

Simon Hyken for communication

21 Jan

Optical fiber

Taught By- Prof. Md isteyaque ashraf

frequency $\gamma = 2100 \times 10^6 \text{ GHz}$
 $2100 \text{ THz} - 2300 \text{ THz}$
opto electronics
optical domain

^{is used}
Splasher for adding Optical fiber
(splashing)

Wavelength of optical fiber

1300 nm to 1500 nm

$$\text{Wavelength} = \frac{\text{Speed of light}}{\text{frequency}}$$

$$\lambda = \frac{c}{f}$$

size of the antenna depend
upon λ

7/4

Wavelength jina chota hoga Antenna
utna chota hoga

Optical fiber

optical fiber is the study of propagation of light through transparent dielectrical waveguide. the fiber optics are used for transmission of data from point-to-point location extensively as the transmission line for terrestrial hardwired system.

optical fiber has higher rate of data transmission capacity. it can transmit higher carrier frequency and hence it has greater (large) bandwidth.

Advantages of optical fiber

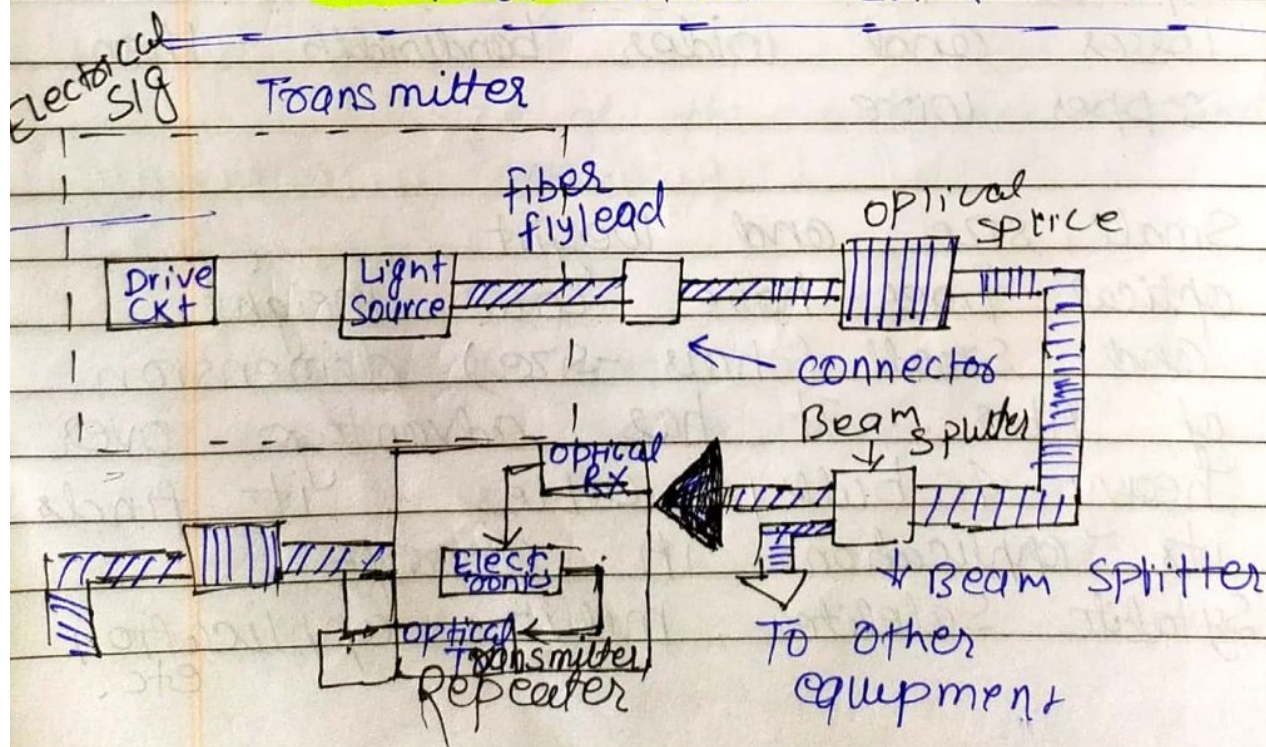
1. Low transmission loss and large bandwidth. optical fiber have low transmission losses and wider bandwidth than copper wire.
2. Small size and weight. optical fiber have low weight and small (hair-size) dimension of fiber. It has advantage over heavy & bulky cables. it finds its application in aircraft, Satellite, Satellite, military application etc.

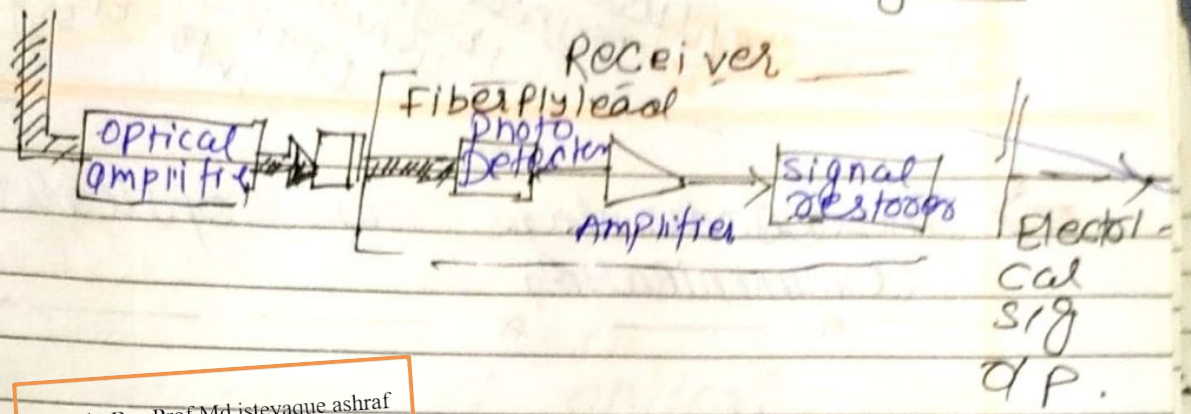
3. Immunity to Interference. —
optical wave-light fiber optics provides immunity to electro-magnetic interference from signal carrying wires and lightning

4. Signal Security.
High degree of data security is achieved in optical fiber as the optical signal is well confined within the waveguide
(प्रारंभिक)

5. Abundant raw material.
Optical The main material for optical fiber is silica which is found in abundant ordinary sand.

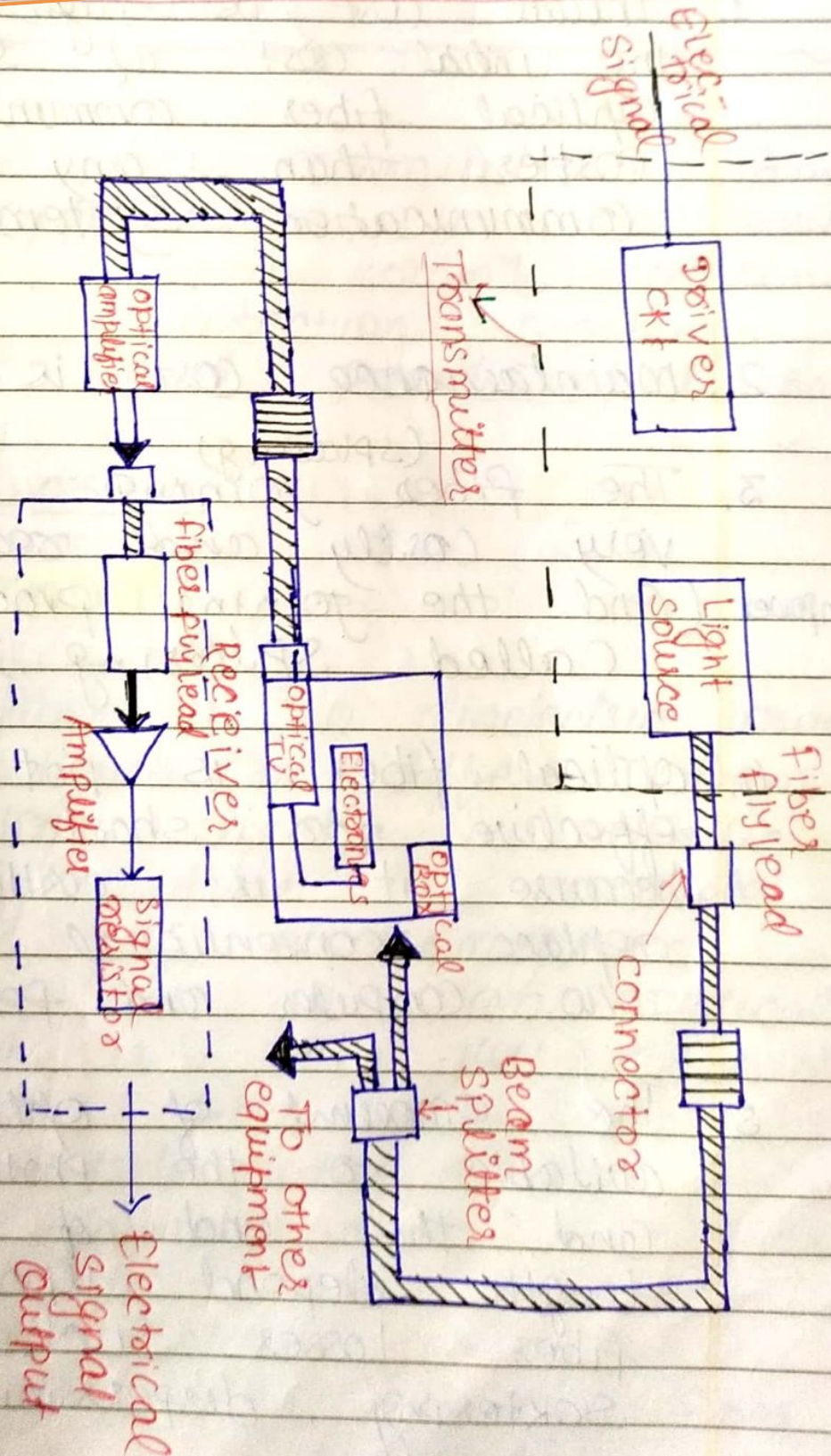
⇒ ELEMENTS OF OPTICAL FIBER COMMUNICATION LINK





Taught By- Prof. Md isteyaque ashraf

Elements of optical fiber communication Link



⇒ Disadvantage of optical fiber communication

1. Initial cost is high.
The initial cost of setup for optical fiber communication is costlier than any other communication system.
2. Maintenance cost is high.
3. The fiber joining process is very costly and requires skilled manpower (And the joining process is called splicing.)
4. Optical fiber is not cost effective for short distance link because it is costlier to replace conventional connectors b/w computer and peripheral peripheral.
5. The amount of optical fiber available to the photodetector and the end of fiber length depend upon various fiber losses such as scattering, dispersion, attenuation.

and Reflection

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⇒ Application of optical fiber Communication

optical fiber communication find its applications in telecommunication, data commⁿ, video-control, protection switching, sensor and power application.

03 Feb

NEW TOPIC

*

RAY MODEL

optical fiber is a dielectric wave current which is different from metallic wave light. In metallic wave light there is complete shielding of electromagnetic radiation but in an optical fiber it is not just confined inside the fiber but also extend outside the fiber. (CRITIC)

The light guided inside the fiber through the basic phenomenon of total internal reflection.

The optical fiber

vacuum to that in matter is called the index of refraction n of the material

$$n = \frac{c}{v} = \frac{\text{Speed of light in air}}{\text{Speed of light in material}}$$

$n = 1$ for Air

$n = 1.33$ for Water

$n = 1.50$ for glass

$n = 2.42$ for diamond

* **Refraction** : Refraction occurs when the light passes from one medium to the other medium and changes its direction at interface. That is it occurs when the density of medium changes for example, Refraction occurs at Air and water interface. (Surface)

V.V.I
⇒

Snell's Law : Snell's Law states that how light ray behaves when it meet the interface of two media having different indexes of Refraction.

Let the two media have Refractive indexes n_1 and n_2 where $n_1 > n_2$

Let, ϕ_1 & ϕ_2 with the angle of incidence and refraction respectively then according to Snell's Law a relation b/w the Refractive index of both material is given by

$$n_1 \sin \phi_1 = n_2 \sin \phi_2$$

$$\frac{n_1}{n_2} = \frac{\sin \phi_2}{\sin \phi_1}$$

(1)

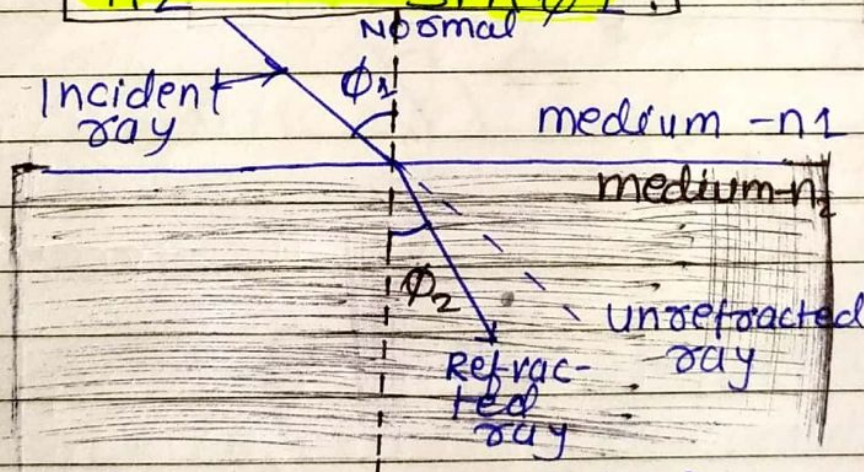


Fig : Refractive model for Snell's Law.

The Refracted ray will be towards normal when $n_1 < n_2$ and will be away from the normal if $n_2 > n_1$ $n_2 < n_1$

Refractive index,

$$n_1 = \frac{c}{v_1}$$

$$n_2 = \frac{c}{v_2}$$

Putting the value of n_1 and n_2 in eqⁿ (1)

$$\frac{\frac{c}{v_1}}{\frac{c}{v_2}} = \frac{\sin \phi_2}{\sin \phi_1}$$

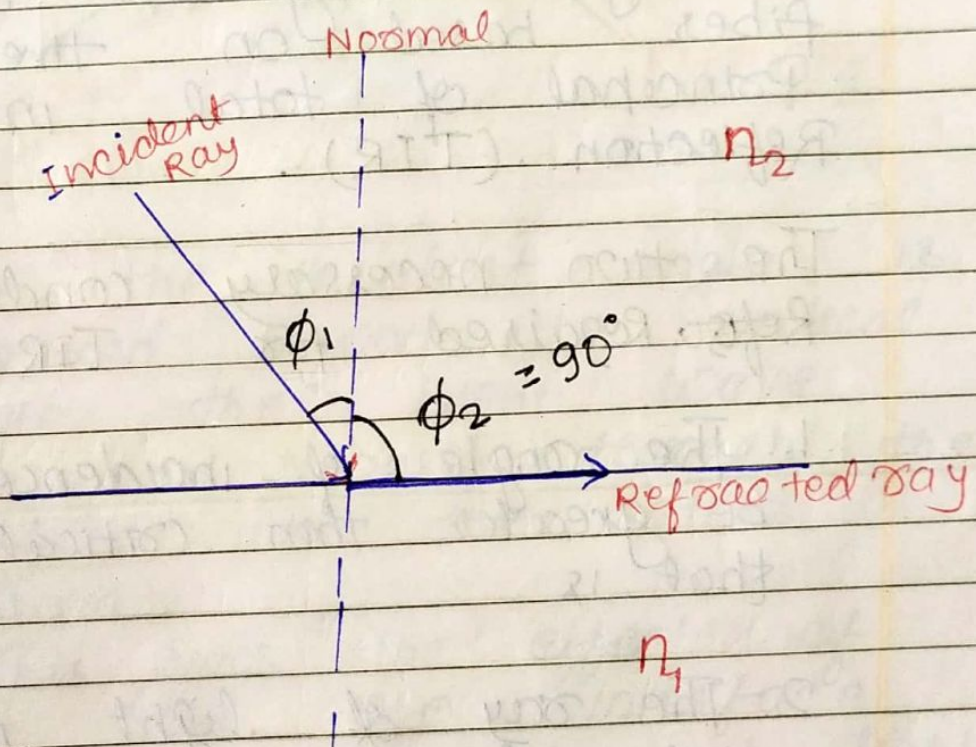
$$\boxed{\frac{v_2}{v_1} = \frac{\sin \phi_2}{\sin \phi_1}}$$

$$\sin \phi_c = \frac{n_2 \sin 90^\circ}{n_1}$$
$$\phi_c =$$

7 Feb 11

⇒ Critical Angle (θ_c) :-

The critical Angle defined as the minimum angle of incidence (ϕ_1) at which the ray strike the interface of two medium and causes angle of refraction (ϕ_2) equal to 90° .



$\phi_1 = \phi_c$
 ϕ_1 is
 critical
 angle

Hence at critical angle

and $\phi_1 = \phi_c$

$\phi_2 = 90^\circ$

Using Snell's Law

$$\Rightarrow \sin \phi_c = \frac{n_2 \sin 90}{n_1}$$

$$\sin \phi_c = \frac{n_2}{n_1}$$

$$\phi_c = \sin^{-1} \frac{n_2}{n_1}$$

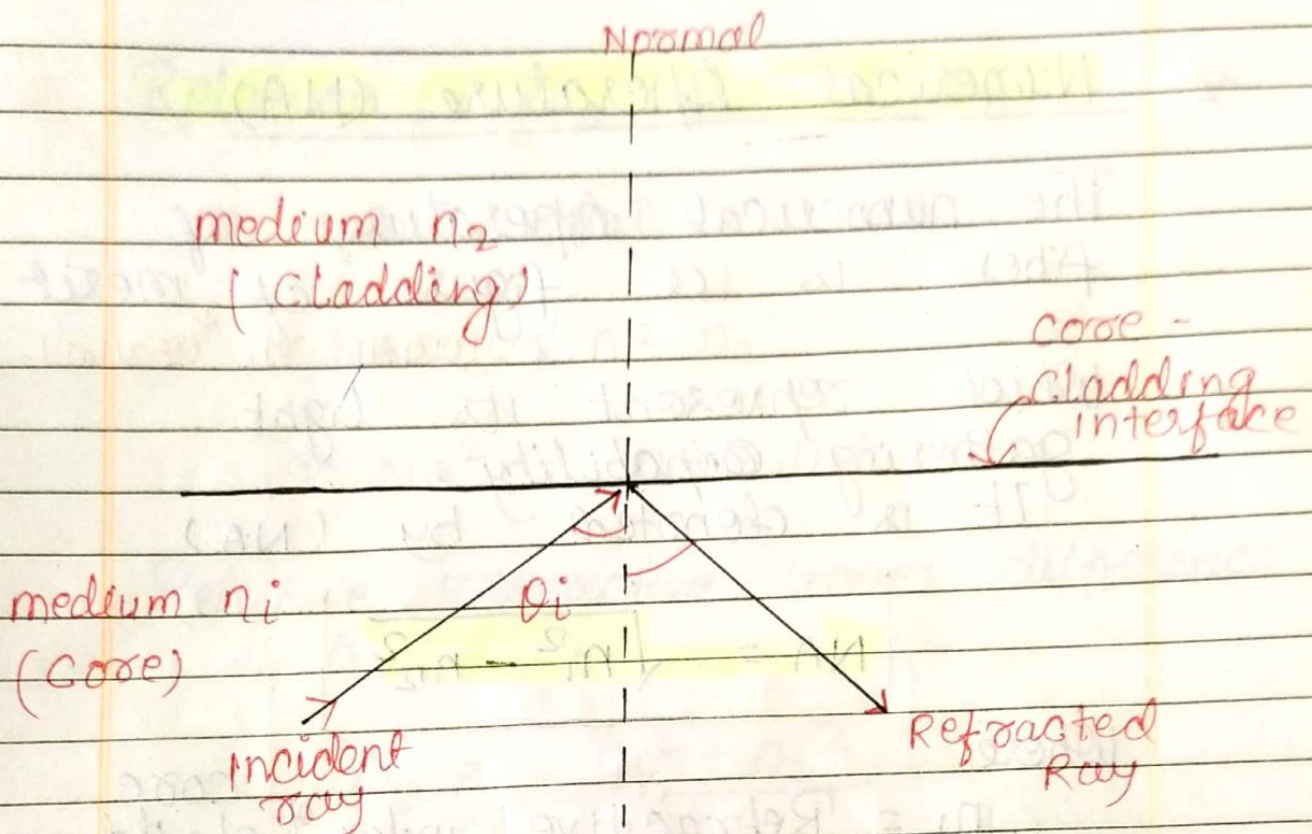
⇒ Total Internal Reflection (TIR)

The Light is guided in optical fiber based on the Principle of total internal Reflection (TIR).

The two necessary condition ~~Refro~~ Required for TIR are

1. The angle of incidence must be greater than critical angle that is
2. The ray of light must Inter Higher Refractive index medium [optically denser (core)] to Low Refractive index [optically rarer (cladding)] medium.

It is shown in figure below.



→ When the incident angle is increased beyond the critical angle the light wave does not pass through the interface into the other medium.

This gives the effect of mirror at the interface with no possibility of light escaping outside the medium. In this condition angle of Reflection (θ_r) is equal to the angle of incidence (θ_i). This phenomenon is known as Total Internal Reflection.

* Numerical Aperture (NA)

The numerical aperture of fiber is a figure of merit
(गुणवत्ता, आ, धरणात्मक)

Which represent its light gathering capability.

It is denoted by (NA)

$$NA = \sqrt{n_1^2 - n_2^2}$$

Where

n_1 = Refractive index of ^{core} cladding

n_2 = Refractive index of cladding

* Relative Refractive index difference (Δ)

Relative Refractive index difference between core and cladding is given by

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

* Relation b/w NA & Δ

We know that

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$\Rightarrow NA^2 = n_1^2 - n_2^2$$

Relative refractive index difference Δ is

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

$$\Delta = \frac{NA^2}{2n_1^2}$$

$$\Rightarrow NA^2 = 2n_1^2 \Delta$$

$$\Rightarrow NA = n_1 \sqrt{2\Delta}$$

* Acceptance angle (θ_0)

It is the maximum incidence angle at which a ray of light strike the fiber axis in order to propagate

Acceptance angle.

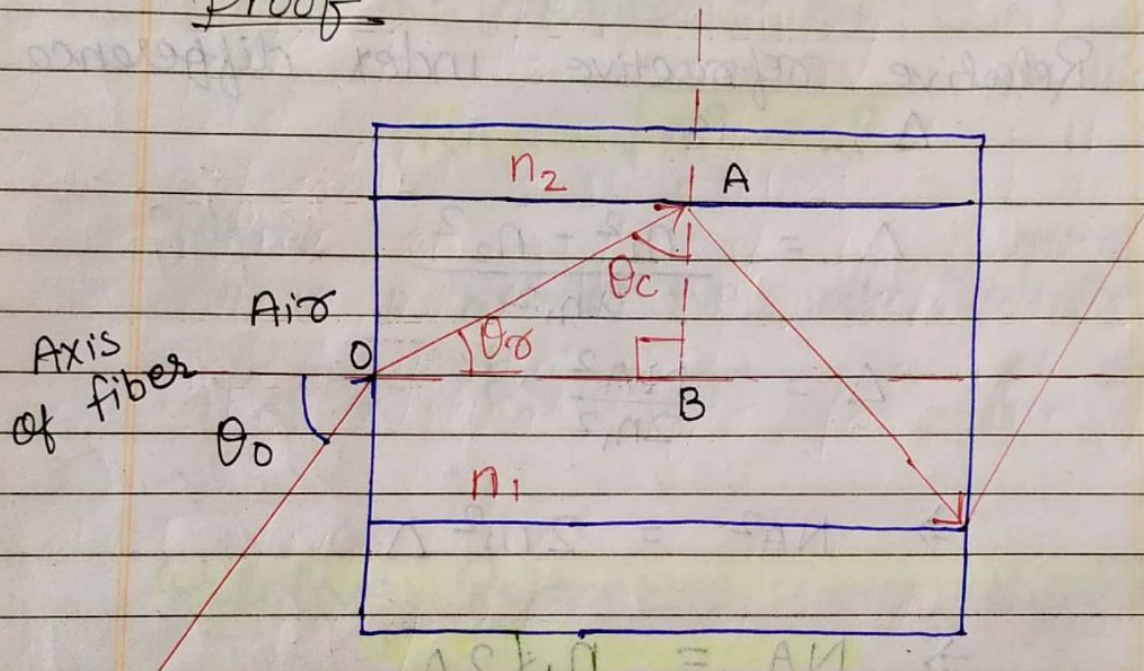
$$\theta_0 = \sin^{-1} \left(\frac{NA}{n} \right)$$

$n = R.I$ outside medium
If air $n = 1$

Where

$$NA = \sqrt{n_1^2 - n_2^2}$$

Proof



Applying Snell's law at
Surrounding medium (n_2) &
fiber core (n_1) interface

$$n_2 \sin \theta_0 = n_1 \sin \theta_c$$

from triangle OAB refracted
angle

$$\theta_c = 90^\circ - \theta_0$$

$$\therefore n_2 \sin \theta_0 = n_1 \sin (90^\circ - \theta_0)$$

$$n \sin \theta_o = n_1 \cos \theta_c \quad \text{--- (1)}$$

from critical angle definition

$$\sin \theta_c = \frac{n_2}{n_1}$$

$$\Rightarrow \cos \theta_c = \sqrt{1 - \sin^2 \theta_c}$$

$$\cos \theta_c = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

$$= \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

$$= \frac{NA}{n_1} \quad \text{--- (2)}$$

Where

$$NA = \sqrt{n_1^2 - n_2^2}$$

Substituting $\cos \theta_c = \frac{NA}{n_1}$ in eqⁿ (1)

$$n \sin \theta_o = \frac{NA}{n_1} \times n_1$$

$$n \sin \theta_o = NA$$

$$\sin \theta_o = \frac{NA}{n}$$

$$\Rightarrow \theta_o = \sin^{-1} \left(\frac{NA}{n} \right)$$

If outside medium is air
then $n = 1$,

we have, $\theta_o = \sin^{-1} (NA)$

$$\theta_0 = \sin^{-1}(NA)$$

1. Light travelling in air strikes a glass plate at an angle $\phi_1 = 33^\circ$. Where ϕ_1 is measured between the incoming ray and glass surface. Upon striking the glass part of the beam is reflected and part is refracted. If the refracted and reflected beams makes an angle of 90° with each other, what is the refractive index of the glass? What is the critical angle for the glass?

Solⁿ: Given that

$$\phi_1 = 33^\circ \text{ and } \phi_2 = 90^\circ$$

Assume refractive index of air = 1
According to Snell's law

$$n_1 \sin \phi_1 = n_2 \sin \phi_2 \quad \text{--- (1)}$$

Suppose

n_1 is refractive index of glass

n_2 is refractive index of air = 1

$$\Rightarrow n_2 \sin \phi_2 = n_1 \sin \phi_1$$

$$n_1 = \frac{n_2 \sin \phi_2}{\sin \phi_1} = \frac{\sin 90^\circ}{\sin 33^\circ} = \frac{\sin 90^\circ}{\sin 33^\circ}$$

Refractive index of glass

$$n_1 = \frac{1.836}{0.5446} = 1.836$$

$$n_1 = 1.836$$

From definition of critical angle $\phi_2 = 90^\circ$ and $\phi_1 = \phi_c$

$$\sin \phi_c = \frac{n_2}{n_1} \sin 90^\circ$$

$$\sin \phi_c = \frac{1}{1.836} \sin 90^\circ = \frac{1}{1.86}$$

$$\sin \phi_c = 0.54$$

$$\phi_c = \sin^{-1}(0.54)$$

$$\phi_c = 32.68^\circ$$

∴ Critical angle $\phi_c = 32.68^\circ$

Q2 A silica optical fiber with a core diameter large enough to be considered by ray theory analysis has a core refractive index of 1.50 and a cladding refractive index of 1.47 determine
 (a) The critical angle at the core-

Cladding interface

(b) The NA for the fiber

(c) The acceptance angle in air for the fiber

Solⁿ: The critical angle ϕ_c at the core cladding interface is given by

$$a) \phi_c = \sin^{-1} \frac{n_2}{n_1}$$

$$= \sin^{-1} \left(\frac{1.47}{1.50} \right)$$

$$\boxed{\phi_c = 78.5^\circ}$$

(b) The Numerical Aperture is given by,

$$NA = (n_1^2 - n_2^2)^{1/2} = \sqrt{n_1^2 - n_2^2}$$

$$= \sqrt{(1.50)^2 - (1.47)^2}$$

$$= \sqrt{2.25 - 2.16}$$

$$\boxed{NA = 0.30}$$

(c) The acceptance angle in air θ_a is given by

$$\theta_a = \sin^{-1} NA$$

$$= \sin^{-1} 0.30$$

$$\boxed{\theta_a = 17.45^\circ}$$

3 A typical relative refractive index difference for an optical fiber designed for long distance transmission is 1%. Estimate the NA and the solid acceptance angle in air for the fiber when the core index is 1.46. further calculate the critical angle at the core-cladding interface within the fiber.

Solⁿ: Given data ;

$$\Delta = 0.01, \quad n_1 = 1.46$$

$$NA = \sqrt{n_1(2\Delta)}$$

$$= \sqrt{1.46(2 \times 0.01)}$$

$$= \sqrt{1.46 \times 0.02} = 0.17$$

$$NA = 0.17$$

for small angles the solid acceptance angle in air is given by -

$$\phi_0 = \pi \theta_a^2 = \pi \sin^2 \theta_a$$

$$\phi = \pi (NA)^2 = \pi (0.17)^2$$

$$= 3.14 \times (0.17)^2 =$$

$$\phi = 0.090 \text{ rad}$$

As WKT, for the relative refractive index difference Δ gives

$$\Delta = \frac{n_1 - n_2}{n_1} = 1 - \frac{n_2}{n_1}$$

Hence, $\frac{n_2}{n_1} = 1 - \Delta$

$$\frac{n_2}{n_1} = 1 - 0.01$$

$$\Rightarrow \boxed{\frac{n_2}{n_1} = 0.99}$$

Hence, the critical angle at core-cladding interface is

$$\phi_c = \sin^{-1} \frac{n_2}{n_1}$$

$$\phi_c = \sin^{-1} (0.99)$$

$$\boxed{\phi_c = 81.89}$$

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Acceptance Cone

Acceptance Cone of the fiber is the cone shaped pattern obtained by Rotating the acceptance angle (θ_0) around the fiber axis

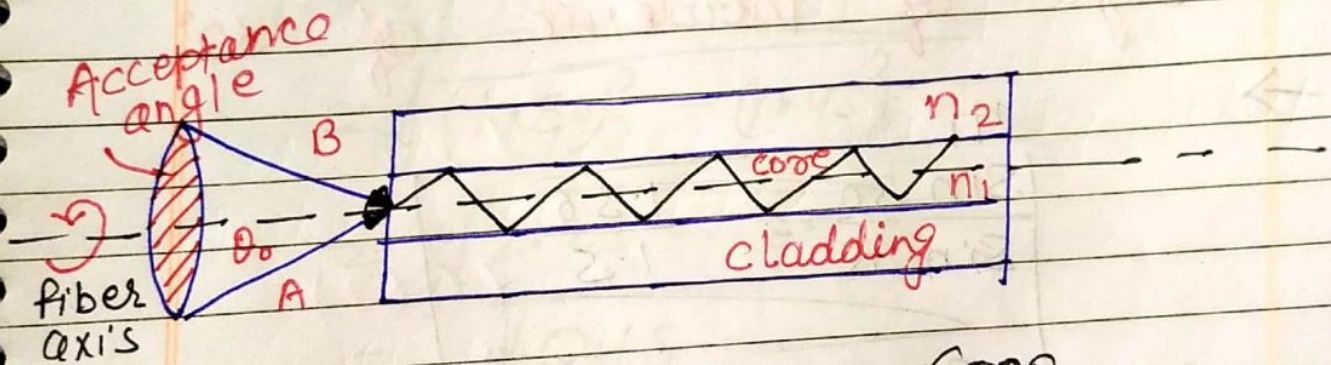


fig: Acceptance Cone

from figure, Total cone angle = $2\theta_0$

→ Acceptance angle;

θ_0 = half of cone angle

Q A light ray is incident from medium one to medium 2 if the refractive index of medium 1 is 1.5 and medium 2 is 1.36 then find out the angle of Refraction for an angle of incidence of 30°

→

$$\frac{n_1 \sin 30}{n_2 \sin \theta} = \frac{1.36}{1.5}$$

$$\sin \theta = \frac{\sin 30}{0.906}$$

Solⁿ:- $n_1 \sin \phi_1 = n_2 \sin \phi_2$

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \phi_1$$

$$\theta_2 = \sin^{-1} \left(\frac{1.5 \sin 30^\circ}{1.36} \right)$$

$$\theta_2 = \sin^{-1} (0.5514)$$

$$\theta_2 = 33.46^\circ$$

Q. Calculate the numerical aperture, acceptance angle, and critical angle of fiber having core R.I = 1.50 and cladding R.I = 1.45

$$\begin{aligned}
 NA &= \sqrt{n_1^2 - n_2^2} \\
 &= \sqrt{(1.50)^2 - (1.45)^2} \\
 &= \sqrt{2.25 - 2.10} \\
 &= \sqrt{0.15}
 \end{aligned}$$

$$NA = 0.387$$

$$\begin{aligned}
 \textcircled{ii} \quad \theta_a &= \sin^{-1} NA \\
 &= \sin^{-1} (0.387)
 \end{aligned}$$

$$\theta_a = 22.767^\circ$$

$$\sin \phi_c = \frac{n_2}{n_1} \sin 90^\circ$$

$$\textcircled{iii} \quad \theta_c = \sin^{-1} \frac{n_2}{n_1}$$

$$= \sin^{-1} \frac{1.45}{1.50}$$

$$\theta_c = 75.016^\circ$$

Q (1) Light ray is incidence from glass to air
Calculate the critical angle

$$\Rightarrow n_1 \sin \phi_c = n_2 \sin 90^\circ$$

$$\sin \phi_c = \frac{n_2}{n_1} \sin 90^\circ$$

$$\sin \phi_c = \frac{1}{1.5}$$

Air = 1
Glass = 1.5
Water = 1.33

$$\sin \phi_c = 0.67$$

$$\phi_c = \sin^{-1}(0.67)$$

$$\phi_c = 42.66^\circ$$

⇒ Types of RAY

The exact path of Ray in optical fiber is determined by the position and angle of ray at which it strikes the core. Th^o

→ There are 3 Types of RAY

- (i) Skew RAYs
- (ii) meridional rays
- (iii) Axial rays

⇒ (i) Skew Rays

→ The skew Rays do not pass through the centre. It gets reflected from the core cladding boundaries and again bounces around the outside of the core. It takes shape similar to spiral or helical path.



The acceptance ^{angle} for skew rays
is larger than meridional rays

To

The Principal
PCE Purnea

Sub : for original certificate

Refractive Indices of optical Materials

Material

Refractive index

Vacuum

1.00

Air

1.000292

Water

1.333

magnesium fluoride

1.390

Polymethyl methacrylate

1.496

Borosilicate crown glass

1.510

Hard crown glass

1.516

Canada Balsam (optical cement)

1.530

Light flint glass

1.579

Polystyrene

1.593

Dense flint glass

1.623

Double extra dense

1.754

flint glass

Densest flint glass

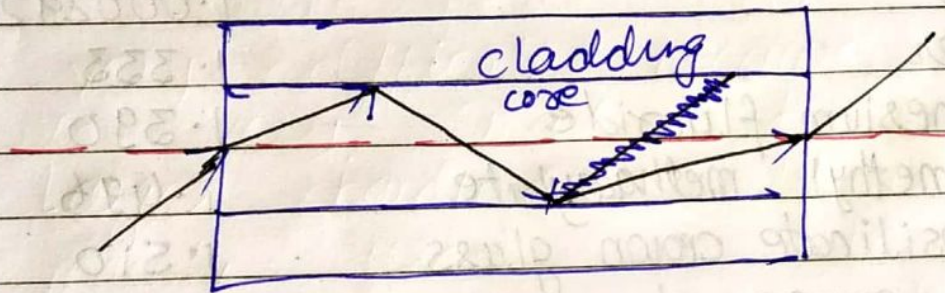
1.96

Diamond

2.4195

(ii) Meridional Ray

The meridional Rays follow a ZIK-ZAK Path through the fiber. When the core surface is parallel it will always be reflected to pass through the centre that is through the meridian plane of the fiber (fiber axis)



(iii) Axial Rays

The axial Rays travel along the axis of the fiber and stay at the axis all the time

V.V.E

Modes of fiber

Light ray propagates as an electromagnetic waves (EMW) along the fiber. The two components that is the electric field and magnetic field form pattern across the fiber. These patterns are called modes of transmission.

Modes: The mode of a fiber tells us about the no of path for a light ray within the optical fiber. It is obtained by solving Maxwell's equation. Modes referred to no of path.

According to no of modes optical fiber can be divided into two types

- (i) Single mode fiber
- (ii) Multi mode fiber

1. Single mode fiber

Single mode fiber allows propagation of light rays by

Only one path. They are best for Retaining the fidelity of each light pulse over longer distance. They do not show dispersion.

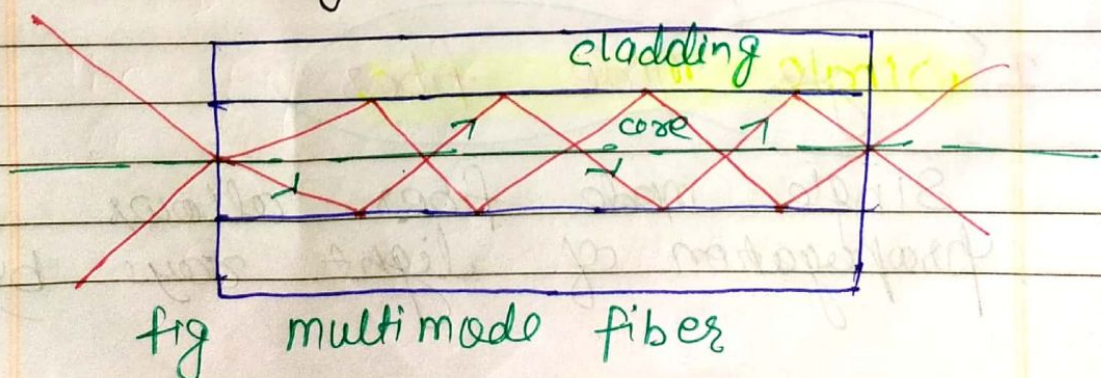
Feb 15 → The radius of core is very small in single mode fiber that is it is order of $10\mu\text{m}$.

The launching of optical power in single mode fiber is very difficult because radius is very small. It supports larger bandwidth. It is mainly used for longer distance.

ii)

Multimode fiber

Allow light rays to propagate by multiple path. It has larger diameter as compared to single mode fiber.



- The Launching of optical power is easier in multimode fiber as compared to single mode fiber.
- It supports laser bandwidth and used for short distance communication
ex: LAN, WAN
- It is not suitable for integrated optical technology
- It is more expensive than single mode fiber

⇒ Fiber Profile Index Profile

A fiber characterise by its profile and by its core and cladding diameter. One way of classifying the fiber cable is according to the index profile. the index profile is graphical representation of value of refractive index across the core diameter. there are two basic type of index profile.

1. Step index fiber
2. Graded index fiber

17 Feb

1. Step index fiber

1. Step index fiber is a cylindrical waveguide with central or inner ~~core~~ core has a uniform refractive index of n_1 , and the core is surrounded by outer cladding with uniform refractive index n_2 . The cladding R.I (n_2) is less than core R.I (n_1) but there is an abrupt change in the R.I at the core-clad cladding interface. (अचानक)

The Refractive index is Plotted on horizontal axis and Radial Distance from the core Plotted on vertical axis

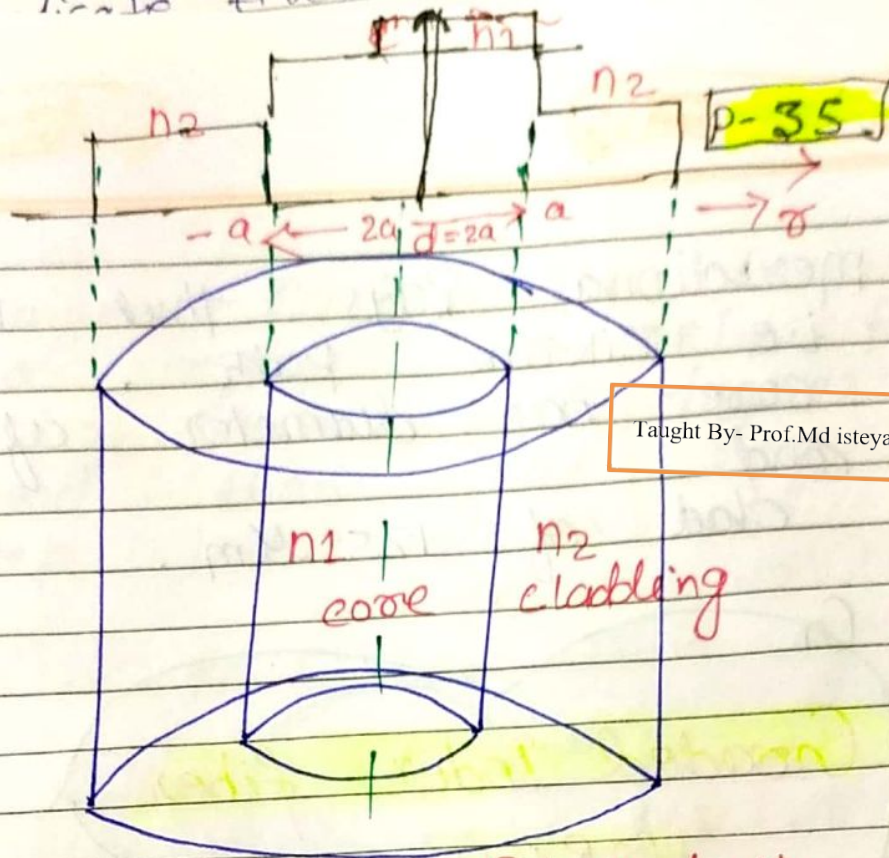


Fig: step index fiber with index profile

$$n(r) = \begin{cases} n_1 ; |r| < a \text{ [core]} \\ n_2 ; |r| \geq a \text{ [cladding]} \end{cases}$$

r = radial distance from fiber axis

a = radius of core

n_1 = R.I of core

n_2 = R.I of cladding

The propagation of light wave within the core of step index fiber takes the path of

meridional Rays that is
i.e. Zik-zak path.
core has diameter of 50-80 μm
and clad of 125 μm .

2. Graded Index fiber (GRIN)

In graded index fiber the R.I is not uniform in the core, it is highest at the centre and decreases smoothly and continuously with distance toward the cladding.

The R.I profile across the core takes the parabolic nature. The core of GRIN fiber is made from many layers of glass.

The velocity of light wave

Changes continuously in the
 Core region. In GRIN fiber
 data travels at higher
 Speed than that of
 Step Index fiber

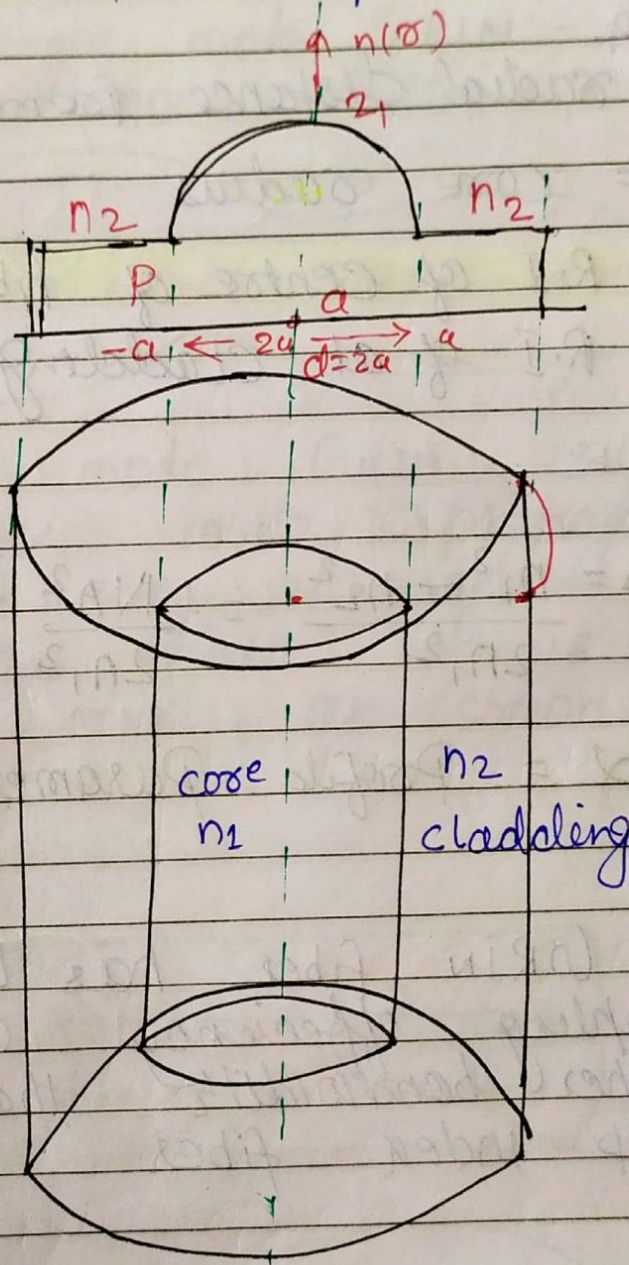


fig GRIN fiber with Index Profile

$$n(r) = \begin{cases} n_1 (1 - 2\Delta \left(\frac{r}{a}\right)^2)^{1/2} ; |r| < a \text{ [core]} \\ n_2 (1 - 2\Delta)^{1/2} = n_2 ; |r| \geq a \text{ [cladding]} \end{cases}$$

Where

r = radial distance from axis

a = core radius

n_1 = R.I of centre of fiber core

n_2 = R.I of cladding

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} = \frac{NA^2}{2n_1^2}$$

Δ = Profile Parameter

The GRIN fiber has lower coupling efficiency and higher bandwidth than the step index fiber.

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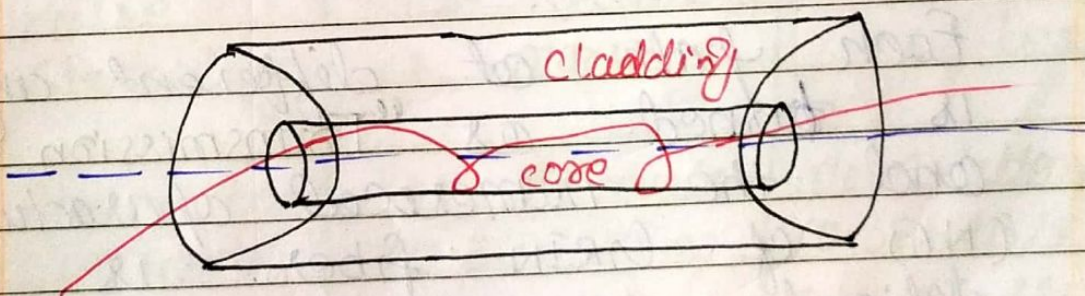
Depending upon no of modes

Graded index optical fiber are classified as

1. Single mode GRIN - fiber
2. Multimode GRIN - fiber

1. Single mode GRIN fiber

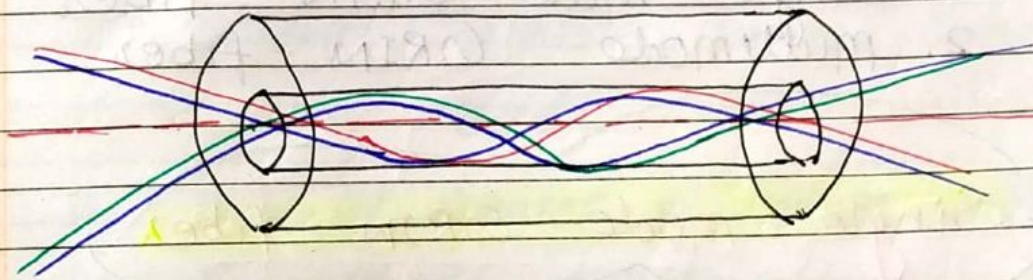
Single mode GRIN supports single mode propagation, Light ray propagates in helical path it is called skew rays as shown in fig.



2. Multimode GRIN fiber

In multimode GRIN fiber

multiple ways can be transmitted simultaneously through the fiber core



In core size in multimode GRIN fiber lies b/w 50 to 100 μm range. the light ray enters the fiber at many different angle. this fiber is mostly used for long distance communication.

Each path at different angle is termed as "Transmission mode" and the numerical aperture (NA) of GRIN fiber is defined as the maximum value of acceptance angle at the fiber axis (θ_0).

S. N	Parameter	Step Index fiber	Graded Index fiber
1.	Data rate	Slow	Fast
2.	Coupling Efficiency	Higher coupling efficiency	Lower coupling Efficiency
3.	PoI of core	Constant through out core	Not constant
4.	Ray path	Zik-Zak path	Helical path
5.	Numerical aperture (NA)	Remains Same	Changes continuously from fiber axis
6.	material used	Normally Plastic or Glass	only glass is used
7.	Bandwidth Efficiency	10-20 MHz/Km	1 GHz / Km
8.	Pulse Spreading	Pulse spreading is more	Pulse spreading is less
9.	Attenuation (कमी अंश)	Less Attenuation (0.34 dB/Km) at 1.3 μ m	More attenuation (0.621 dB/Km) at 1.3 μ m
10	Light source	LED	LED, Laser
11	Application	Local network application	Network

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V- numbers and numbers of guided modes

V- numbers - V number decides number of modes of an optical fiber. It is also called normalised frequency. It provides relationship between operating wavelength (λ) core radius (a) and numerical aperture (NA) of the optical fiber.

V- number is defined as,

$$V = \sqrt{U^2 + W^2}$$

Where,

U = Radial Propagation Constant

W = Cladding decay parameter

Radial propagation is defined as,

$$V = a \sqrt{n_1^2 k^2 - \beta^2}$$

Where,

a = Radius of core

n_1 = Refractive index of core

$$k = \frac{2\pi}{\lambda}$$

β = propagation constant

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Cladding decay parameter is defined as.

$$W = \alpha \sqrt{\beta^2 - n_2^2 K^2}$$

Where,

n_2 = Refractive index of cladding

So

V-numbers will be

$$V = \sqrt{\alpha^2 (n_1^2 K^2 - \beta^2) + \alpha^2 (\beta^2 - n_2^2 K^2)}$$

$$V = \sqrt{\alpha^2 n_1^2 K^2 - \alpha^2 \beta^2 + \alpha^2 \beta^2 - \alpha^2 n_2^2 K^2}$$

$$V = \sqrt{\alpha^2 n_1^2 K^2 - \alpha^2 n_2^2 K^2}$$

$$V = \alpha K \sqrt{n_1^2 - n_2^2}$$

$$V = \frac{2\pi\alpha}{\lambda} \sqrt{n_1^2 - n_2^2}$$

Numerical Aperture,

$$NA = \sqrt{n_1^2 - n_2^2} = n_1 (2\Delta)^{1/2}$$

$$V = \frac{2\pi\alpha}{\lambda} NA$$

$$V = \frac{2\pi\alpha}{\lambda} n_1 (2\Delta)^{1/2}$$

V-number is useful find the no of guided modes in multimode fiber.

⇒ no of guided modes $[M_g]$

No of guided modes in multimode step index fiber is, $M_g = \frac{V^2}{2}$

No of guided modes in multimode Graded index fiber (GRIN) is,

$$M_g = \left(\frac{\alpha}{\alpha + 2} \right) \frac{V^2}{2}$$

Where,

α = Profile parameter

For Parabolic variation of Refractive index (R.I) profile

$$\alpha = 2.$$

then

$$M_g = \frac{V^2}{4}$$

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⇒ Relationship between No of modes and V-Numbers

The number of mode is given by m

$$m = \frac{2A}{\lambda^2} \Omega$$

Where,

$$A = \text{Area of core} = \pi a^2$$

$$\Omega = \text{Solid Acceptance angle} = \pi \theta_a^2$$

We know that Numerical aperture

$$NA = \sqrt{n_1^2 - n_2^2} = \sin \theta_a$$

For small value of θ_a

$$\sin \theta_a \approx \theta_a$$

So, NA will be

$$\sqrt{n_1^2 - n_2^2} = \theta_a$$

So,

no of modes will be

$$m = \frac{2\pi a^2}{\lambda^2} \pi (n_1^2 - n_2^2)^{\frac{1}{2}}$$

$$= \frac{2\pi^2 a^2}{\lambda^2} (n_1^2 - n_2^2)$$

$$= \frac{1}{2} \left[\frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} \right]^2$$

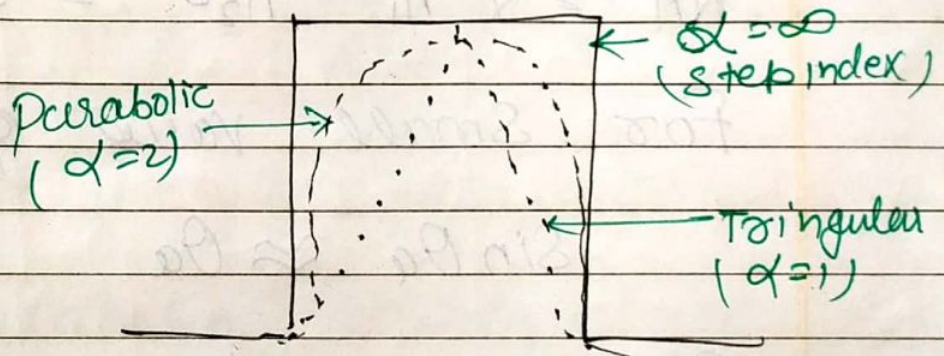
$$\left(V = \frac{2\pi a}{\lambda} \text{ NA} \right)$$

$$= \frac{V^2}{2} \text{ (no of modes for step index)}$$

for GRIN fiber

$$m_g = \left(\frac{\alpha}{\alpha+2} \right) \frac{V^2}{2}$$

Where $\alpha = \text{R.I profile}$



Parabolic, $\alpha = 2$

$$m_g = \left(\frac{\alpha}{\alpha+2} \right) \frac{V^2}{2}$$

$$\frac{2}{2+2} \left(\frac{V^2}{2} \right)$$

$$m_g = \frac{2}{4} \left(\frac{V^2}{2} \right) = \frac{V^2}{4}$$

Triangular, $\alpha = 1$

$$m_g = \left(\frac{\alpha}{\alpha+2} \right) \frac{V^2}{2}$$

$$= \left(\frac{1}{1+2} \right) \frac{v^2}{2}$$

$$mg = \frac{1}{3} \left(\frac{v^2}{2} \right) = \frac{v^2}{6}$$

$$\boxed{mg = \frac{v^2}{6}}$$

Step
Index
Put $\alpha = \infty$

$$= \left(\frac{\frac{\alpha}{\infty}}{\frac{\alpha}{\infty} + 2} \right) \frac{v^2}{2}$$

$$= \left(\frac{1}{1 + \frac{2}{\infty}} \right) \frac{v^2}{2}$$

$$= \left(\frac{1}{1+0} \right) \frac{v^2}{2}$$

$$mg = \frac{1}{1} \left(\frac{v^2}{2} \right) = \frac{v^2}{2}$$