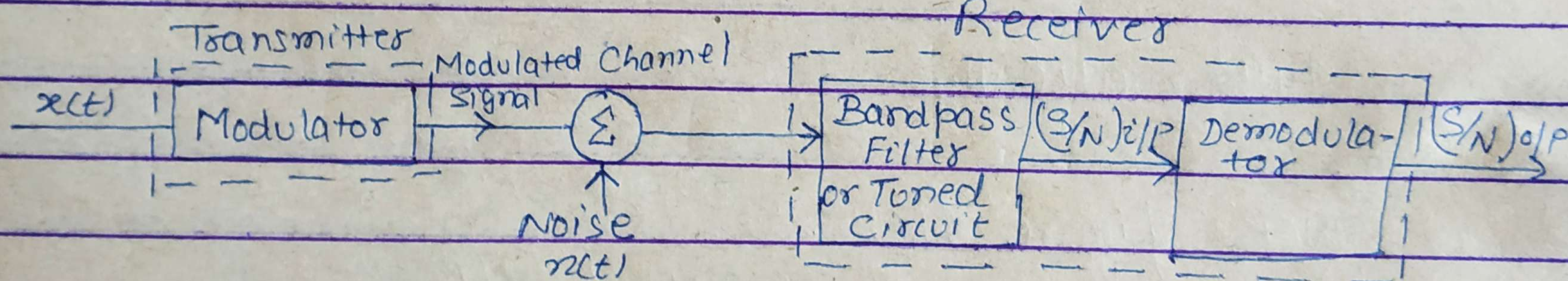
Noise in Communication System

In communication system, the message signal travels from the transmitter to the receiver through a medium called channel. Now, noise is present in every communication system. The channel introduces an additive noise in the message signal and thus the message which is received at the receiver is distorted. Since, the receiver detects both message signal and the noise signal, it will reproduce a message signal which contains noise.

The noise calculation in a communication system is carried out in the form of a parameter which is known as figure of merit, denoted by 'γ'.

$$\gamma = \frac{\text{Output signal to noise ratio}}{\text{Input signal to noise ratio}} = \frac{(SNR)_{o/p}}{(SNR)_{i/p}}$$

Communication system with higher 'γ' will have a better noise performance i.e the adverse effect of noise is small.



Generalised equivalent model of a communication system for noise calculation.

Few assumptions to calculate the Figure of Merit for various communication system

i) Channel Noise is always white and Gaussian :- The channel noise n(t) is always white noise. This means that it is uniformly distributed over the entire band of frequencies under consideration.

The power spectrum density of channel noise will be uniform over the frequency range under consideration.

$\therefore$ Total Noise power $= \frac{\eta}{2} \times$ Bandwidth.

ii> Channel Noise is always additive :- The disturbing effect of channel noise is always additive.

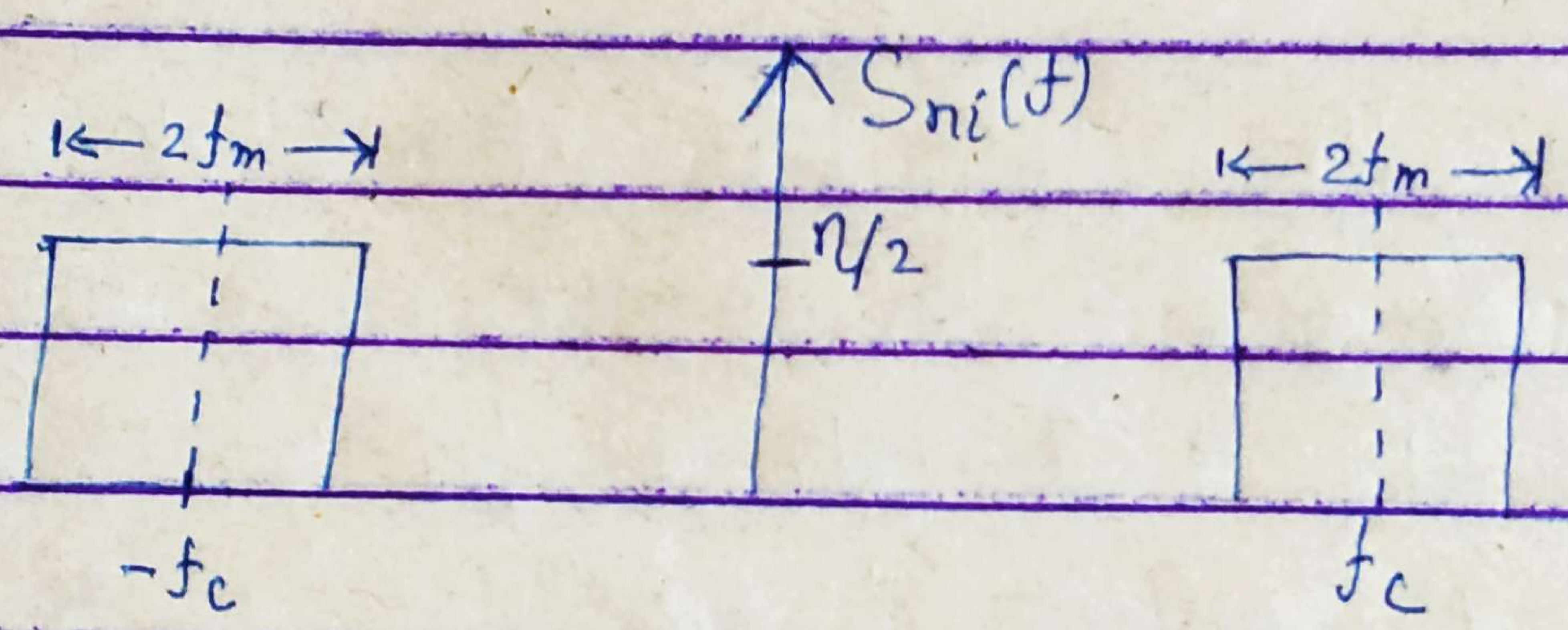
iii> The Noise at the Input of demodulator is a Bandpass noise :- Receiver is a tuned circuit which works as a bandpass filter (BPF). The function of this tuned circuit is to allow only a narrowband signal centred about carrier frequency $\pm f_c$ and rejects all other frequencies. This means that the noise signal lying outside this range is also rejected and thus the bandwidth of the noise signal at the input of detector will be same as that of the incoming modulated signal.

But the assumption (i) says that the channel noise is white in nature. So, the noise at the input of the demodulator will have a

Constant or uniform power spectrum density over the passband. This power spectrum density of the white noise at the input of demodulator is given by

$$S_{ni} = \eta/2$$

the passband for amplitude modulation is twice the maximum modulating frequency f_m around the carrier frequency f_c



iv) The Noise Input Power N_i :- To calculate the figure of merit γ, we evaluate the noise input power N_i for passband or for bandwidth of incoming modulated signal ($2f_m$ in case of AM).

Using assumption (i), the total Noise power N

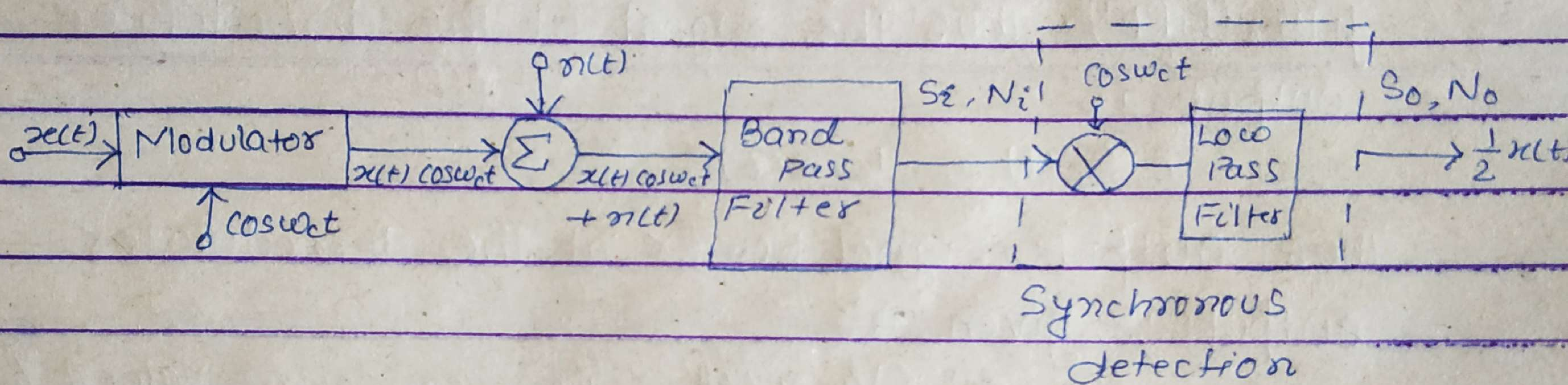
$$N = \frac{\eta}{2} \times \text{Bandwidth}$$

∴ Input noise power N_i at the input of demodulator for AM

$$N_i = \frac{\eta}{2} \times 2f_m = \eta f_m$$

Noise Figure Calculation in Double Sideband Suppressed Carrier (DSB-SC) System

In order to calculate Figure of merit for DSB-SC system, we need to redraw the DSB-SC system from noise point of view.



i) Input Signal Power :- The modulated (DSB-SC) signal at the input of the demodulator is given as

$$S_i(t) = x(t) \cos\omega_c t$$

where $x(t)$ is modulating signal and $\cos\omega_c t$ is the carrier signal.

input signal power S_i is given as the mean square value of signal $s(t) = x(t)\cos\omega_c t$

Hence $$S_i = \overline{\{x(t)\cos\omega_c t\}^2}$$

$$S_i = \frac{1}{2}\overline{x^2(t)}$$

ii> Output Signal Power :- In synchronous detection method, the incoming DSB-SC signal at the detector input is multiplied by a synchronous carrier $\cos\omega_c t$. Thus, the signal at the multiplier output is

$$e(t) = x(t)\cos\omega_c t \cdot \cos\omega_c t$$
$$= \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos 2\omega_c t$$

This signal is finally passed through the low pass filter (LPF). Thus the signal at the demodulator output is

$$S_o(t) = \frac{1}{2}x(t)$$

Thus output signal power at the demodulator output is given as

$$S_o = \frac{1}{4}\overline{x^2(t)}$$ i.e mean square value of $\frac{1}{2}x(t)$

iii> Input Noise Power :- The noise at the input of the receiver is white and Gaussian in nature. This noise after passing through the bandpass filter (BPF) is converted into a band pass noise.

The mathematical expression for band pass noise is given as

$$n_i(t) = n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t$$

where $n_c(t)$ is Inphase noise component
$n_s(t)$ is quadrature noise component

$n_c(t)$ and $n_s(t)$ are equal in magnitude
i.e $|n_c(t)| = |n_s(t)|$
however only difference is
$n_c(t)$ and $n_s(t)$ are ~~out~~ $\pi/2$ phase
Shifted.

∴ input noise power at the input of demodulator is given as

$$N_i = \overline{n_i^2(t)} = \overline{n_c^2(t)} = \overline{n_s^2(t)}$$

iv) Output Noise Power :- During synchronous detection, Signal as well as noise will also get multiplied by synchronous carrier $\cos\omega_c t$.
Therefore mutiplied noise is given as

$$n_d(t) = \{n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t\}\cos\omega_c t$$
$$n_d(t) = n_c \cos^2\omega_c t - n_s(t)\sin\omega_c t \cos\omega_c t$$
$$n_d(t) = \frac{1}{2} n_c(t)(2\cos^2\omega_c t) - \frac{1}{2} n_s(t)(2\sin\omega_c t\cos\omega_c t)$$
$$n_d(t) = \frac{1}{2} n_c(t)(1+\cos 2\omega_c t) - \frac{1}{2} n_s(t)\sin 2\omega_c t$$
$$n_d(t) = \frac{1}{2} n_c(t) + \frac{1}{2} n_c(t)\cos 2\omega_c t - \frac{1}{2} n_s(t)\sin 2\omega_c t$$

This multiplied noise signal will pass through Low pass Filter. $\cos 2\omega_c t$ & $\sin 2\omega_c t$ component will be filtered out and the final noise $n_o(t)$

at the output of the demodulator will be

$$n_o(t) = \frac{1}{2} n_c(t)$$

So, the output noise power N_o will be

N_o = mean square of $n_o(t)$

$$N_o = \frac{1}{4} \overline{n_c^2(t)}$$

Figure of merit $\gamma = \frac{(SNR)_{o/p}}{(SNR)_{i/p}}$

$$\boxed{\cancel{\gamma = \frac{S_o N_i}{N_o S_i}} \qquad \cancel{\gamma = \frac{1/2\, \overline{x^2(t)}}{1/4\, \overline{x^2(t)}} \Big/ \frac{\overline{n_c^2(t)}}{1/4\, \overline{n_c^2(t)}} = \frac{4}{2}}}$$

$$\gamma = \frac{S_o \cdot N_i}{S_i \cdot N_o}$$

$$\frac{S_o}{S_i} = \frac{1/4\, \overline{x^2(t)}}{1/2\, \overline{x^2(t)}} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{N_i}{N_o} = \frac{\overline{n_c^2(t)}}{1/4\, \overline{n_c^2(t)}} = 4$$

$$\gamma = \frac{S_o}{S_i} \times \frac{N_i}{N_o} = \frac{1}{2} \times 4 = 2$$

Noise Figure Calculation of Single Sideband Suppressed Carrier (SSB-SC) System

The receiver of an SSB-SC system is similar to that of DSB-SC system for synchronous detection except for the fact that the bandwidth of the bandpass filter of SSB-SC receiver is ½ of that required for DSB-SC.

i) Input Signal Power :- The incoming signal to the SSB-SC receiver is given as

$$S_{SSB}(t) = x(t)\cos\omega_c t \pm x_h(t)\sin\omega_c t$$

where $x(t)$ is the modulating signal
$x_h(t)$ is hilbert transformed signal of $x(t)$

Magnitude of $x(t)$ is ~~eqal~~ equal to $x_h(t)$
i.e $|x(t)| = |x_h(t)|$

therefore,
Signal power available at the input of demodulator is given by

$$S_i = \overline{S^2_{SSB}(t)}$$

$$= \overline{(x(t)\cos\omega_c t \pm x_h(t)\sin\omega_c t)^2}$$

$$= \frac{1}{2}\overline{x^2(t)} + \frac{1}{2}\overline{x_h^2(t)}$$

$$S_i = \overline{x^2(t)} \qquad \{\because |x(t)| = |x_h(t)|$$

ii> Output Signal Power :- The incoming signal is multiplied by a synchronous local carrier $\cos\omega_c t$, therefore, the output will be

$$e(t) = \{x(t)\cos\omega_c t \pm x_h(t)\sin\omega_c t\}\cos\omega_c t$$

$$e(t) = x(t)\cos^2\omega_c t \pm x_h(t)\sin\omega_c t\cos\omega_c t$$

$$e(t) = \frac{1}{2}x(t)\,2\cos^2\omega_c t \pm \frac{1}{2}x_h(t)\,2\sin\omega_c t\cos\omega_c t$$

$$e(t) = \frac{1}{2}x(t)(1+\cos 2\omega_c t) \pm \frac{1}{2}x_h(t)\sin 2\omega_c t$$

$$e(t) = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos 2\omega_c t \pm \frac{1}{2}x_h(t)\sin 2\omega_c t$$

finally this $e(t)$ will be passed through the low pass filter; all the terms containing $\cos 2\omega_c t$ & $\sin 2\omega_c t$ will be rejected.

So, final demodulated output signal is

$$S_o(t) = \tfrac{1}{2}x(t)$$

Hence, Output signal power of the demodulator

$$S_o = \tfrac{1}{4}\overline{x^2(t)}$$

iii> Input Noise Power :- The noise at the input of the receiver is white and Gaussian. This noise after passing through the bandpass filter (BPF) is converted into a bandpass noise.

The input bandpass noise is given as

$$n_i(t) = n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t$$

The input noise power at the input of demodulator is given as

$$N_i = \overline{n_i^2(t)} = \overline{n_c^2(t)} = \overline{n_s^2(t)}$$

iv) Output Noise Power :- Noise signal at the input of the demodulator is a bandpass noise and is expressed as

$$n(t) = n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t$$

- noise $n(t)$ is multiplied by synchronous carrier signal $\cos\omega_c t$ in demodulator, multiplied noise signal is given as

$$n_d(t) = \{n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t\}\cos\omega_c t$$

$$n_d(t) = n_c(t)\cos^2\omega_c t - n_s(t)\sin\omega_c t\cos\omega_c t$$

$$n_d(t) = \tfrac{1}{2}n_c(t)\,2\cos^2\omega_c t - \tfrac{1}{2}n_s(t)\,2\sin\omega_c t\cos\omega_c t$$

$$n_d(t) = \tfrac{1}{2}n_c(t)(1+\cos 2\omega_c t) - \tfrac{1}{2}n_s(t)\sin 2\omega_c t$$

$$n_d(t) = \tfrac{1}{2}n_c(t) + \tfrac{1}{2}n_c(t)\cos 2\omega_c t - \tfrac{1}{2}n_s(t)\sin 2\omega_c t$$

this noise signal would pass through low pass filter, which will reject $\cos 2\omega_c t$ & $\sin 2\omega_c t$ term.

So, noise signal available at the output of demodulator will be

$$n_o(t) = \tfrac{1}{2}n_c(t)$$

So Output noise signal power

$$N_o = \tfrac{1}{4}\overline{n_c^2(t)}$$

$$\therefore \gamma = \frac{S_o}{S_i}\cdot\frac{N_i}{N_o} = \frac{1}{4}\times 4 = 1$$