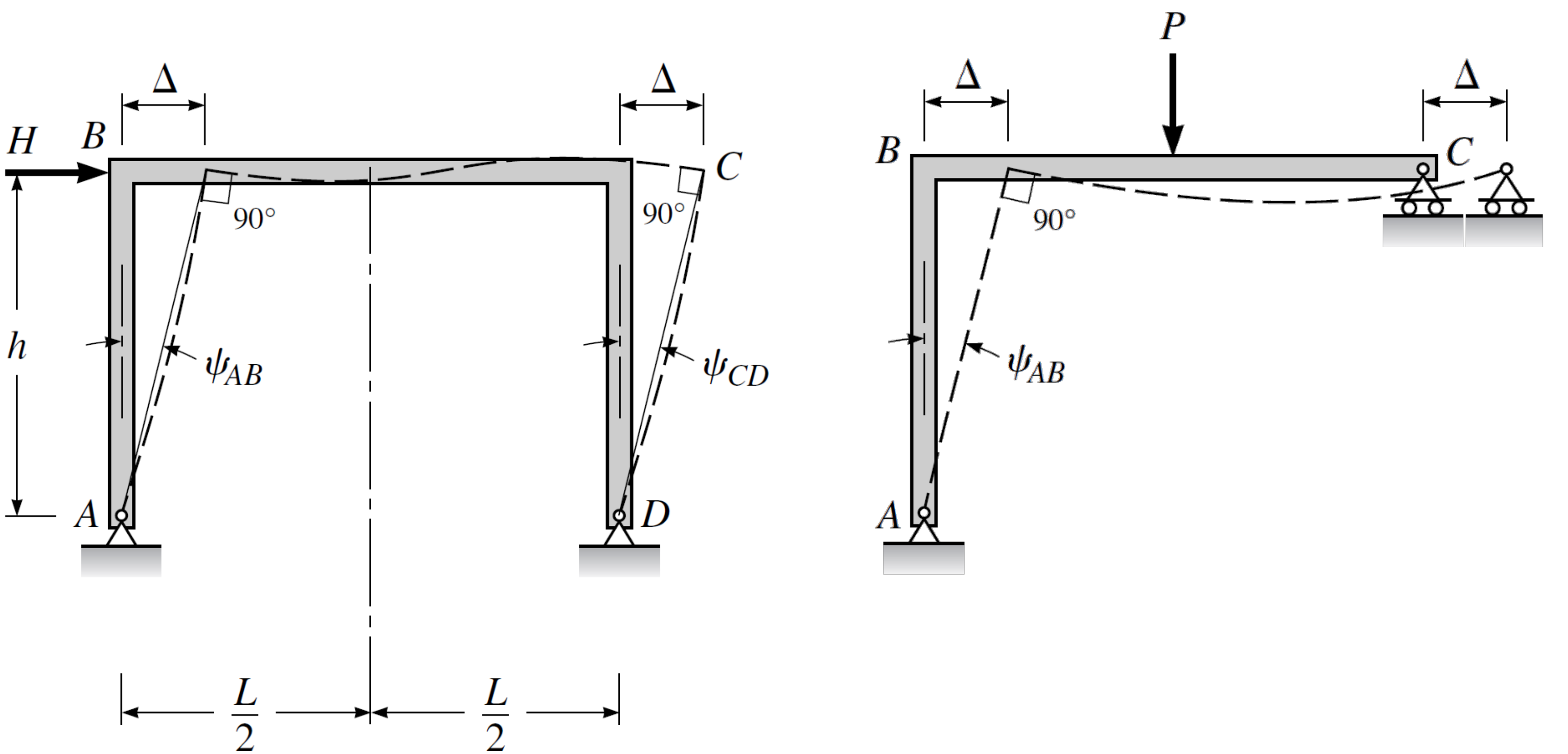


All joints restrained against displacement; all chord rotations ψ equal zero

Due to symmetry of structure and loading, joints free to rotate but not translate; chord rotations equal zero

Figure 12.7

Analysis of Structures by the Slope-Deflection Method

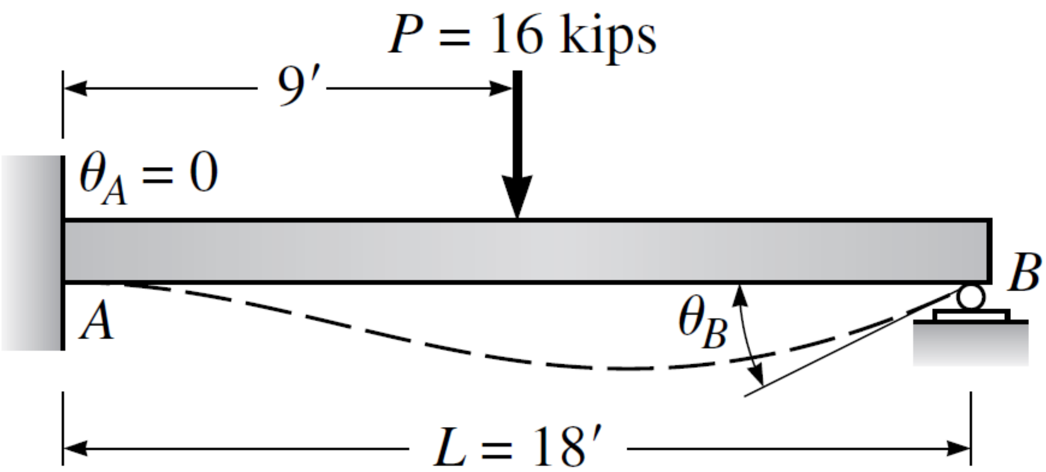


Unbraced frames with chord rotations

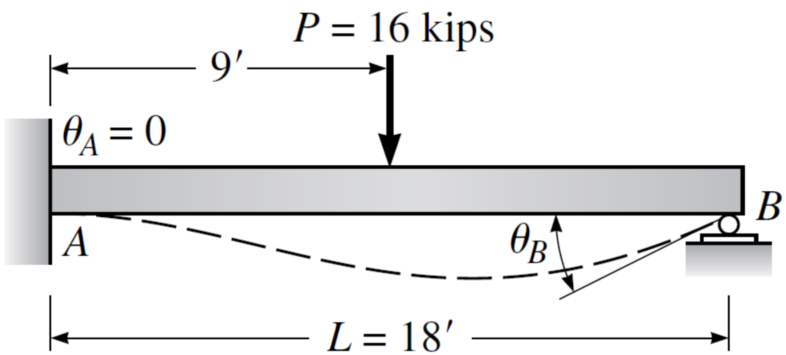
Figure 12.7 (continued)

Example 12.2

Using the slope-deflection method, determine the member end moments in the indeterminate beam shown in Figure 12.8*a*. The beam, which behaves elastically, carries a concentrated load at midspan. After the end moments are determined, draw the shear and moment curves. If I = 240 in^4 and E = 30,000 kips/in^2, compute the magnitude of the slope at joint B.

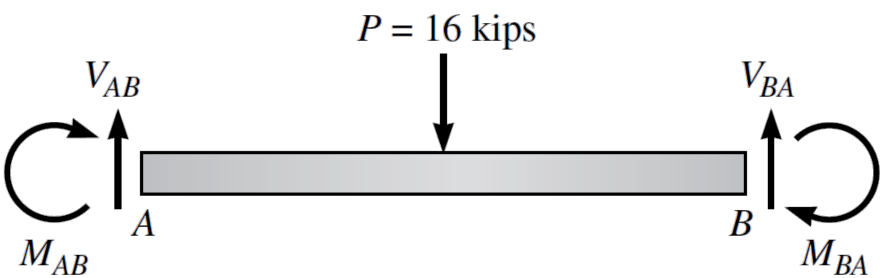


Example 12.2 Solution



- Since joint A is fixed against rotation, $\theta_A = 0$; therefore, the only unknown displacement is θ_B. Using the slope-deflection equation

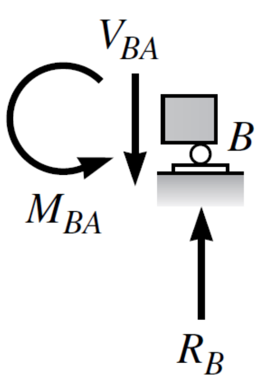
$$M_{NF} = \frac{2EI}{L}(2\theta_N + \theta_F - 3\psi_{NF}) + \text{FEM}_{NF}$$



- The member end moments are:

$$M_{AB} = \frac{2EI}{L}(\theta_B) - \frac{PL}{8}$$

$$M_{BA} = \frac{2EI}{L}(2\theta_B) + \frac{PL}{8}$$



- To determine θ_B, write the equation of moment equilibrium at joint B

$$\circlearrowright + \quad \Sigma M_B = 0$$

$$M_{BA} = 0$$

Example 12.2 Solution (continued)

- Substituting the value of M_{BA} and solving for θ_B give

$$\frac{4EI}{L}\theta_B + \frac{PL}{8} = 0$$

$$\theta_B = -\frac{PL^2}{32EI}$$

where the minus sign indicates both that the *B* end of member *AB* and joint *B* rotate in the counterclockwise direction

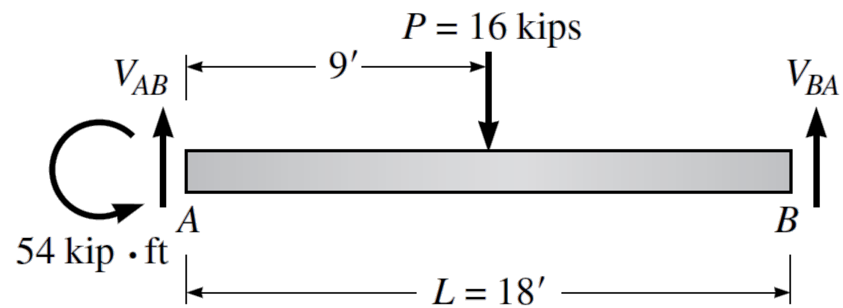
- To determine the member end moments,

$$M_{AB} = \frac{2EI}{L}\left(\frac{-PL^2}{32EI}\right) - \frac{PL}{8} = -\frac{3PL}{16} = -54 \text{ kip}\cdot\text{ft} \qquad \textbf{Ans.}$$

$$M_{BA} = \frac{4EI}{L}\left(\frac{-PL^2}{32EI}\right) + \frac{PL}{8} = 0$$

Example 12.2 Solution (continued)

- To complete the analysis, apply the equations of statics to a free body of member *AB*



Free body used to compute end shears

$$\circlearrowright^{+} \quad \Sigma M_A = 0$$

$$0 = (16\text{ kips})(9\text{ ft}) - V_{BA}(18\text{ ft}) - 54\text{ kip}\cdot\text{ft}$$

$$V_{BA} = 5\text{ kips}$$

$$\overset{+}{\uparrow} \quad \Sigma F_y = 0$$

$$0 = V_{BA} + V_{AB} - 16$$

$$V_{AB} = 11\text{ kips}$$

- To evaluate θ_B, express all variables in units of inches and kips.

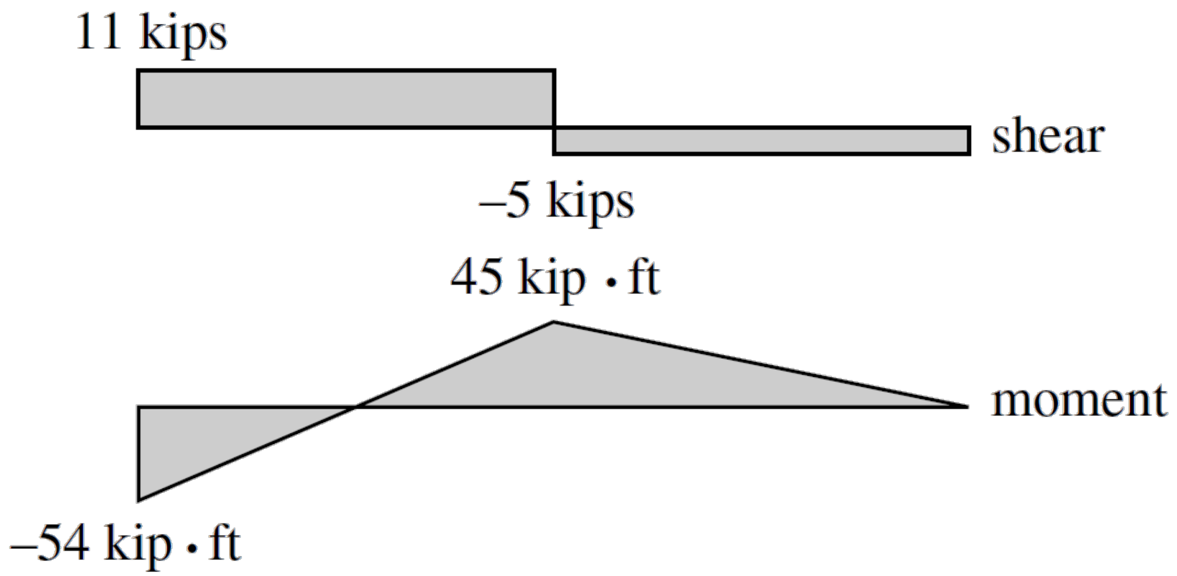
$$\theta_B = -\frac{PL^2}{32EI} = -\frac{16(18 \times 12)^2}{32(30{,}000)240} = -0.0032\text{ rad}$$

Example 12.2 Solution (continued)

- Expressing θ_B in degrees

$$\frac{2\pi \text{ rad}}{360^\circ} = \frac{-0.0032}{\theta_B}$$

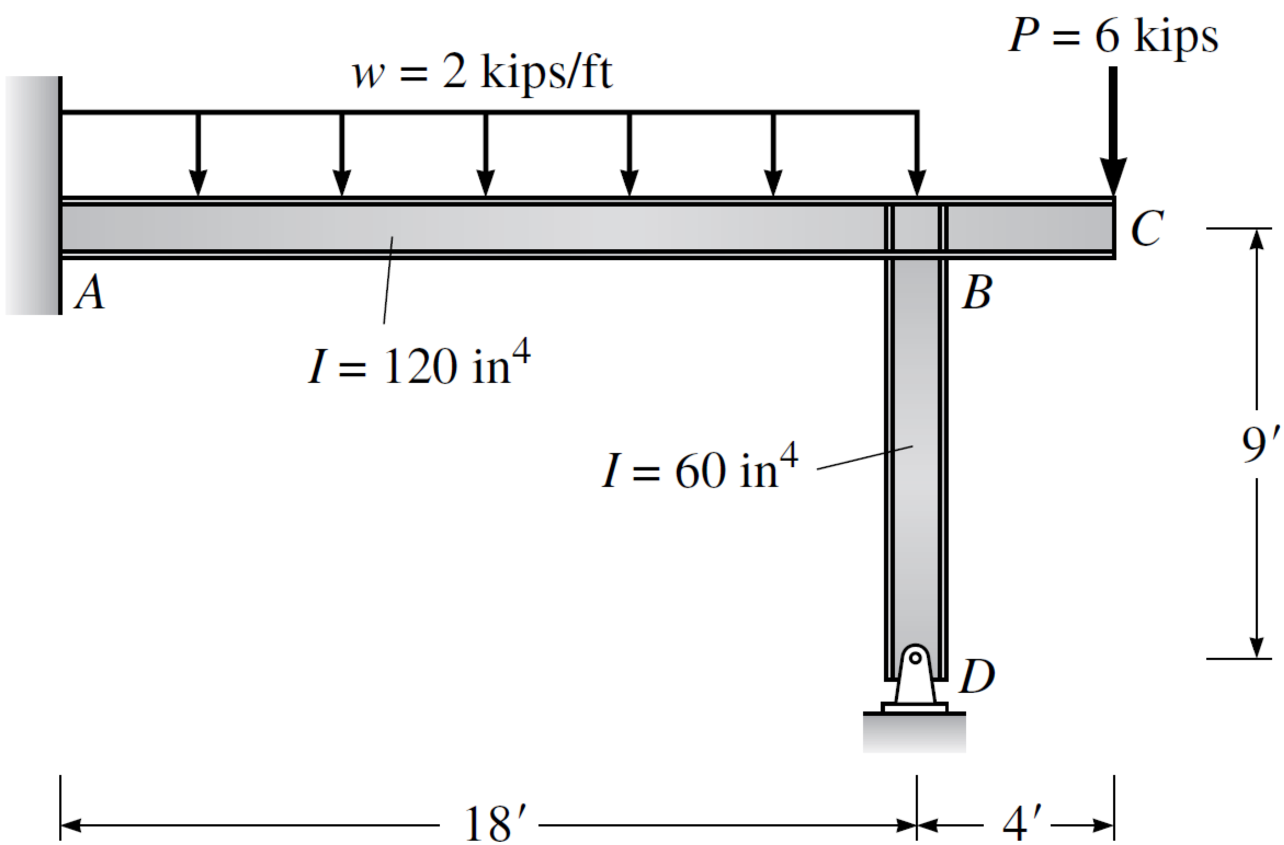
$$\theta_B = -0.183^\circ \qquad \textbf{Ans.}$$



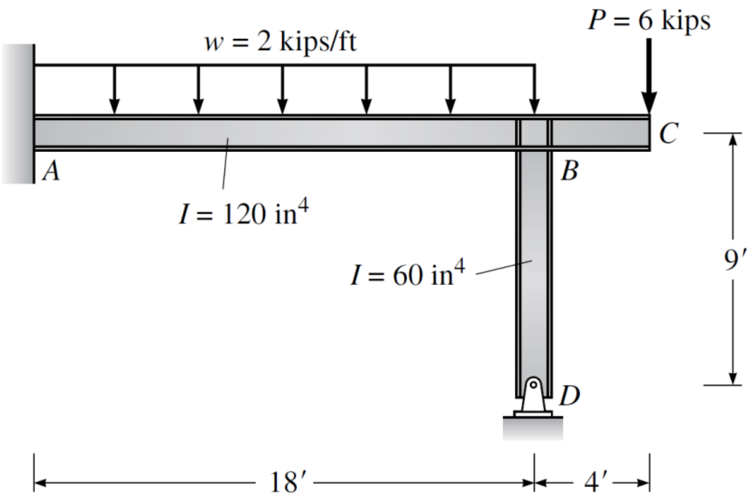
Shear and moment curves

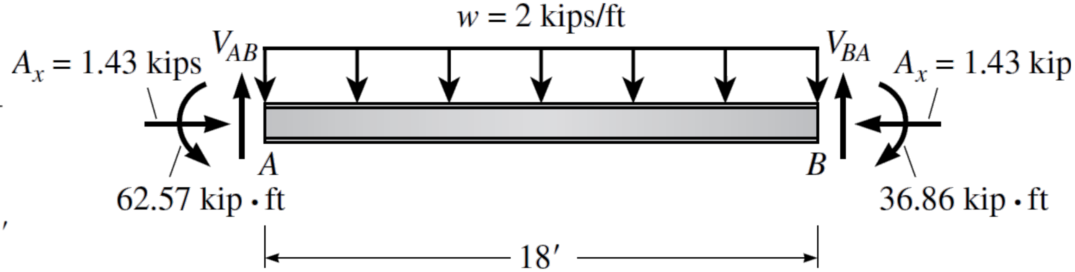
Example 12.3

Using the slope-deflection method, determine the member end moments in the braced frame shown in Figure 12.9*a*. Also compute the reactions at support *D*, and draw the shear and moment curves for members *AB* and *BD*.



Example 12.3 Solution





- Use the slope-deflection equation

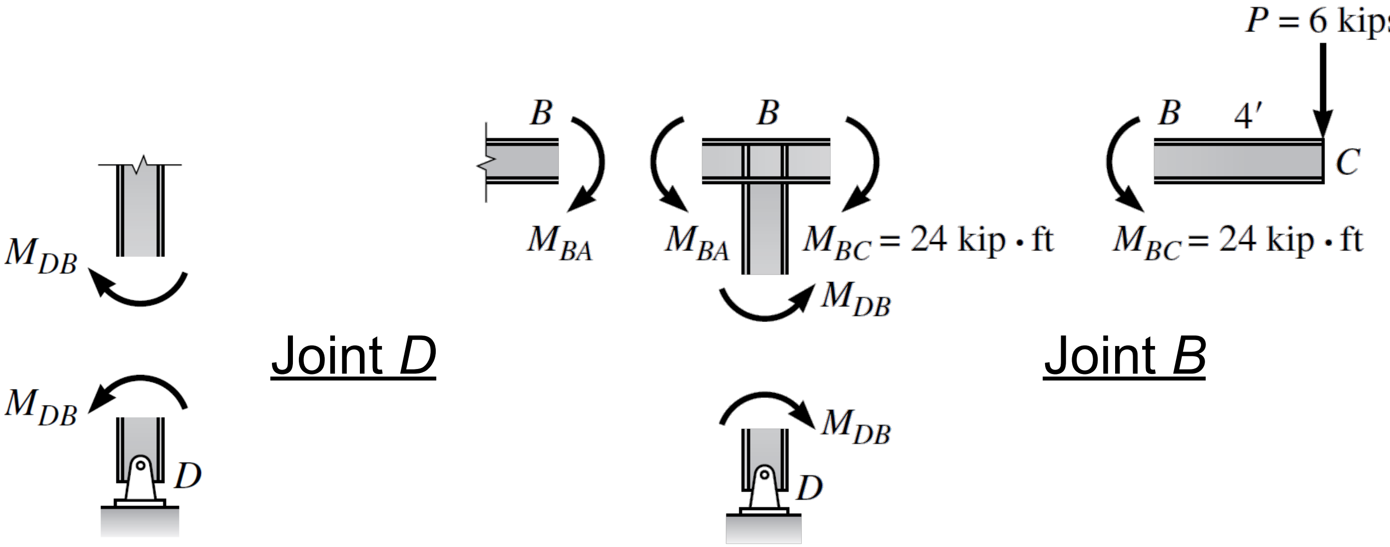
$$M_{NF} = \frac{2EI}{L}(2\theta_N + \theta_F - 3\psi_{NF}) + \text{FEM}_{NF}$$

- The fixed-end moments produced by the uniform load on member *AB*

$$\text{FEM}_{AB} = -\frac{wL^2}{12}$$

$$\text{FEM}_{BA} = +\frac{wL^2}{12}$$

Example 12.3 Solution (continued)



- Express the member end moments as

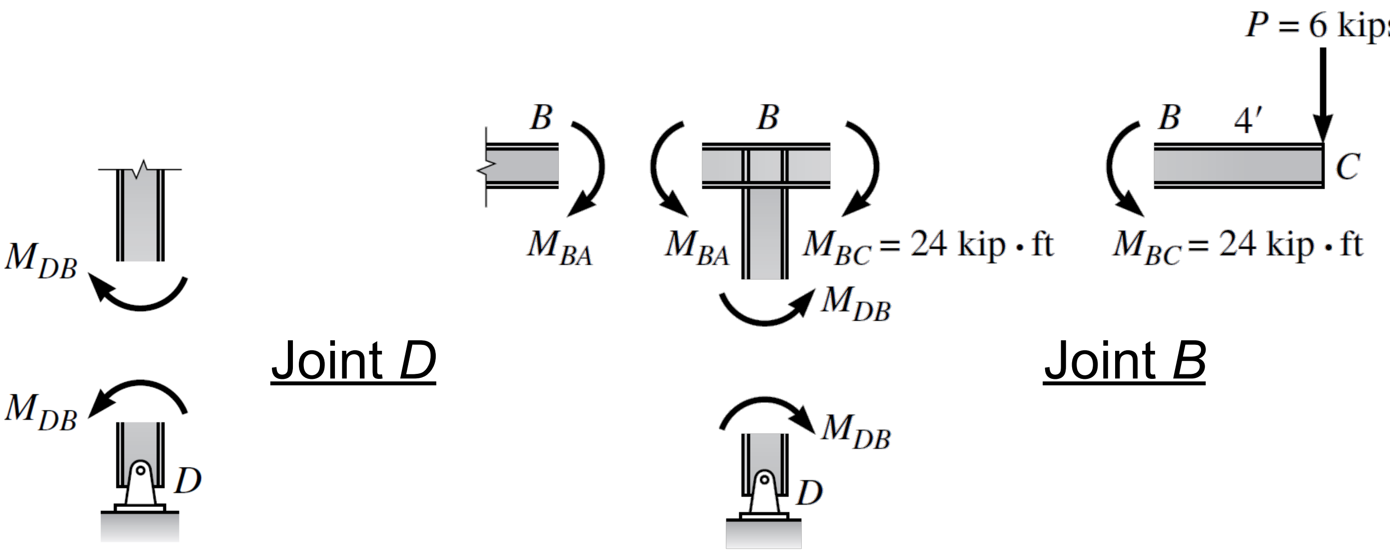
$$M_{AB} = \frac{2E(120)}{18(12)}(\theta_B) - \frac{2(18)^2(12)}{12} = 1.11E\theta_B - 648$$

$$M_{BA} = \frac{2E(120)}{18(12)}(2\theta_B) + \frac{2(18)^2(12)}{12} = 2.22E\theta_B + 648$$

$$M_{BD} = \frac{2E(60)}{9(12)}(2\theta_B + \theta_D) = 2.22E\theta_B + 1.11E\theta_D$$

$$M_{DB} = \frac{2E(60)}{9(12)}(2\theta_D + \theta_B) = 2.22E\theta_D + 1.11E\theta_B$$

Example 12.3 Solution (continued)



- To solve for the unknown joint displacements θ_B and θ_D, write equilibrium equations at joints *D* and *B*.

At joint *D* (see Fig. 12.9*b*): $\overset{+}{\circlearrowleft} \quad \Sigma M_D = 0$

$$M_{DB} = 0$$

At joint *B* (see Fig. 12.9*c*): $\overset{+}{\circlearrowleft} \quad \Sigma M_B = 0$

$$M_{BA} + M_{BD} - 24(12) = 0$$

Example 12.3 Solution (continued)

- Express the moments in terms of displacements; write the equilibrium equations as

At joint D: $$2.22E\theta_D + 1.11E\theta_B = 0$$

At joint B: $$(2.22E\theta_B + 648) + (2.22E\theta_B + 1.11E\theta_D) - 288 = 0$$

- Solving equations simultaneously gives

$$\theta_D = \frac{46.33}{E}$$

$$\theta_B = -\frac{92.66}{E}$$

Example 12.3 Solution (continued)

- To establish the values of the member end moments, the values of θ_B and θ_D are substituted

$$M_{AB} = 1.11E\left(-\frac{92.66}{E}\right) - 648$$

$$= -750.85 \text{ kip} \cdot \text{in} = -62.57 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

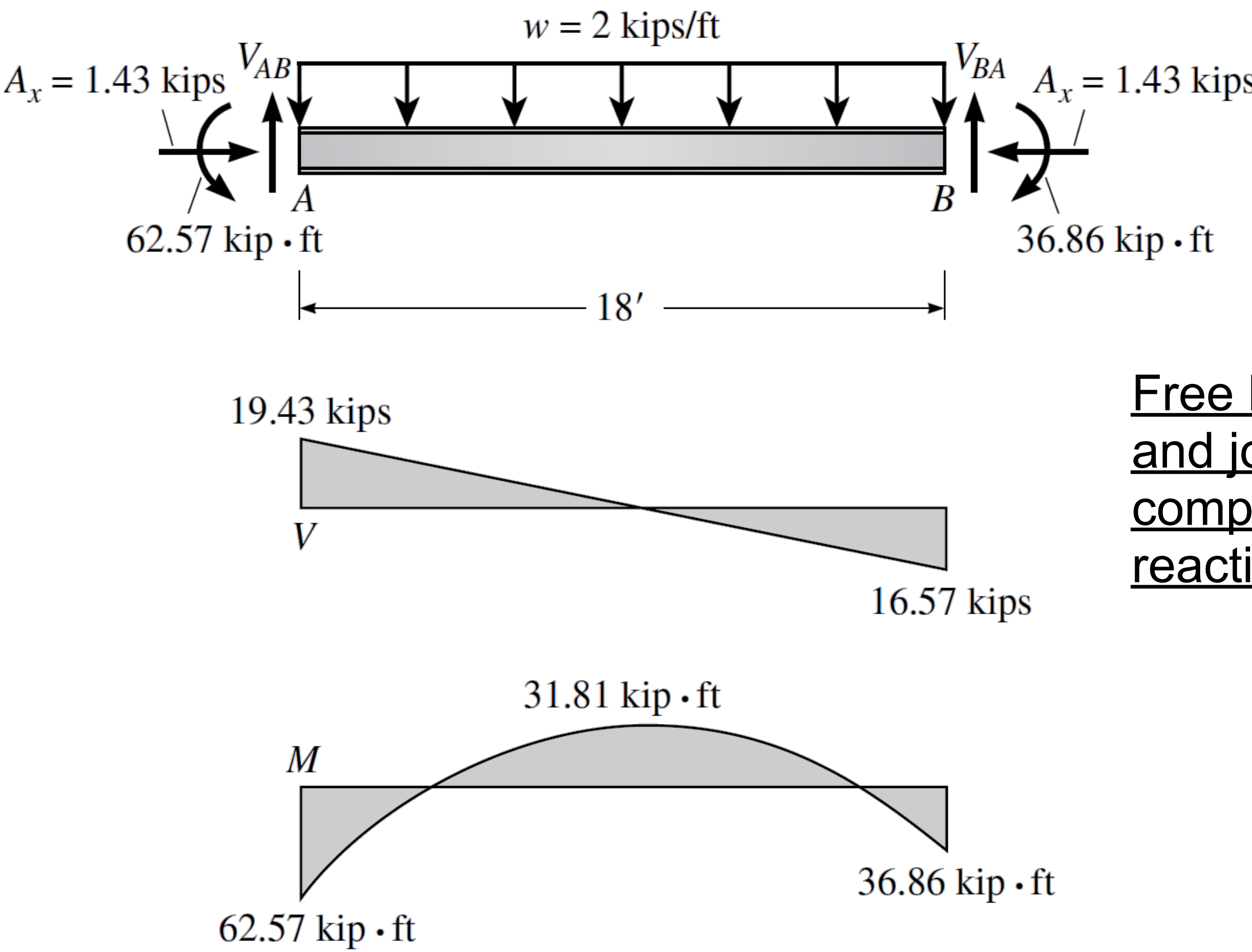
$$M_{BA} = 2.22E\left(-\frac{92.66}{E}\right) + 648$$

$$= 442.29 \text{ kip} \cdot \text{in} = +36.86 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

$$M_{BD} = 2.22E\left(-\frac{92.66}{E}\right) + 1.11E\left(\frac{46.33}{E}\right)$$

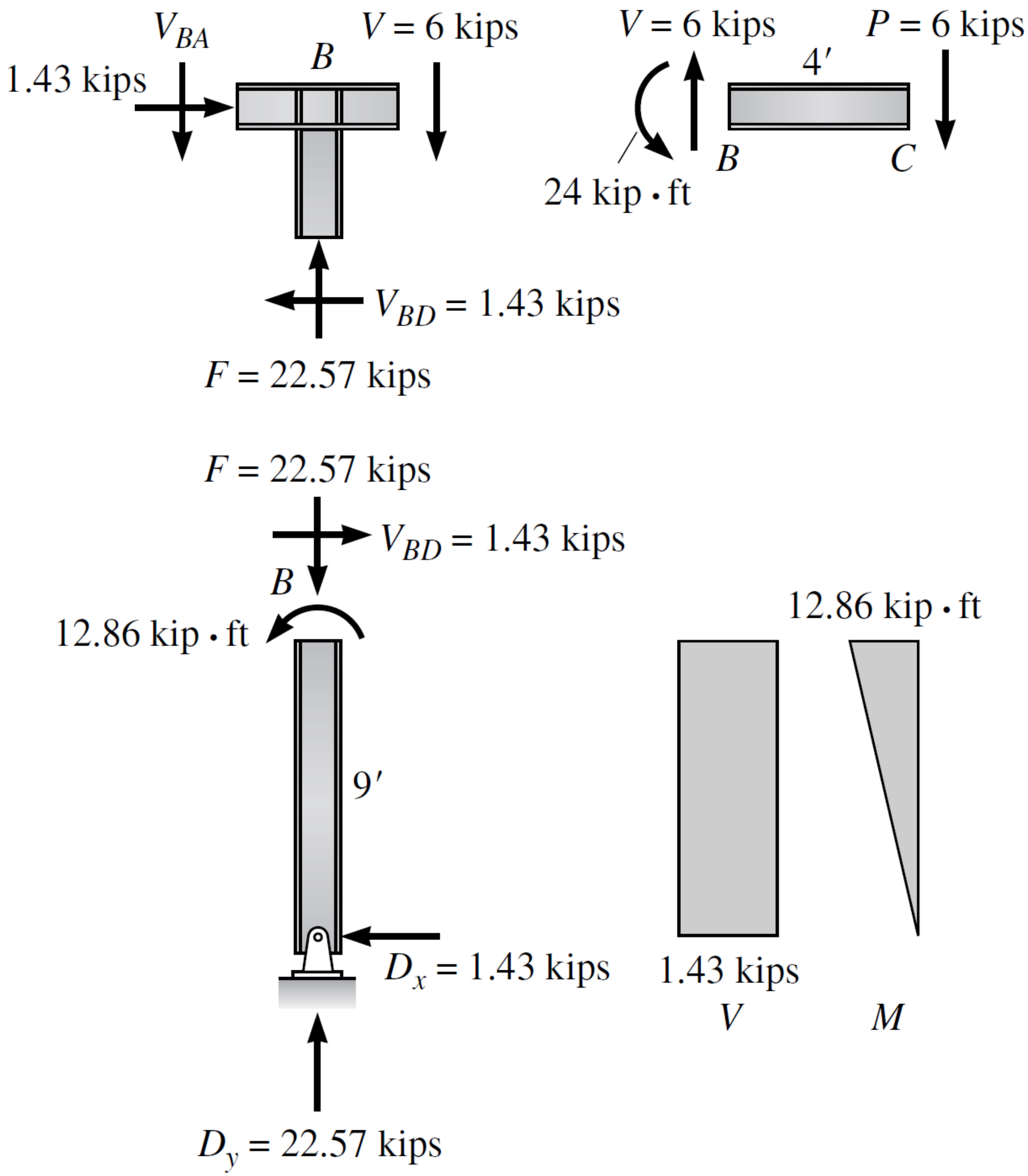
$$= -154.28 \text{ kip} \cdot \text{in} = -12.86 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

Example 12.3 Solution (continued)



Free bodies of members and joints used to compute shears and reactions

Example 12.3 Solution (continued)

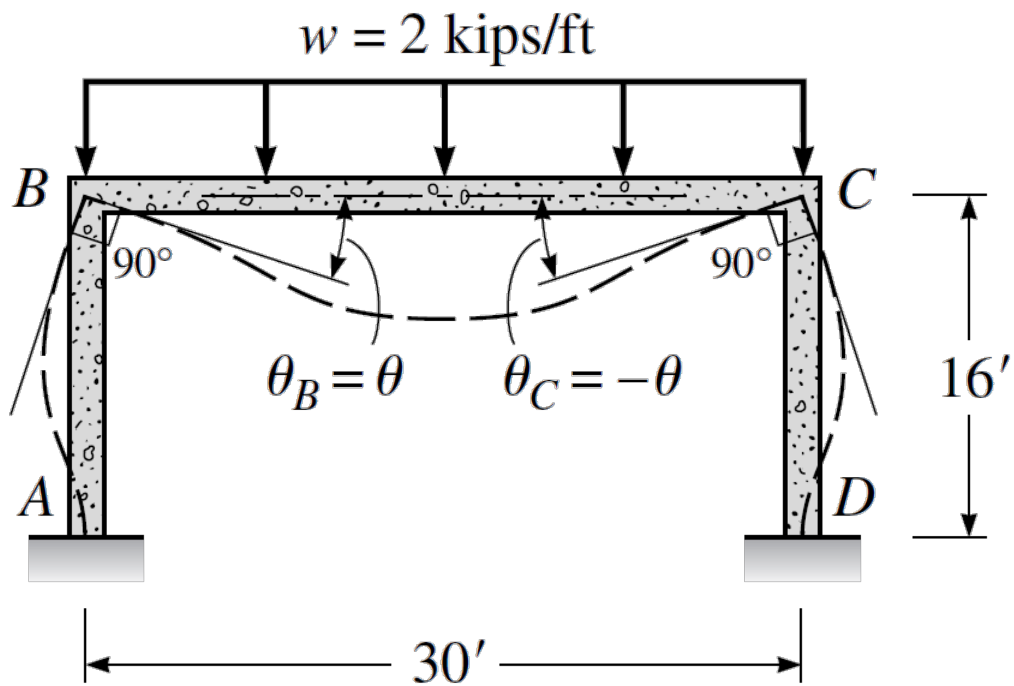


Free bodies of members and joints used to compute shears and reactions

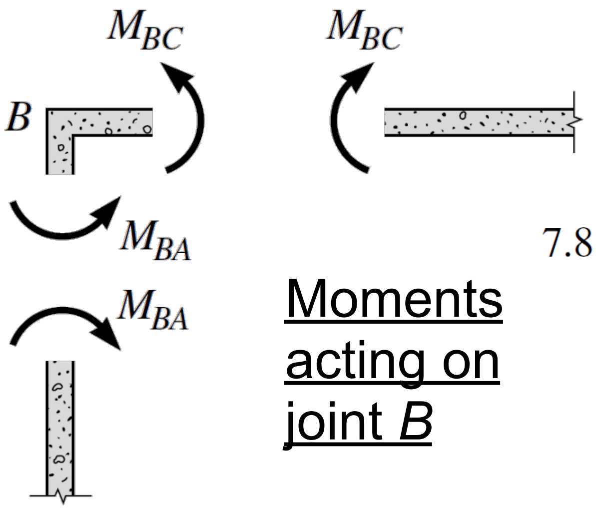
Example 12.4

Use of Symmetry to Simplify the Analysis of a Symmetric Structure with a Symmetric Load

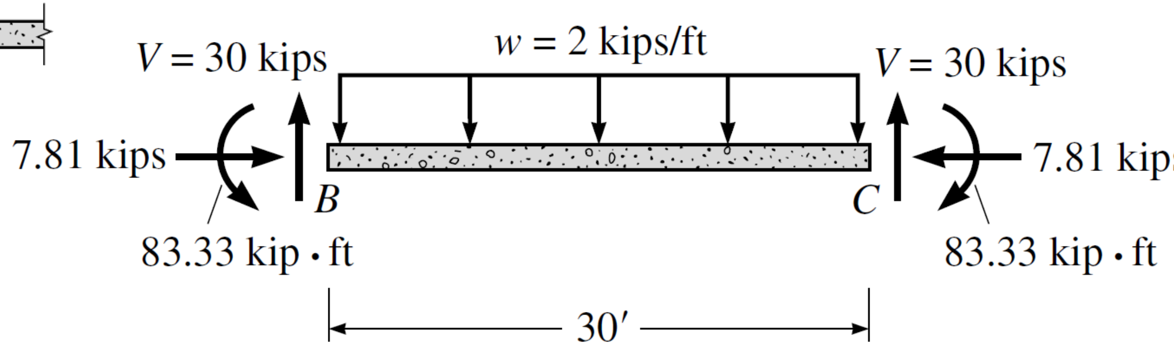
Determine the reactions and draw the shear and moment curves for the columns and girder of the rigid frame shown in Figure 12.10*a.* Given: $I_{AB} = I_{CD} = 120\ \text{in}^4$, $I_{BC} = 360\ \text{in}^4$, and E is constant for all members.



Example 12.4 Solution



Moments acting on joint *B*



- Expressing member end moments with Equation 12.16, reading the value of fixed-end moment for member *BC* from Figure 12.5*d*, and substituting $\theta_B = \theta$ and $\theta_C = -\theta$,

$$M_{AB} = \frac{2E(120)}{16(12)}(\theta_B) = 1.25E\theta_B$$

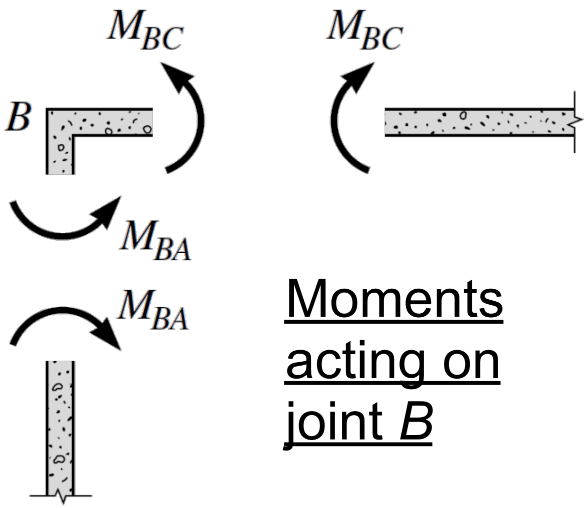
$$M_{BA} = \frac{2E(120)}{16(12)}(2\theta_B) = 2.50E\theta_B$$

$$M_{BC} = \frac{2E(360)}{30(12)}(2\theta_B + \theta_C) - \frac{wL^2}{12}$$

$$= 2E[2\theta + (-\theta)] - \frac{2(30)^2(12)}{12}$$

$$= 2E\theta - 1800$$

Example 12.4 Solution (continued)



Moments acting on joint *B*

- Writing the equilibrium equation at joint *B* yields

$$M_{BA} + M_{BC} = 0$$

- Substituting Equations 2 and 3 into Equation 4 and solving for θ produce

$$2.5E\theta + 2.0E\theta - 1800 = 0$$

$$\theta = \frac{400}{E}$$

Example 12.4 Solution (continued)

- Substituting the value of θ given by Equation 5 into Equations 1, 2, and 3 gives

$$M_{AB} = 1.25E\left(\frac{400}{E}\right)$$

$$= 500 \text{ kip} \cdot \text{in} = 41.67 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

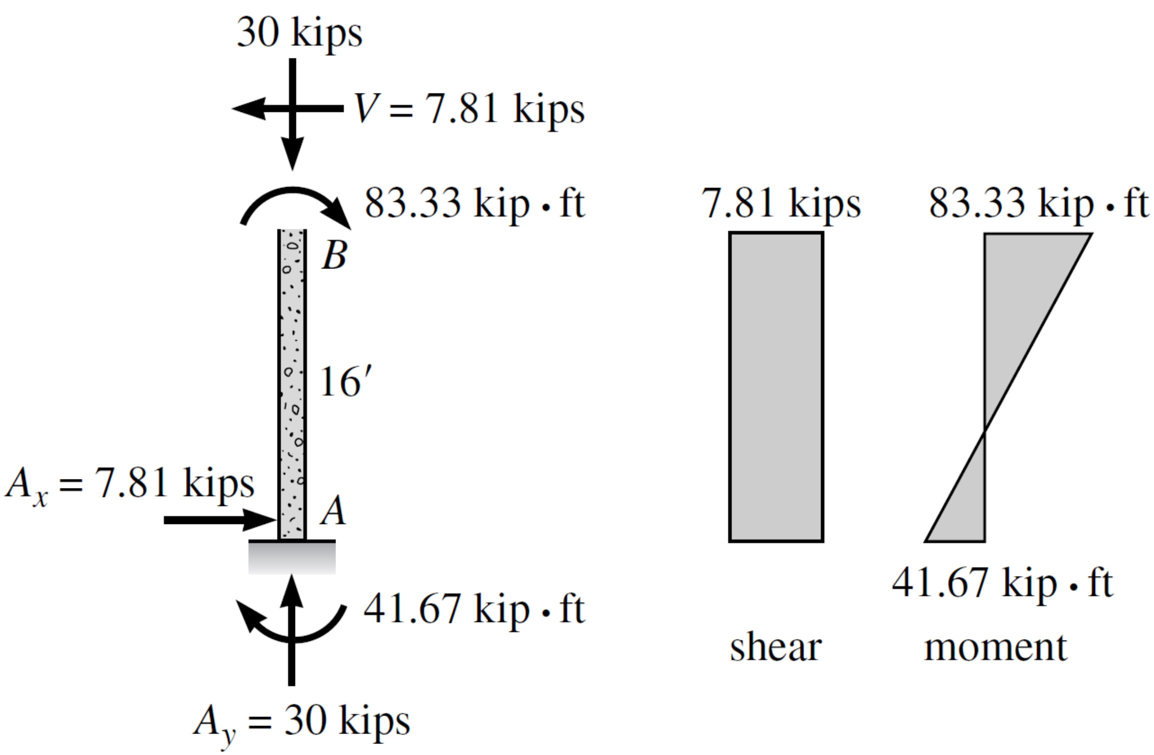
$$M_{BA} = 2.5E\left(\frac{400}{E}\right)$$

$$= 1000 \text{ kip} \cdot \text{in} = 83.33 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

$$M_{BC} = 2E\left(\frac{400}{E}\right) - 1800$$

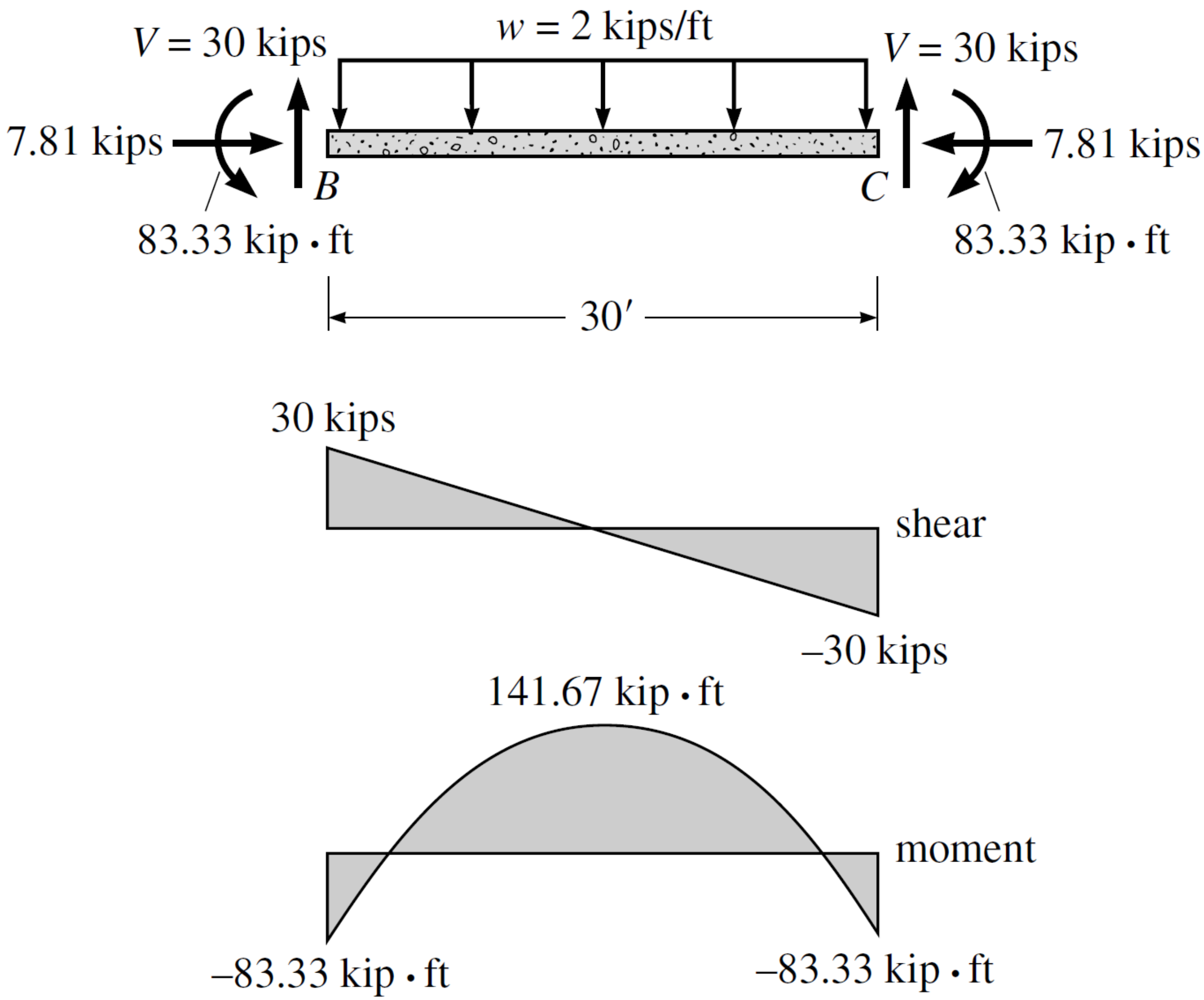
$$= -1000 \text{ kip} \cdot \text{in} = -83.33 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

Example 12.4 Solution (continued)



Free bodies of girder *BC* and column *AB* used to compute shears; final shear and moment curves also shown

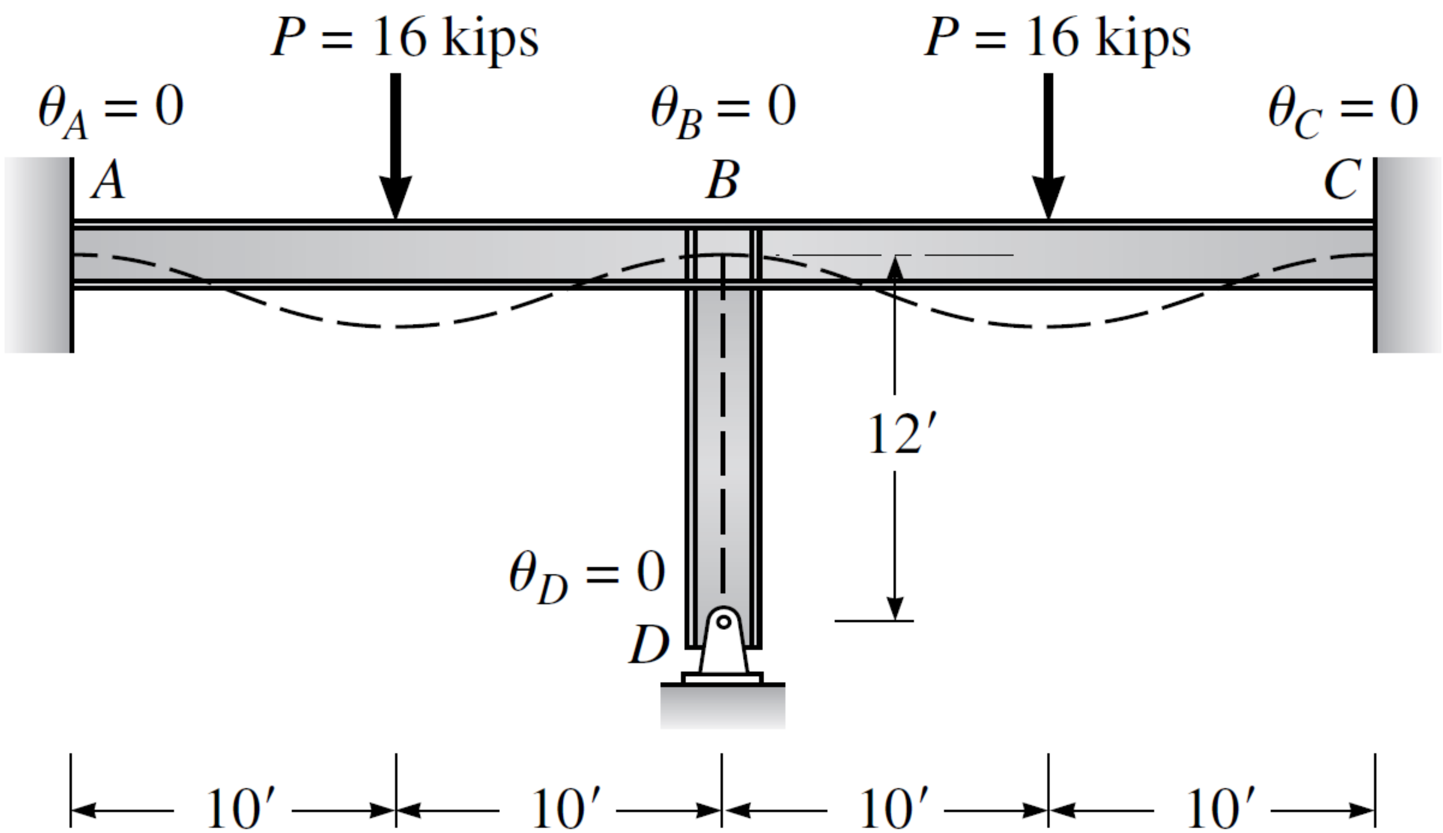
Example 12.4 Solution (continued)



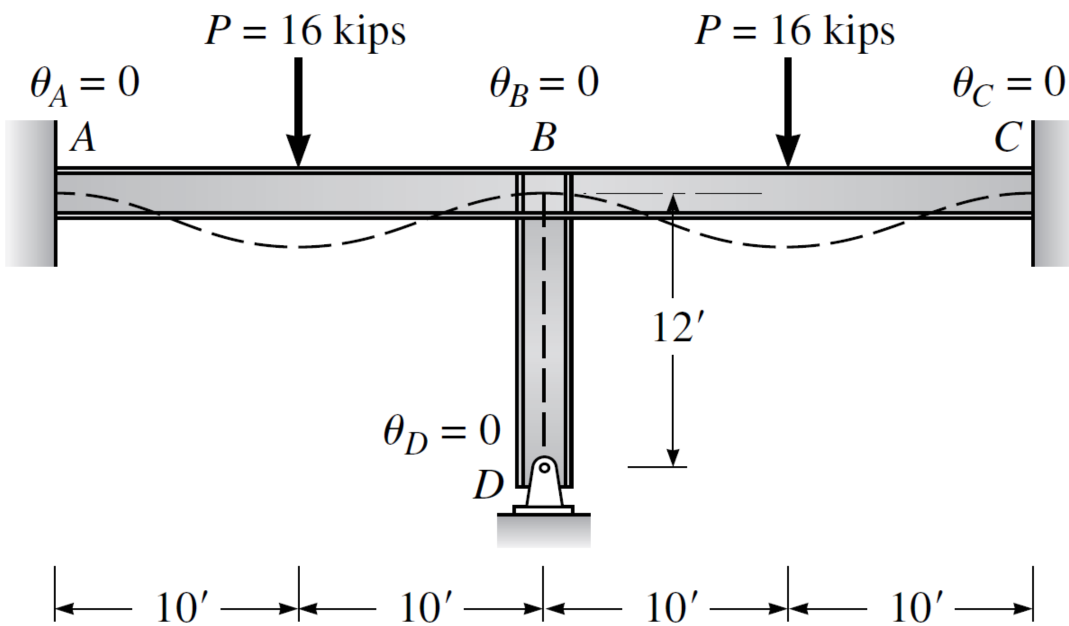
Free bodies of girder *BC* and column *AB* used to compute shears; final shear and moment curves also shown

Example 12.5

Using symmetry to simplify the slope-deflection analysis of the frame in Figure 12.11*a*, determine the reactions at supports *A* and *D*. *EI* is constant for all members.



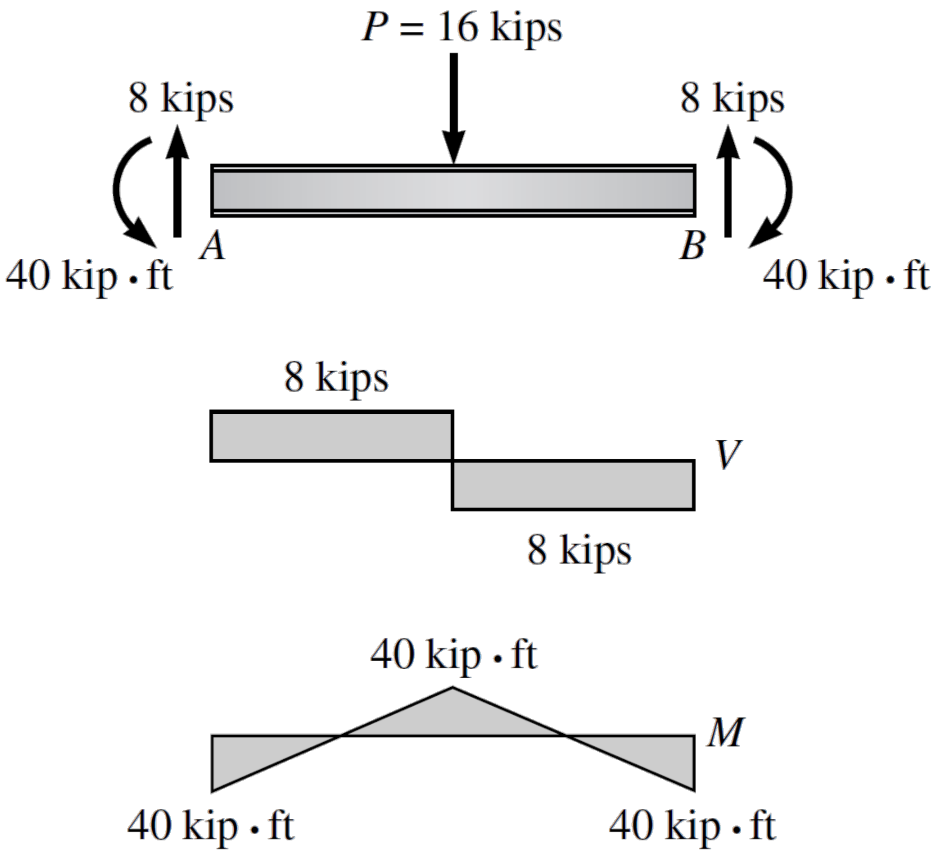
Example 12.5 Solution

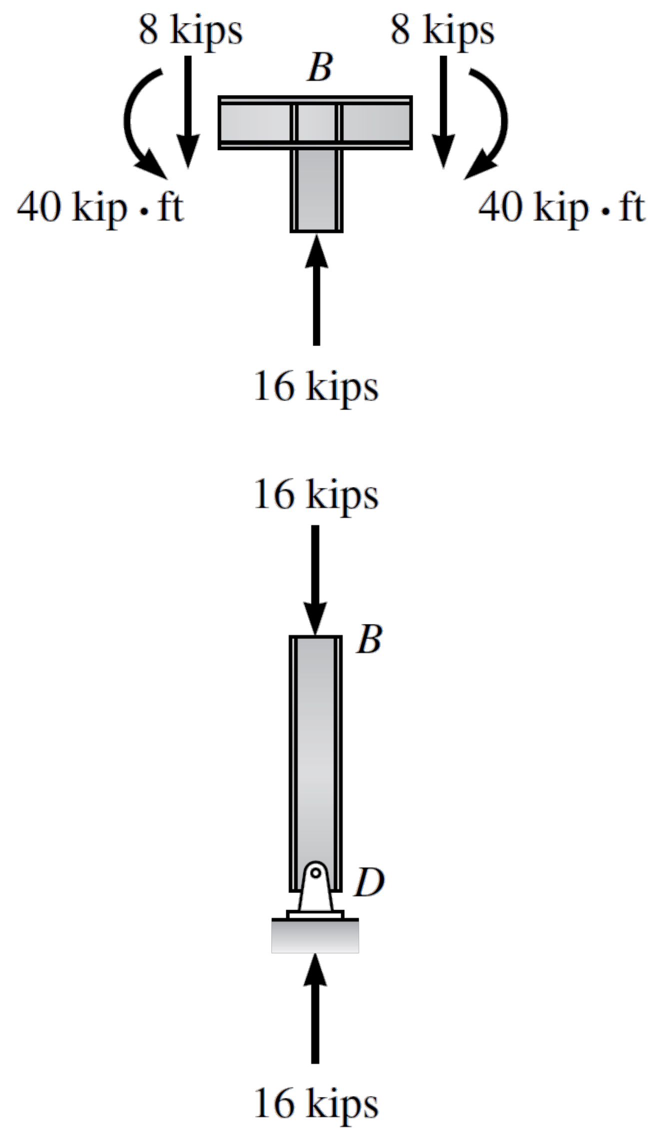


- Since all joint and chord rotations are zero, the member end moments at each end of beams *AB* and *BC* are equal to the fixed-end moments *PL*/8 given by Figure 12.5*a*:

$$\text{FEM} = \pm\frac{PL}{8} = \frac{16(20)}{8} = \pm 40\ \text{kip}\cdot\text{ft} \qquad \textbf{Ans.}$$

Example 12.5 Solution (continued)

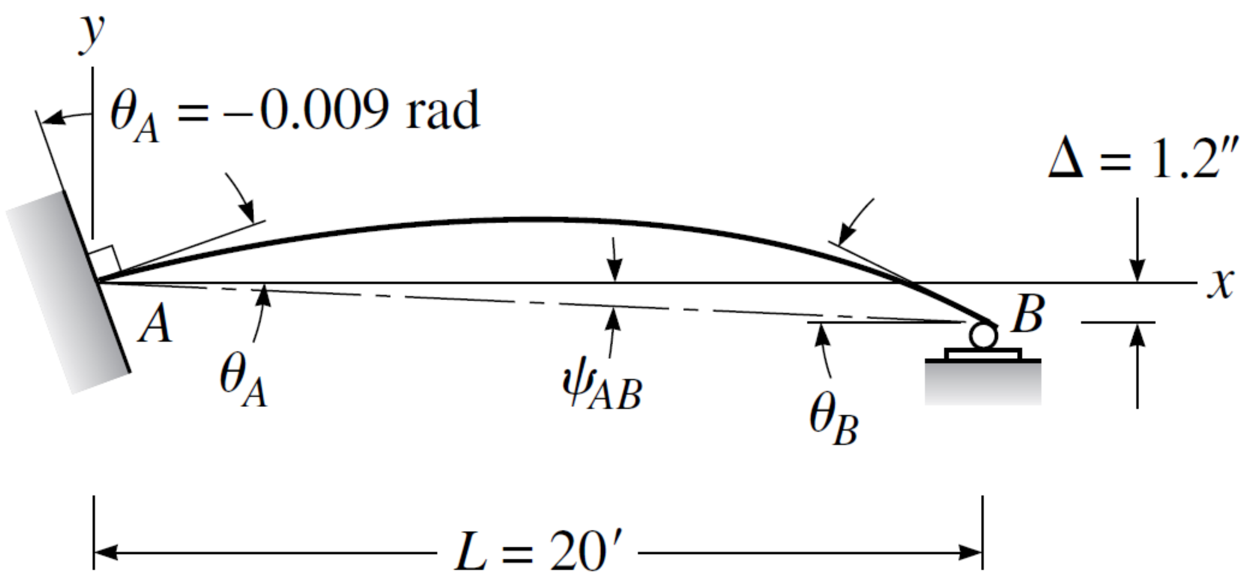




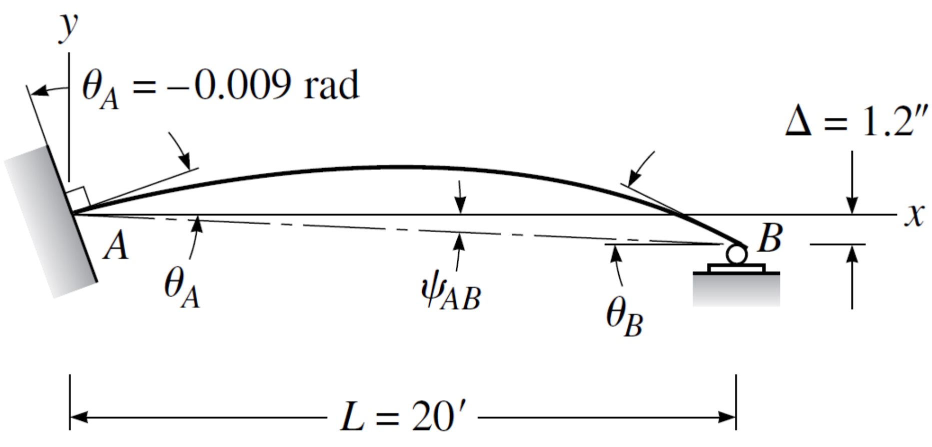
Free body of beam *AB*, joint *B*, and column *BD*. Final shear and moment diagrams for beam *AB*.

Example 12.6

Determine the reactions and draw the shear and moment curves for the beam in Figure 12.12. The support at *A* has been accidentally constructed with a slope that makes an angle of 0.009 rad with the vertical *y*-axis through support *A*, and *B* has been constructed 1.2 in below its intended position. Given: *EI* is constant, $I = 360$ in^4, and $E = 29{,}000$ kips/in^2.



Example 12.6 Solution



- θ_A = -0.009 rad. The settlement of support *B* relative to support *A* produces a clockwise chord rotation

$$\psi_{AB} = \frac{\Delta}{L} = \frac{1.2}{20(12)} = 0.005 \text{ radians}$$

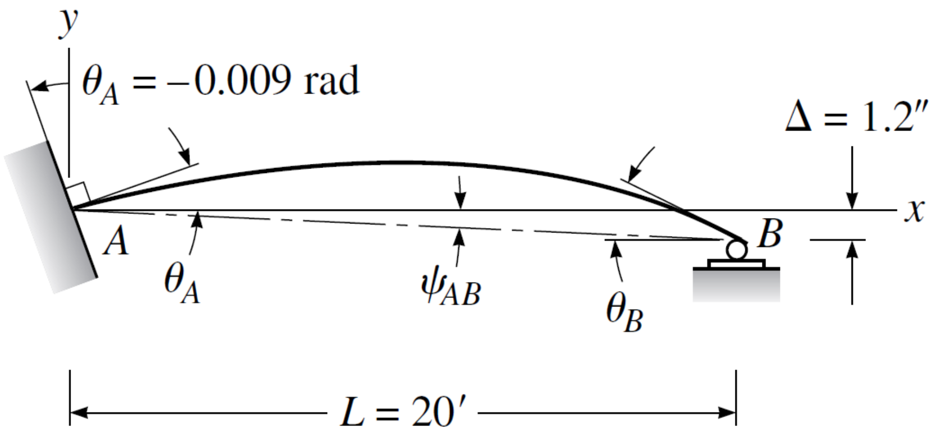
- Angle θ_B is the only unknown displacement. Expressing member end moments with the slope-deflection equation

$$M_{AB} = \frac{2EI_{AB}}{L_{AB}}(2\theta_A + \theta_B - 3\psi_{AB}) + \text{FEM}_{AB}$$

$$M_{AB} = \frac{2E(360)}{20(12)}[2(-0.009) + \theta_B - 3(0.005)]$$

$$M_{BA} = \frac{2E(360)}{20(12)}[2\theta_B + (-0.009) - 3(0.005)]$$

Example 12.6 Solution (continued)



- Writing the equilibrium equation at joint *B* yields

$$\overset{+}{\circlearrowleft} \quad \Sigma M_B = 0$$

$$M_{BA} = 0$$

- Substituting Equation 2 into Equation 3 and solving for θ_B yield

$$3E(2\theta_B - 0.009 - 0.015) = 0$$

$$\theta_B = 0.012 \text{ radians}$$

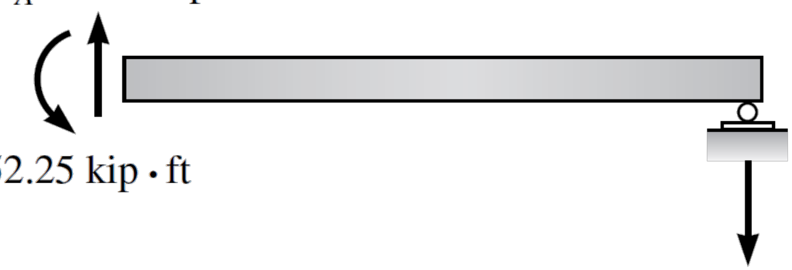
Example 12.6 Solution (continued)

- To evaluate M_{AB}, substitute θ_B into Equation 1:

$$M_{AB} = 3(29{,}000)[2(-0.009) + 0.012 - 3(0.005)]$$

$$= -1827 \text{ kip} \cdot \text{in} = -152.25 \text{ kip} \cdot \text{ft}$$

- Complete the analysis by using the equations of statics to compute the reaction at *B* and the shear at *A*.



$$\circlearrowright^{+} \quad \Sigma M_A = 0$$

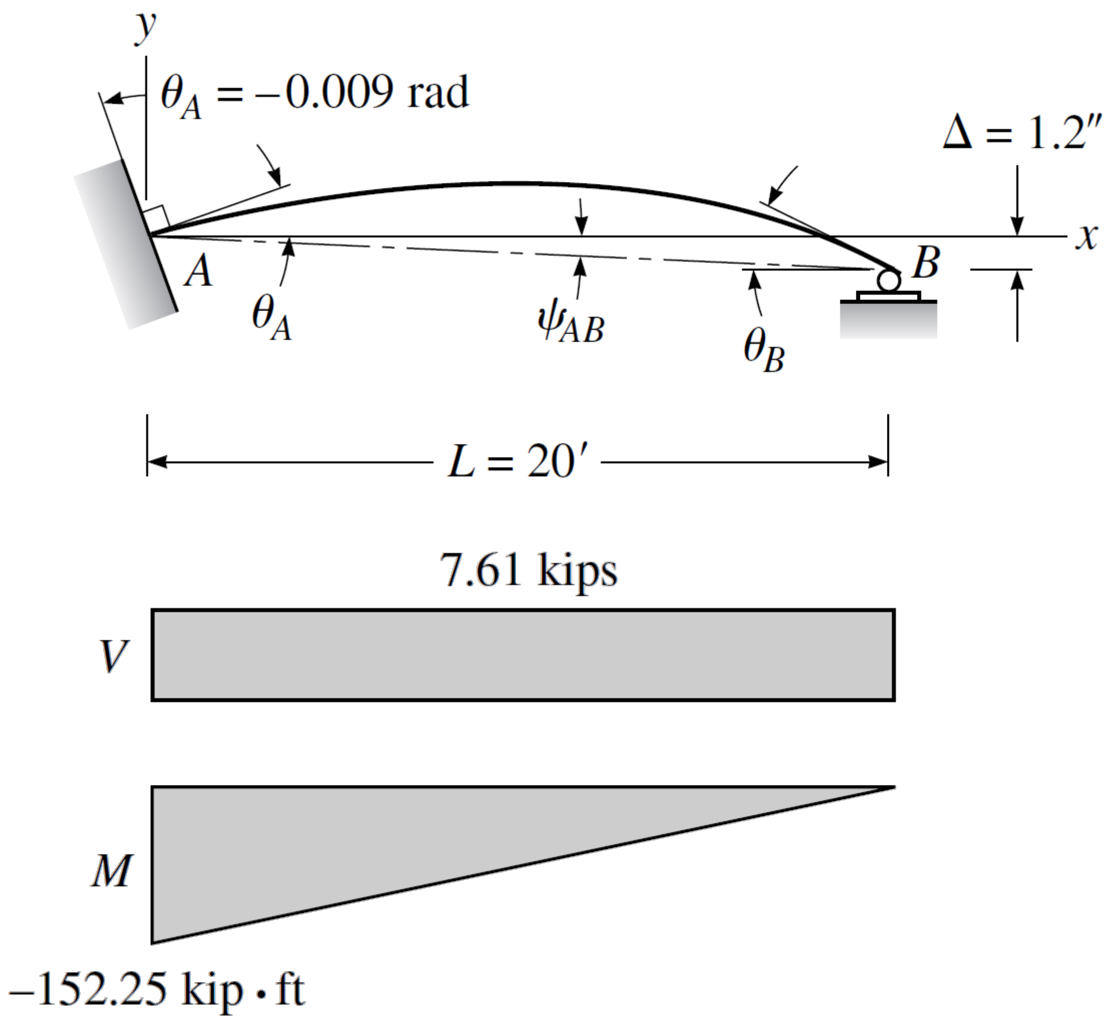
$$0 = R_B(20) - 152.25$$

$$R_B = 7.61 \text{ kips} \qquad \textbf{Ans.}$$

$$\overset{+}{\uparrow} \quad \Sigma F_y = 0$$

$$V_A = 7.61 \text{ kips} \qquad \textbf{Ans.}$$

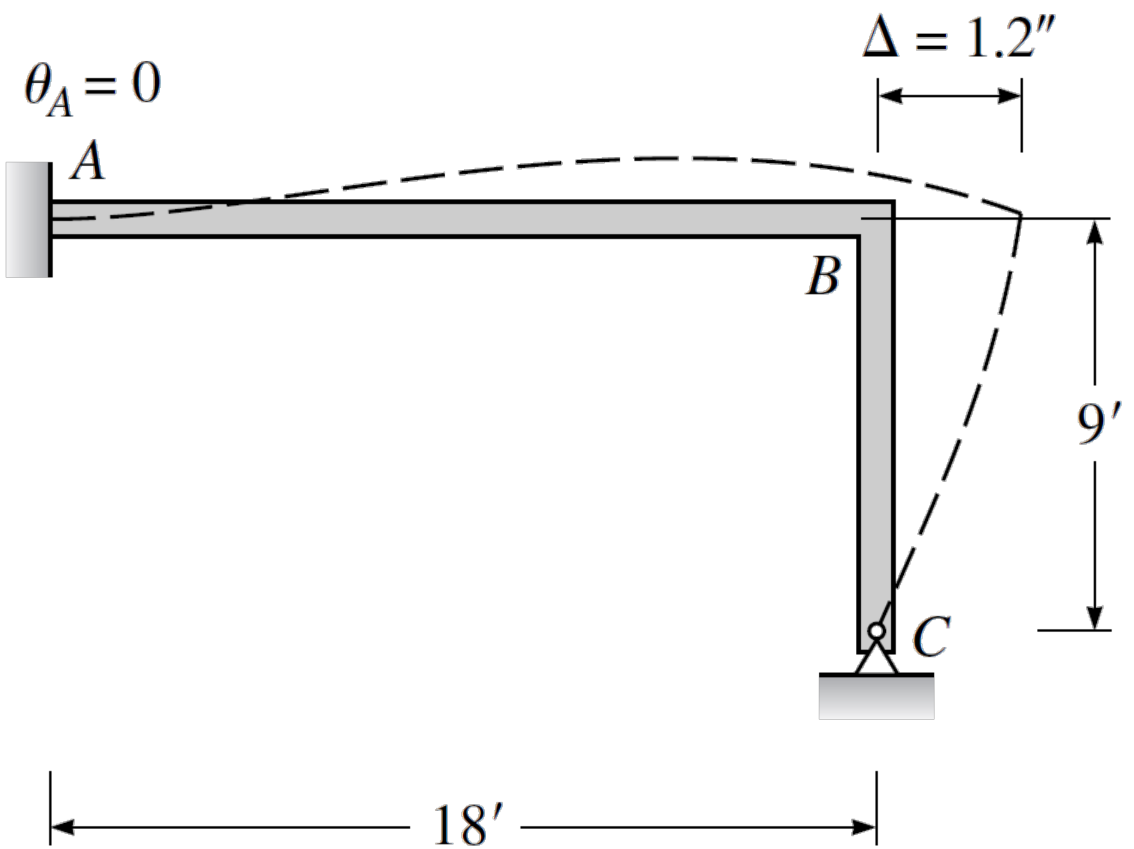
Example 12.6 Solution (continued)



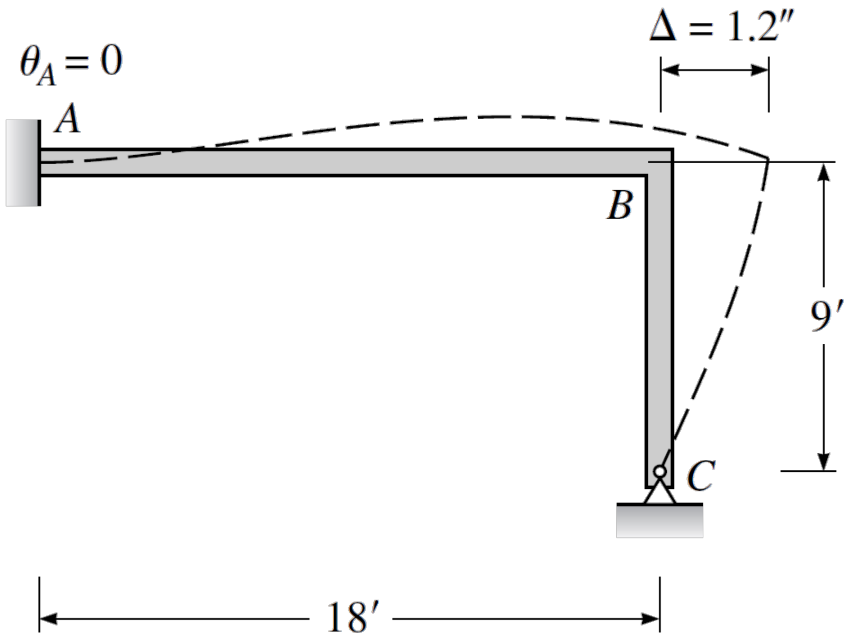
Shear and moment curves

Example 12.7

Although the supports are constructed in their correct position, girder *AB* of the frame shown in Figure 12.13 is fabricated 1.2 in too long. Determine the reactions created when the frame is connected into the supports. Given: *EI* is a constant for all members, $I = 240$ in^4, and $E = 29{,}000$ kips/in^2.



Example 12.7 Solution



- The chord rotation ψ_{BC} of column *BC* equals

$$\psi_{BC} = \frac{\Delta}{L} = \frac{1.2}{9(12)} = \frac{1}{90} \text{ rad}$$

- Since the ends of girder *AB* are at the same level, ψ_{AB} = 0. The unknown displacements are θ_B and θ_C

Example 12.7 Solution (continued)

- Using the slope-deflection equation (Equation 12.16), express member end moments in terms of the unknown displacements. Because no loads are applied to the members, all fixed-end moments equal zero.

$$M_{AB} = \frac{2E(240)}{18(12)}(\theta_B) = 2.222E\theta_B$$

$$M_{BA} = \frac{2E(240)}{18(12)}(2\theta_B) = 4.444E\theta_B$$

$$M_{BC} = \frac{2E(240)}{9(12)}\left[2\theta_B + \theta_C - 3\left(\frac{1}{90}\right)\right]$$

$$= 8.889E\theta_B + 4.444E\theta_C - 0.1481E$$

$$M_{CB} = \frac{2E(240)}{9(12)}\left[2\theta_C + \theta_B - 3\left(\frac{1}{90}\right)\right]$$

$$= 8.889E\theta_C + 4.444E\theta_B - 0.1481E$$

Example 12.7 Solution (continued)

- Writing equilibrium equations gives

Joint C: $$M_{CB} = 0$$

Joint B: $$M_{BA} + M_{BC} = 0$$

- Substituting and solving for θ_B and θ_C yield

$$8.889E\theta_C + 4.444E\theta_B - 0.1481E = 0$$

$$4.444E\theta_B + 8.889E\theta_B + 4.444E\theta_C - 0.1481E = 0$$

$$\theta_B = 0.00666 \text{ rad}$$

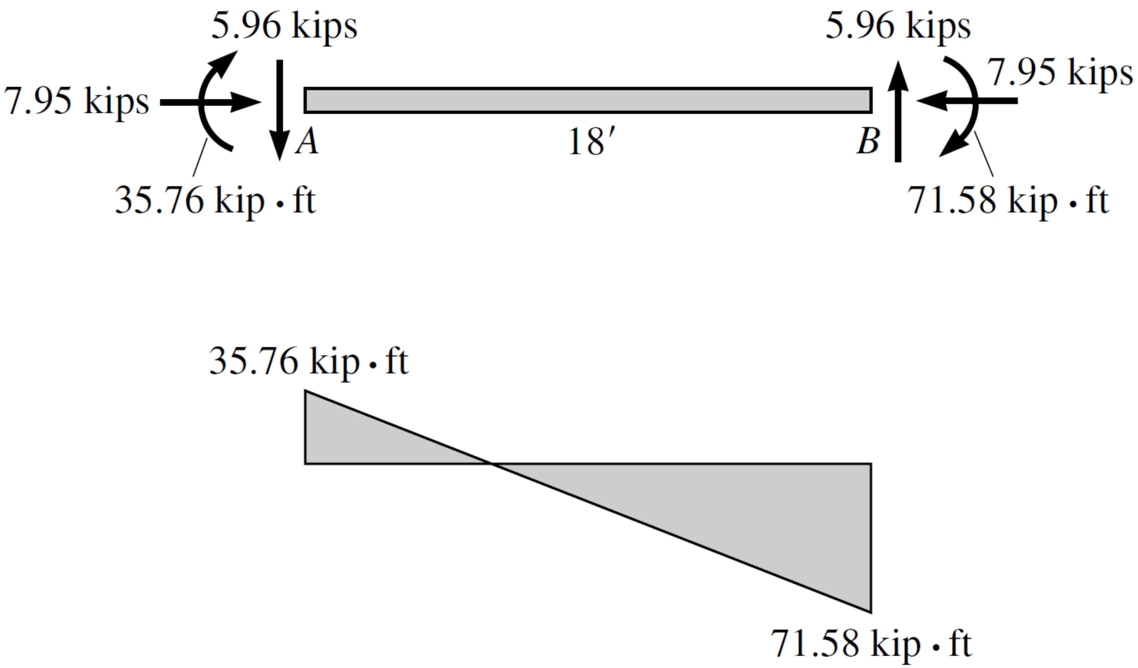
$$\theta_C = 0.01332 \text{ rad}$$

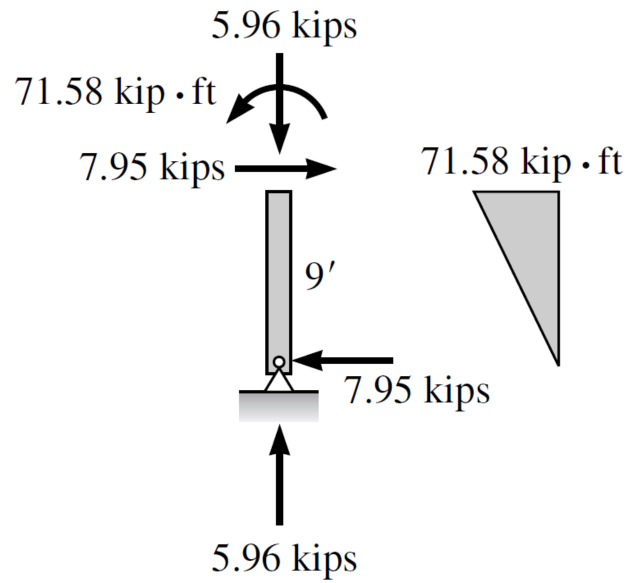
- Substituting θ_C and θ_B into Equations 1 to 3 produces

$$M_{AB} = 35.76 \text{ kip} \cdot \text{ft} \qquad M_{BA} = 71.58 \text{ kip} \cdot \text{ft}$$

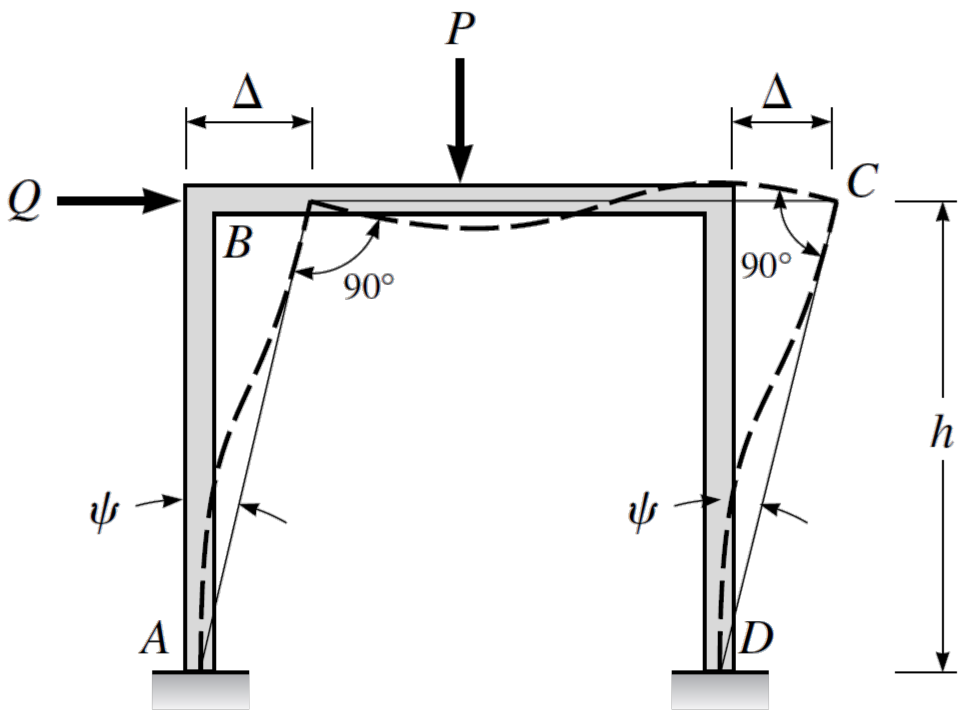
$$M_{BC} = -71.58 \text{ kip} \cdot \text{ft} \qquad M_{CB} = 0 \qquad \textbf{Ans.}$$

Example 12.7 Solution (continued)





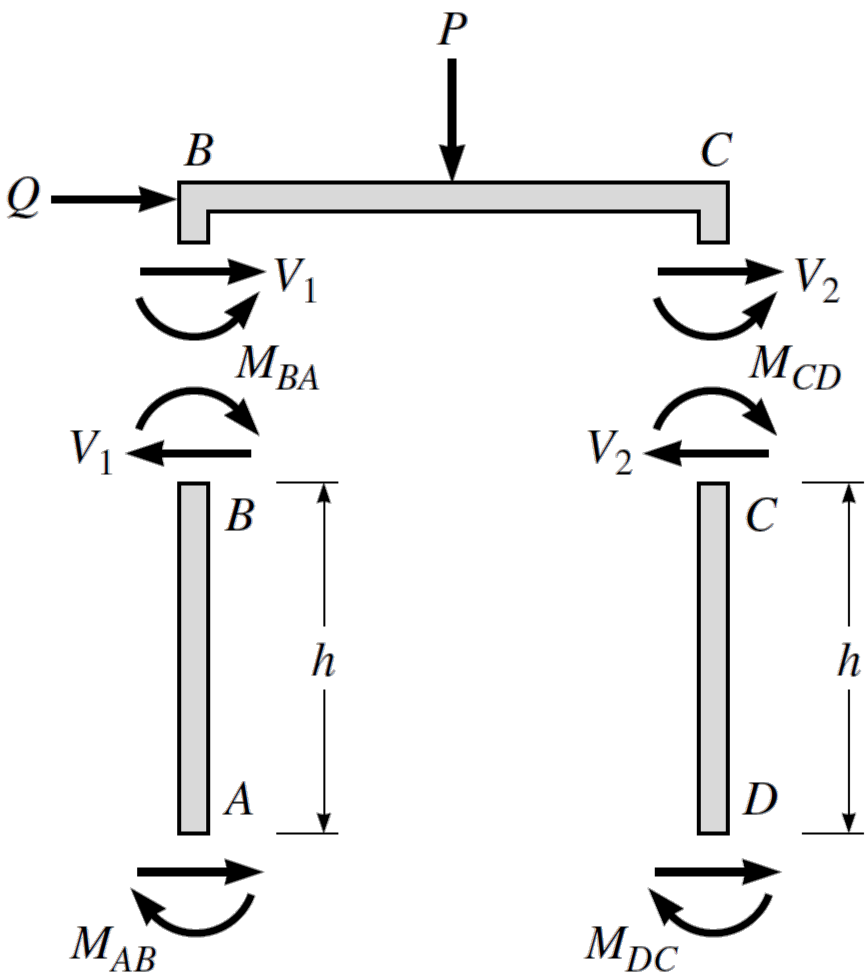
§12.5 Analysis of Structures That Are Free to Sidesway



Unbraced frame, deflected shape shown to an exaggerated scale by dashed lines, column chords rotate through a clockwise angle ψ

Figure 12.14

§12.5 Analysis of Structures That Are Free to Sidesway

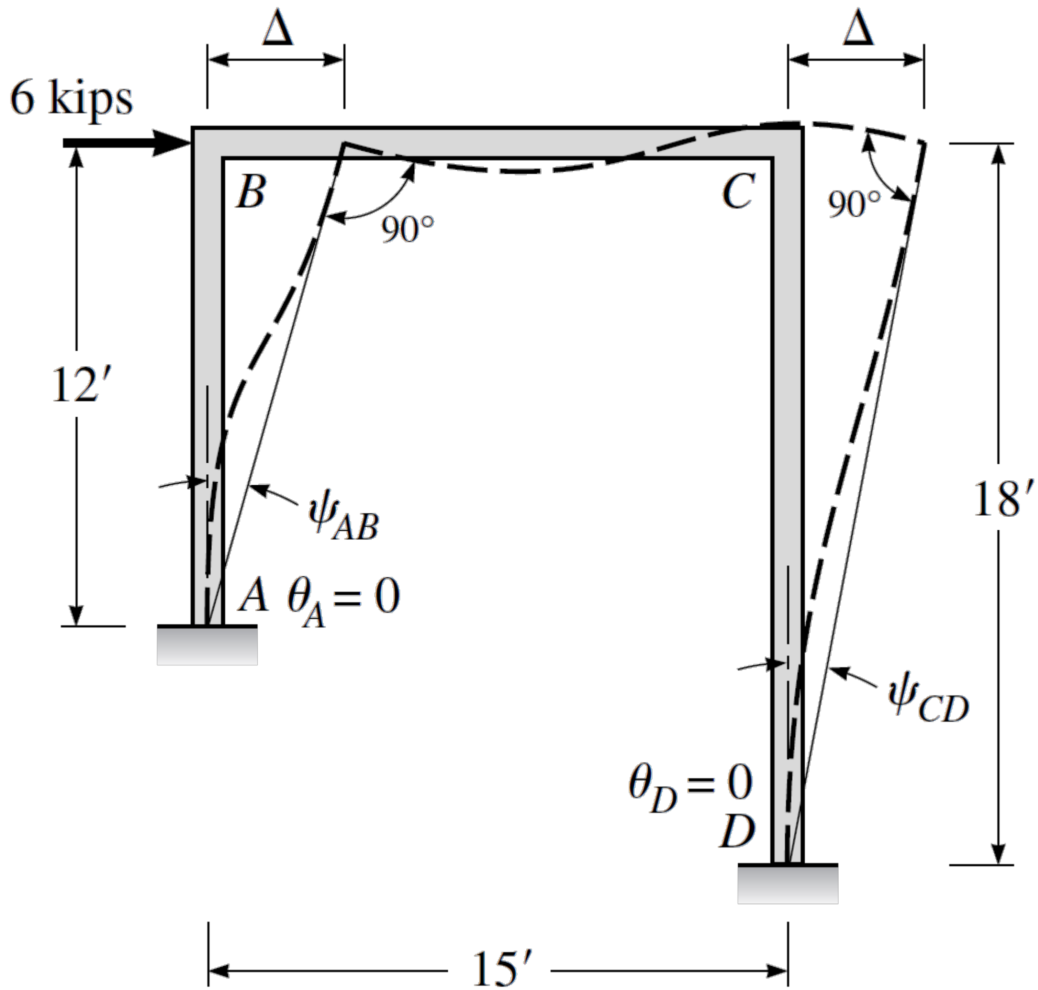


Free-body diagrams of columns and girders; unknown moments shown in the positive sense, that is, clockwise on ends of members

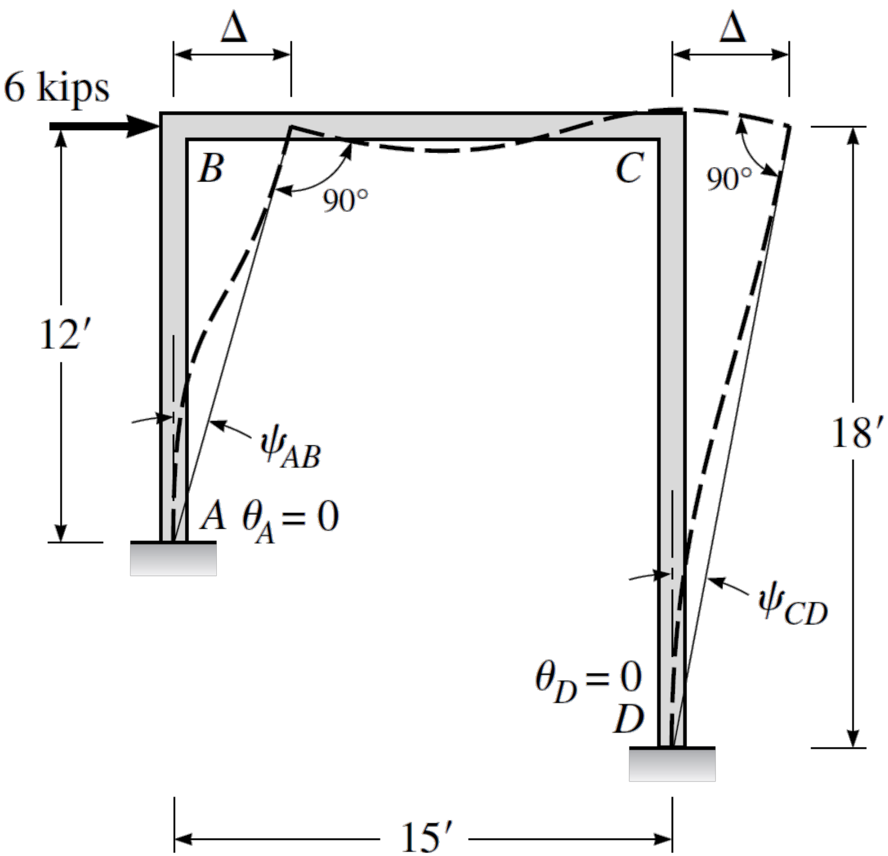
Figure 12.14 (continued)

Example 12.8

Analyze the frame in Figure 12.15*a* by the slope-deflection method. *E* is constant for all members; I_{AB} = 240 in^4, I_{BC} = 600 in^4, and I_{CD} = 360 in^4.



Example 12.8 Solution



- Identify the unknown displacements θ_B, θ_C, and Δ. Express the chord rotations ψ_{AB} and ψ_{CD} in terms of Δ:

$$\psi_{AB} = \frac{\Delta}{12} \qquad \text{and} \qquad \psi_{CD} = \frac{\Delta}{18}$$

$$\text{so} \qquad \psi_{AB} = 1.5\psi_{CD}$$

- Compute the relative bending stiffness of all members.

$$K_{AB} = \frac{EI}{L} = \frac{240E}{12} = 20E$$

$$K_{BC} = \frac{EI}{L} = \frac{600E}{15} = 40E$$

$$K_{CD} = \frac{EI}{L} = \frac{360E}{18} = 20E$$

Example 12.8 Solution (continued)

- Set 20*E* = *K,* then

$$K_{AB} = K \qquad K_{BC} = 2K \qquad K_{CD} = K$$

- Express member end moments in terms of displacements: $M_{NF} = (2EI/L)(2\theta_N + \theta_F - 3\psi_{NF}) + FEM_{NF}$. Since no loads are applied to members between joints, all $FEM_{NF} = 0$.

$$M_{AB} = 2K_{AB}(\theta_B - 3\psi_{AB}) \qquad M_{CB} = 2K_{BC}(2\theta_C + \theta_B)$$

$$M_{BA} = 2K_{AB}(2\theta_B - 3\psi_{AB}) \qquad M_{CD} = 2K_{CD}(2\theta_C - 3\psi_{CD})$$

$$M_{BC} = 2K_{BC}(2\theta_B + \theta_C) \qquad M_{DC} = 2K_{CD}(\theta_C - 3\psi_{CD})$$

- Use Equations 1 to express ψ_{AB} in terms of ψ_{CD}, and use Equations 2 to express all stiffness in terms of the parameter *K*.

$$M_{AB} = 2K(\theta_B - 4.5\psi_{CD}) \qquad M_{CB} = 4K(2\theta_C + \theta_B)$$

$$M_{BA} = 2K(2\theta_B - 4.5\psi_{CD}) \qquad M_{CD} = 2K(2\theta_C - 3\psi_{CD})$$

$$M_{BC} = 4K(2\theta_B + \theta_C) \qquad M_{DC} = 2K(\theta_C - 3\psi_{CD})$$

Example 12.8 Solution (continued)

- The equilibrium equations are:

Joint B: $$M_{BA} + M_{BC} = 0$$

Joint C: $$M_{CB} + M_{CD} = 0$$

Shear equation (see Eq. 12.21): $$\frac{M_{BA} + M_{AB}}{12} + \frac{M_{CD} + M_{DC}}{18} + 6 = 0$$

- Substitute Equations 4 into Equations 5, 6, and 7 and combine terms.

$$12\theta_B + 4\theta_C - 9\psi_{CD} = 0$$

$$4\theta_B + 12\theta_C - 6\psi_{CD} = 0$$

$$9\theta_B + 6\theta_C - 39\psi_{CD} = -\frac{108}{K}$$

Example 12.8 Solution (continued)

- Solving the equations simultaneously gives

$$\theta_B = \frac{2.257}{K} \qquad \theta_C = \frac{0.97}{K} \qquad \psi_{CD} = \frac{3.44}{K}$$

Also, $$\psi_{AB} = 1.5\psi_{CD} = \frac{5.16}{K}$$

Since all angles are positive, all joint rotations and the sidesway angles are clockwise.

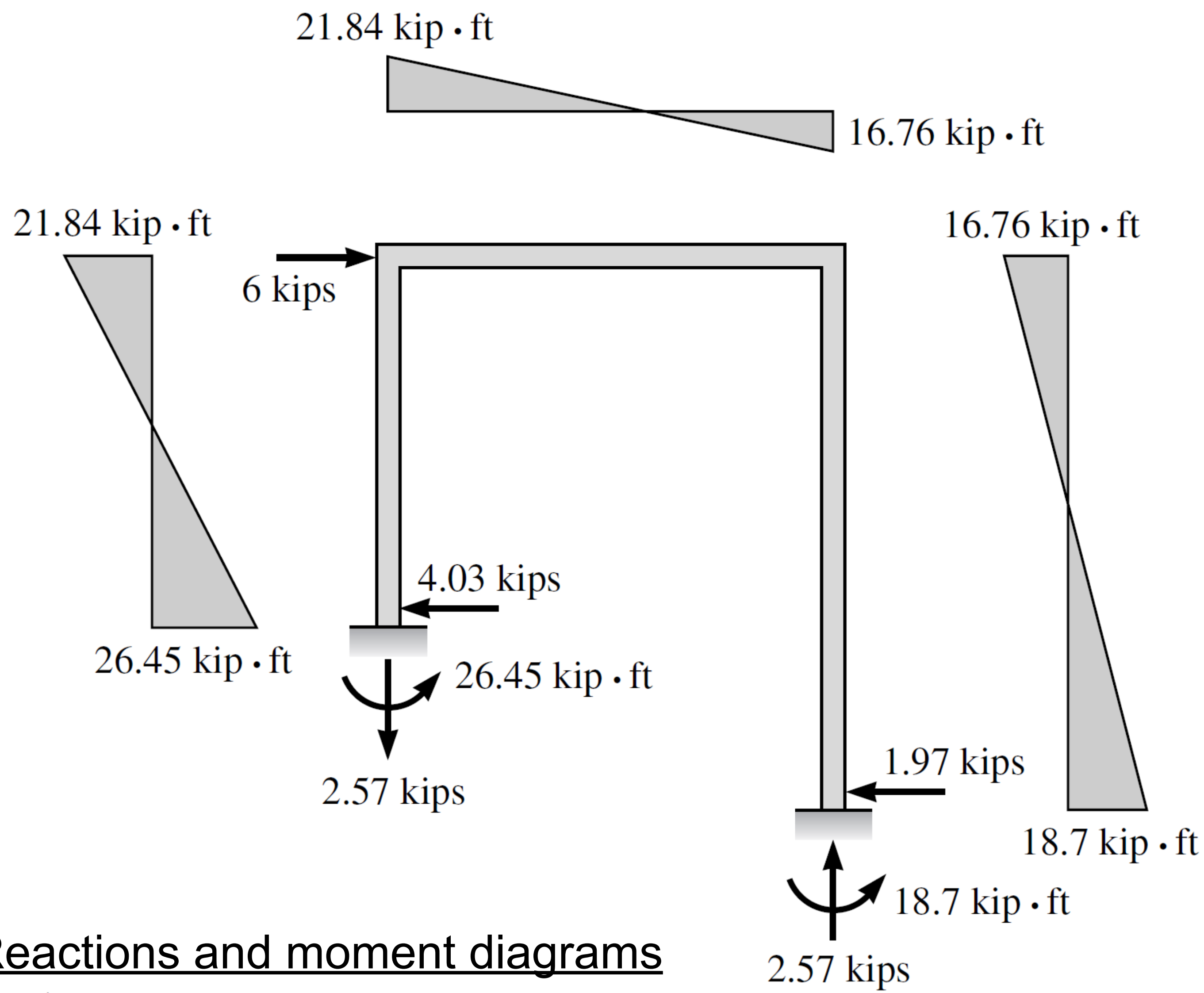
- Substituting the values of displacement above into Equations 4, establish the member end moments.

$$M_{AB} = -26.45 \text{ kip} \cdot \text{ft} \qquad M_{BA} = -21.84 \text{ kip} \cdot \text{ft}$$

$$M_{BC} = 21.84 \text{ kip} \cdot \text{ft} \qquad M_{CB} = 16.78 \text{ kip} \cdot \text{ft}$$

$$M_{CD} = -16.76 \text{ kip} \cdot \text{ft} \qquad M_{DC} = -18.7 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

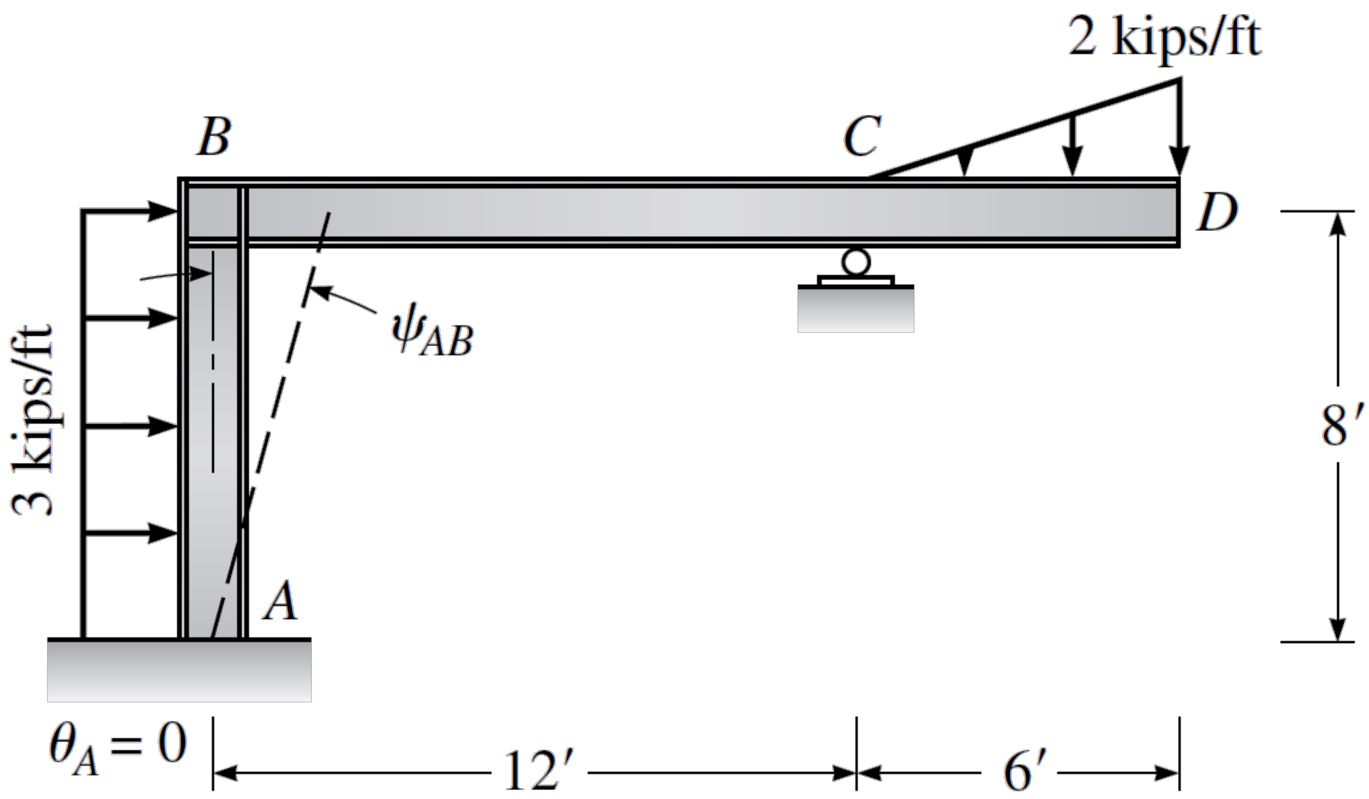
Example 12.8 Solution (continued)



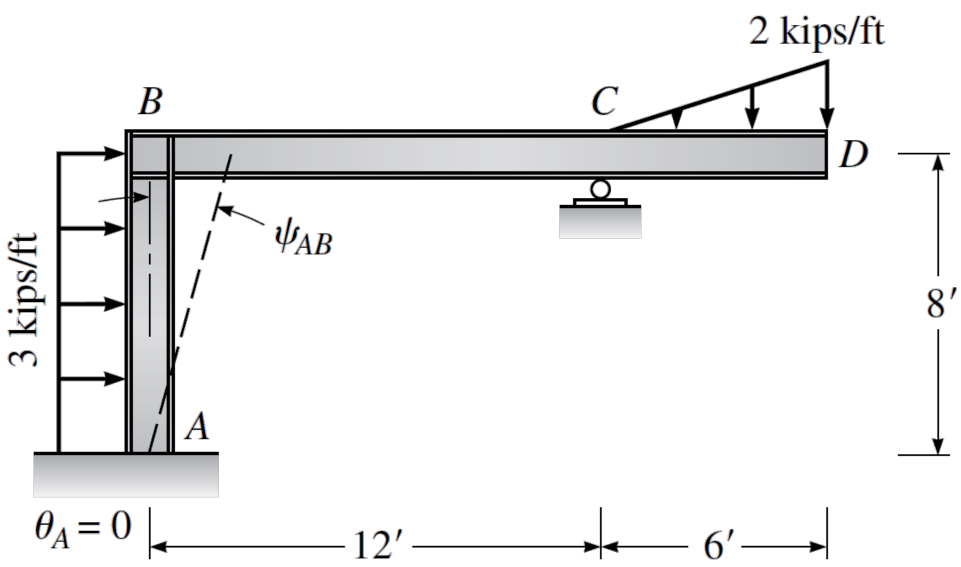
Reactions and moment diagrams

Example 12.9

Analyze the frame in Figure 12.16*a* by the slope-deflection method.
Given: *EI* is constant for all members.



Example 12.9 Solution



- Express member end moments in terms of displacements with Equation 12.16 (all units in kip-feet).

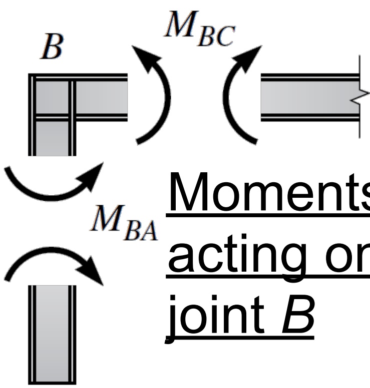
$$M_{AB} = \frac{2EI}{8}(\theta_B - 3\psi_{AB}) - \frac{3(8)^2}{12}$$

$$M_{BA} = \frac{2EI}{8}(2\theta_B - 3\psi_{AB}) + \frac{3(8)^2}{12}$$

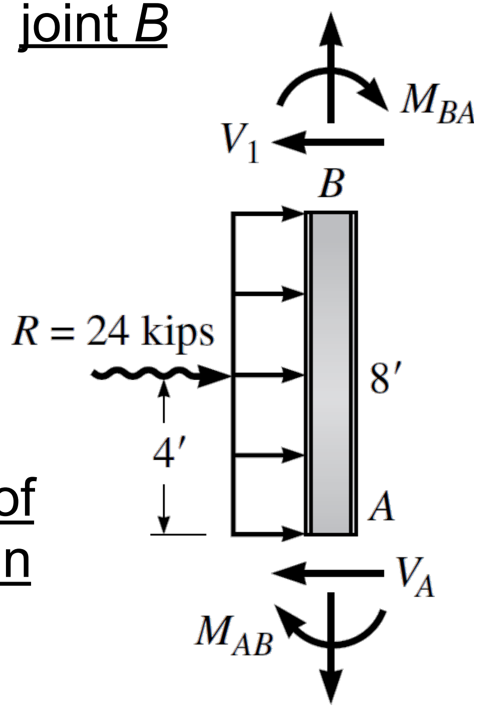
$$M_{BC} = \frac{2EI}{12}(2\theta_B + \theta_C)$$

$$M_{CB} = \frac{2EI}{12}(2\theta_C + \theta_B)$$

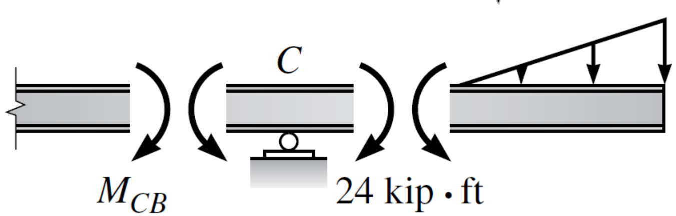
Example 12.9 Solution (continued)



Moments acting on joint *B*



Free body of column *AB*



Moments acting on joint *C*

- Write the joint equilibrium equations at *B* and *C*. Joint *B*:

$$\overset{+}{\circlearrowleft} \quad \Sigma M_B = 0: \qquad M_{BA} + M_{BC} = 0$$

- Joint C:

$$\overset{+}{\circlearrowleft} \quad \Sigma M_C = 0: \qquad M_{CB} - 24 = 0$$

- Shear equation:

$$\circlearrowright^{+} \quad \Sigma M_A = 0$$

$$M_{BA} + M_{AB} + 24(4) - V_1(8) = 0$$

- Solving for V_1 gives

$$V_1 = \frac{M_{BA} + M_{AB} + 96}{8}$$

Example 12.9 Solution (continued)

Free body of girder used to establish third equilibrium equation

- Isolate the girder and consider equilibrium in the horizontal direction.

$$\xrightarrow{+} \quad \Sigma F_x = 0: \quad \text{therefore} \quad V_1 = 0$$

- Substitute Equation 4*a* into Equation 4*b*:

$$M_{BA} + M_{AB} + 96 = 0$$

- Express equilibrium equations in terms of displacements by substituting Equations 1 into Equations 2, 3, and 4. Collecting terms and simplifying,

$$10\theta_B - 2\theta_C - 9\psi_{AB} = -\frac{192}{EI}$$

$$\theta_B - 2\theta_C = \frac{144}{EI}$$

$$3\theta_B - 6\psi_{AB} = -\frac{384}{EI}$$

Example 12.9 Solution (continued)

- Solution of the equations

$$\theta_B = \frac{53.33}{EI} \qquad \theta_C = \frac{45.33}{EI} \qquad \psi_{AB} = \frac{90.66}{EI}$$

- Establish the values of member end moments by substituting the values of θ_B, θ_C, and ψ_{AB} into Equations 1.

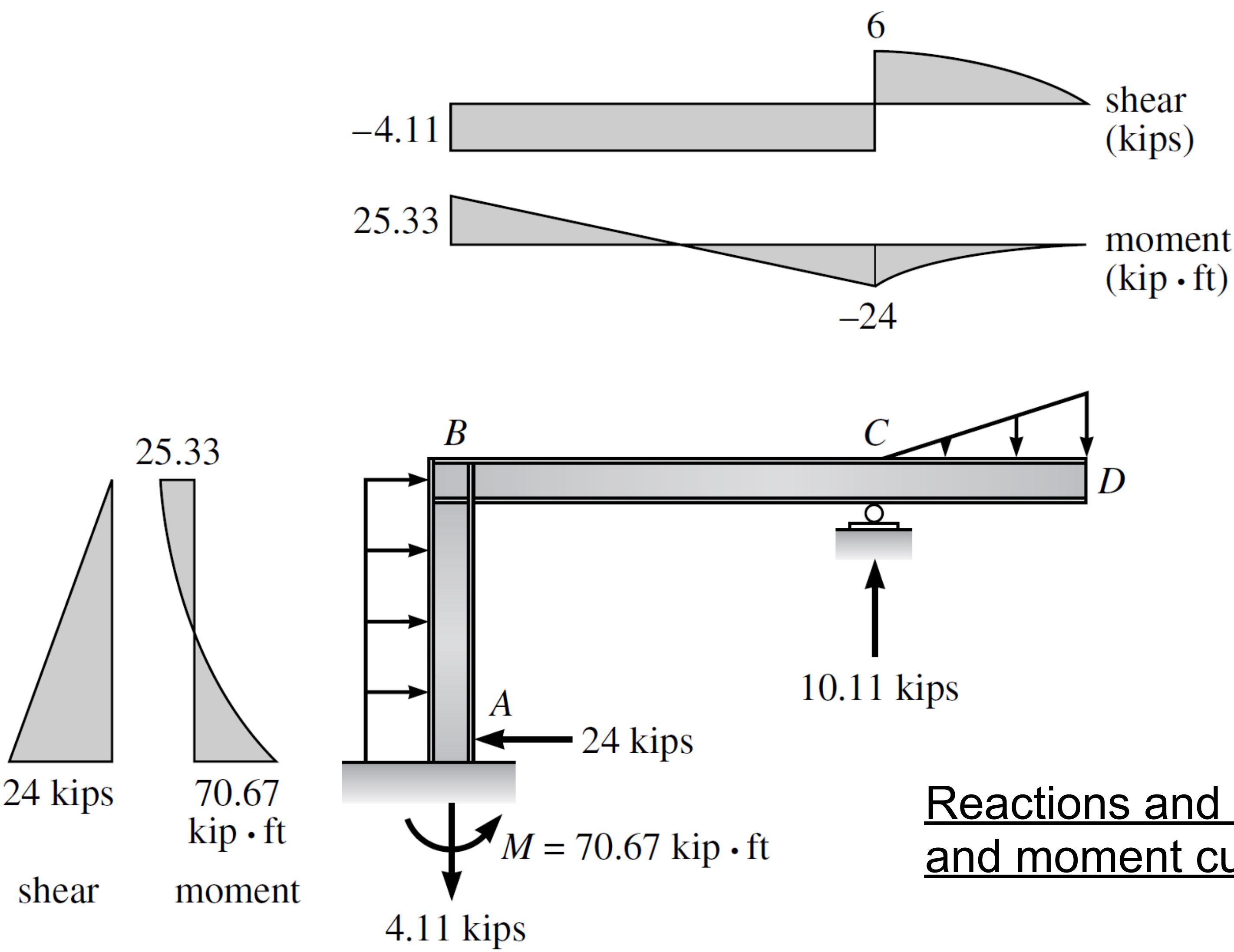
$$M_{AB} = \frac{2EI}{8}\left[\frac{53.33}{EI} - \frac{(3)(90.66)}{EI}\right] - 16 = -70.67 \text{ kip} \cdot \text{ft}$$

$$M_{BA} = \frac{2EI}{8}\left[\frac{(2)(53.33)}{EI} - \frac{(3)(90.66)}{EI}\right] + 16 = -25.33 \text{ kip} \cdot \text{ft}$$

$$M_{BC} = \frac{2EI}{12}\left[\frac{(2)(53.33)}{EI} + \frac{45.33}{EI}\right] = 25.33 \text{ kip} \cdot \text{ft}$$

$$M_{CB} = \frac{2EI}{12}\left[\frac{(2)(45.33)}{EI} + \frac{53.33}{EI}\right] = 24 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

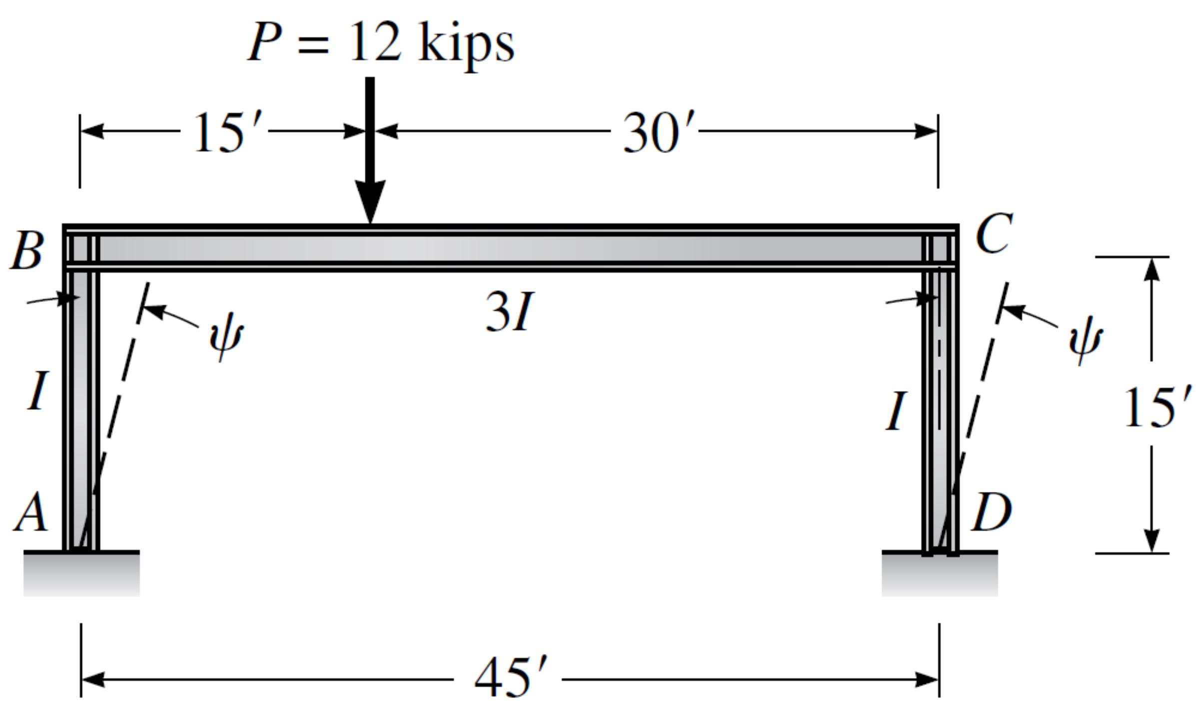
Example 12.9 Solution (continued)



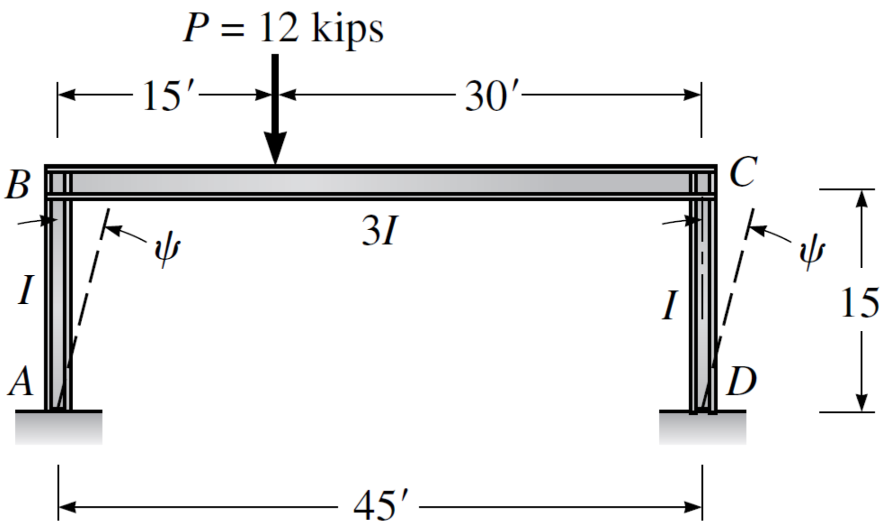
Reactions and shear and moment curves

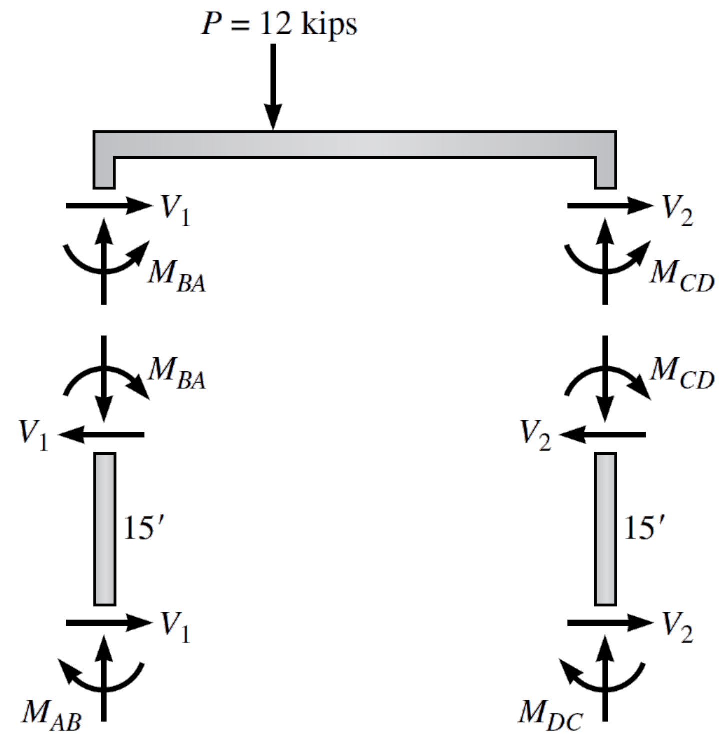
Example 12.10

Analyze the frame in Figure 12.17*a* by the slope-deflection method. Determine the reactions, draw the moment curves for the members, and sketch the deflected shape. If I = 240 in^4 and E = 30,000 kips/in^2, determine the horizontal displacement of joint B.



Example 12.10 Solution





- Express member end moments in terms of displacements with the slope-deflection equation.

$$M_{NF} = \frac{2EI}{L}(2\theta_N + \theta_F - 3\psi_{NF}) + \text{FEM}_{NF} \qquad (12.16)$$

$$\text{FEM}_{BC} = -\frac{Pb^2a}{L^2} = \frac{12(30)^2(15)}{(45)^2} = -80 \text{ kip}\cdot\text{ft}$$

$$\text{FEM}_{CD} = \frac{Pa^2b}{L^2} = \frac{12(15)^2(30)}{(45)^2} = 40 \text{ kip}\cdot\text{ft}$$

Example 12.10 Solution (continued)

- To simplify slope-deflection expressions, set $EI/15 = K$.

$$M_{AB} = \frac{2EI}{15}(\theta_B - 3\psi) \qquad = 2K(\theta_B - 3\psi)$$

$$M_{BA} = \frac{2EI}{15}(2\theta_B - 3\psi) \qquad = 2K(2\theta_B - 3\psi)$$

$$M_{BC} = \frac{2EI}{45}(2\theta_B + \theta_C) - 80 = \frac{2}{3}K(2\theta_B + \theta_C) - 80$$

$$M_{CB} = \frac{2EI}{45}(2\theta_C + \theta_B) + 40 = \frac{2}{3}K(2\theta_C + \theta_B) + 40$$

$$M_{CD} = \frac{2EI}{15}(2\theta_C - 3\psi) \qquad = 2K(\theta_C - 3\psi)$$

$$M_{DC} = \frac{2EI}{15}(\theta_C - 3\psi) \qquad = 2K(\theta_C - 3\psi)$$

Example 12.10 Solution (continued)

- The equilibrium equations are:

Joint B: $$M_{BA} + M_{BC} = 0$$

Joint C: $$M_{CB} + M_{CD} = 0$$

- Shear equation:

$$\rightarrow + \quad \Sigma F_x = 0 \qquad V_1 + V_2 = 0$$

where $$V_1 = \frac{M_{BA} + M_{AB}}{15} \qquad V_2 = \frac{M_{CD} + M_{DC}}{15}$$

- Substituting V_1 and V_2 given by Equations 4*b* into 4*a* gives

$$M_{BA} + M_{AB} + M_{CD} + M_{DC} = 0$$

Alternatively, set $Q = 0$ in Equation 12.21 to produce Equation 4.

Example 12.10 Solution (continued)

- Express equilibrium equations in terms of displacements by substituting Equations 1 into Equations 2, 3, and 4. Combining terms and simplifying give

$$8K\theta_B + K\theta_C - 9K\psi = 120$$

$$2K\theta_B + 16K\theta_C - 3K\psi = -120$$

$$K\theta_B + K\theta_C - 4K\psi = 0$$

- Solving the equations simultaneously,

$$\theta_B = \frac{410}{21K} \qquad \theta_C = -\frac{130}{21K} \qquad \psi = \frac{10}{3K}$$

- Substituting the values of the θ_B, θ_C, and ψ into Equations 1,

$$M_{AB} = 19.05 \text{ kip} \cdot \text{ft} \qquad M_{BA} = 58.1 \text{ kip} \cdot \text{ft}$$

$$M_{CD} = -44.76 \text{ kip} \cdot \text{ft} \qquad M_{DC} = -32.38 \text{ kip} \cdot \text{ft} \qquad (6)$$

$$M_{BC} = -58.1 \text{ kip} \cdot \text{ft} \qquad M_{CB} = 44.76 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

Example 12.10 Solution (continued)

- Compute the horizontal displacement of joint *B*. Use Equation 1 for M_{AB}. Express all variables in units of inches and kips.

$$M_{AB} = \frac{2EI}{15(12)}(\theta_B - 3\psi)$$

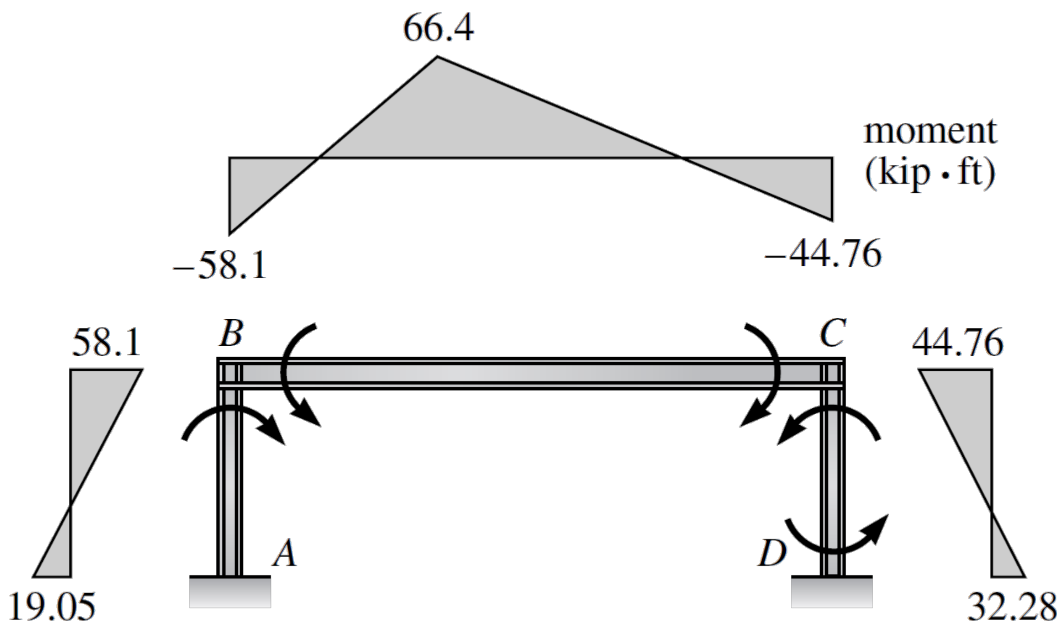
- From the values in Equation 5 (p. 485), $\theta_B = 5.86\psi$; substituting into Equation 7,

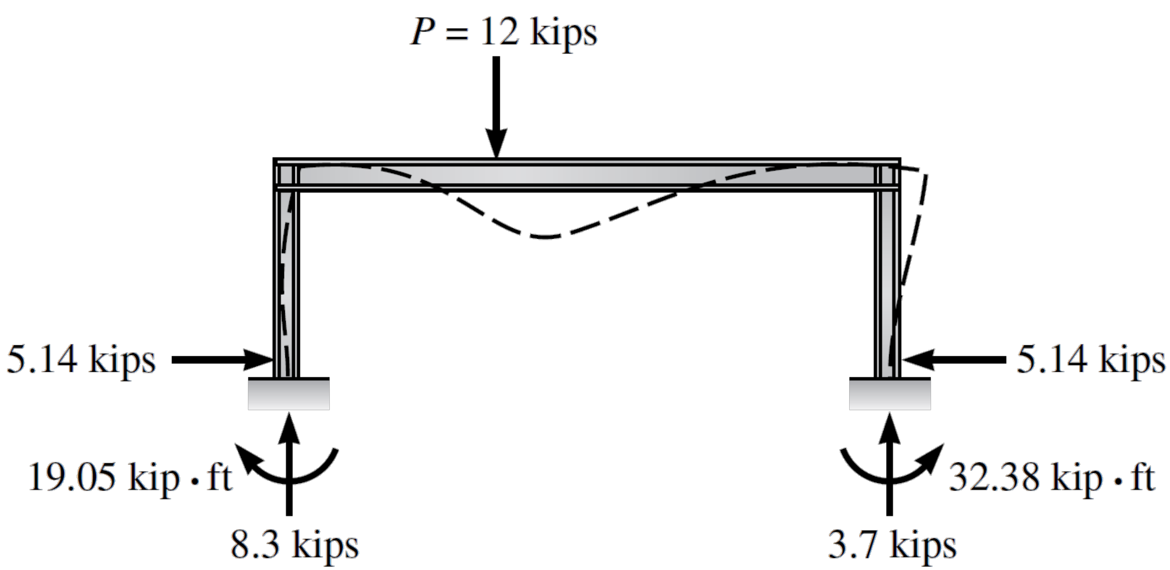
$$19.05(12) = \frac{2(30{,}000)(240)}{15(12)}(5.86\psi - 3\psi)$$

$$\psi = 0.000999 \text{ rad}$$

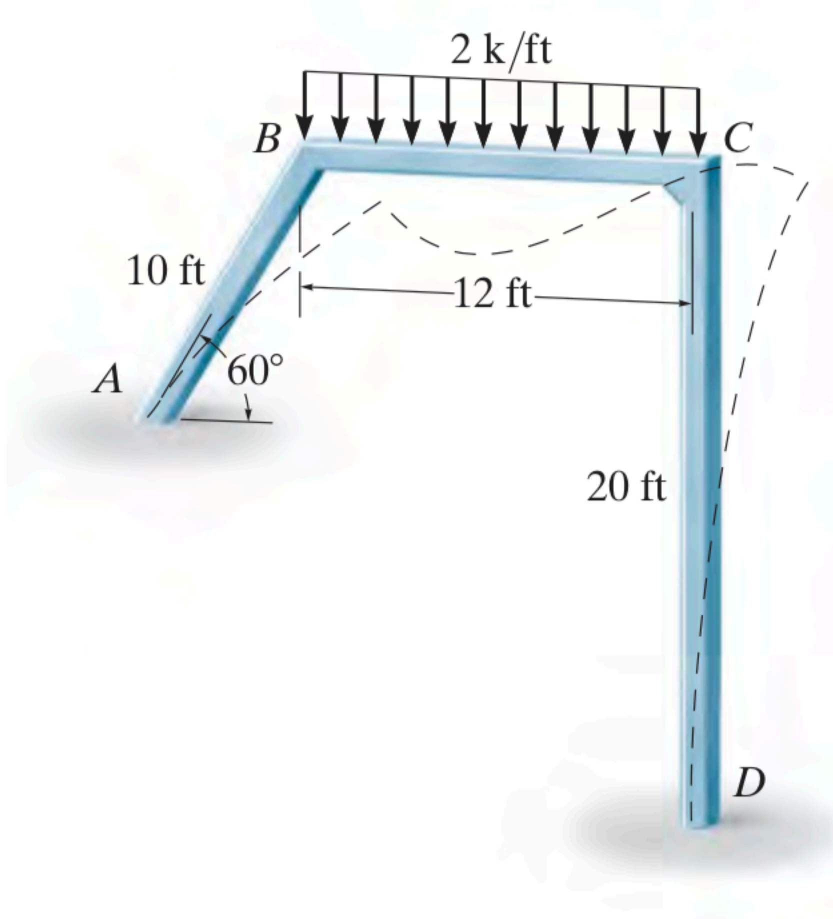
$$\psi = \frac{\Delta}{L} \qquad \Delta = \psi L = 0.000999(15 \times 12) = 0.18 \text{ in} \qquad \textbf{Ans.}$$

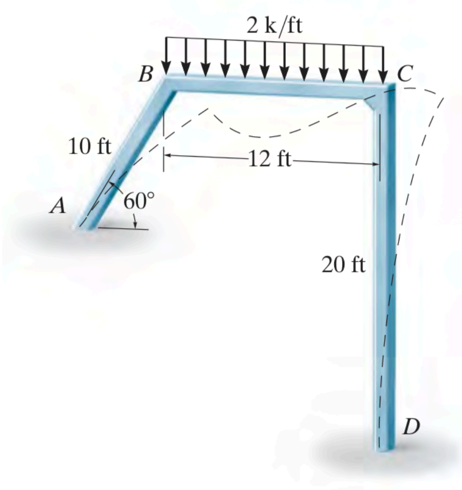
Example 12.10 Solution (continued)

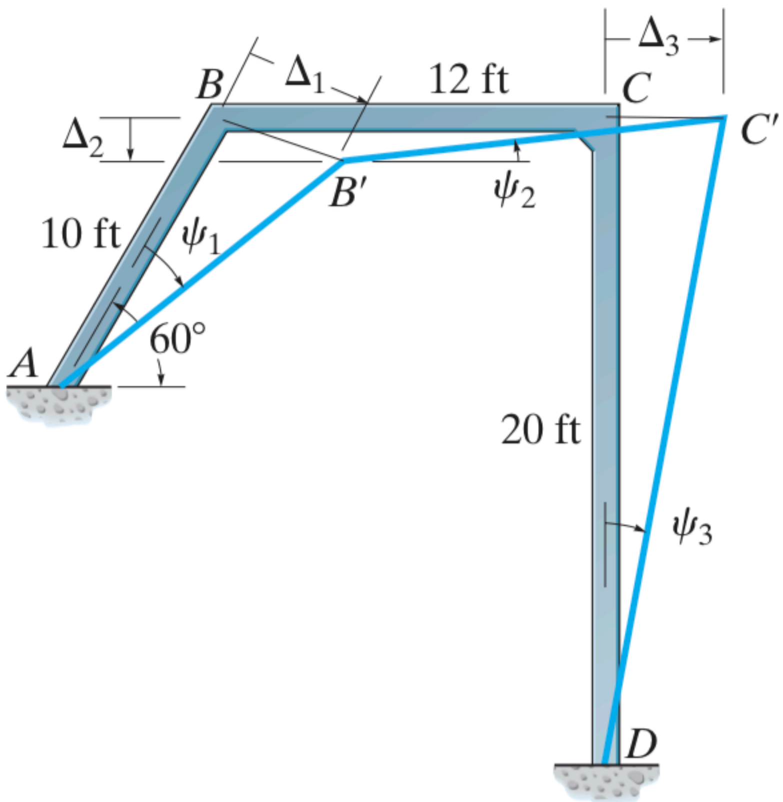




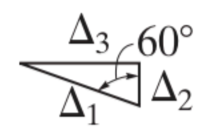
Determine the moments at each joint of the frame shown in Fig. 11–22*a*. *EI* is constant for each member.




2 k/ft
B
C
10 ft
12 ft
A
60°
20 ft
D


Δ3
B
Δ1
12 ft
C
C′
Δ2
B′
ψ2
10 ft
ψ1
60°
A
20 ft
ψ3
D

(b)


Δ3
60°
Δ2
Δ1

(c)

$$(\text{FEM})_{BC} = -\frac{wL^2}{12} = -\frac{2(12)^2}{12} = -24\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{CB} = \frac{wL^2}{12} = \frac{2(12)^2}{12} = 24\ \text{k}\cdot\text{ft}$$

$$\psi_1 = \frac{\Delta_1}{10} \qquad \psi_2 = -\frac{\Delta_2}{12} \qquad \psi_3 = \frac{\Delta_3}{20}$$

As shown in Fig. 11–22*c*, the three displacements can be related. For example, $\Delta_2 = 0.5\Delta_1$ and $\Delta_3 = 0.866\Delta_1$. Thus, from the above equations we have

$$\psi_2 = -0.417\psi_1 \qquad \psi_3 = 0.433\psi_1$$

$$M_{AB} = 2E\left(\frac{I}{10}\right)[2(0) + \theta_B - 3\psi_1] + 0 \tag{1}$$

$$M_{BA} = 2E\left(\frac{I}{10}\right)[2\theta_B + 0 - 3\psi_1] + 0 \tag{2}$$

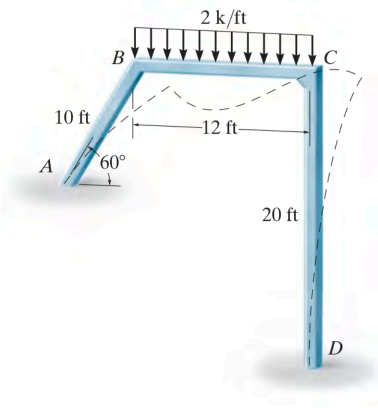
$$M_{BC} = 2E\left(\frac{I}{12}\right)[2\theta_B + \theta_C - 3(-0.417\psi_1)] - 24 \tag{3}$$

$$M_{CB} = 2E\left(\frac{I}{12}\right)[2\theta_C + \theta_B - 3(-0.417\psi_1)] + 24 \tag{4}$$

$$M_{CD} = 2E\left(\frac{I}{20}\right)[2\theta_C + 0 - 3(0.433\psi_1)] + 0 \tag{5}$$

$$M_{DC} = 2E\left(\frac{I}{20}\right)[2(0) + \theta_C - 3(0.433\psi_1)] + 0 \tag{6}$$

Equations of Equilibrium. Moment equilibrium at joints B and C yields



$$M_{BA} + M_{BC} = 0 \quad (7)$$

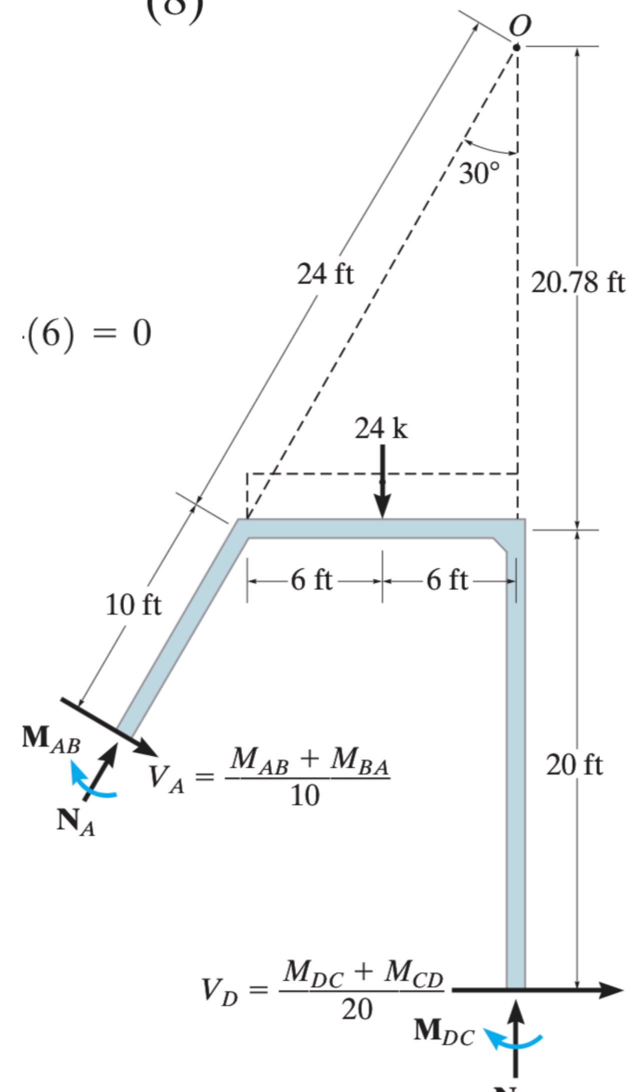
$$M_{CD} + M_{CB} = 0 \quad (8)$$

$(6) = 0$

$$\curvearrowleft + \Sigma M_O = 0;$$

$$M_{AB} + M_{DC} - \left(\frac{M_{AB} + M_{BA}}{10}\right)(34) - \left(\frac{M_{DC} + M_{CD}}{20}\right)(40.78) - 24(6) = 0$$

$$-2.4M_{AB} - 3.4M_{BA} - 2.04M_{CD} - 1.04M_{DC} - 144 = 0 \quad (9)$$



(d)

$$0.733\theta_B + 0.167\theta_C - 0.392\psi_1 = \frac{24}{EI}$$

$$0.167\theta_B + 0.533\theta_C + 0.0784\psi_1 = -\frac{24}{EI}$$

$$-1.840\theta_B - 0.512\theta_C + 3.880\psi_1 = \frac{144}{EI}$$

Solving these equations simultaneously yields

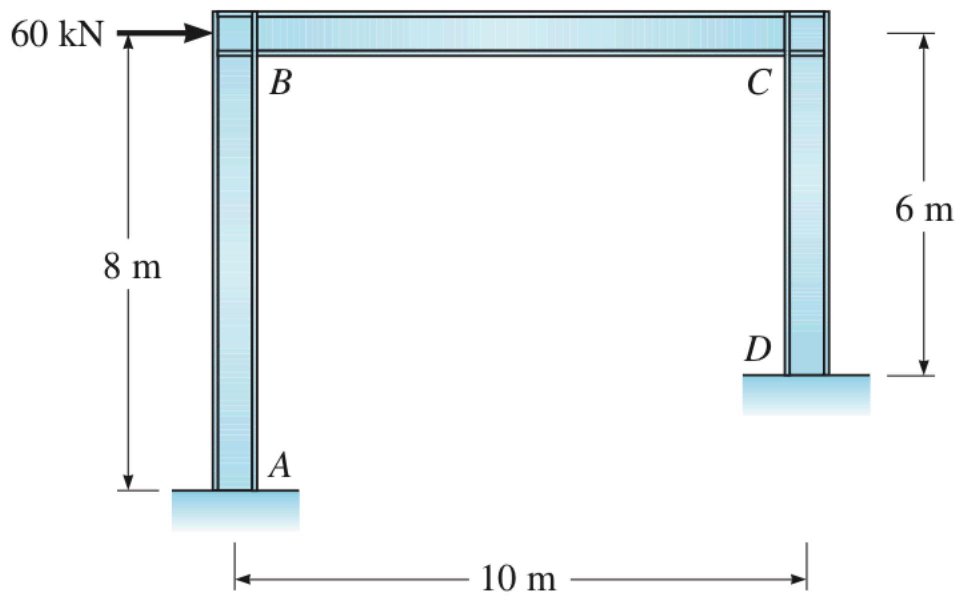
$$EI\theta_B = 87.67 \qquad EI\theta_C = -82.3 \qquad EI\psi_1 = 67.83$$

Substituting these values into Eqs. (1)–(6), we have

$M_{AB} = -23.2\ \text{k}\cdot\text{ft}$ $M_{BC} = 5.63\ \text{k}\cdot\text{ft}$ $M_{CD} = -25.3\ \text{k}\cdot\text{ft}$ *Ans.*

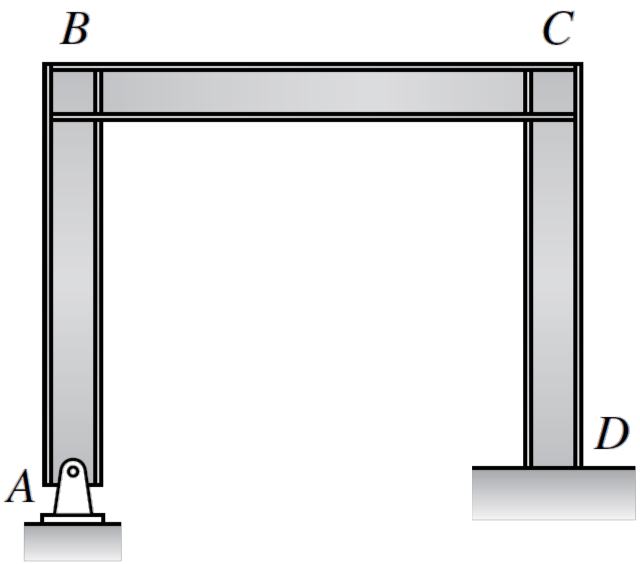
$M_{BA} = -5.63\ \text{k}\cdot\text{ft}$ $M_{CB} = 25.3\ \text{k}\cdot\text{ft}$ $M_{DC} = -17.0\ \text{k}\cdot\text{ft}$ *Ans.*

Determine all relations at points *A* and *D* in Figure shown. *EI* is constant.



§12.6 Kinematic Indeterminacy

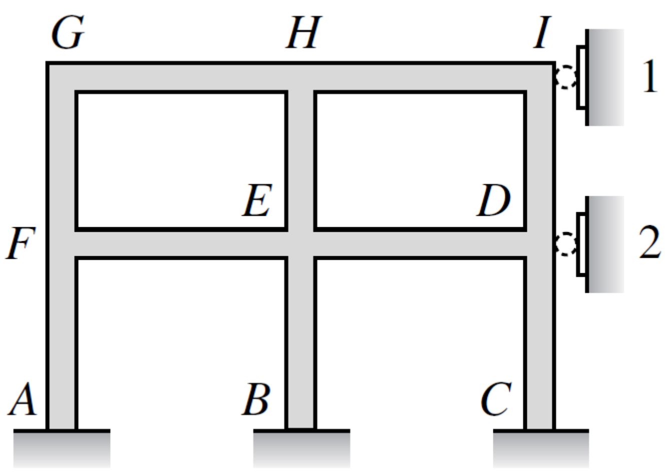
Indeterminate first degree, neglecting axial deformations



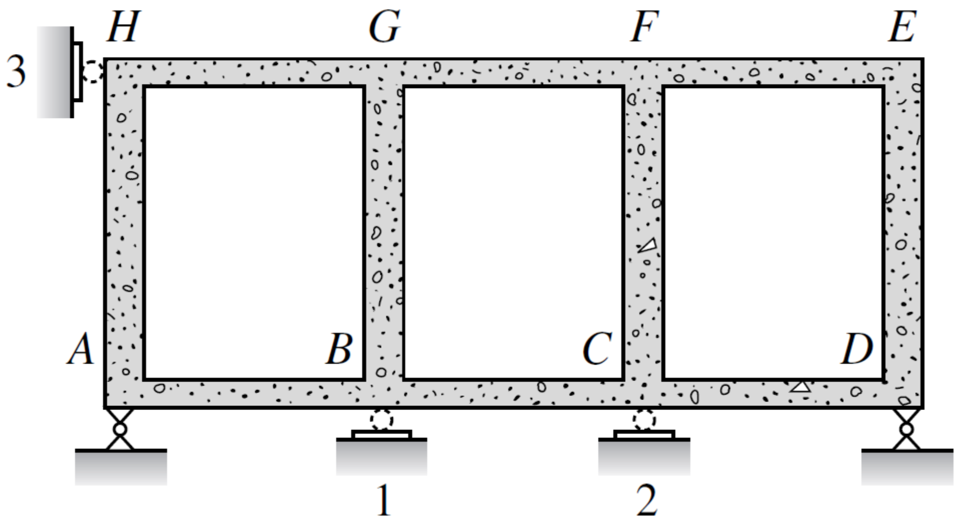
Indeterminate fourth degree

Figure 12.18 Evaluating degree of kinematic indeterminacy

§12.6 Kinematic Indeterminacy



Indeterminate eighth degree, imaginary rollers added at points 1 and 2



Indeterminate eleventh degree, imaginary rollers added at points 1, 2, and 3

Figure 12.18 Evaluating degree of kinematic indeterminacy (continued)

Figure 10.17: Evaluating degree of kinematic indeterminacy: (*a*) indeterminate first degree, neglecting axial deformations; (*b*) indeterminate fourth degree; (*c*) indeterminate eighth degree, imaginary rollers added at points 1 and 2; (*d*) indeterminate eleventh degree, imaginary rollers added at points 1, 2, and 3.