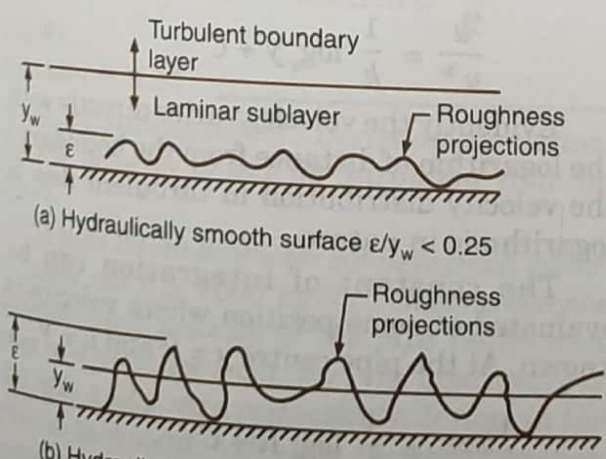
9.16. HYDRAULICALLY SMOOTH AND ROUGH PIPES

Any surface over which a fluid flows is never perfectly smooth ; the surface has innumerable irregularities which vary in shape, size and spacing.

Absolute roughness of a pipe surface refers to the average height of the bumps/

irregularities (ε) on the pipe surface. When all the projections are of the same size and shape, the roughness is known as uniform granular wall roughness. Physically the roughness of a pipe-surface increases with growth in absolute roughness. Frequently, the surface condition is quantitatively specified in terms of *relative roughness* which is defined as the ratio of the absolute roughness ε and the pipe radius R. Two pipes are said to have the same hydraulic roughness when they have equal value of friction coefficient f for flows at equal Reynolds numbers. Pipe A with small relative roughness is considered to be hydraulically smooth in comparison to pipe B with a higher value of relative roughness ; it is irrespective of the fact that pipe A may have a greater value of absolute roughness.

Hydraulic behaviour of a pipe surface is characterised by the absolute roughness and the thickness y_w of the laminar sub-layer. Experiments indicates that even for the most highly turbulent flows, there does exit a region of small thickness immediately adjacent to the pipe surface in which the fluid flows much slower and without mixing. The fluid is of laminar character and the thin layer is called the *laminar sub-layer*.



(a) Hydraulically smooth surface $\varepsilon/y_w < 0.25$

(b) Hydraulically rough surface $\varepsilon/y_w > 6.0$

Fig. 9.37. Hydraulically smooth and rough surfaces

- When $\dfrac{\varepsilon}{y_w} < 0.25$ the surface is called *hydraulically smooth*. The surface irregularities are well immersed in the laminar sublayer ; they do not shed eddies and therefore, can have no effect on the main turbulent flow. The resistance to flow depends only on the flow Reynolds number.

- When $\dfrac{\varepsilon}{y_w} > 6.0$ the pipe surface is classified *hydraulically rough*. The surface irregularities protrude through the sublayer and generate additional turbulence in the main flow. The resistance to flow is governed essentially by the relative roughness.

- When $0.25 < \dfrac{\varepsilon}{y_w} < 6.0$ the pipe surface is called *boundary in transition*. The friction factor is a function of both the Reynolds number and the relative roughness.

Laminar sublayer diminishes in thickness with growth in Reynolds number. Accordingly a pipe may be hydraulically smooth at low Reynolds number but rough at high Reynolds number.

Two other well-known criteria for prescribing the smoothness and roughness of a pipe surface are :

- For hydraulically smooth surface,

$$\frac{u^* \varepsilon}{\nu} < 3 \; ; \quad \frac{Re\sqrt{4f}}{(R/\varepsilon)} < 17$$

- For hydraulically rough surfaces

$$\frac{u^* \varepsilon}{\nu} > 70 \; ; \quad \frac{Re\sqrt{4f}}{(R/\varepsilon)} < 400$$

- For surface behaving as in transition,

$$3 < \frac{u^* \varepsilon}{\nu} < 70 \; ; \quad 17 < \frac{Re\sqrt{4f}}{(R/\varepsilon)} < 400$$

EXAMPLE 9.52

How would you distinguish between hydraulically smooth and rough boundaries?

A pipeline carrying water has average height of irregularities projecting from the surface of the boundary of the pipe as 0.155 mm. If the shear stress developed is 4.9 N/m², indicate the type of the boundary ;

smooth or rough. Take kinematic viscosity of water as 0.01 stokes.

Solution : Kinematic viscosity, ν

$$= 0.01 \text{ stokes}$$
$$= 0.01 \text{ cm}^2/\text{s} = 0.01 \times 10^{-4} \text{ m}^2/\text{s}$$

By definition shear velocity u^*

$$= \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{4.9}{1000}} = 0.07 \text{ m/s}$$

Roughness Reynolds number

$$= \frac{u^* \varepsilon}{\nu}$$
$$= \frac{0.07 \times (0.155 \times 10^{-3})}{0.01 \times 10^{-4}} = 10.85$$

Since $\frac{u^* \varepsilon}{\nu}$ lies between 3 to 70, the boundary of the pipe surface is in transition.

EXAMPLE 9.53

Water flows through a 30 cm diameter pipe with characteristic roughness ε = 0.02 mm. There occurs a pressure drop of 4.0 kN/m² over a length of 50 m. State whether the pipe will act as hydrodynamically smooth, in transition or rough. Assume suitable value for the mass density and kinematic viscosity of water.

Solution : An equilibrium between the accelerating force due to pressure difference and the retarding force due to shear stress on pipe wall gives

$$(p_1 - p_2)\, \pi R^2 = \tau_0 \times 2\pi Rl$$

or

$$\tau_0 = (p_1 - p_2)\frac{R}{2l}$$
$$= \frac{4 \times 10^3 \times 0.15}{2 \times 50} = 6 \text{ N/m}^2$$

$$u^* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{6}{1000}} = 0.0775 \text{ m/s}$$

$$y_w = \frac{11.6\,\nu}{u^*} = \frac{11.6 \times 10^{-6}}{0.0775}$$
$$= 149.68 \times 10^{-6} \text{ m} \approx 0.1497 \text{ mm}$$

$$\frac{\varepsilon}{y_w} = \frac{0.02}{0.1497} = 0.1336$$

Since $\frac{\varepsilon}{y_w} < 0.25$, the pipe surface will behave as hydraulically smooth.

9.17. PRANDTL UNIVERSAL VELOCITY DISTRIBUTION

From Prandtl mixing length theory, turbulent shear stress is prescribed by the relation,

$$\tau = \rho\, l_m^2 \left(\frac{du}{dy}\right)^2$$

Prandtl assumed that near a pipe wall, shear stress is constant and equal to the wall shear stress τ_0 ; and that mixing length is given by $l_m = ky$. With these assumptions, the above relation can be written:

$$\tau_0 = \rho\, k^2 y^2 \left(\frac{du}{dy}\right)^2$$

$$du = \sqrt{\frac{\tau_0}{\rho}} \times \frac{1}{k} \times \frac{dy}{y} = \frac{u^*}{k}\frac{dy}{y}$$

This expression can be integrated to give

$$\frac{u}{u^*} = \frac{1}{k} \log_e y + C \qquad ...(9.40)$$

Evidently the velocity varies directly with the logarithm of distance from the boundary; the velocity distribution in turbulent flow is logarithmic in nature.

The constant of integration can be evaluated at some position where velocity is known. At the pipe centre $y = R$ and $u = V_{max}$

$$\frac{V_{max}}{u^*} = \frac{1}{k} \log_e R + C$$

or

$$C = \frac{V_{max}}{u^*} - \frac{1}{k} \log_e R$$

Equation 9.40 then takes the form :

$$\frac{u}{u^*} = \frac{1}{k} \log_e y + \frac{V_{max}}{u^*} - \frac{1}{k} \log_e R$$

$$= \frac{V_{max}}{u^*} + \frac{1}{k} \log_e \frac{y}{R}$$

$$\therefore \quad u = V_{max} + \frac{u^*}{k} \log_e \frac{y}{R}$$

Changing from natural to common logarithms and taking $k = 0.4$

$$u = V_{max} + 5.75\, u^* \log_{10} \frac{y}{R} \quad ...(9.41)$$

Equation 9.41 is known as *Prandtl universal velocity distribution* for flow in smooth and rough pipes. The equation is, however, valid only over the turbulent core and not close to the pipe wall where the velocity is very small, *i.e.*, where the flow is viscous and no longer turbulent.

Equation 9.41 may be recast in the non-dimensional form :

$$\frac{V_{max} - u}{u^*} = 5.75 \log_{10} \frac{R}{y} \quad ...(9.42)$$

The term $(V_{max} - u)$ is called the **velocity defect** and the equation 9.42 is referred to as the **velocity deficiency equation**. Evidently, except in a very small region near the wall, $\frac{V_{max} - u}{u^*}$ is a function of $\frac{y}{R}$.

EXAMPLE 9.54

What is a velocity defect ? Derive an expression for the velocity defect for turbulent flow in pipes.

A pipe 20 cm diameter conveys fresh water of mass density 1000 kg/m³. Measurements indicate that velocity at the pipe centre and that at a distance 5 cm from the pipe centre is 2.5 m/s and 2 m/s respectively. If flow in the pipe is turbulent, calculate the shear friction velocity and the wall shear stress.

Solution : Maximum velocity occurs at the pipe centre.

$$V_{max} = 2.5 \text{ m/s}, \; R = 10 \text{ cm} ;$$
$$u = 2 \text{ m/s}, y = 10 - 5 = 5 \text{ cm}$$

Substitute the given data in the velocity-deficiency equation,

$$\frac{V_{max} - u}{u^*} = 5.75 \log_{10} \left(\frac{R}{y}\right)$$

$$\frac{2.5 - 2}{u^*} = 5.75 \log_{10} \left(\frac{10}{5}\right)$$

Solution gives : $u^* =$ **0.289 m/s**

By definition, $u^* = \sqrt{\frac{\tau_0}{\rho}}$

$\therefore$ Wall shear stress, τ_0

$$= u^{*2} \rho$$
$$= (0.289)^2 \times 1000 = \mathbf{83.5 \; N/m^2}$$

9.18. VELOCITY DISTRIBUTION IN SMOOTH PIPES

(*i*) **Laminar sublayer :** Even with highly turbulent flows, there does exist a peripheral annulus of fluid adjacent to the wall which is in laminar motion. In this viscous sublayer velocity rises from zero at the wall to a finite value over a minute distance y_w equivalent to the thickness of sublayer ; here the velocity gradient is rather sharp. Assuming the velocity distribution to be essentially linear, the shear stress at the pipe wall may be computed as :

$$\tau_0 = \mu \frac{du}{dy} = \mu \frac{u}{y}$$

Dividing both sides by ρ :

$$\frac{\tau_0}{\rho} = \frac{\mu}{\rho} \frac{u}{y}$$

Making the substitution $\sqrt{\frac{\tau_0}{\rho}} = u^*$ (the friction velocity) and $\frac{\mu}{\rho} = \nu$ (the kinematic viscosity)

$$u^{*2} = \nu \frac{u}{y}$$

$$\text{or} \quad \frac{u^* y}{\nu} = \frac{u}{u^*} \quad ...(9.43)$$

The $\frac{u}{u^*}$ and $\frac{u^* y}{\nu}$ are dimensionless terms : $\frac{u}{u^*}$ is a ratio of the point velocity, and $\frac{u^* y}{\nu}$ is modified Reynolds number involving the friction velocity, the distance from the wall and the kinematic viscosity of fluid. At the outer edge of the laminar sublayer ;

$$y = y_w \quad \text{and} \quad u = u_w$$

$$\therefore \quad \frac{u_w}{u^*} = \frac{u^* y_w}{\nu}$$

For turbulent flow in smooth pipes,

$$\frac{u^* y_w}{\nu} = 11.6,$$

and, therefore, for the laminar sublayer:

$$\frac{u_w}{u^*} = \frac{u^* y_w}{\nu} = 11.6 = \text{a constant } C \qquad ...(9.44)$$

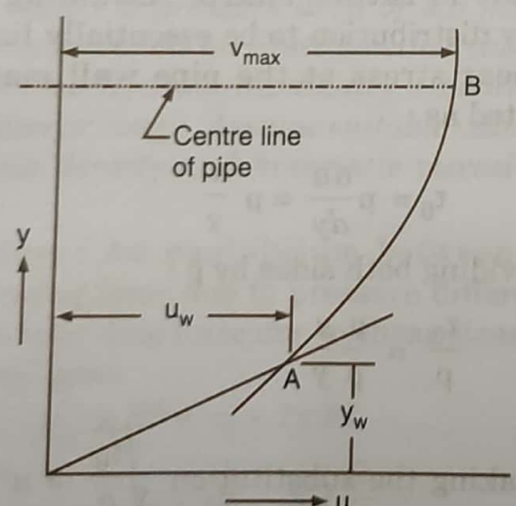


Fig. 9.38. Viscous sublayer

(*ii*) **Buffer zone** : The fully laminar flow at the viscous sublayer, and the fully turbulent core of flow do not merge suddenly ; the change is gradual through a transition zone called the buffer zone. Experiments indicate that in the range $5 < \frac{u^* y}{\nu} < 0$ both the molecular and turbulent shear (momentum change) are significant. Theoretical analysis of the velocity profile is, however, made by neglecting the buffer zone, and presuming a value of $\frac{u^* y_w}{\nu} = 11.6$ for the laminar thickness. From Prandtl universal velocity distribution, we have :

$$u = V_{max} + 5.75\, u^* \log_{10} \frac{y}{R}$$

$$\text{or} \quad \frac{u}{u^*} = \frac{V_{max}}{u^*} + 5.75 \log_{10} \frac{y}{R} \qquad ...(9.45)$$

At the commencement of turbulent core

$$y = y_w \quad \text{and} \quad u = u_w$$

$$\therefore \quad \frac{u_w}{u^*} = \frac{V_{max}}{u^*} + 5.75 \log_{10} \frac{y_w}{R}$$

$$\text{But} \quad \frac{u_w}{u^*} = \frac{u^* y_w}{\nu} = 11.6 = \text{a constant } C$$

$$\therefore \quad C = \frac{V_{max}}{u^*} + 5.75 \log_{10} \frac{C\nu / u^*}{R}$$

$$= \frac{V_{max}}{u^*} + 5.75 \log_{10} C + 5.75 \log_{10} \frac{\nu}{u^* R}$$

$$\text{or} \quad \frac{V_{max}}{u^*} = C - 5.75 \log_{10} C - 5.75 \log_{10} \frac{\nu}{u^* R}$$

$$= (C - 5.75 \log_{10} C) + 5.75 \log_{10} \frac{R u^*}{\nu}$$

Collecting all the constant terms in constant A,

$$\frac{V_{max}}{u^*} = A + 5.75 \log_{10} \frac{R u^*}{\nu}$$

Substituting this value of $\frac{V_{max}}{u^*}$ in equation 9.45, we get

$$\frac{u}{u^*} = A + 5.75 \log_{10} \frac{R u^*}{\nu} + 5.75 \log_{10} \frac{y}{R}$$

$$= A + 5.75 \log_{10} \frac{y u^*}{\nu}$$

which is a logarithmic equation for the turbulent core outside the laminar sublayer. A best fit with experimental data is obtained when $A = 5.50$. Therefore :

$$\frac{u}{u^*} = 5.5 + 5.75 \log_{10} \frac{y u^*}{\nu} \quad ...(9.46)$$

Equation 9.46 is known as karman–Prandtl equation for the velocity distribution in the fully developed turbulent core of flow in hydraulically smooth pipes.

Figure 9.39 gives a plot of $\frac{u}{u^*} = \frac{u^* y}{\nu}$ for the laminar sublayer, and

$\frac{u}{u^*} = 5.5 + 5.75 \log_{10} \frac{u^* y}{\nu}$ for fully developed turbulent core.

There is a close agreement between the experimental and theoretical velocity distributions, except in the transitional region $8 < \frac{u^* y}{\nu} < 30$. The actual conditions are represented by the dotted line.

In the turbulent region prescribed by $70 < \frac{yu^*}{\nu} < 700$, equation 9.46 is generally approximated by :

$$\frac{u}{u^*} = 8.74 \left(\frac{yu^*}{\nu}\right)^{1/7} \quad ...(9.47)$$

which is called the **one-seventh power law.** At the centre line of the pipe $y = R$ and $u = V_{max}$ and, therefore,

$$\frac{V_{max}}{u^*} = 8.74 \left(\frac{u^* R}{\nu}\right)^{1/7} \quad ...(9.48)$$

Dividing equation 9.47 by equation 9.48, one obtains the so called Blasius one-seventh power velocity distribution equation,

$$\frac{u}{V_{max}} = \left(\frac{y}{R}\right)^{1/7} = \left(1 - \frac{r}{R}\right)^{1/7} \quad ...(9.49)$$

On the basis of a series of experiments conducted by Nikurdase for determining laws for velocity distribution for turbulent flow in smooth pipes, equation 9.49 can be rewritten by replacing the index 1/7 by $1/n$.

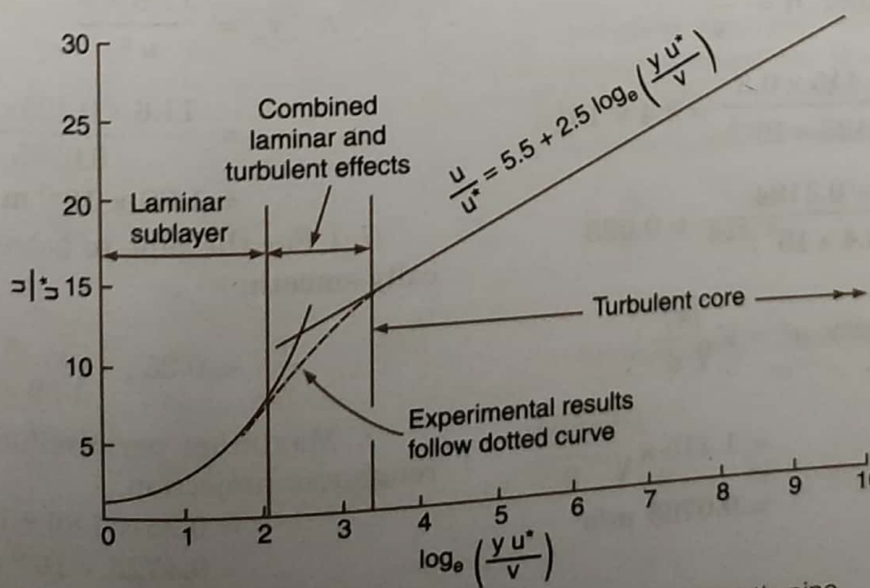


Fig. 9.39. Velocity distribution in a hydraulically smooth pipe

$$\frac{u}{V_{max}} = \left(\frac{y}{R}\right)^{1/7} = \left(1-\frac{r}{R}\right)^{1/n} \quad ...(9.50)$$

Variation of n with Reynolds number is given in Table 9.4.

Table 9.4. Variation of index n with Reynolds number

Reynolds number	4×10^3	3.3×10^4	1.1×10^5	1.1×10^6	2×10^6	3.2×10^6
n	6.0	6.6	7.0	8.8	10.0	10.8

EXAMPLE 9.55

An oil of specific gravity 0.85 and kinematic viscosity 0.125 cm²/s flows in a 30 cm diameter smooth pipe at the rate of 0.1 m³/s. Calculate (i) flow velocity at a point 10 cm from the pipe centre, (ii) shear friction velocity, (iii) thickness of laminar sublayer, and (iv) maximum permissible height of the roughness projection for the pipe to act as smooth.

Use Blasius equation $4f = \frac{0.3164}{(Re)^{0.25}}$ *to compute the friction coefficient.*

Solution : Mean velocity, V

$$= \frac{Q}{A} = \frac{0.1}{\frac{\pi}{4}(0.3)^2} = 1.415 \text{ m/s}$$

Reynolds number, $R = \frac{Vd}{\nu}$

$$= \frac{1.415 \times 0.3}{0.125 \times 10^{-4}} = 3.4 \times 10^4$$

$$\therefore \quad 4f = \frac{0.3164}{(3.4 \times 10^4)^{0.25}} = 0.023$$

(*i*) Shear velocity, $u^* = V\sqrt{\frac{4f}{8}}$

$$= 1.415 \times \sqrt{\frac{0.023}{8}}$$

$$= \mathbf{0.0765 \text{ m/s}}$$

(*ii*) For turbulent flow in smooth pipes,

$$\frac{u}{u^*} = 5.5 + 5.75 \log_{10} \frac{u^* y}{\nu}$$

At $y = (R - r) = (15 - 10) = 5$ cm $= 0.05$ cm

$$\therefore \quad \frac{u}{0.0765} = 5.5 + 5.75 \log_{10}\left(\frac{0.0765 \times 0.05}{0.125 \times 10^{-4}}\right)$$

$$= 19.79$$

and $u = 0.0765 \times 19.79 = 1.51$ m/s

Thus the velocity at a point, 10 cm from the pipe centre equals 1.5 m/s

(*iii*) Thickness of laminar sublayer is prescribed by :

$$\frac{u_w}{u^*} = \frac{y_w u^*}{\nu} = 11.6$$

$$\therefore \quad y_w = \frac{11.6 \times \nu}{u^*}$$

$$= \frac{11.6 \times 0.125 \times 10^{-4}}{0.0765}$$

$$= \mathbf{1.89 \times 10^{-3} \text{ m}}$$

(*iv*) For the pipe to behave as hydraulically smooth,

$$\frac{\varepsilon}{y_w} < 0.25 ; \quad \frac{\varepsilon}{1.89 \times 10^{-3}} < 0.25$$

$\therefore$ Maximum permissible height of the roughness projection,

$$\varepsilon = 0.25 \times 1.89 \times 10^{-3}$$

$$= \mathbf{0.4725 \times 10^{-3} \text{ m}}$$

EXAMPLE 9.56

What do you understand by smooth pipe flow? Distinguish it from rough pipe flow.

The friction coefficient for turbulent flow through rough pipes can be determined by Karman-Prandtl equation :

$$\frac{1}{\sqrt{4f}} = 2\log_{10}\frac{R_0}{k} + 1.74$$

where f is friction coefficient, R_0 is pipe radius and k is average roughness of pipe.

The two reservoirs with surface level difference of 20 m are to be connected by 1 m diameter pipe 6 km long. What will be the discharge when a cast iron pipe of roughness k = 0.3 mm is used ? What will be the percentage increase in discharge if the cast iron pipe were to be replaced by a steel pipe of roughness k = 0.1 mm ? Neglect all local losses.

Solution : Case (*i*) *When the pipe is made of cast iron :*

$$\frac{1}{\sqrt{4f}} = 2\log_{10}\frac{0.5}{0.0003} + 1.74$$

$$= 2 \times 3.2218 + 1.74 = 8.1837$$

∴ Friction coefficient

$$f = \frac{1}{4}\left(\frac{1}{8.1837}\right)^2 = 0.00373$$

The difference in reservoir levels represents the total head loss h_f. Invoking the relation,

$$h_f = \frac{f\,l\,Q^2}{3d^5}$$

$$20 = \frac{0.00373 \times 6000 \times Q^2}{3 \times (1)^5}$$

Solution gives : Q = **1.637 m³/s**

Case (*ii*) *When the pipe is made of steel :*

$$\frac{1}{\sqrt{4f}} = 2\log_{10}\frac{0.5}{0.0001} + 1.74$$

$$= 2 \times 3.699 + 1.74 = 9.138$$

∴ Friction coefficient,

$$f = \frac{1}{4}\left(\frac{1}{9.138}\right)^2 = 0.003$$

Substituting the values in the relation,

$$h_f = \frac{f\,l\,Q^2}{3d^5}$$

$$20 = \frac{0.003 \times 6000 \times Q^2}{3 \times (1)^5}$$

Solution gives : Q = **1.826 m³/s**

∴ Percentage increase in discharge

$$= \frac{1.826 - 1.637}{1.637} \times 100$$

$$= \mathbf{11.54\%}$$

9.19. VELOCITY DISTRIBUTION FOR ROUGH PIPES

The Prandtl velocity distribution is universal, *i.e.*, it applies equally to smooth and rough surfaces.

$$u = V_{max} + 5.75\,u^* \log_{10}\frac{y}{R};$$

$$\frac{u}{u^*} = \frac{V_{max}}{u^*} + 5.75\log_{10}\frac{y}{R} \quad ...(9.51)$$

At the outer edge of the viscous sublayer, *i.e.*, at the commencement of turbulent core $y = y_w$ and $\mu = \mu_w$. Therefore,

$$\frac{u_w}{u^*} = \frac{V_{max}}{u^*} + 5.75\log_{10}\frac{y_w}{R}$$

Prandtl assumed that for rough surface, y_w is dependent on the wall roughness ; $y_w = \alpha\,\varepsilon$ where ε is the mean height of the wall bumps/protrusions and α is constant

$$\frac{u_w}{u^*} = \frac{V_{max}}{u^*} + 5.75\log_{10}\frac{\alpha\varepsilon}{R} \quad ...(9.52)$$

Eliminating $\frac{V_{max}}{u^*}$ from equations 9.51 and 9.52.

$$\frac{u}{u^*} = \frac{u_w}{u^*} - 5.75\log_{10}\frac{\alpha\varepsilon}{R} + 5.75\log_{10}\frac{y}{R}$$

$$= \left(\frac{u_w}{u^*} - 5.75 \log_{10} \alpha\right)$$

$$+ 5.75 \log_{10} \frac{R}{\varepsilon} + 5.75 \log_{10} \frac{y}{R}$$

$$= A + 5.75 \log_{10} \frac{y}{\varepsilon}$$

Evidently the velocity distribution in a rough pipe is not influenced by the Reynolds number.

A best fit with experimental data is obtained with constant $A = 8.5$.

$$\therefore \quad \frac{u}{u^*} = 8.5 + 5.75 \log_{16} \frac{y}{\varepsilon} \quad ...(9.53)$$

EXAMPLE 9.57

What do you understand by the hydraulically smooth and rough pipes ?

A pipeline 12 cm in diameter and 100 m long conveys water at the rate of 0.075 m^3/s. The average height of the surface protrusions is 0.012 cm and the coefficient of friction is 0.005. Calculate the loss of head, wall shearing stress, centre line velocity and nominal thickness of laminar sublayer.

For water $\rho = 1000\ kg/m^3$, $\nu = 0.01$ stokes

Solution : Mean velocity, V

$$= \frac{Q}{A} = \frac{0.075}{\frac{\pi}{4}(0.12)^2} = 6.64 \text{ m/s}$$

(*i*) From Darcy equation, the head loss is given as :

$$h_f = \frac{4 f l V^2}{2gd}$$

$$= \frac{4 \times 0.005 \times 1000 \times 6.64^2}{2 \times 9.81 \times 0.12}$$

$$= 373.4 \text{ m}$$

(*ii*) Shearing velocity, $u^* = V\sqrt{\frac{4f}{8}}$

$$= 6.64 \times \sqrt{\frac{4 \times 0.005}{8}}$$

$$= 0.332 \text{ m/s}$$

Since $u^* = \sqrt{\frac{\tau_0}{\rho}}$, the wall shearing stress is τ_0 is given by,

$$\tau_0 = u^{*2} \times \rho$$

$$= 0.332^2 \times 1000 = 110.22 \text{ N/m}^2$$

(*iii*) Assuming the flow to be rough turbulent,

$$\frac{u}{u^*} = 8.50 + 5.75 \log_{10} \frac{y}{\varepsilon}$$

As the pipe centre :

$$y = R = 6 \text{ cm and } u = V_{max}$$

$$\frac{V_{max}}{u^*} = 8.50 + 5.75 \log_{10} \frac{6}{0.012} = 24$$

$$\therefore \quad V_{max} = 0.332 \times 24 = 7.968 \text{ m/s}$$

(*iv*) Thickness of laminar sublayer is prescribed by :

$$\frac{u_w}{u^*} = \frac{y_w u^*}{\nu} = 11.6$$

$$\therefore \quad y_w = 11.6 \times \frac{\nu}{u^*} = 11.6 \times \frac{0.01 \times 10^{-4}}{0.332}$$

$$= 34.9 \times 10^{-6} \text{ m}$$

$$\text{Parameter } \frac{\varepsilon}{y_w} = \frac{0.012}{34.9 \times 10^{-6}} = 343$$

which is greater than 60. Hence an assumption of rough turbulent flow is valid.

9.20. AVERAGE VELOCITY DISTRIBUTION FOR SMOOTH AND ROUGH PIPES

The average velocity V_{av} through a pipe can be obtained by first finding the total discharge Q and then dividing the total discharge by the cross-sectional area of the pipe.

Consider an annular elementary ring of radius r and thickness dr at a distance y from the pipe wall, and let the point velocity in this elemental area be u.

Discharge through this ring is,

$$dQ = \text{area of the ring} \times \text{flow velocity}$$
$$= 2\pi r\, dr \times u$$

Total discharge, Q

$$= \int dQ = \int_{\delta}^{R} 2\pi u r\, dr$$

Average flow velocity V_{av}

$$= \frac{\text{total discharge through the pipe}}{\text{cross-sectional area of the pipe}}$$

$$= \frac{1}{\pi R^2} \int_{\delta=0}^{R} 2\pi u r\, dr$$

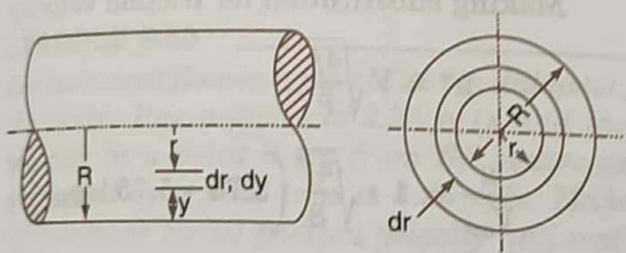


Fig. 9.40. Average velocity for turbulent flow

The distance of the ring from pipe wall is,

$$y = R - r; \quad dy = -dr$$

$$\therefore V_{av} = \frac{1}{\pi R^2} \int_{\delta}^{R} -2\pi u (R - y)\, dy$$

$$= \frac{1}{\pi R^2} \int_{R}^{\delta} 2\pi u (R - y)\, dy \qquad ...(9.54)$$

(*a*) **For smooth pipes :** The point velocity u for turbulent flow in a smooth pipe is given by :

$$\frac{u}{u^*} = 5.5 + 5.75 \log_{10} \frac{yu^*}{\nu}$$

Making this substitution in equation 9.54, we get :

$$V_{av} = \frac{1}{\pi R^2} \int_{R}^{\delta=0} 2\pi u^* \left(5.5 + 5.75 \log_{10} \frac{yu^*}{\nu}\right) \times (R - y)\, dy$$

The above expression is valid only for the turbulent zone and as such integration cannot be carried out to $y = 0$. However, that hardly matters as the flow in the laminar sublayer is insignificant and can be neglected without introducing any appreciable error. The result of integration is :

$$V_{av} = u^* \left(1.75 + 5.75 \log_{10} \frac{Ru^*}{\nu}\right)$$

$$\text{or} \quad \frac{V_{av}}{u^*} = 1.75 + 5.75 \log_{10} \frac{Ru^*}{\nu} \qquad ...(9.55)$$

(*b*) **For rough pipes :** The point velocity u for turbulent flow in a rough pipe is given by :

$$\frac{u}{u^*} = \left(8.50 + 5.75 \log_{10} \frac{y}{\varepsilon}\right)$$

Making this substitution in equation 9.54, we get

$$V_{av} = \frac{1}{\pi R^2} \int_{R}^{0} 2\pi u^* \left(8.50 + 5.75 \log_{10} \frac{y}{\varepsilon}\right) \times (R - y)\, dy$$

The result of integration is

$$V_{av} = u^* \left(4.75 + 5.75 \log_{10} \frac{R}{\varepsilon}\right)$$

$$\text{or} \quad \frac{V_{av}}{u^*} = 4.75 + 5.75 \log_{10} \frac{R}{\varepsilon} \qquad ...(9.56)$$

(*c*) **Difference of point velocity and average velocity for smooth and rough pipes :**

(*i*) For turbulent flow through smooth pipes, the velocity at any point is

$$\frac{u}{u^*} = 5.5 + 5.75 \log_{10} \frac{yu^*}{\nu}$$

and the average velocity is

$$\frac{V_{av}}{u^*} = 1.75 + 5.75 \log_{10} \frac{Ru^*}{\nu}$$

$\therefore$ Difference between u and V_{av} for a smooth pipe is,

$$\frac{u}{u^*} - \frac{V_{av}}{u^*} = \left[5.5 + 5.75 \log_{10} \frac{yu^*}{\nu}\right] - \left[1.75 + 5.75 \log_{10} \frac{Ru^*}{\nu}\right]$$

$$= (5.5 - 1.75) + 5.75\left(\log_{10} \frac{yu^*}{\nu} - \log_{10} \frac{Ru^*}{\nu}\right)$$

$$= 3.75 + 5.75 \log_{10} \times \left(\frac{yu^*}{\nu} \div \frac{Ru^*}{\nu}\right)$$

or
$$\frac{u - V_{av}}{u^*} = 3.75 + 5.75 \log_{10} \frac{y}{R} \qquad ...(9.57\ a)$$

(*ii*) For turbulent flow through rough pipes, the velocity at any point is

$$\frac{u}{u^*} = 8.5 + 5.75 \log_{10} \frac{y}{\varepsilon}$$

and the average velocity is

$$\frac{V_{av}}{u^*} = 4.75 + 5.75 \log_{10} \frac{R}{\varepsilon}$$

∴ Difference between u and V_{av} for a rough pipe is

$$\frac{u}{u^*} - \frac{V_{av}}{u^*} = \left[8.5 + 5.75 + \log_{10} \frac{y}{\varepsilon}\right] - \left[4.75 + 5.75 + \log_{10} \frac{R}{\varepsilon}\right]$$

$$= (8.5 - 4.75) + 5.75\left(\log_{10} \frac{y}{\varepsilon} - \log_{10} \frac{R}{\varepsilon}\right)$$

$$= 3.75 + 5.75 \log_{10} \left(\frac{y}{\varepsilon} \div \frac{R}{\varepsilon}\right)$$

or
$$\frac{u - V_{av}}{u^*} = 3.75 + 5.75 \log_{10} \frac{y}{R} \qquad ...(9.57\ b)$$

A look at equations 9.57(*a*) and 9.57(*b*) shows that the average velocity distribution is same both for the smooth and rough pips. This equality stems from the fact that the mechanism of turbulent flow beyond the laminar sublayer is independent of the surface conditions and that is precisely the reason that surface roughness does not appear in the expression for average velocity distribution.

The average velocity distribution law as prescribed by equation 9.57 may be recast as

$$\frac{u}{u^*} - \frac{V_{av}}{u^*} = 3.75 + 5.75 \log_{10} \frac{y}{R}$$

Multiplying both sides by $\frac{u^*}{V_{av}}$, we get

$$\frac{u}{V_{av}} = 1 + \frac{u^*}{V_{av}}\left(3.75 + 5.75 \log_{10} \frac{y}{R}\right)$$

Making substitution for friction velocity

$$u^* = V\sqrt{\frac{4f}{8}},$$

$$\frac{u}{V_{av}} = 1 + \sqrt{\frac{4f}{8}}\left(3.75 + 5.75 \log_{10} \frac{y}{R}\right)$$

$$= 1 + \sqrt{4f}\left(1.32 + 2.0 \log_{10} \frac{y}{R}\right)$$

The numerical constants appearing in the above expression require slight modification in order to conform with experiments of Nikuradse,

$$\frac{u}{V_{av}} = 1 + \sqrt{4f}\left(1.43 + 2.15 \log_{10} \frac{y}{R}\right) \qquad ...(9.58)$$

At the centre of pipe : $y = R$ and $u = V_{max}$

Then

$$\frac{V_{max}}{V_{av}} = 1 + 1.43\sqrt{4f} \qquad ...(9.59)$$

The ratio $\frac{V_{av}}{V_{max}} = \frac{1}{1 + 1.43\sqrt{4f}}$ is sometimes referred to as **pipe coefficient** or **pipe factor**.

EXAMPLE 9.58

For turbulent flow in pipes, compute the distance from the pipe wall at which the velocity is equal to the average velocity of flow.

or $-7.475 \log_{10} \in + 5.75 \log_{10} \in$

$= -2.55 + 5.75 \times 0.477$

or $-1.725 \log_{10} \in = 0.1934$

or $\log_{10} \in = \dfrac{0.1934}{-1.725} = -0.1121$

That gives : Pipe roughness $\in$
$= 0.7725$ cm

9.21. FRICTION FACTOR FOR SMOOTH AND ROUGH PIPES

(*i*) The average velocity distribution for smooth pipes is governed by equation 9.55 :

$$\frac{V_{av}}{u^*} = 1.75 + 5.75 \log_{10} \frac{R\,u^*}{\nu}$$

Substituting the value of friction velocity

$$u^* = V_{av} \sqrt{\frac{4f}{8}}$$

$$\frac{V_{av}}{V_{av}\sqrt{4f/8}} = 1.75 + 5.75 \log_{10} \frac{R V_{av} \sqrt{4f}}{\nu \sqrt{8}}$$

$$= 1.75 + 5.75 \log_{10} \times \left(\frac{V_{av} D}{\nu} \sqrt{4f} \div 2\sqrt{8} \right)$$

or
$$\frac{1}{\sqrt{4f}} = \frac{1}{\sqrt{8}} [1.75 + 5.75 \log_{10} Re \sqrt{4f} - 5.75 \log_{10} 2\sqrt{8}\,]$$

$$= 0.618 + 2.03 \log_{10} Re \sqrt{4f} - 2.03 \log_{10} 2\sqrt{8}$$

$$= 0.618 + 2.03 \log_{10} Re \sqrt{4f} - 1.528$$

$$= 2.03 \log_{10} Re \sqrt{4f} - 0.91$$

To secure better fit with experimental data of Nikuradse, the final formula is taken as

$$\frac{1}{\sqrt{4f}} = 2.00 \log_{10} Re \sqrt{4f} - 0.80 \quad ...(9.60)$$

The average velocity distribution for turbulent flow in rough pipes is governed by equation 9.56.

$$\frac{V_{av}}{u^*} = 4.75 + 5.75 \log_{10} \frac{R}{\in}$$

Substituting the value of friction velocity $u^* = V_{av} \sqrt{\dfrac{4f}{8}}$, and then simplification results

$$\frac{1}{\sqrt{4f}} = 1.68 + 2.03 \log_{10} \frac{R}{\in}$$

The accepted formula for rough pipes based on experimental results is

$$\frac{1}{\sqrt{4f}} = 1.74 + 2.00 \log_{10} \frac{R}{\in} \quad ...(9.61)$$

Equation 9.61 is known as karman-Prandtl resistance equation for turbulent flow in rough pipes. Apparently the friction coefficient depends only upon the roughness and is independent of Reynolds number. This stems from the fact that shear turbulence in rough pipes is essentially due to surface roughness.

Finally a semi-empirical equation linking up smooth with rough turbulence has been developed by Colebrook and White.

$$\frac{1}{\sqrt{4f}} = 2.0 \log_{10} \frac{R}{\in} + 1.74 - 2.0 \log_{10} \left(1 + 18.7 \frac{R/\varepsilon}{R_e \sqrt{4f}} \right) \quad ...(9.62)$$

When ε/R is very small, equation 9.62 reduces to equation 9.58 for smooth pipes. At high Reynolds number, the second term on the right hand side can be neglected and the resulting equation corresponds to the rough pipe law ; equation 9.61.

EXAMPLE 9.61

The following data pertains to the test conducted for determining the equivalent sand grain roughness of a certain pipe :

9.22. FRICTION FACTOR CHARTS

Consider the friction factors $\lambda = 4f$ to depend upon the fluid density ρ, the flow velocity V, pipe diameter d, fluid viscosity μ and the roughness ∈. Then dimensional analysis would reveal that:

$$\lambda = 4f = \psi\left(R_e, \frac{\epsilon}{d}\right)$$

i.e., the friction factor λ is a function of the Reynolds number and the relative roughness.

The relationship between λ, R_e, $\frac{\epsilon}{d}$ is graphically represented in Fig. 9.41. The plot is the outcome of extensive experimental work done by Nikuradse on artificially roughened pipes. Several different pipes of 2.5 cm, 5 cm diameter were coated with sand grains which had been segregated by sieving so as to obtain different sizes of grains of uniform (0.1 to 1.6 mm). These pipes were tested in a broad range of relative roughness $\frac{\epsilon}{R} = \frac{1}{500}$ to $\frac{1}{15}$ and Reynolds number $R_e = 500$ to 10^6.

The following information can be gleaned from the plot shown in Fig. 9.41.

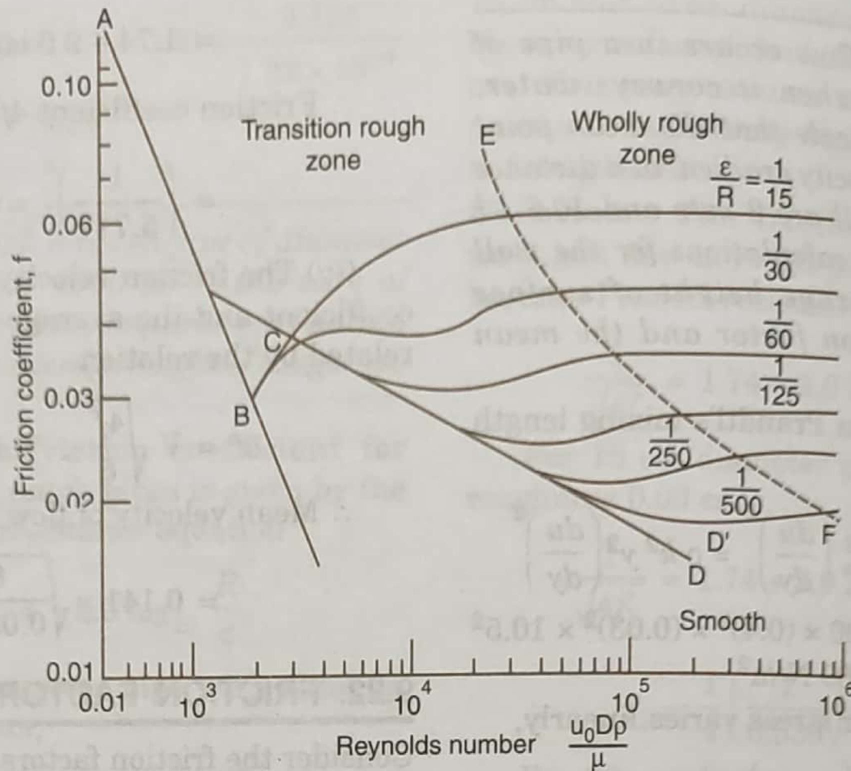


Fig. 9.41. Friction factor versus Reynolds number and relative roughness

(*i*) For values of $R_e < 2000$, the friction factor varies inversely with Reynolds number and is independent of the pipe roughness. The relationship between R_e and f is represented by a straight line AB, $4f = \frac{64}{R_e}$ and the region corresponds to that of laminar flow.

(*ii*) In the region $2000 < R_e < 4000$, the flow changes from laminar to turbulent flow and in this transition region, there is an abrupt rise in the friction factor (curve BC). This region is, however, not of much practical significance owing to the transient and unstable nature of flow.

(*iii*) Except for very large values of relative roughness, there is a range of Reynolds within the turbulent flow regime wherein the friction coefficient is independent of relative roughness (curve CD and CD′). The surface projections remain embedded in the laminar sublayer and cannot affect the friction factor. Experiments indicate that for the range $4 \times 10^3 < R < 10^5$, the friction factor is closely approximated by the Blasius relation, $4f = \frac{0.3164}{(R_e)^{0.25}}$. For $R_e < 10^5$, the friction factor corresponds to the karman-Prandtl resistance equation : $\frac{1}{\sqrt{4f}} = 2.0 \log_{10} (R_e \sqrt{4f}) - 0.80$ for turbulent flow in smooth pipes.

(*iv*) When the flow is turbulent and both the Reynolds number and relative roughness are large, the friction factor is dependent only upon the relative roughness. This is indicated by horizontal portion of the various ϵ/R curves. The situation corresponds to hydraulically rough pipes for which the variation of friction factor with relative roughness is prescribed by the karman-Prandtl resistance equation :

$1/\sqrt{4f} = 1.74 + 2.0 \log_{10} (R/\epsilon)$ for turbulent flow in rough pipes. Greater the height of protuberances/protrusions (*i.e.*, greater the value of parameters ϵ/R), the lower is the Reynolds number at which an originally smooth pipe begins to follow the pattern of a rough pipe. In other words a pipe with the roughest surface causes the earliest break away from the line CD of smooth boundary region. This aspect is reflected by the dotted line EF which marks the lower limit of rough pipe zone.

(v) Between the smooth and rough pipe regions, there is a transition zone wherein the friction coefficient is a function of both the relative roughness and the Reynolds number.

Nikuradse results for artificially roughened pipes do not hold good for commercial pipes in which both the size and distribution of protuberances is non-uniform. His results, however, provide a basis for establishing an equivalent roughness parameter for commercial pipes. Series of experiments are conducted on the commercial pipes at sufficient high Reynolds number corresponding to the horizontal portions of the Nikuradse curves. From the measured values of the loss of head h_f, a limiting value of the friction factor f is determined from the Darcy equation. This value of f is substituted in the equation : $1/\sqrt{4f} = 1.74 + 2.0 \log_{10} (R/\epsilon)$, and value of equivalent sand grain roughness (R/ϵ) is computed. With the roughness parameter thus established, Moody summarised the work on commercial pipes in the form of curves between f and R_e. The Moody diagram (Fig. 9.42) is similar to the Nikuradse plot except for the transition zone ; the difference can be attributed to the non-uniformity of roughness in commercial pipes. In the artificial roughened pipes of Nikuradse, laminar sublayer either covers all the artificial roughness or allows it to protrude uniformly. In commercial pipes with roughness varying in height spacing and location, there is every likelihood of some protuberances projecting outside the sublayer.

A logarithmic plot of friction factor versus Reynolds number for various values of relative roughness as depicted in the Nikuradse and Moody plots is often referred to as Stanton diagrams.

Moody has suggested the following approximate relation for evaluating the friction factor within ± 5% for $4000 < R_e < 10^7$ and for the values of $\frac{\epsilon}{R}$ upto 0.005.

$$f = 0.001375 \left[1 + \left(10{,}000\,\frac{\epsilon}{R} + \frac{10^6}{R_e}\right)^{1/3}\right] \qquad ...(9.63)$$

The roughness of a pipe depends upon the pipe material, its manufacturing process, erosion or encrustation with time, fluid flowing and the environment etc. With the passage of time, the roughness increases due to rusting and accumulation of sediments on the pipe surface. An increase in roughness with age is generally prescribed by the following linear relation suggested by Colebrook and White

$$\varepsilon = \varepsilon_0 + \alpha t \qquad ...(9.64)$$

where ε_0 is the roughness of the new material, ε is the roughness after any time t and α is the coefficient to be determined by experiments.

EXAMPLE 9.66

What do you understand by commercial pipes ? How would you determine the equivalent roughness of a commercial pipe ?

A 20 cm diameter pipe has a relative roughness (R/ε) of 100. After 10 years of service, the relative roughness drops to 80. Determine the magnitude of the rate of roughness increase.

Solution : At the time of installation,

$$\varepsilon_0 = \frac{R}{100} = \frac{20}{100} = 0.2 \text{ cm}$$

After 10 years of service,

$$\varepsilon_{10} = \frac{20}{80} = 0.25 \text{ cm}$$

An increase is roughness with time t follows the linear law suggested by Colebrook and White,

$$\varepsilon_\tau = \varepsilon_0 + \alpha t \; ; \; \varepsilon_{10} = \varepsilon_0 + 10\,\alpha$$

$$\therefore \quad \alpha = \frac{\varepsilon_{10} - \varepsilon_0}{10} = \frac{0.5 - 0.2}{10}$$

$$= \mathbf{0.0005\ cm} \text{ per year}$$

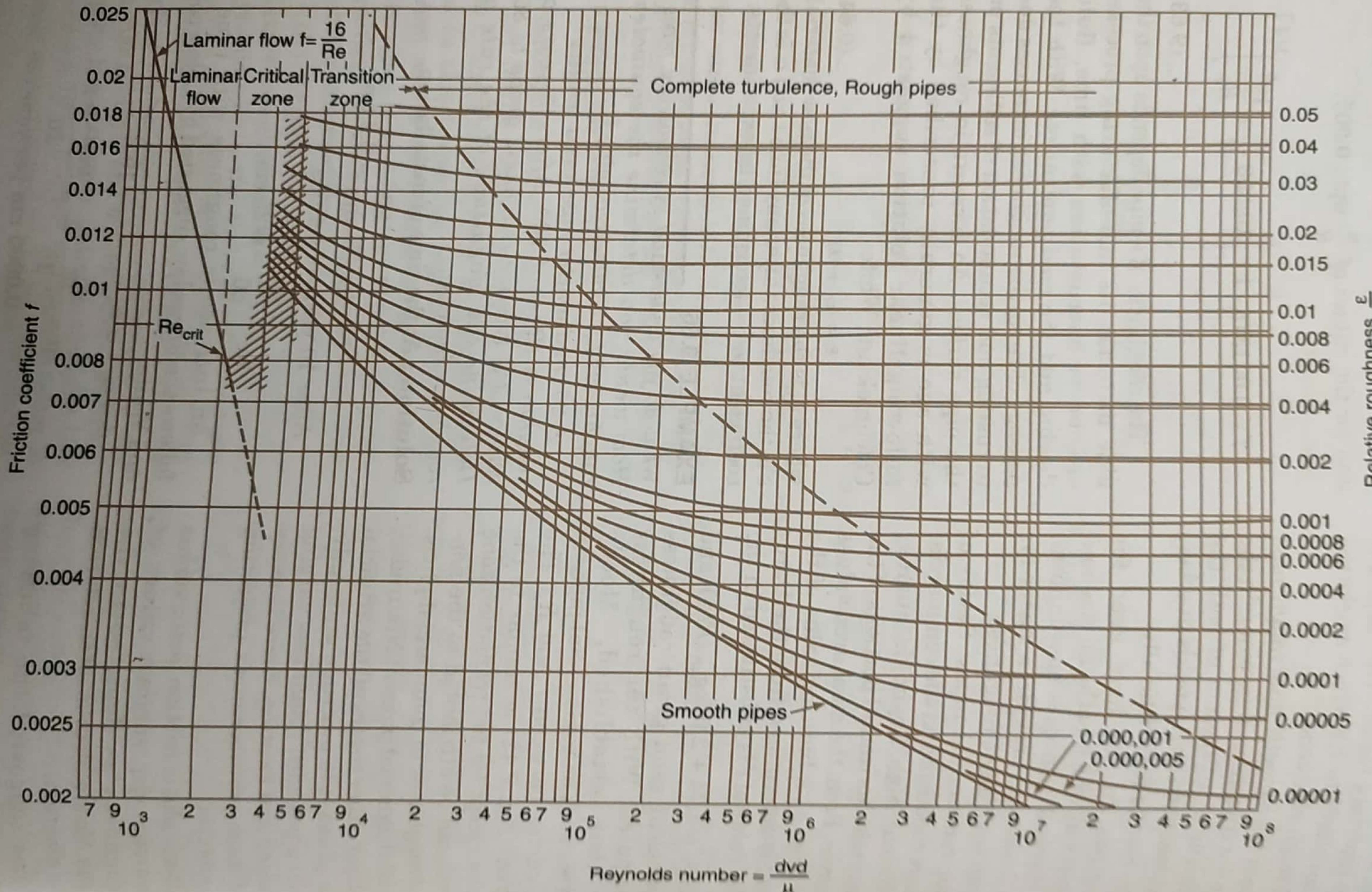


Fig. 9.42. Moody diagram for friction factors