Civil Engineering Design (1)

Design of Reinforced Concrete Columns

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Contents

1. Introduction

1.1 Background

The two main parameters governing column design are:

- Bracing: if the column can sway additional moments are generated through the $P-\delta$ effect. This does not affect braced columns
- Slenderness ratio: The effective length divided by the lateral dimension of the column. Low values indicate a crushing failure, while high values denote buckling.

Bracing

The two kinds of bracing are:

- Braced:

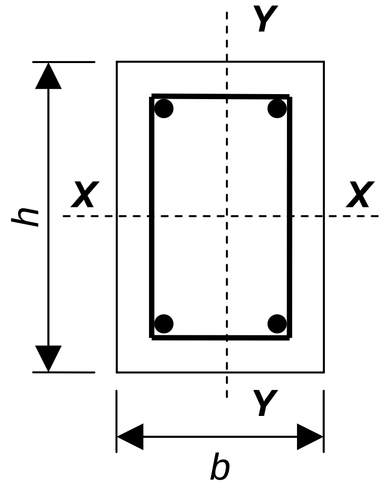
  Lateral loads on the structure are resisted by elements other than the columns in the structure; e.g. lift cores, shear walls. Hence these columns mainly take axial load; however, bending moments may result in the columns due to unsymmetrical arrangements of loads.
- Unbraced:

  Lateral loads are resisted by the bending action of the columns. The axial loads are also taken by the columns.

Slenderness ratio

The slenderness ratio is the ratio of the effective length l_e to the lateral dimension of the column in that direction. We have two directions on plan, and we can have two ratios:

- Long side of column: $\text{slenderness ratio} = l_{ex}/h$;
- Short side of column: $\text{slenderness ratio} = l_{ey}/b$.



Standard Column

The effective length is calculated from a table in BS8110 and depends on the end-conditions of the column, for each axis:

Table 3.19 — Values of β for braced columns

End condition at top	End condition at bottom		
	1	2	3
1	0.75	0.80	0.90
2	0.80	0.85	0.95
3	0.90	0.95	1.00

Table 3.20 — Values of β for unbraced columns

End condition at top	End condition at bottom		
	1	2	3
1	1.2	1.3	1.6
2	1.3	1.5	1.8
3	1.6	1.8	—
4	2.2	—	—

3.8.1.6.2 *End conditions*

The four end conditions are as follows.

a) *Condition 1.* The end of the column is connected monolithically to beams on either side which are at least as deep as the overall dimension of the column in the plane considered. Where the column is connected to a foundation structure, this should be of a form specifically designed to carry moment.

b) *Condition 2.* The end of the column is connected monolithically to beams or slabs on either side which are shallower than the overall dimension of the column in the plane considered.

c) *Condition 3.* The end of the column is connected to members which, while not specifically designed to provide restraint to rotation of the column will, nevertheless, provide some nominal restraint.

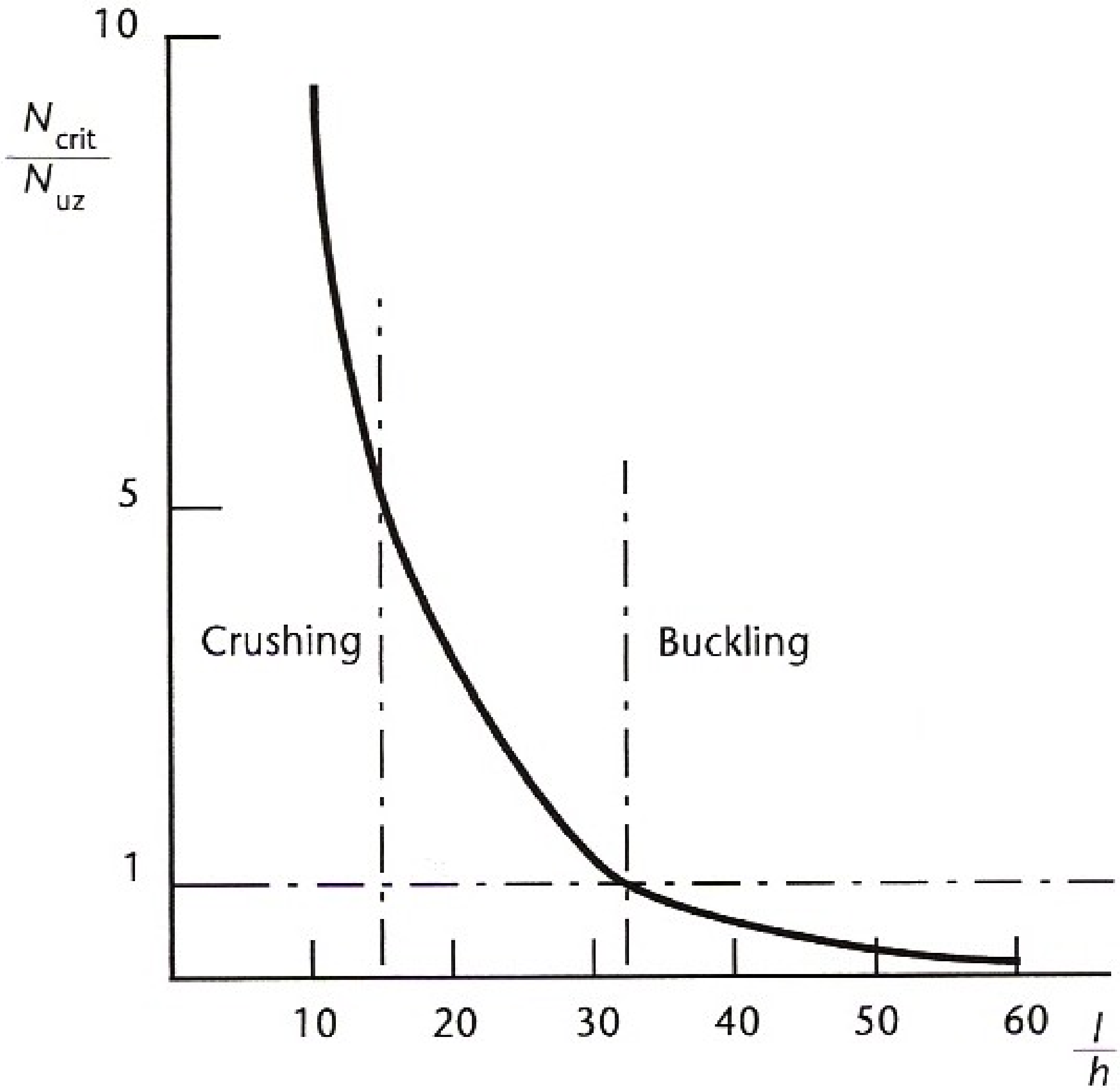
d) *Condition 4.* The end of the column is unrestrained against both lateral movement and rotation (e.g. the free end of a cantilever column in an unbraced structure).

1.2 Failure Modes

We define:

- N_{uz} as the crushing load of a perfectly axially loaded column;
- N_{crit} as Euler's buckling load for the column.

The following graph is typical and shows how for different slenderness ratios, different forms of failure are possible:



From the figure:

- For $l/h \leq 15$ the crushing capacity is much lower than the buckling capacity and so the column crushes.
- For $l/h > 32$ the buckling capacity is less than the crushing capacity and so the column buckles.
- For l/h values in between, the failure mode is not clear and depends on imperfections in the column and the way the load is applied.

Therefore, BS8110 defines:

- Short columns: $l/h \leq 15$ for a braced column and $l/h \leq 10$ for an unbraced column. These ratios must be met for both axes.
- Slender columns: any column not meeting the criteria for short columns.

As this module is short, we will only consider short, braced columns.

1.3 Design Aspects

At SLS, the axial compression tends to close up the cracks, which is beneficial. Hence crack widths do not normally need to be checked in columns (crack widths are checked in beams and slabs in water retaining structures and bridges, but don't need to be for columns). Further, deflections do not normally need to be checked in columns.

It is ULS conditions that govern the design of columns, i.e., their strength.

Areas of reinforcement are given for reference:

Cross Sectional areas of groups of bars (mm^2)

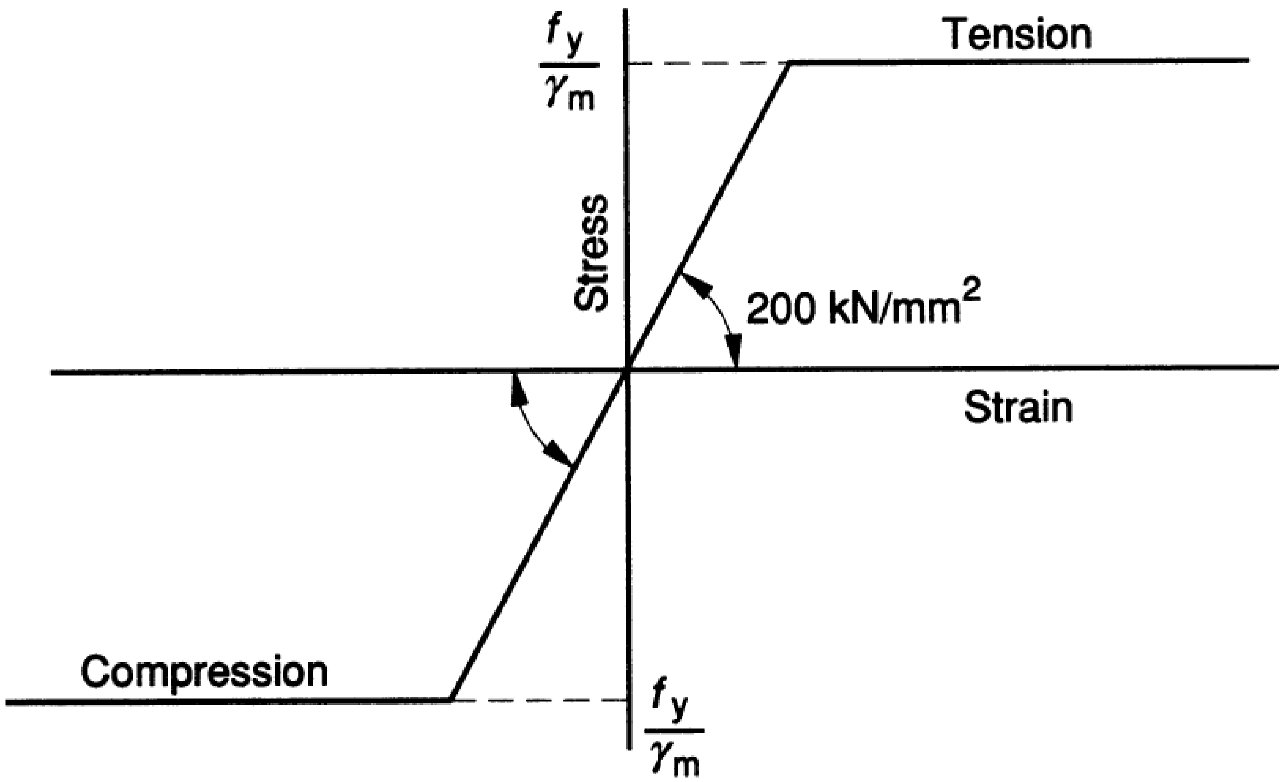
Bar Size (mm)		6	8	10	12	16	20	25	32	40
Number of bars	1	28	50	79	113	201	314	491	804	1257
	2	57	101	157	226	402	628	982	1608	2513
	3	85	151	236	339	603	942	1473	2413	3770
	4	113	201	314	452	804	1257	1963	3217	5027
	5	141	251	393	565	1005	1571	2454	4021	6283
	6	170	302	471	679	1206	1885	2945	4825	7540
	7	198	352	550	792	1407	2199	3436	5630	8796
	8	226	402	628	905	1608	2513	3927	6434	10053
	9	254	452	707	1018	1810	2827	4418	7238	11310
	10	283	503	785	1131	2011	3142	4909	8042	12566
Circumference		18.8	25.1	31.4	37.7	50.3	62.8	78.5	100.5	125.7

1.4 Background Information

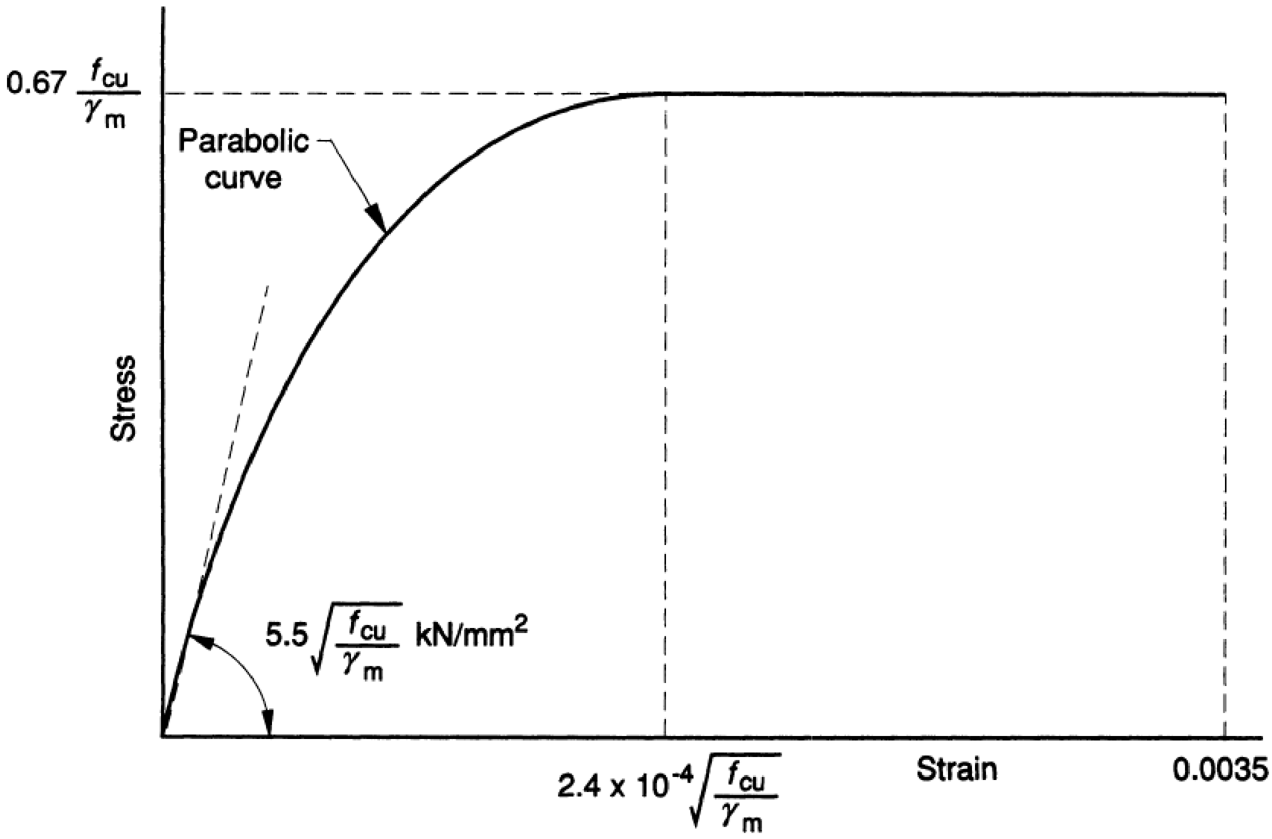
We can carry out a section analysis for columns just as we do for beams.

Review section analysis for beams as precursor to that for columns.

We need to use the material properties from BS8110. As a reminder, we have:



Stress-Strain Curve for Steel from BS8110



Stress-Strain Curve for Concrete from BS8110

Remember that the 0.67 is to relate cube strength to bending strength and is not a factor of safety (which is γ_m).

2. Short Braced Axially Loaded Columns

2.1 Development

The design of such columns is straightforward. The ultimate force is the sum of the stress × areas of the steel and concrete:

$$N_{uz} = \left(0.67\frac{f_{cu}}{\gamma_m}\right)A_c + \left(\frac{f_y}{\gamma_m}\right)A_{sc}$$

For concrete $\gamma_m = 1.5$ and for steel $\gamma_m = 1.05$ (this is due to change back shortly to 1.15). Therefore we have:

$$N_{uz} = 0.45 f_{cu} A_c + 0.95 f_y A_{sc}$$

Because this relies on a perfect axial load which is virtually impossibly to achieve in practice, a small allowance for an eccentricity of $\approx 0.05h$ is made to give:

$$N_{uz} = 0.4 f_{cu} A_c + 0.8 f_y A_{sc}$$

For a rectangular section this is equal to:

$$N_{uz} = 0.4 f_{cu} bh + A_{sc}\left(0.8 f_y - 0.4 f_{cu}\right)$$

Notice that we have effectively reduced the stress in the steel to account for the concrete that isn't present, but which we have allowed for in the first term. We will do this again.

2.2 Example

Problem

A short braced column is 300 mm square and supports 1700 kN at the ultimate limit state. The characteristic material strengths are $f_y = 400$ N/mm^2 and $f_{cu} = 30$ N/mm^2, design the steel for the column.

Solution

2.3 When loaded by Beams

Short braced columns that support an approximately (within 15%) symmetrical arrangement of beams can be design using:

$$N_{uz} = 0.35 f_{cu} A_c + 0.7 f_y A_{sc}$$

For a rectangular section this is equal to:

$$N_{uz} = 0.35 f_{cu} bh + A_{sc}\left(0.7 f_y - 0.35 f_{cu}\right)$$

Redesign the previous problem for the case when it is loaded by such beams:

3. Short Braced Columns Resisting Axial Load and Moment

3.1 Introduction

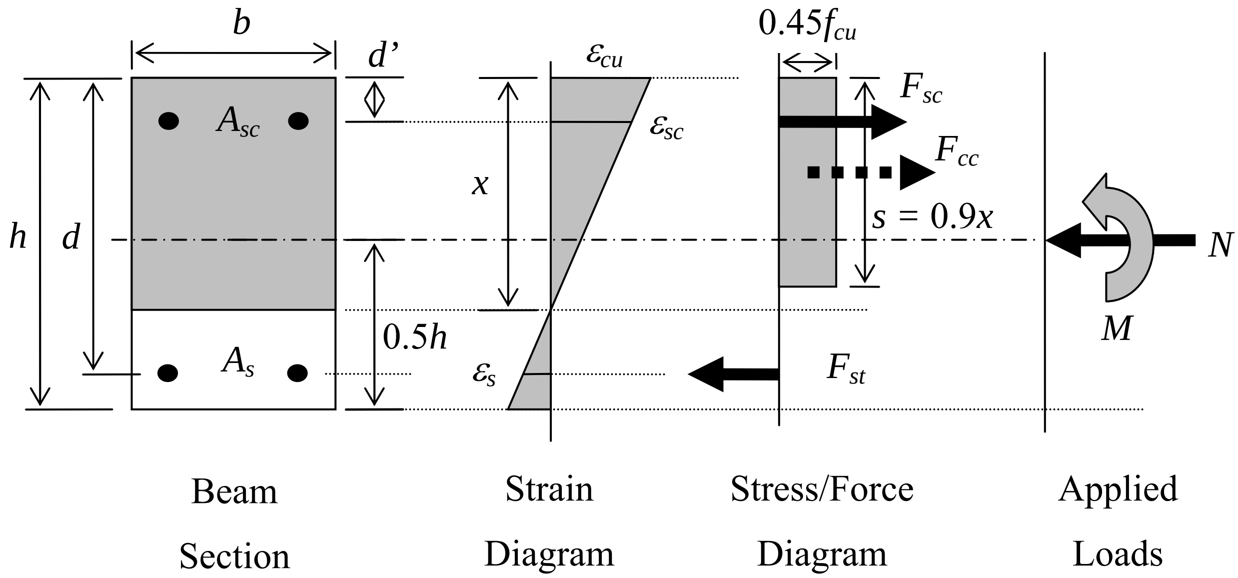
This problem is not so straightforward. We must account for the various possible positions of the neutral axis, as the bending might be large compared to the axial load. Or indeed, vice-verse, in which case we should get our result for short braced axially-only loaded columns.

Here, we consider the general case for rectangular concrete columns subject to moment M and axial load N.

Remember: the basis is that the strain diagram is linear, and the ultimate strain of concrete is $\varepsilon_{cu} = 0.0035$.

3.2 Case 1: 0.9x ≤ h

In this case the equivalent rectangular stress block is inside the section and so part of the section is in compression, and part in tension. This occurs when a relatively small axial force and a large moment are applied.



Taking $\sum F_x = 0$, we see:

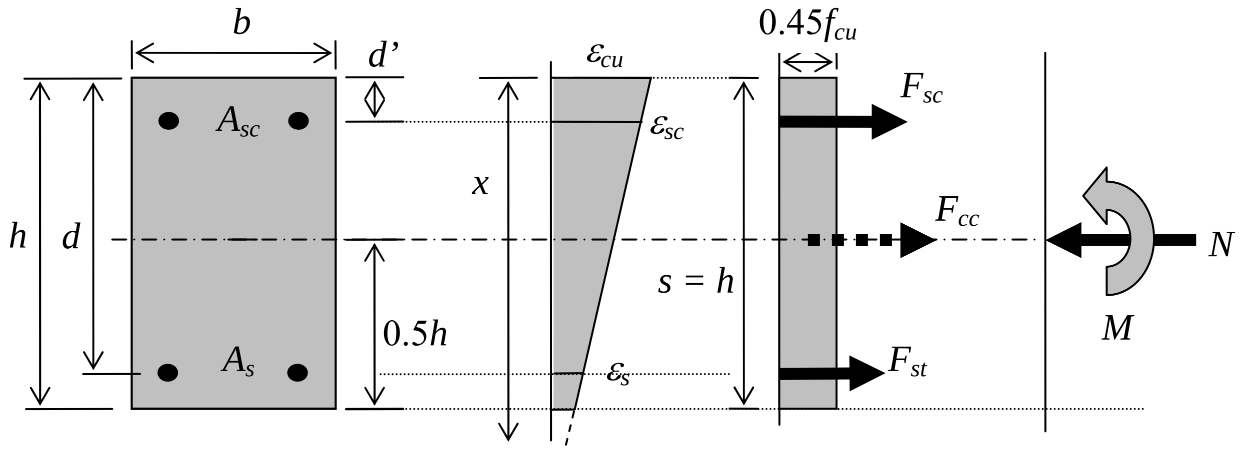
$$N = F_{cc} + F_{sc} - F_{st}$$

For moment equilibrium,: $\sum M$ about centre line $= 0$

$$M = F_{cc}\left(\frac{h}{2} - \frac{s}{2}\right) + F_{sc}\left(\frac{h}{2} - d'\right) + F_{st}\left(d - \frac{h}{2}\right)$$

3.3 Case 2: 0.9x > h

In this case the equivalent rectangular stress block is outside the section and so all of the section is in compression. This occurs when a relatively large axial force and a small moment are applied.



Beam Section | Strain Diagram | Stress/Force Diagram | Applied Loads

Taking $\sum F_x = 0$:

$$N = F_{cc} + F_{sc} - F_{st}$$

Moment equilibrium:

$$M = F_{sc}\left(\frac{h}{2} - d'\right) - F_{st}\left(d - \frac{h}{2}\right)$$

3.4 Combination of Cases

Noting that the direction of F_{st} changes, and taking its positive direction to be that of compression, we combine the two cases:

$$N = F_{cc} + F_{sc} - F_{st}$$
$$M = F_{cc}\left(\frac{h}{2} - \frac{s}{2}\right) + F_{sc}\left(\frac{h}{2} - d'\right) + F_{st}\left(d - \frac{h}{2}\right)$$

Thus when F_{st} is in tension and is negative, we get Case 1. Conversely when it is positive and $\frac{h}{2} = \frac{s}{2}$ we get Case 2.

Forces:

The forces in the expressions are derived from stress × area:

$$F_{cc} = 0.45 f_{cu} bs$$
$$F_{sc} = \left(f_{sc} - 0.45 f_{cu}\right) A_{sc}$$
$$F_{st} = \left(f_s - 0.45 f_{cu}\right) A_s$$

In which the area displaced by the reinforcement has been taken into account by reducing the stress accordingly. Note that for Case 1, the force F_{st} does not need to be reduced in this manner.

The stress at any level z can be got from:

$$\varepsilon_z = 0.0035\left(\frac{x - z}{x}\right)$$

Hence, the strain in the most compressed steel is:

$$\varepsilon_{sc} = 0.0035\left(\frac{x-d'}{x}\right) = 0.0035\left(1-\frac{d'}{x}\right)$$

And in the least compressed steel is:

$$\varepsilon_{s} = 0.0035\left(\frac{x-d}{x}\right) = 0.0035\left(1-\frac{d}{x}\right)$$

Remember that the strain in the steel is limited to:

$$\frac{f_y/\gamma_m}{E_s} = \frac{460/1.05}{200\times10^3} = 0.0022 \approx 0.002$$

and at that strain the stress is f_y. Hence:

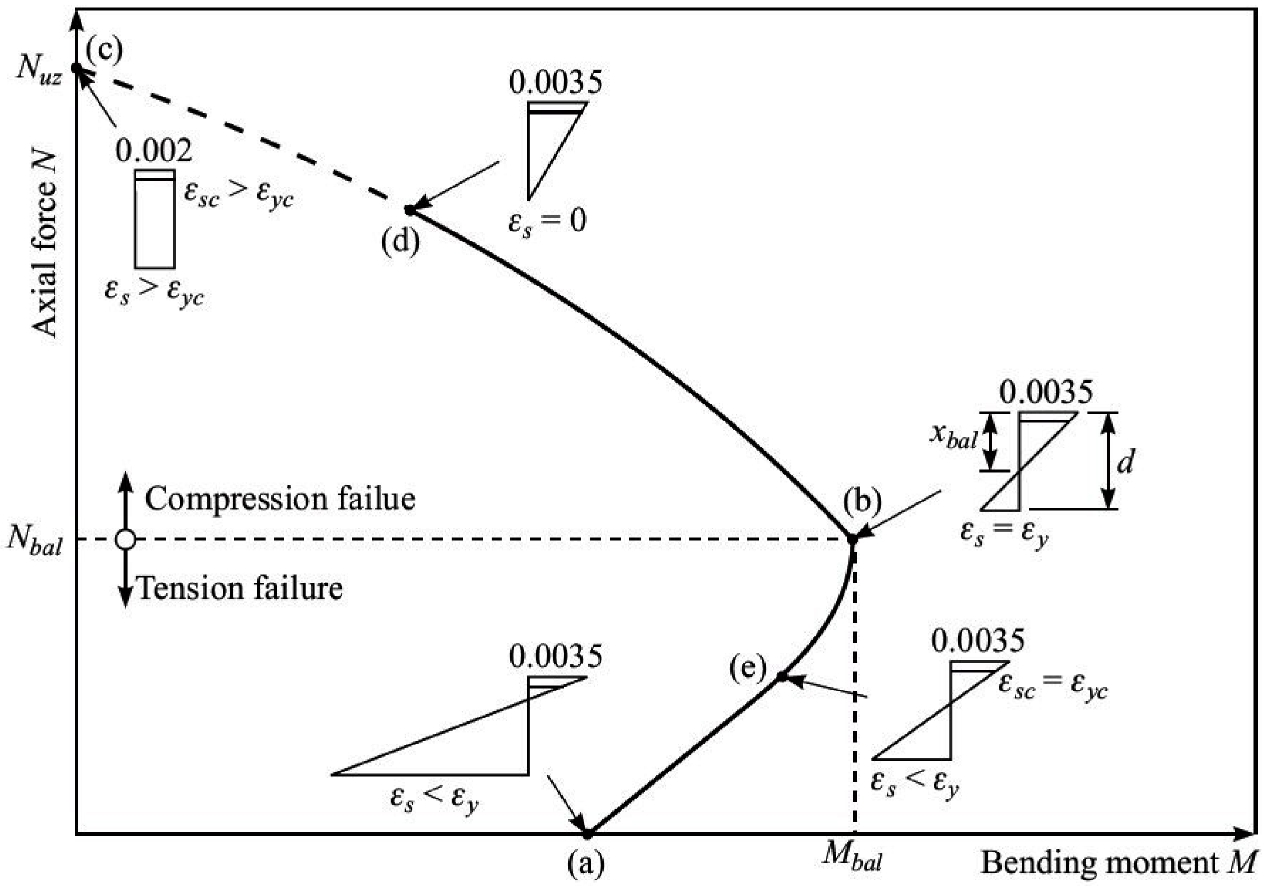
$$f_s = \begin{cases} E_s\varepsilon_s & \varepsilon_s < 0.002 \\ f_y & \varepsilon_s \geq 0.002 \end{cases}$$

Therefore, from the set of equations presented, for a given value of *x* an (*M*, *N*) couple results. For a range of values of *x* the interaction chart can be plotted.

It is very worthwhile for your understanding to program these equations in Excel (say) and plot some such equations.

3.5 Description of Interaction Diagram

For a range of values of x/h, a set of points results, each representing a combination of axial force and moment. Any combination of applied moment and axial force that fall inside this curve is therefore safe against failure. A number of important points can be identified on a typical interaction diagram as indicated:



(a) Pure bending

This point represents that of a beam in bending. Note that the presence of a small axial force will generally increases the moment capacity of a beam.

(b) Balance point

This is the point where the concrete reaches its ultimate strain at the same time the tension reinforcement yields. For combinations of *N* and *M* that fall below the balance point, the failure mode is ductile with the reinforcement yielding before the concrete fails in compression. Otherwise the failure mode is brittle: the concrete crushes without yielding of the tension reinforcement. Unfortunately the failure mode in a column cannot be controlled by reinforcement quantities as it can in beams.

(c) Pure axial compression

At this point, the column is subjected to an axial force only with $M = 0$. The capacity of the section is equal to N_{uz}. Note that the 'tension' reinforcement yields in compression for this case.

(d) Zero strain in the tension reinforcement

Moving from point (b) to point (c) it can be seen that the neutral axis increases from x_{bal} to infinity as *N* increases. The strain in the tension reinforcement changes from yielding in tension to yielding in compression, passing through zero at point (d). Moving from points (d) to (c) the neutral axis will fall outside the section and the strain distribution will eventually change from triangular to uniform.

Between points (b) and (c) an increase in axial load *N* will lead to a smaller moment capacity M at failure. Conversely, below the balance point an increase in *N* increases will increase the moment capacity of the section.

(e) Yielding of the compression reinforcement

As the axial force *N* increases and the neutral axis *x* increases, the strain in compression reinforcement will often change from elastic to yielding. This will clearly be influenced by the strength of the reinforcement and its position within the section. This point will typically correspond to a change in slope of the interaction diagram as shown at point (e).

Minimum *N*

Note 1:

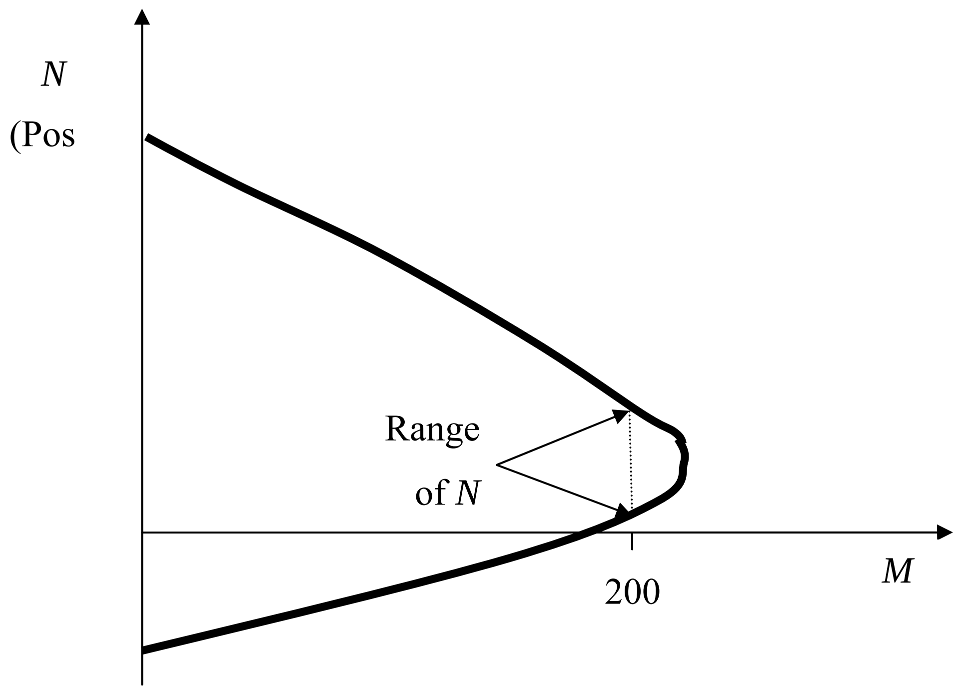
All columns can take some small tension. This may be needed for example in a pile that needs to resist uplift. However, you need to be cautious that you do not have a cracking problem at SLS.

Note 2:

In many cases you will find that you are in the lower part of the Interaction Diagram. When this is the case, it is important to note that the MOST CRITICAL case is the *minimum* value of *N* combined with maximum coexisting *M*.

Example:

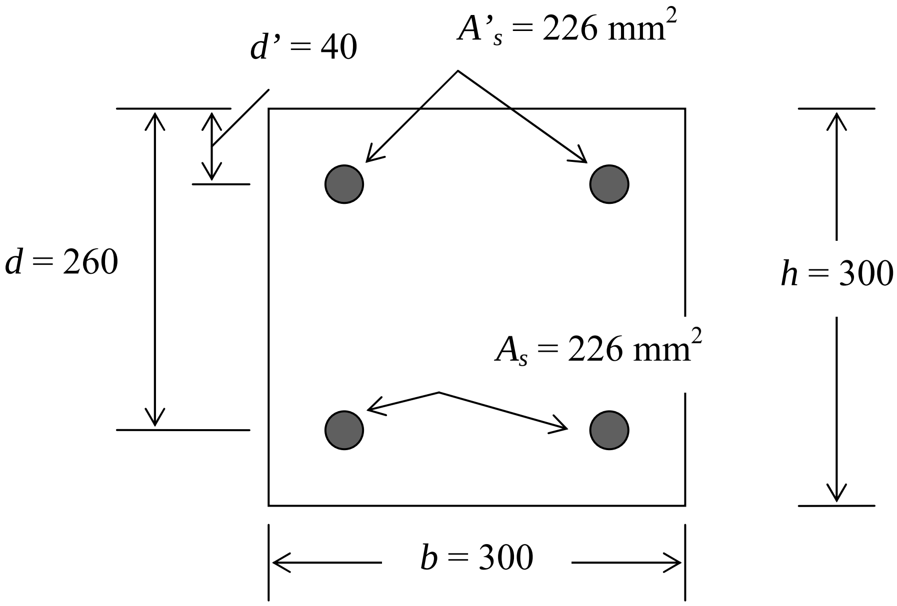
N varies from 100 to 300 kN & *M* from 0 to 200 kNm. The critical case is (*N, M*) = (100, 200).



3.6 Example

Problem

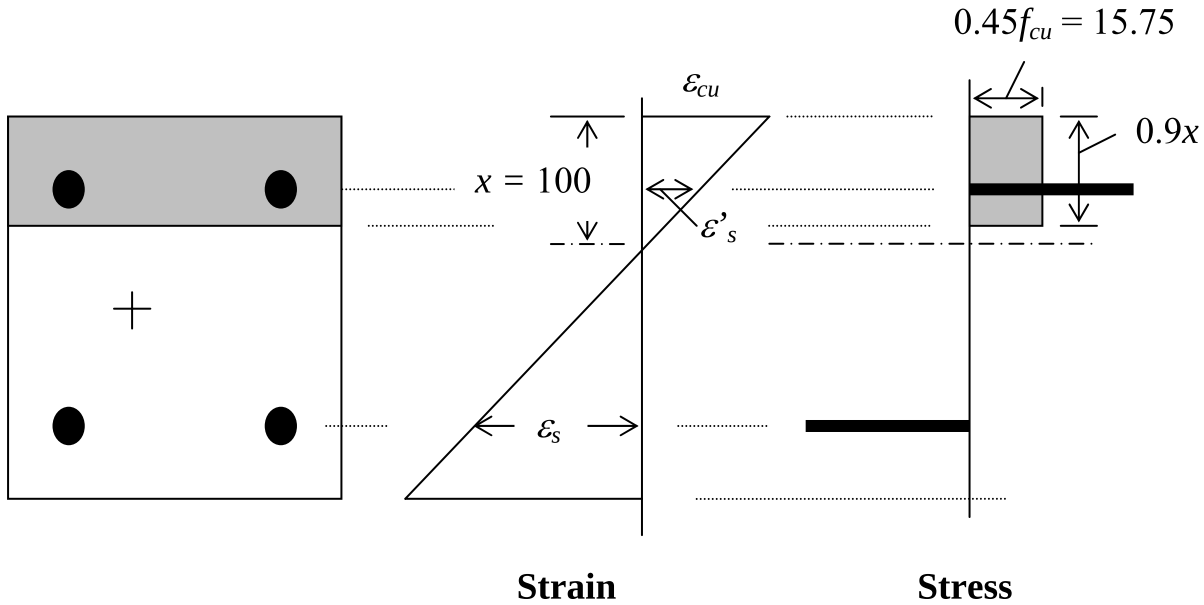
Derive points on the Interaction Diagram for the column illustrated. The concrete cube strength is f_{cu} = 35 N/mm^2.



Solution

We begin by taking several locations for the position of the neutral axis.

Point 1: Let $x = 100$ mm



The definition of failure is the time when the concrete crushes (yielding of steel is not a problem), i.e., failure is when maximum concrete strain = ε_{cu} = 0.0035 (BS8110 definition).

Force in concrete = (15.75 N/mm^2)(300×90) = 425 250 N

= 425 kN

Force in 'displaced concrete' = 15.75×226 = 3650 N

= 4 kN

Strain in compression reinforcement is found by similar triangles:

$$\frac{\varepsilon_s'}{60} = \frac{\varepsilon_{cu}}{100} \Rightarrow \varepsilon'_s = 0.0021$$

The yield strain of steel is, $\dfrac{0.95 f_y}{E_s} = \dfrac{0.95 \times 460}{200,000} = 0.00219$

Hence the compression steel has yielded. The force in the compression reinforcement is therefore $= (0.95f_y)(226)$

= 98 762 N

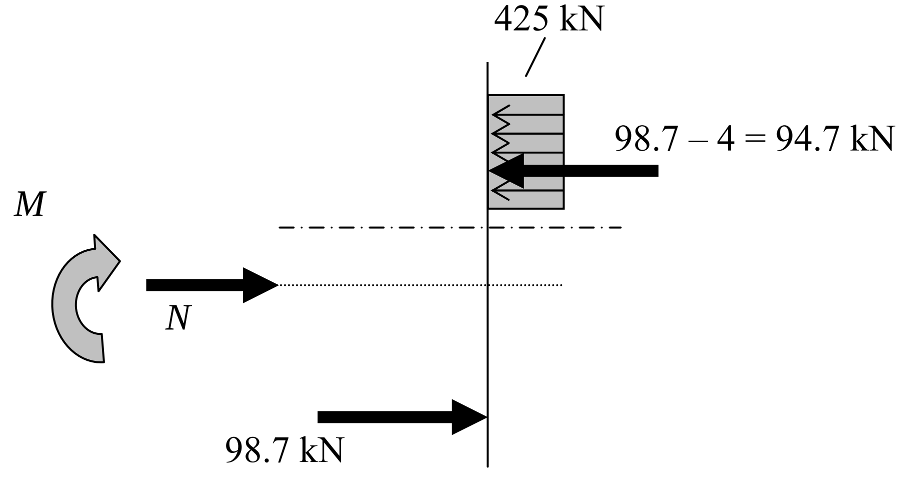
= 98.7 kN

The strain in the tension steel is also found by similar triangles:

$$\frac{\varepsilon_s}{160} = \frac{\varepsilon_{cu}}{100} \Rightarrow \varepsilon_s = 0.0056$$

Therefore this steel has also yielded. Hence, force in tension steel = $(0.95f_y)(226)$

= 98.7 kN.



Forces

Now apply equilibrium of axial forces: In order for x to equal 100, there must be an applied net compressive force of,

$$N = 94.7 + 425 - 98.7 = 421 \text{ kN}$$

Similarly moment equilibrium gives the applied moment that will result in this state of stress. Summing moment about the point of application of N gives:

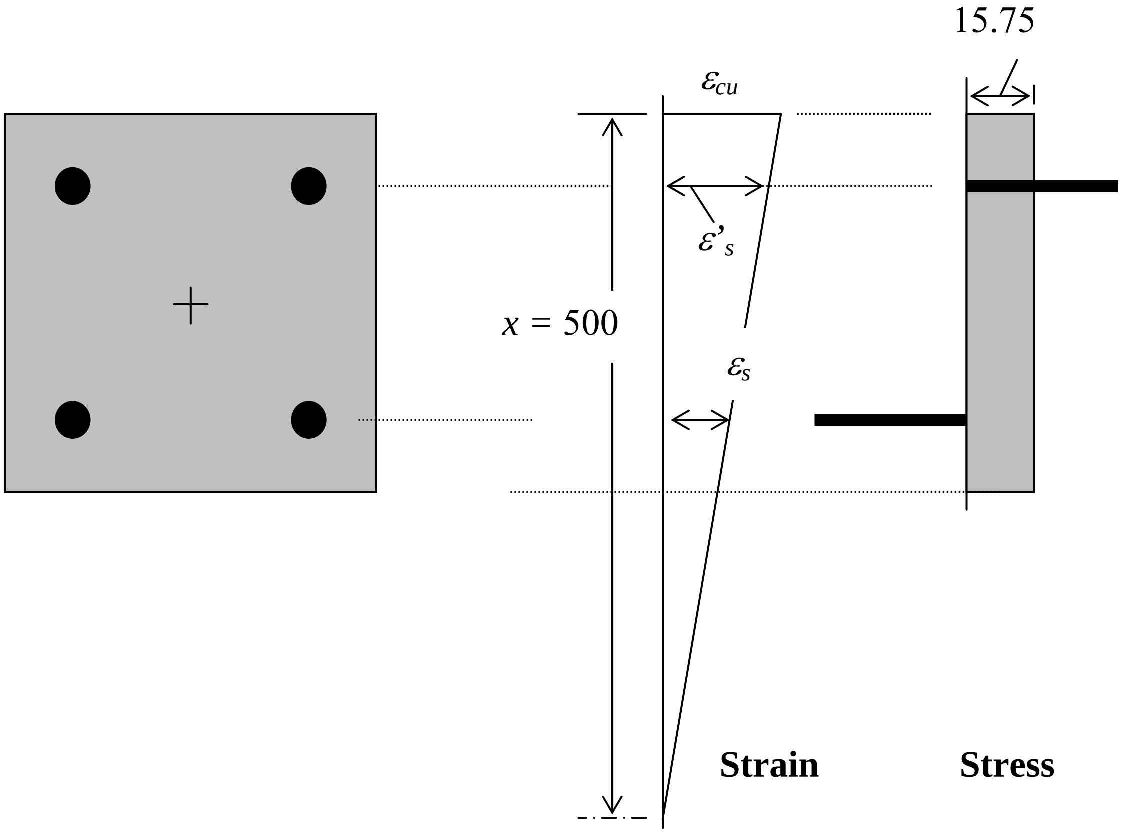
$$\begin{aligned} M &= 425\times10^3(150 - 90/2) + 94.7\times10^3 (150 - 40) \\ &\qquad + 98.7\times10^3 (260 - 150) \\ &= 66\times10^6 \text{ Nmm} = 66 \text{ kNm.} \end{aligned}$$

Hence the first point in the Interaction Diagram is $(N, M) = (421, 66)$.

Point 2: Let x = 500 mm

Force in concrete = (15.75 N/mm^2)(300×300) = 1417 500 N

= 1418 kN



Force in displaced concrete

= 15.75×226 = 3650 N

= 4 kN

Both layers of reinforcement are in compression and concrete is displaced in both cases (previously the bottom layer displaced concrete in tension that carried no load).

Strain in top reinforcement is found by similar triangles:

$$\frac{\varepsilon_s'}{460} = \frac{\varepsilon_{cu}}{500} \Rightarrow \varepsilon'_s = 0.00322$$

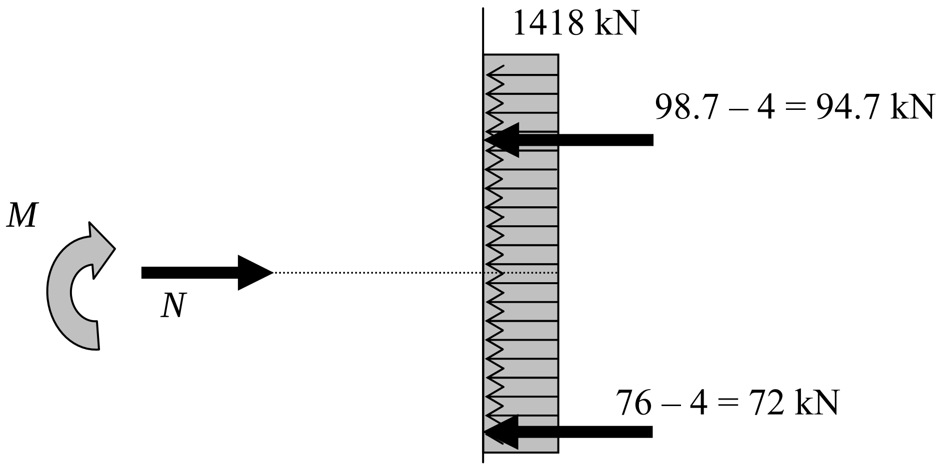
Therefore this steel has yielded. Hence, force in top steel = $(0.95f_y)(226)$ = 98.7 kN.

Strain in bottom reinforcement:

$$\frac{\varepsilon_s}{(500-260)} = \frac{\varepsilon_{cu}}{500} \Rightarrow \varepsilon'_s = 0.00168$$

(i.e., steel *not* yielded)

Hence, stress in bottom steel is (0.00168)(200,000) = 336 N/mm^2 and the force is (336)(226) = 75 936 N = 76 kN.



Forces

Equilibrium of axial forces:

$$N = 94.7 + 72 + 1418 = 1585 \text{ kN}$$

Moment equilibrium:

$$M = 1418\times10^{3}(150 - 150) + 94.7\times10^{3}\,(150 - 40)$$

$$- 72\times10^{3}\,(260 - 150)$$

$$= 2.49\times10^{6}\text{ Nmm} = 2.5\text{ kNm.}$$

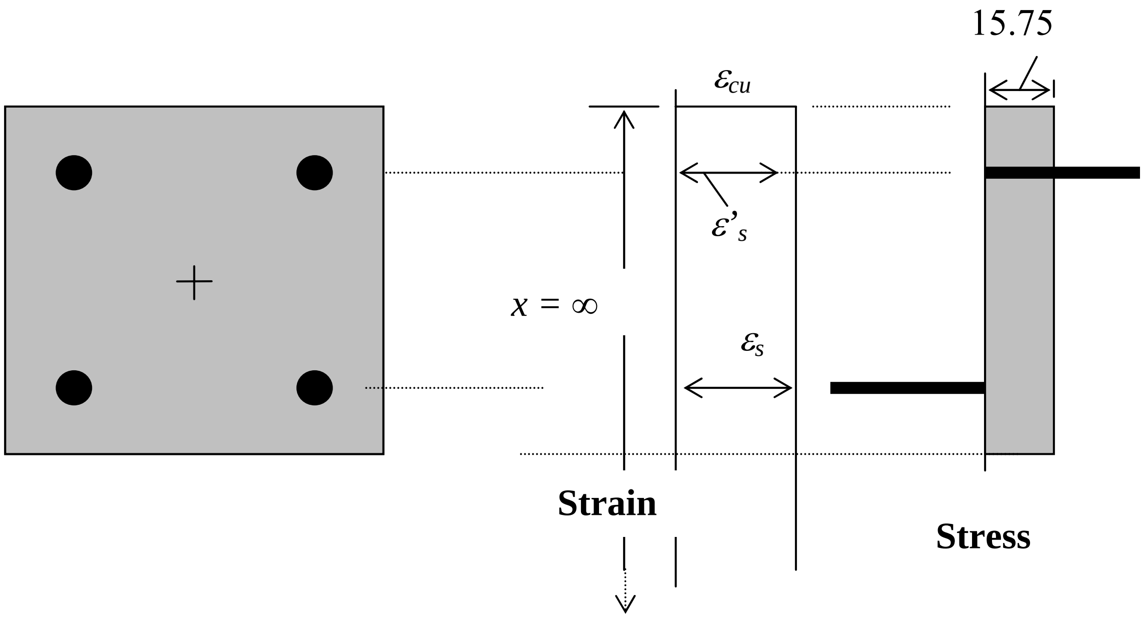
Hence the second point in the Interaction Diagram is (N, M) = (1585, 2).

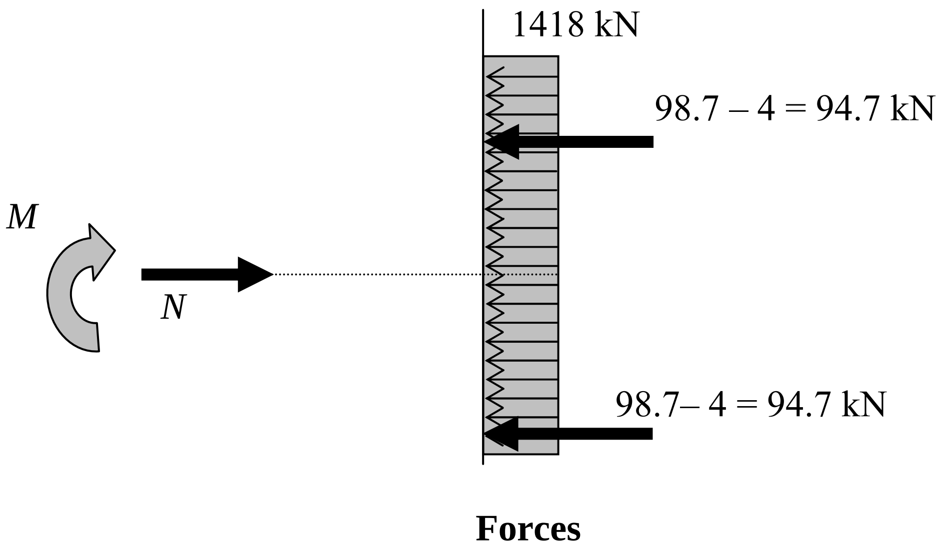
<u>Point 3: Let $x = \infty$</u>

Force in concrete $= (15.75\ \text{N/mm}^2)(300\times300) = 1417\ 500\ \text{N}$

$= 1418\ \text{kN}$

Force in displaced concrete = 4 kN. Steel in all layers has yielded. Hence, force in top/bottom steel = $(0.95f_y)(226) = 98.7$ kN.





Equilibrium of axial forces:

$$N = 95 + 95 + 1418 = 1608 \text{ kN}$$

Moment equilibrium:

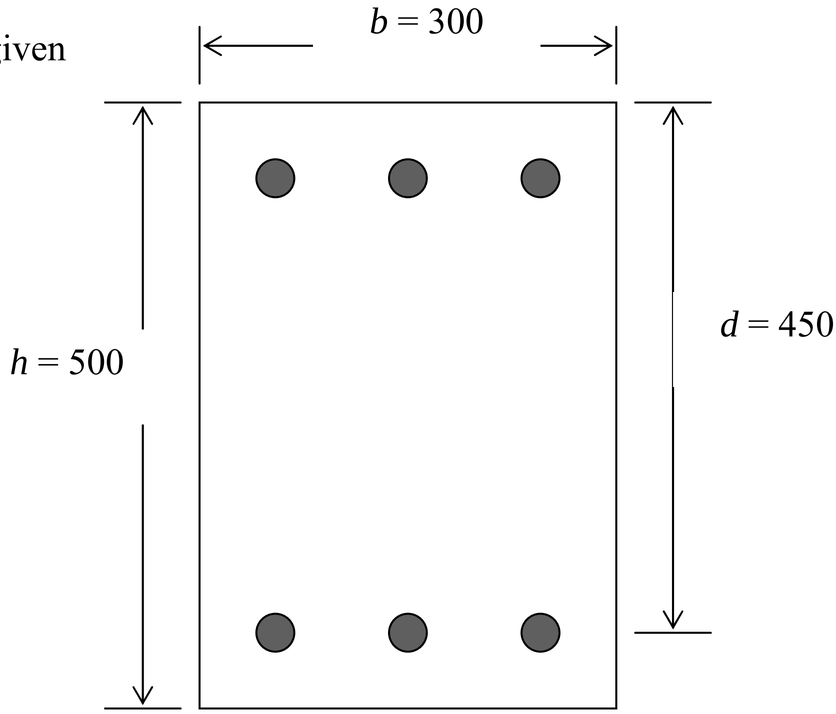
$$M = 1418\times10^3(150 - 150) + 95\times10^3 (150 - 40) - 95\times10^3 (260 - 150) = 0$$

Hence a third point in the Interaction Diagram is (N, M) = (1608, 0).

Similarly other points are found until the complete diagram can be found.

3.7 Design Using Standard Interaction Diagrams

Find the area of reinforcement required in the column shown given that:



$M = 450$ kNm

$N = 500$ kN

$f_{cu} = 40$ N/mm^2

$f_y = 460$ N/mm^2.

$d/h = 450/500 = 0.9$

Chart No. 39 applies.

$$\frac{M}{bh^2} = \frac{450 \times 10^6}{300 \times 500^2} = 6$$

$$\frac{N}{bh} = \frac{500 \times 10^6}{300 \times 500} = 3.33$$

I.A. Diagram suggests 3% reinforcement.

$A_{sc} = 0.03(300 \times 500) = 4500$ mm^2

6 T32s gives $6(\pi 32^2/4) = 4825$ mm^2.

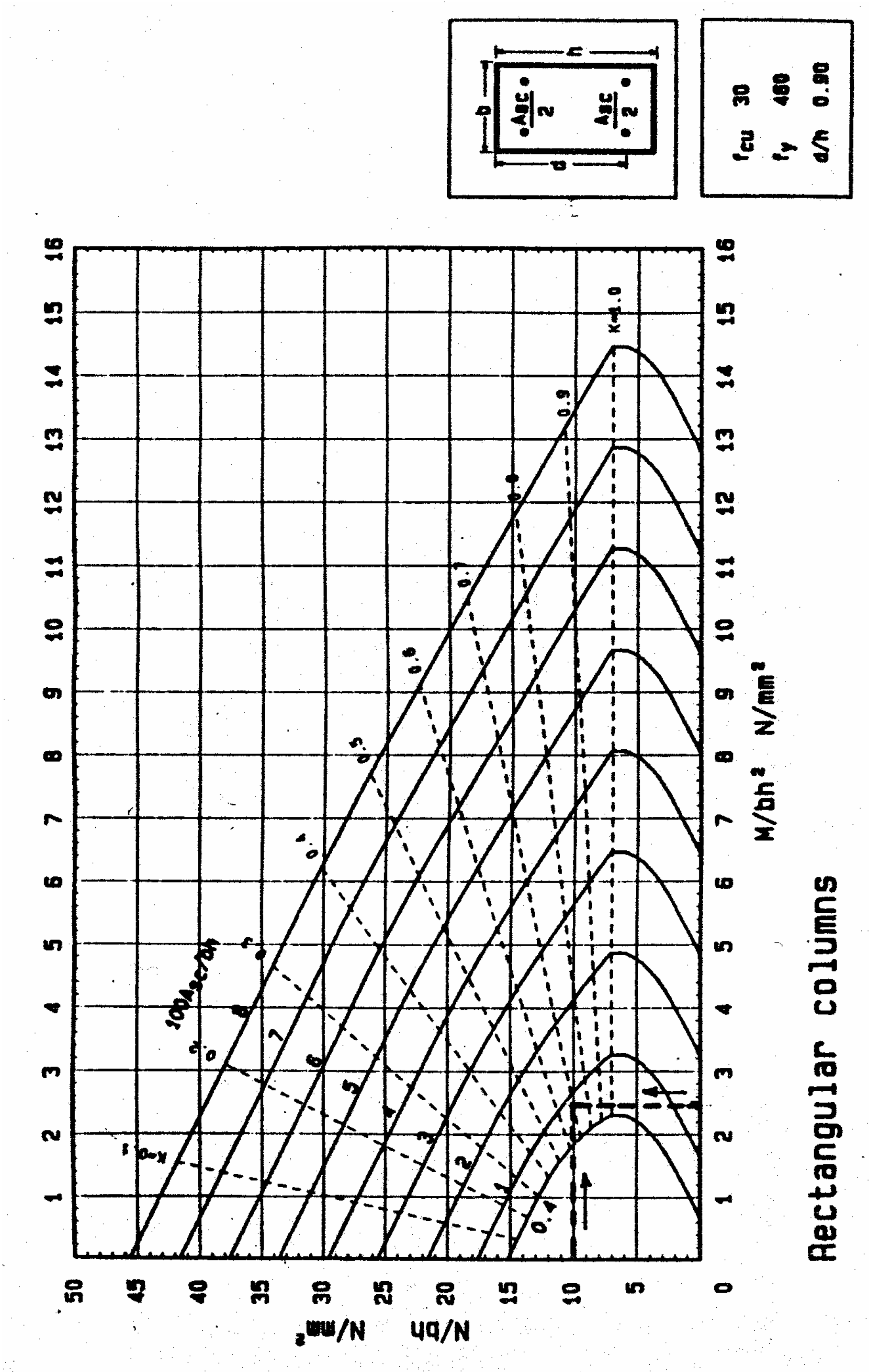
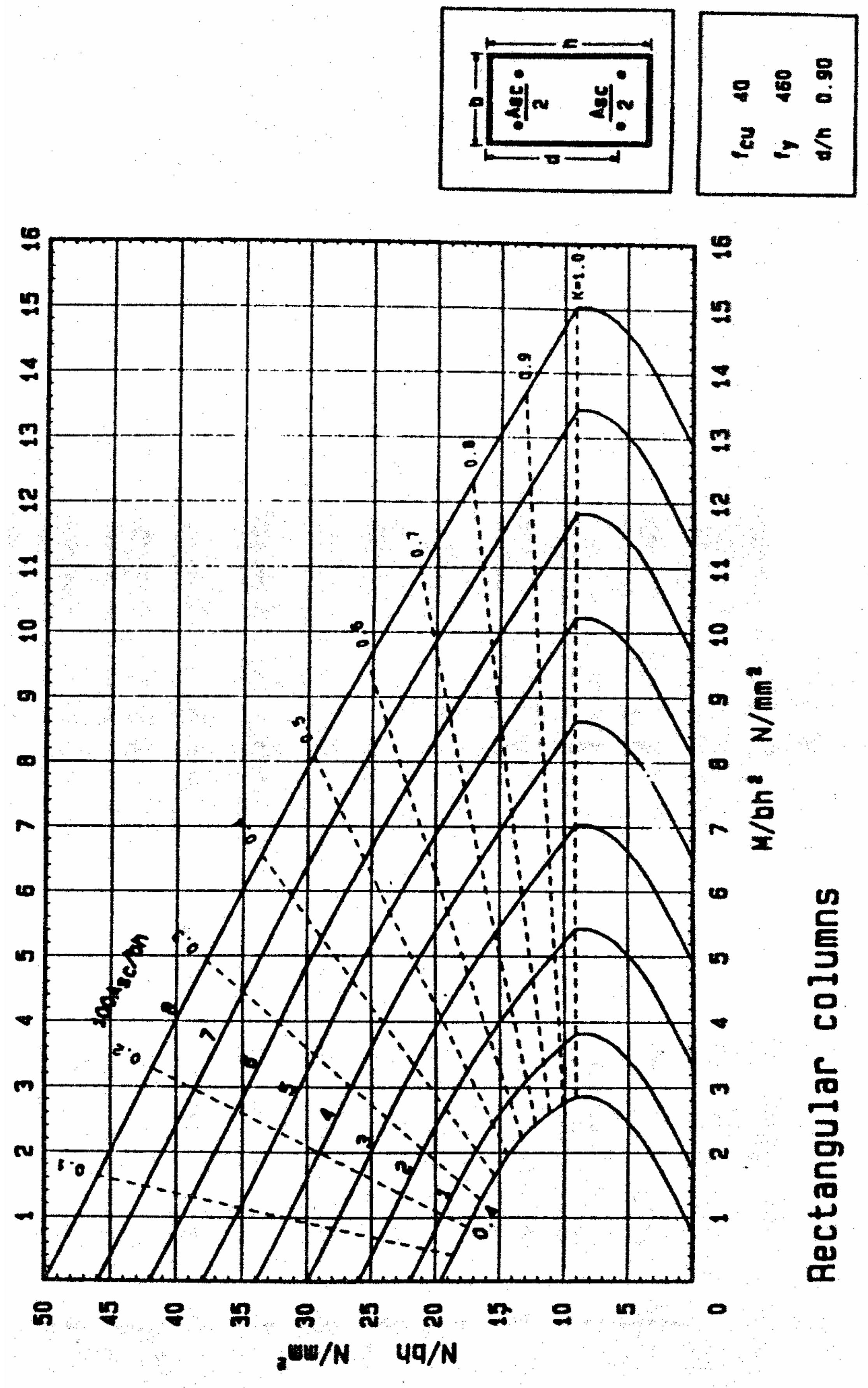

Asc/2
Asc/2
b
h
d
fcu 30
fy 460
d/h 0.90
100Asc/bh
K=1.0
M/bh² N/mm²
N/bh N/mm²
Rectangular columns

Chart No. 29



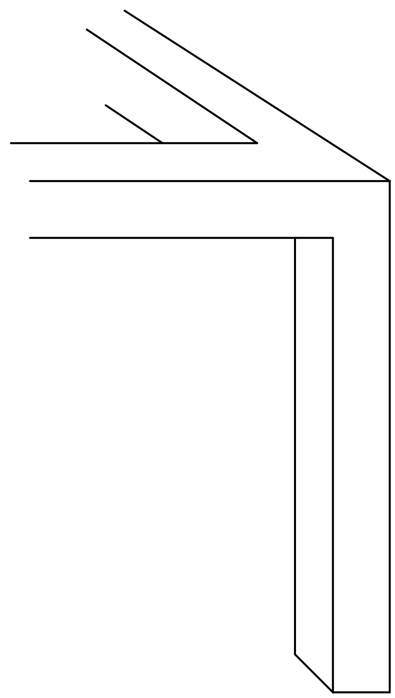
Rectangular columns
M/bh² N/mm²
N/bh N/mm²
fcu 40
fy 460
d/h 0.90
K=1.0
100Asc/bh
Asc/2

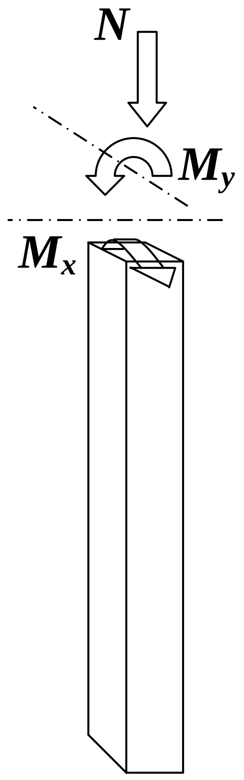
Chart No. 39

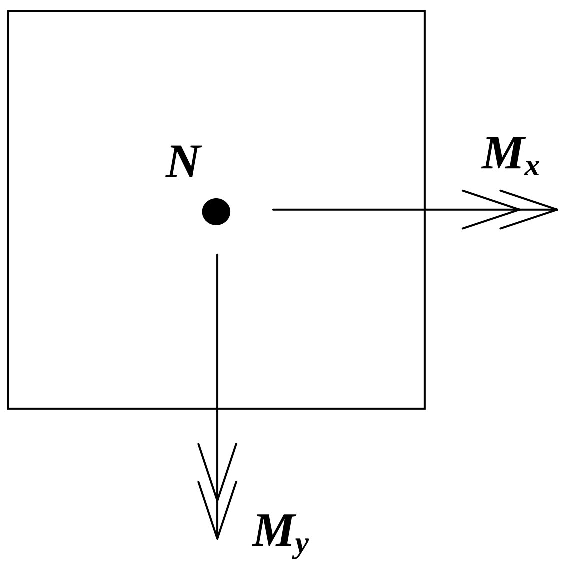
4. Biaxial Bending

4.1 Introduction

Up to now we have only considered uniaxial bending, i.e., axial force plus moment about one axis. In real (3-D) structures, biaxial bending is common, i.e.:







There are 3 ways to design columns for biaxial bending.

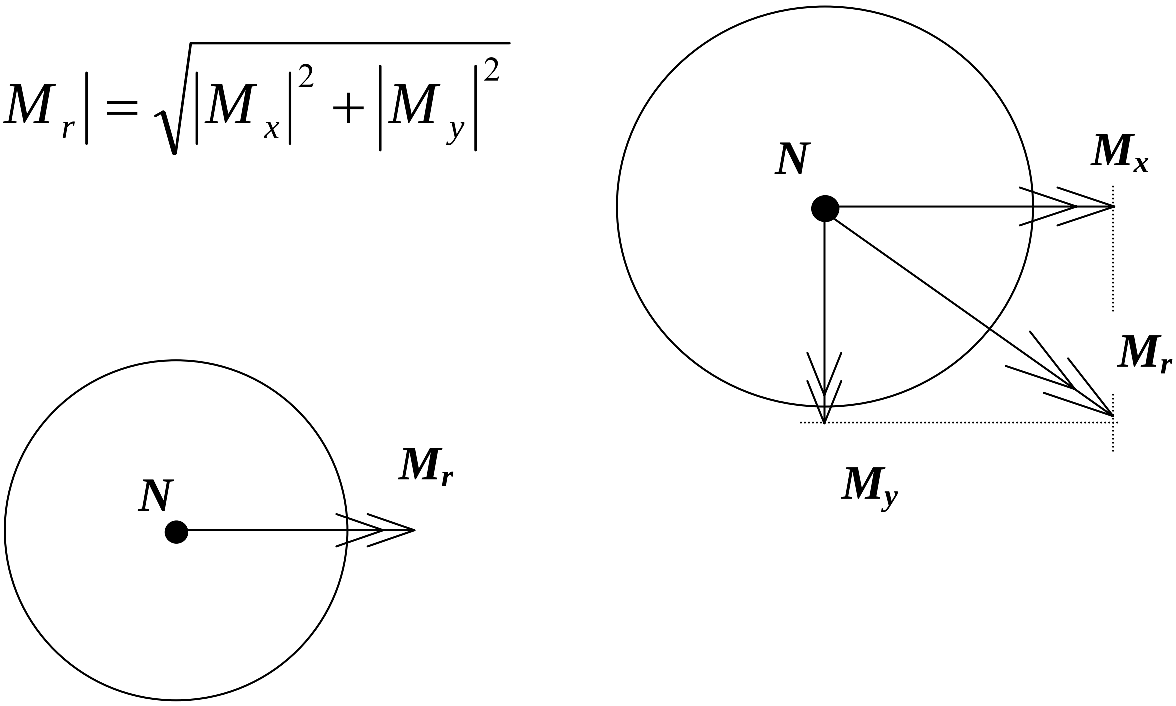
4.2 Method 1 – Resolution of Moments

Moment is a vector so we can add the two vectors, M_x and M_y and design the column to resist the resultant.

Example 1 – Circular Column

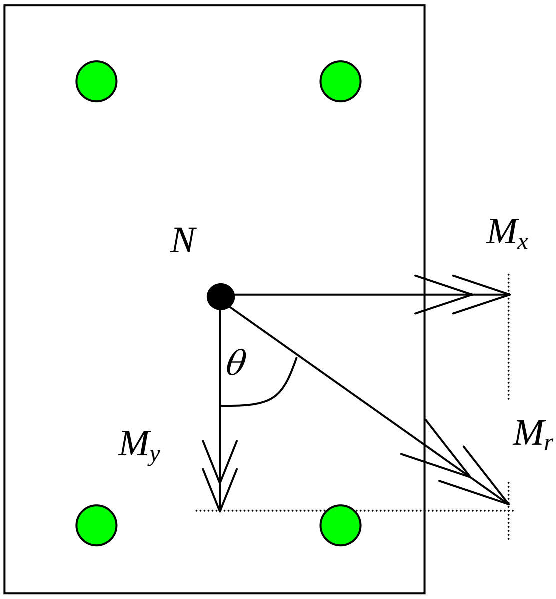
The vector sum of M_x and M_y is M_r. So, simply design the column to resist M_r where:

$$\left|M_r\right| = \sqrt{\left|M_x\right|^2 + \left|M_y\right|^2}$$



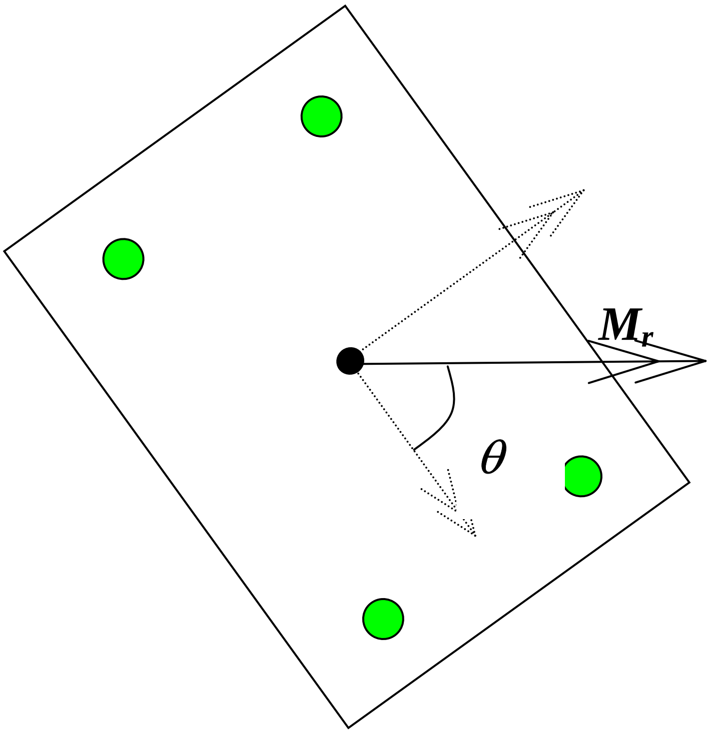
Example 2 – Non-circular Column

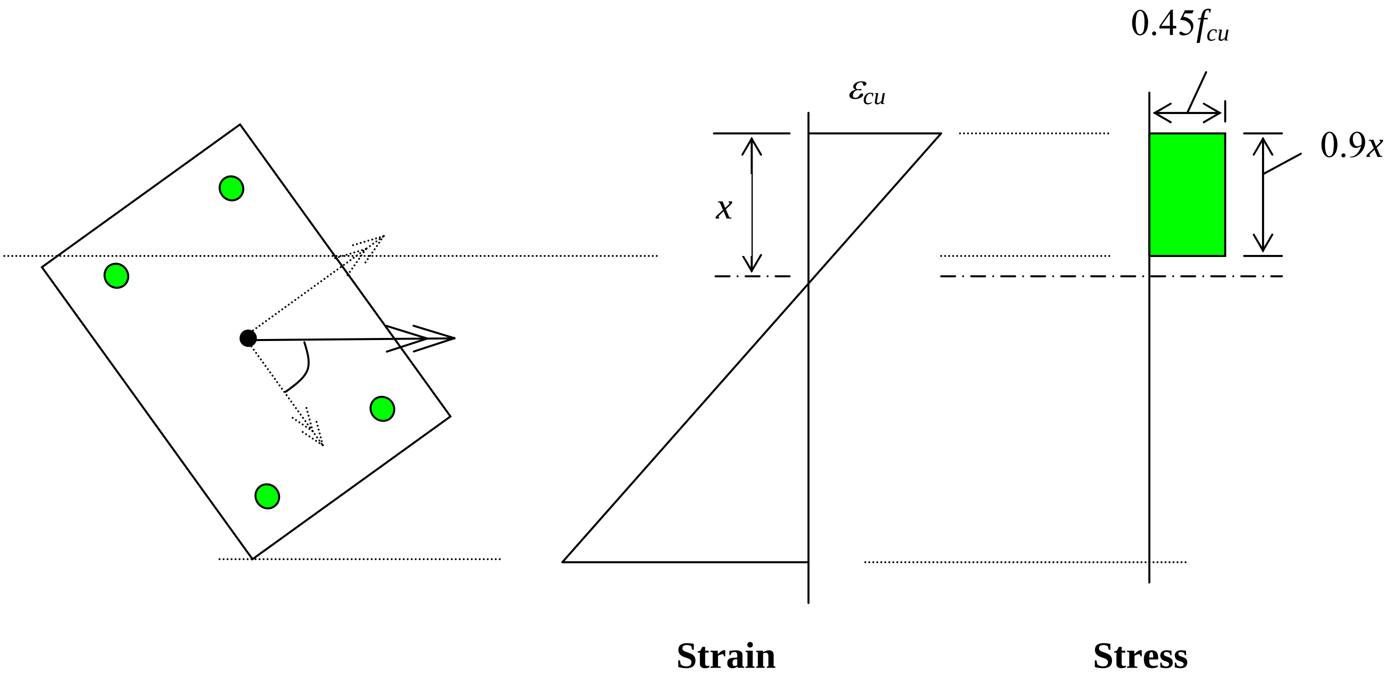
As for the circular column, resolve moments and design to resist M_r:



$$\left|M_r\right| = \sqrt{\left|M_x\right|^2 + \left|M_y\right|^2}$$

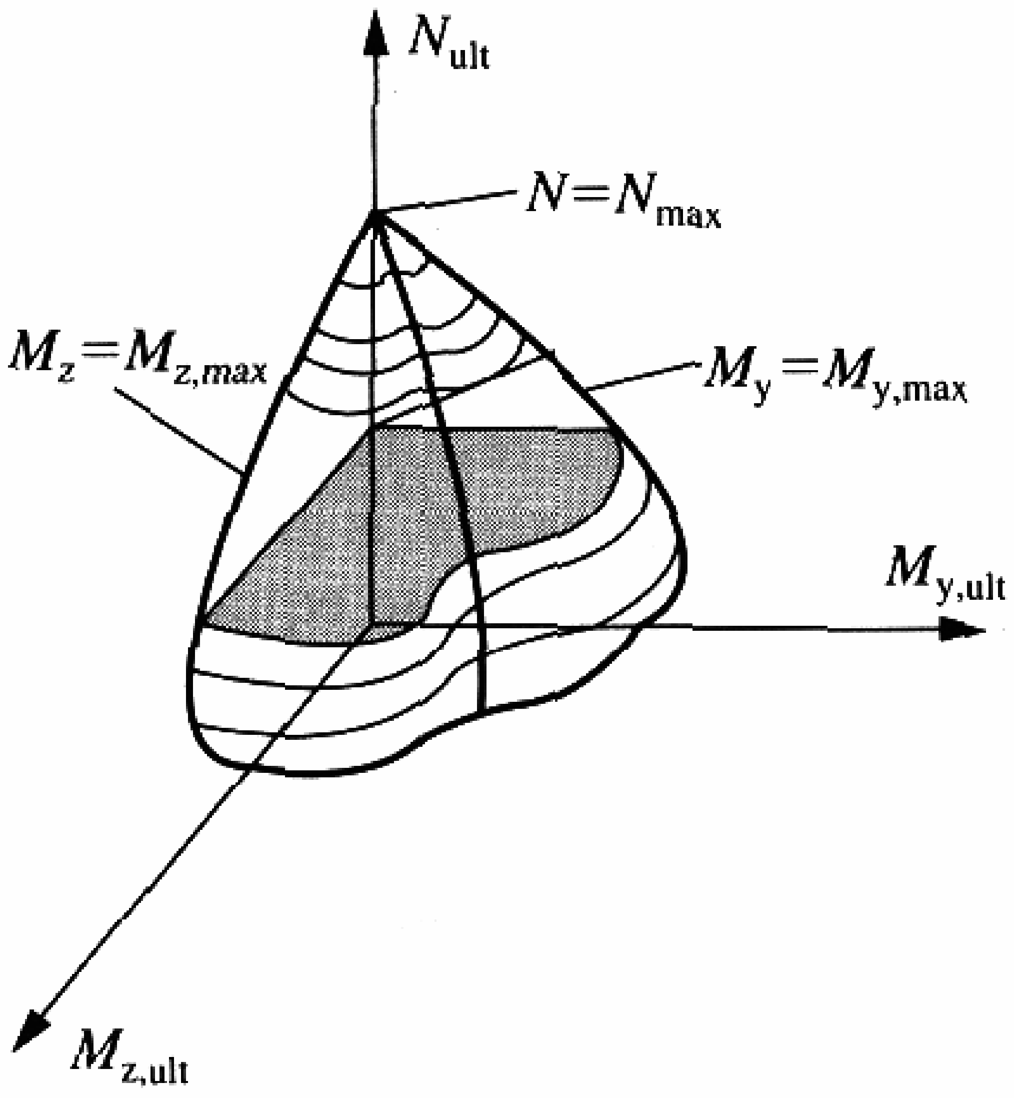
$$\theta = \tan^{-1}\left(\left|M_x\right| \big/ \left|M_y\right| \right)$$



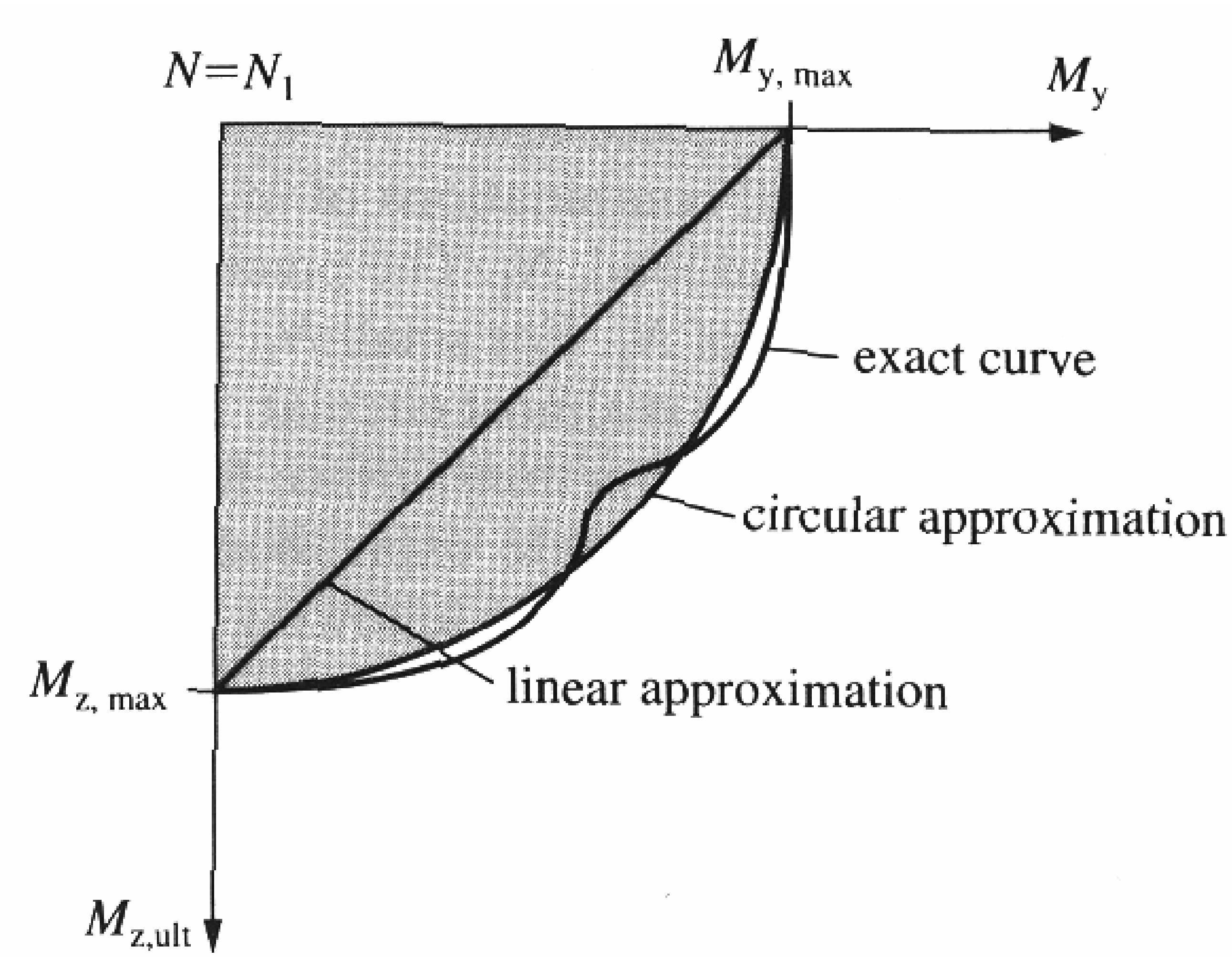

0.45f_{cu}
ε_{cu}
0.9x
x
Strain
Stress

4.3 Method 2 – 3-D Interaction Diagrams

It is possible to do an interaction diagram for M_x and M_y and N. Points inside the surface are safe while points outside are beyond the capacity of the column.



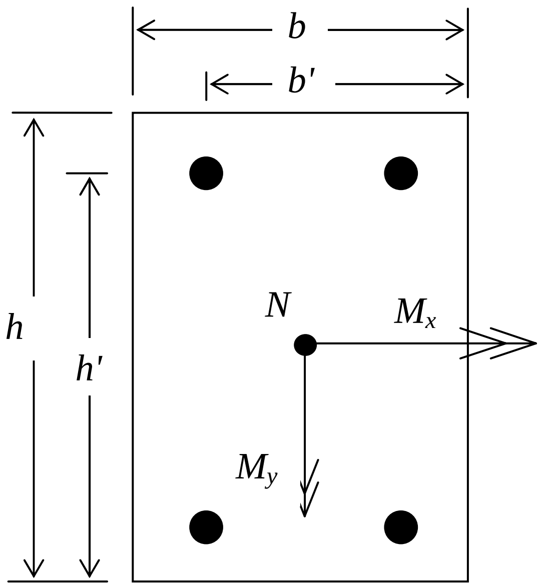
Note: The linear approximation is safe (in our notation: $M_x/M_{x\ max} + M_y/M_{y\ max} \leq 1$ while the circular approximation is not safe but is closer to reality $((M_x/M_{x\ max})^2 + (M_y/M_{y\ max})^2 \leq 1)$



4.4 Method 3 – Code Methods

Codes of practice provide some formulas for combining the effects of N, M_x and M_y.

For example, BS8110 specifies that you design for an increased moment about the dominant axis, given the following column:



- For $\frac{M_x}{h'} \geq \frac{M_y}{b'}$, M_x dominates. Therefore design for

$$M_x' = M_x + \beta \frac{h'}{b'} M_y$$

- For $\frac{M_x}{h'} < \frac{M_y}{b'}$, M_y dominates. Hence, design for

$$M_y' = M_y + \beta \frac{b'}{h'} M_x$$

In which, β is given by:

$\frac{N}{bhf_{cu}}$	0	0.1	0.2	0.3	0.4	0.5	≥0.6
β	1.00	0.88	0.77	0.65	0.53	0.42	0.30

Important Note:

This only applies to symmetrically reinforced rectangular sections.

4.5 Example

Problem

Design a 300 × 350 mm column for an axial load of 1200 kN and moments of 75 kNm about the *x*-axis and 80 kNm about the *y*-axis. The distance from the face to the centroid of the reinforcement may be taken as 60 mm.

Solution