The velocity vector of a fluid is given by
$\vec{V} = 2xy\vec{i} + 6y^2\vec{j} - 9t\vec{k}$ then
find
(i) convective accⁿ fluid particle at the Pt (1,2,3)
(ii) Temporal accⁿ of the particle at time t = 5 sec.

Solⁿ

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$= 2xy \times 2y + 6y^2 \times \frac{\partial (2xy)}{\partial y} - 9t \cancel{\frac{\partial (2xy)}{\partial z}}^{0}$$

$$= 4xy^2 + 12xy^2$$

$$= 16xy^2$$

$$= 16 \times 1 \times 2^2$$

$$= 64\ m/sec^2$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$= \cancel{2xy\frac{\partial (6y^2)}{\partial x}}^{0} + 6y^2 \times 12y + \cancel{9t\frac{\partial (6y^2)}{\partial z}}^{0}$$

$$= 72y^3$$

$$= 72 \times 2^3$$

$$= 576\ m/sec^2$$

(ii)

$$a_z = u\frac{\partial \vec{\omega}}{\partial x} + v\frac{\partial \vec{\omega}}{\partial y} + \omega\frac{\partial \vec{\omega}}{\partial z}$$

$$= 0 + 0 + 0$$

$$= 0$$

convective accl^n vector is given by

$a_x = 64\ m/s^2$ and $a_y = 576\ m/sec^2$

$a_z = 0$

$$\vec{a}_{con} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

$$= \sqrt{(64)^2 + (576)^2}$$

$$= 579.54\ m/sec^2$$

Direction of $\vec{a}$ convective

$$\tan\theta = \frac{a_y}{a_x} = \frac{576}{64}$$

$$\theta = \tan^{-1}\left(\frac{\cancel{576}^{9}}{\cancel{64}}\right)$$

$$= \tan^{-1}(9)$$

$$= 83.65^\circ$$

$$= 83^\circ + (.65 \times 60)'$$

$$= 83^\circ 39'$$

(ii) local accl^n

$$\vec{a_T} = \frac{\partial u}{\partial t}\vec{i} + \frac{\partial v}{\partial t}\vec{J} + \frac{\partial w}{\partial t}\vec{k}$$

$$= 0 + 0 + (-9)\vec{k}$$

$$\vec{a_T} = -9\vec{k}$$

$$|\vec{a_T}| = 9\ m/s^2$$

Total accl^n = $579.54 + 9$

$$= \boxed{588.54\ m/sec^2}$$

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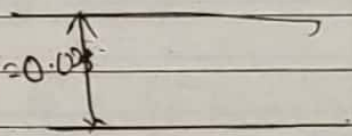
Q2. A plate 0.025 millimetre distance from a fixed plate moves at 60cm/s and a required a force of 2N/m² to maintain this speed. Determine the fluid viscosity b/w the plate.

Ans

Soln

Given, $\tau = 2 N/m^2$



$u = 60 cm/s$

$= 60 \times 10^{-2}$

$y = 0.025$

$= 0.025 \times 10^{-3} m/s$

$$\tau = \mu \frac{du}{dy}$$

$$2 = \mu \times \frac{du}{dy}$$

$$\mu = 2 \times \frac{1}{\frac{60 \times 10^{-2}}{0.025 \times 10^{-3}}}$$

$$= \frac{2 \times 10^{2} \times 0.025}{60 \times 10^{-3} \times 10^{-3}}$$

$$= \frac{5}{6} \times 10^{-4}$$

$$= 8.3 \times 10^{-5} Ns/m^2$$

classmate

Date

Page

Q2 A flow is described by the stream function $\psi = 2\sqrt{3}\,xy$ locate the point at which velocity vector has magnitude 4 units and makes an angle 150° with the x axis. also find velocity potential.

Solⁿ.

$$u = -\frac{\partial\psi}{\partial y}$$

$$u = -2\sqrt{3}x$$

$$v = \frac{\partial\psi}{\partial x}$$

$$= 2\sqrt{3}y$$

$$\vec{V} = u\vec{i} + v\vec{j}$$

$$= -2\sqrt{3}x\,\vec{i} + 2\sqrt{3}y\,\vec{j}$$

$$|\vec{V}| = \sqrt{(-2\sqrt{3}x)^2 + (2\sqrt{3}y)^2}$$

$$= \sqrt{12x^2 + 12y^2}$$

$$= \sqrt{12(x^2+y^2)}$$

$$= \sqrt{12}\cdot\sqrt{x^2+y^2}$$

$$= 2\sqrt{3}(\sqrt{x^2+y^2})$$

$$2\sqrt{3}(\sqrt{x^2+y^2}) = 4$$

$$\sqrt{3}(\sqrt{x^2+y^2}) = \frac{\cancel{4}\,2}{\cancel{2}}$$

squaring both sides

$$3(x^2+y^2) = 4$$

$$x^2+y^2 = 4/3 \quad \text{--- (1)}$$

$vdy + udx = 0$

$\theta = \tan^{-1}\left(\frac{v}{u}\right)$

$\tan\theta = \frac{v}{u}$

$\tan 150^\circ = \frac{\cancel{2\sqrt{3}}y}{-\cancel{2\sqrt{3}}x}$

$\frac{\cancel{+}1}{\sqrt{3}} = \cancel{+}y/x$

$x = \sqrt{3}y$ —— (1)

Put the value x in eqn (2),

$(\sqrt{3}y)^2 + y^2 = 4/3$

$3y^2 + y^2 = 4/3$

$4y^2 = \frac{4}{3}$

$y^2 = \frac{\cancel{4}\,1}{\cancel{12}\,3} = \frac{1}{3}$

$y = \frac{1}{\sqrt{3}}$

$x = \cancel{\sqrt{3}} \times \frac{1}{\cancel{\sqrt{3}}}$

$x = 1$

$(x, y) = \left(1, \frac{1}{\sqrt{3}}\right)$ Ans

$(x, y) = \left(1, \frac{1}{\sqrt{3}}\right)$

$\psi = -\frac{\partial\phi}{\partial x}$

2nd part,

$$u = -\frac{\partial\psi}{\partial y} = -\frac{\partial\phi}{\partial x}$$

$$\frac{\partial\psi}{\partial y} = \frac{\partial\phi}{\partial x}$$

$$\Rightarrow \frac{\partial}{\partial y}(2\sqrt{3}xy) = \frac{\partial\phi}{\partial x}$$

$$= 2\sqrt{3}x = \frac{\partial\phi}{\partial x}$$

$$\Rightarrow \frac{\partial\phi}{\partial x} = 2\sqrt{3}x$$

$$\partial\phi = 2\sqrt{3}x\,\partial x.$$

Integrating both sides, we have

$$\int\partial\phi = \int 2\sqrt{3}x\,\partial x$$

$$\therefore \phi = \cancel{2}\sqrt{3}\,\frac{x^2}{\cancel{2}} + c$$

here $c = f(y)$

$$\phi = \sqrt{3}x^2 + f(y) \quad \text{--- (1)}$$

$\psi = 2\sqrt{3}xy$

Again,

$$v = -\frac{\partial\phi}{\partial y} = \frac{\partial\psi}{\partial x}$$

$$\Rightarrow -\frac{\partial}{\partial y}\{\sqrt{3}x^2 + f(y)\} = 2\sqrt{3}y$$

$$= 0 - \frac{\partial}{\partial y}f(y) = 2\sqrt{3}y$$

$$\Rightarrow -f'(y) = 2\sqrt{3}y$$

$$f'(y) = -2\sqrt{3}y$$

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Date ______
Page ______

$f(y) = -\frac{\cancel{2}\sqrt{3}y^2}{\cancel{2}}$

$\Rightarrow f(y) = -\sqrt{3}y^2$

from eq^n (1),

$\phi = \sqrt{3}x^2 - \sqrt{3}y^2$

$\phi = \sqrt{3}\{x^2 - y^2\}$ Ans

Q4 ~~The~~ The stream function for a 2-D flow is given by $\psi = 2xy$. calculate the velocity and acc^n at the point P(2, 3). Also find the velocity potential function $\{\phi\}$.

Sol^n

$f(t, x, y)$

$\psi = 2xy$

$u = -\frac{\partial \psi}{\partial y} = -2x = -2 \times 2 = -4$

$v = \frac{\partial \psi}{\partial x} = 2y = 2 \times 3 = 6$

$V = \sqrt{(4)^2 + (6)^2}$

$= \sqrt{16 + 36}$

$= \sqrt{52}$ ~~m/s~~ unit

$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$

$= -2x\frac{\partial}{\partial x}(-2x) + 2y\frac{\partial}{\partial y}\{-2x\}$ $\cancel{0}$

classmate

Date

Page

$= -2x \times 2$

$= -4x$

$= -4 \times (2)$

$= -8$ unit

$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}$

$= -2x \cancel{\frac{\partial}{\partial x} 2y}^{0} + 2y \frac{\partial 2y}{\partial y}$

$= 2y \times 2$

$= 2 \times 3 \times 2$

$= 12$ unit

$a_t = \sqrt{(12)^2 + (-8)^2}$

$= \sqrt{144 + 64}$

$= \sqrt{208}$

$= 4\sqrt{13}$ unit

$u = \frac{-\partial \psi}{\partial y} = \frac{-\partial \phi}{\partial x}$

$\frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x}$

$= \frac{\partial}{\partial y}\{2xy\} = \frac{\partial \phi}{\partial x}$

$\Rightarrow 2x = \frac{\partial \phi}{\partial x}$

$\partial \phi = 2x \, \partial x.$

Integrating both sides,

$$\int \partial\phi = \int 2x\,\partial x$$

$$\Rightarrow \phi = \cancel{2}\cdot\frac{x^2}{\cancel{2}} + c$$

$$\phi = x^2 + f(y)$$

$$v = \frac{\partial\psi}{\partial x} = -\frac{\partial\phi}{\partial y}$$

$$\frac{\partial\psi}{\partial x} = -\frac{\partial\phi}{\partial y}$$

$$\Rightarrow \frac{\partial\{2xy\}}{\partial x} = -\frac{\partial\phi}{\partial y}$$

$$-\frac{\partial}{\partial y}\{x^2 + f(y)\} = 2y$$

$$\Rightarrow -f'(y) = 2y$$

integrating.

$$\Rightarrow f(y) = -\cancel{2}\cdot\frac{y^2}{\cancel{2}}$$

$$f(y) = -y^2$$

$$\boxed{\phi = x^2 - y^2}$$