5.10. KINETIC ENERGY CORRECTION FACTOR α AND MOMENTUM CORRECTION FACTOR β

With real fluids, the flow velocity is different for different fluid particles at a cross-section, and thus the assumption of one-dimensional uniform flow is no longer valid. Velocity distribution across a section depends upon the nature of flow (laminar or turbulent) and upon the smoothness/roughness of the pipe surface. Total kinetic energy at any section can, however, be obtained by integrating the kinetic energy of an element area dA wherein the local velocity u can be presumed to be uniform.

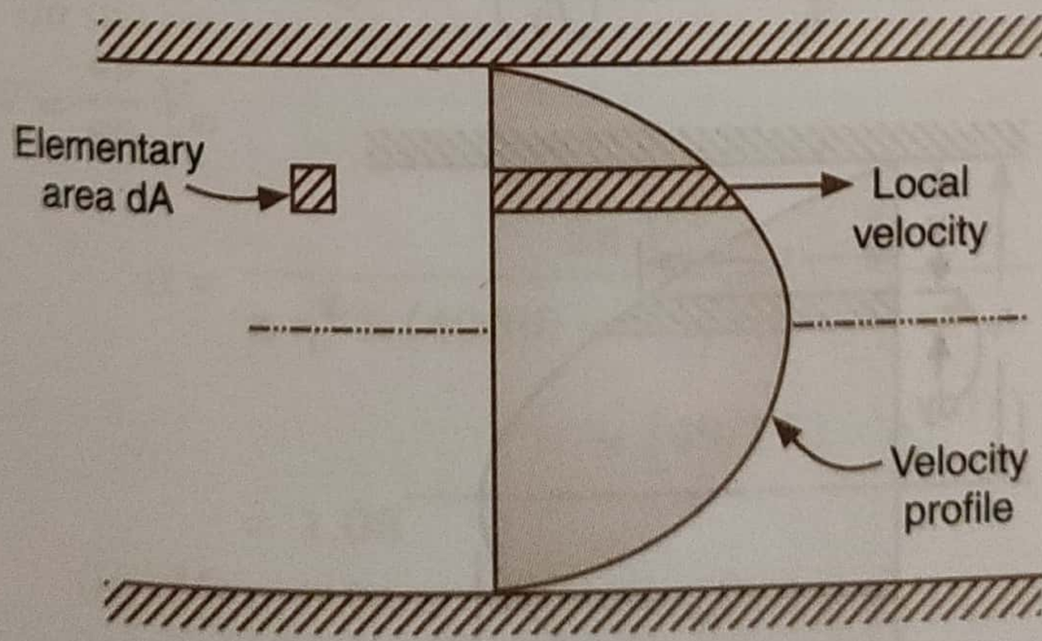

Fig. 5.36.

Reference Fig. 5.36, the kinetic energy of the fluid contained in the infinitesimal area dA which carries mass dm is :

$$= dm \frac{u^2}{2}$$

$$= (\rho\, dA\, u) \frac{u^2}{2} = \frac{\rho\, dA\, u^3}{2} \quad ...(5.34)$$

For constant-density flow the total kinetic energy of the fluid per unit time flowing in the pipe will become :

$$= \frac{\rho}{2} \int_A u^3\, dA \quad ...(5.35)$$

This expression can be evaluated if the velocity distribution across the cross-section is presumed.

However in engineering problems, the total kinetic energy is generally prescribed in terms of average velocity V and a factor α called the *kinetic energy correction factor*. The average flow velocity is obtained either by noting the discharge per unit time and dividing it by the cross-sectional area, or by using the expression :

$$V = \frac{1}{A} \int_A u\, dA$$

Total kinetic energy calculated on the basis of average velocity and factor α is :

$$= \alpha\, (\text{mass}) \times \frac{V^2}{2}$$

$$= \alpha\, (\rho\, AV) \frac{V^2}{2} = \frac{\rho}{2} \alpha\, AV^3 \quad ...(5.36)$$

Comparing equations (5.35) and (5.36), we get

$$\alpha = \frac{1}{AV^3} \int_A u^3\, dA \quad ...(5.37)$$

Evidently the kinetic energy term in the Bernoulli's equation becomes equivalent to $\alpha \frac{V^2}{2g}$.

Practical values of α for circular ducts are :

(*i*) $\alpha = 2$ for laminar flow with parabolic velocity distribution

(*ii*) $\alpha = 1.02$ to 1.15 for general turbulent flow

The effect of non-uniform velocity distribution on the momentum flux is taken care of by using a factor β, called the *momentum correction factor*. Momentum flux per unit time through the elementary area dA would be

$$= \text{mass} \times \text{velocity}$$
$$= (\rho\, dA\, u)\, u = \rho\, dA\, u^2$$

Then total momentum flux per unit time through the entire cross-section is $\int \rho u^2\, dA$.

Momentum flux calculated on the basis of average velocity V and the momentum correction factor β would be

$$= \beta\,(\rho V A)\, V = \beta \rho V^2 A$$

Equating the two expressions for the total momentum flux,

$$\beta \rho V^2 A = \int \rho u^2\, dA$$

$$\beta = \frac{1}{AV^2} \int u^2\, dA \qquad \text{...(5.38)}$$

Evidently in the flow of real fluids, the impulse momentum equations would get modified and take the form :

$$F_x = \frac{wQ}{g}\,(\beta_1 V_1 \cos\theta_1 - \beta_2 V_2 \cos\theta_2)$$

$$F_y = \frac{wQ}{g}\,(\beta_1 V_1 \sin\theta_1 - \beta_2 V_2 \sin\theta_2) \qquad \text{...(5.39)}$$

Practical values of β are :

(*i*) β = 1.33 for laminar flow with parabolic velocity distribution

(*ii*) β = 1.01 to 1.07 for general turbulent flow

Since majority of the flow situations are turbulent in character, the usual practice is to assign unit values to α and β.

EXAMPLE 5.43

The velocity distribution for turbulent flow in a pipe is given approximately by Prandtl's one-seventh power law.

$$v = V_m \left(\frac{y}{r_0}\right)^{1/7}$$

where v is the local velocity of flow at distance y from the pipe wall, V_m is the maximum flow velocity at the centre line of the pipe and r_0 is the pipe radius.

Find the average velocity, kinetic energy correction factor and the momentum correction factor.

Solution : Consider an elementary ring of thickness dy at a distance y from the pipe wall *i.e.,* at distance $(r_0 - y)$ from the pipe axis.

Flow rate through the elemental ring

$$= \text{elemental area} \times \text{local velocity}$$
$$= 2\pi (r_0 - y)\, dy\, v$$

∴ Total flow

$$Q = \int_0^{r_0} 2\pi v\,(r_0 - y)\, dy$$

$$= \int_0^{r_0} 2\pi V_m \left(\frac{y}{r_0}\right)^{1/7} (r_0 - y)\, dy$$

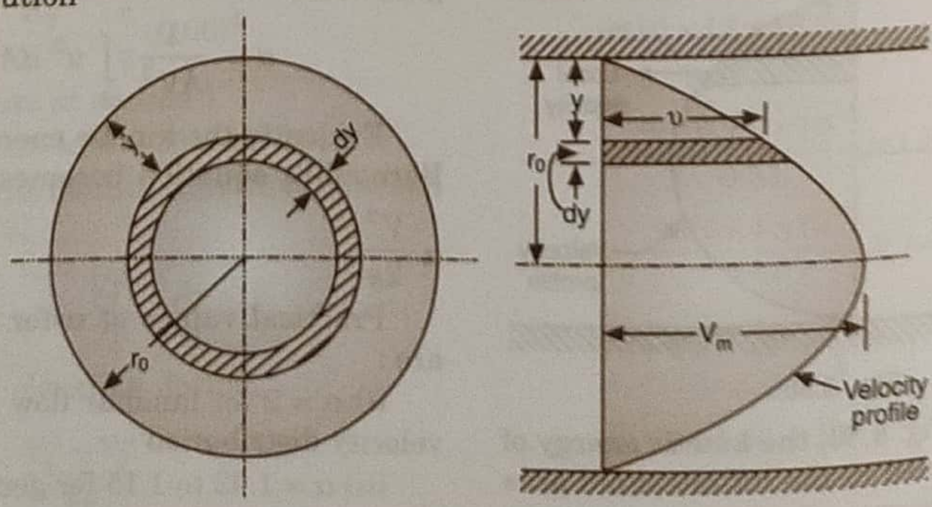

Fig. 5.37. Cross-section and the velocity distribution through a circular pipe

$$= \frac{2\pi V_m}{r_0^{1/7}} \int_0^{r_0} y^{1/7} (r_0 - y)\, dy$$

$$= \frac{2\pi V_m}{r_0^{1/7}} \left| \frac{7}{8} \times r_0\, y^{8/7} - \frac{7}{15} \times y^{15/7} \right|_0^{r_0}$$

$$= \frac{2\pi V_m}{r_0^{1/7}} \left(\frac{7}{8} \times r_0^{15/7} - \frac{7}{15} \times r_0^{15/7} \right)$$

$$= 2\pi V_m \left(49/120 \times r_0^2 \right) \quad \text{...}(i)$$

If V is the average flow velocity, then

$$Q = AV = \pi r_0^2 V \quad \text{...}(ii)$$

From (*i*) and (*ii*),

Average velocity

$$V = \frac{2\pi V_m \left(49/120 \times r_0^2\right)}{\pi r_0^2} = \mathbf{\frac{49}{60}}\, V_m$$

(*ii*) Kinetic energy correction factor,

$$\alpha = \frac{1}{AV^3} \int_0^{r_0} v^3\, dA$$

$$= \frac{1}{AV^3} \int_0^{r_0} V_m^3 \left(\frac{y}{r_0}\right)^{3/7} 2\pi (r_0 - y)\, dy$$

$$= \frac{2\pi V_m^3}{AV^3\, r_0^{3/7}} \int_0^{r_0} y^{3/7} (r_0 - y)\, dy$$

$$= \frac{2\pi V_m^3}{AV^3\, r_0^{3/7}} \left(\frac{49}{170} \times r_0^{17/7} \right)$$

Substituting the values $A = \pi r_0^2$ and

$$V = \frac{49}{60} V_m$$

$$\alpha = \frac{2\pi V_m^3}{\pi r_0^2 \times (49/60 \times V_m)^3\, r_0^{3/7}} \times \left(49/170 \times r_0^{17/7}\right)$$

$$= \mathbf{1.06}$$

(*iii*) Momentum correction factor β,

$$\beta = \frac{1}{AV^2} \int v^2\, dA$$

$$= \frac{1}{AV^2} \int_0^{r_0} V_m^2 \left(\frac{y}{r_0}\right)^{2/7} 2\pi (r_0 - y)\, dy$$

$$= \frac{2\pi V_m^2}{AV^2 r_0^{2/7}} \int_0^{r_0} y^{2/7} (r_0 - y)\, dy$$

$$= \frac{2\pi V_m^2}{AV^2 r_0^{2/7}} \left(\frac{49}{144} \times r_0^{16/7} \right)$$

Substituting the values $A = \pi r_0^2$ and

$V = 49/60 \times V_m$

$$\beta = \frac{2\pi V_m^2}{\pi r_0^2 \times (49/60 \times V_m)^2 r_0^{2/7}} \times \left(49/144 \times r_0^{16/7}\right)$$

$$= 1.02$$