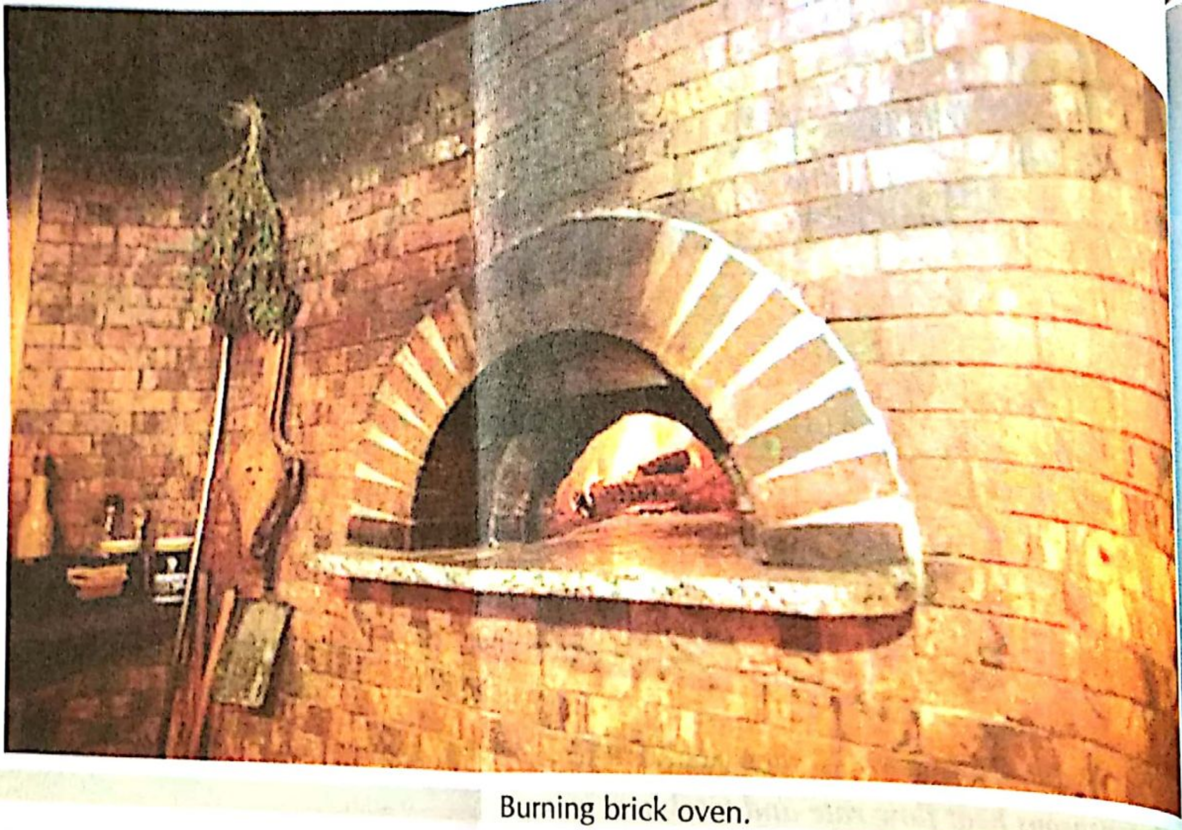
[illegible] ...[4.8 (a)]

... in terms of non-dimensional B_i and F_0 number.

Example 4.1. *A 50 cm × 50 cm copper slab 6.25 mm thick has a uniform temperature of 300°C. Its temperature is suddenly lowered to 36°C. Calculate the time required for the plate to reach the temperature of 108°C.*

Take $\rho = 9000\ kg/m^3$; $c = 0.38\ kJ/kg°C$; $k = 370\ W/m°C$ *and* $h = 90\ W/m^2°C$



Burning brick oven.

Solution. Surface area of plate, $A_s = 2 \times 0.5 \times 0.5 = 0.5\ m^2$ (two sides)

Volume of plate, $V = 0.5 \times 0.5 \times 0.00625 = 0.0015625\ m^3$

Characteristic length, $L_c = \frac{V}{A_s} = \frac{0.0015625}{0.5} = 0.003125\ m$

Biot number, $Bi = \frac{hL_c}{k} = \frac{90 \times 0.003125}{370} = 7.6 \times 10^{-4}$

Since Bi is *less than 0.1*, hence lumped capacitance method (Newtonian heating or cooling) may be applied for the solution of the problem.

The temperature distribution is given by

$$\frac{t - t_a}{t_i - t_a} = \exp\left[\frac{-hA_s}{\rho Vc}\tau\right] \quad \text{...[Eqn. (4.4)]}$$

Substituting the values, we get

$$\frac{108 - 36}{300 - 36} = \exp\left[-\frac{90 \times 0.5}{9000 \times 0.0015625 \times (0.38 \times 1000)}\tau\right] = e^{-0.00842\tau}$$

$$0.2727 = e^{-0.00842\tau} = \frac{1}{e^{0.00842\tau}}$$

or, $$e^{0.00842\tau} = \frac{1}{0.2727} = 3.667$$

or, $$0.00842\ \tau = ln\ 3.667 = 1.2994$$

or, $$\tau = \frac{1.2994}{0.00842} = \mathbf{154.32\ s}\ \textbf{(Ans.)}$$

Example 4.2. *An aluminium alloy plate of 400 mm × 400 mm × 4 mm size at 200°C is suddenly quenched into liquid oxygen at – 183°C. Starting from fundamentals or deriving the necessary expression determine the time required for the plate to reach a temperature of – 70°C. Assume $h = 20000\ kJ/m^2\text{-}h\text{-}°C$, $c_p = 0.8\ kJ/kg°C$, and $\rho = 3000\ kg/m^3$.*

(AMIE Winter, 1997)

Solution. Surface area of the plate, $A_s = 2 \times \frac{400}{1000} \times \frac{400}{1000} = 0.32 \text{ m}^2$

Volume of the plate, $V = \frac{400}{1000} \times \frac{400}{1000} \times \frac{4}{1000} = 0.00064 \text{ m}^3$

Characteristic length, $L_c = \frac{V}{A_s} = \frac{0.00064}{0.32} = 0.002 \text{ m}$

k for aluminium, at low temperatures may be taken as 214 W/m°C or 770.4 kJ/mh°C.

$\therefore$ Biot number, $Bi = \frac{hL_c}{k} = \frac{20000 \times 0.002}{770.4} = 0.0519$

Since Bi is less than 0.1, hence lumper capacitance method may be applied for the solution of the problem.

The temperature distribution is given by

$$\frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA_s}{\rho V c}\tau\right] \quad \text{...[Eqn. (4.4)]}$$

(For derivation of this relation please refer to Article 4.2)

$$\frac{-70-(-183)}{200-(-183)} = \exp\left[-\frac{20000 \times 0.32}{3000 \times 0.00064 \times 0.8} \cdot \tau\right]$$

or, $$0.295 = e^{-4166.67\,\tau}$$

$$= \frac{1}{e^{4166.67\,\tau}}$$

or, $$e^{4166.67\,\tau} = \frac{1}{0.295} = 3.389$$

or, $$4166.67\,\tau = ln\ 3.389 = 1.2205$$

$\therefore$ $$\tau = \frac{1.2205}{4166.67} \times 3600 = \mathbf{1.054\ s} \ \textbf{(Ans.)}$$

Example 4.3. *A solid copper sphere of 10 cm diameter [$\rho = 8954$ kg/m^3, $c_p = 383$ J/kg K, $k = 386$ W/m K], initially at a uniform temperature $t_i = 250$°C, is suddenly immersed in a well-stirred fluid which is maintained at a uniform temperature $t_a = 50$°C. The heat transfer coefficient between the sphere and the fluid is $h = 200$ W/m^2 K. Determine the temperature of the copper block at $\tau = 5$ min after the immersion.* **(AMIE Winter, 1998)**

Solution. *Given* : $D = 10$ cm $= 0.1$ m; $\rho = 8954$ kg/m^3; $c_p = 383$ J/kg K; $k = 386$ W/m K; $t_i = 250$°C; $t_a = 50$°C; $h = 200$ W/m^2 K; $\tau = 5$ min $= 300$ s.

Temperature of the copper block, t :

The characteristic length of the sphere is,

$$L_c = \frac{\text{Volume }(V)}{\text{Surface area }(A_s)} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} = \frac{D}{6} = \frac{0.1}{6} = 0.0167 \text{ m}$$

Biot number, $$Bi = \frac{hL_c}{k} = \frac{200 \times 0.01667}{386} = 8.64 \times 10^{-3}$$

Since Bi is less than 0.1, hence lump capacitance method (Newtonian heating or cooling) may be applied for the solution of the problem.

The temperature distribution is given by

$$\frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA_s}{\rho V c} \cdot \tau\right] \quad \text{...[Eqn. (4.4)]}$$

Substituting the value, we get

$$\frac{t-50}{250-50} = \exp\left[-\frac{200}{8954 \times 0.01667 \times 383} \times 300\right] = 0.35$$

$$\left(\because \frac{A_s}{V} = \frac{L}{L_c} = \frac{1}{0.01667}\right)$$

$\therefore$ $t = (250 - 50) \times 0.35 + 50 =$ **120°C (Ans.)**

Example 4.4. *An average convective heat transfer coefficient for flow of 90°C air over a flat plate is measured by observing the temperature time history of a 40 mm thick copper slab (ρ = 9000 kg/m³, c = 0.38 kJ/kg°C, k = 370 W/m°C) exposed to 90°C air. In one test run, the initial temperature of the plate was 200°C, and in 4.5 minutes the temperature decreased by 35°C. Find the heat transfer coefficient for this case. Neglect internal thermal resistance.*

Solution. *Given* : $t_a = 90°C$; $L = 40$ mm or 0.04 m; $\rho = 9000$ kg/m³; $c = 0.38$ kJ/kg°C; $t_i = 200°C$; $t = 200 - 35 = 165°C$; $\tau = 4.5$ min = 270s

Characteristic length, $L_c = \frac{L}{2} = \frac{0.04}{2} = 0.02$ m

$$\frac{hA_s}{\rho Vc} = \frac{h}{\rho (V/A_s)c} = \frac{h}{\rho c L_c} = \frac{h}{9000 \times (0.38 \times 1000) \times 0.02} = 1.462 \times 10^{-5}\, h$$

Now,
$$\frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA_s}{\rho Vc}\tau\right] \quad \text{...[Eqn. (4.4)]}$$

or,
$$\frac{165 - 90}{200 - 90} = e^{-(1.462 \times 10^{-5} h) \times (270)} = e^{-0.003947h} = \frac{1}{e^{0.003947\, h}}$$

or,
$$0.682 = \frac{1}{e^{0.003947\, h}} \text{ or } e^{0.003947 h} = 1.466$$

or,
$$0.003947\, h = ln\, 1.466 = 0.3825$$

$\therefore$
$$h = \frac{0.3825}{0.003947} = \mathbf{96.9\ W/m^2\,°C\ (Ans.)}$$

Example 4.5. *The heat transfer coefficients for the flow of air at 28°C over a 12.5 mm diameter sphere are measured by observing the temperature-time history of a copper ball of the same dimension. The temperature of copper ball (c = 0.4 kJ/kg K and ρ = 8850 kg/m³) was measured by two thermocouples, one located in the centre and other near the surface. Both the thermocouples registered the same temperature at a given instant. In one test the initial temperature of the ball was 65°C and in 1.15 minute the temperature decreased by 11°C. Calculate the heat transfer coefficient for this case.*

(AMIE Winter, 2001)

Solution. *Given* : $t_a = 28°C$; R (sphere) $= \frac{12.5}{2} = 6.25$ mm = 0.00625 m; $c = 0.4$ kJ/kg°C; $\rho = 8850$ kg/m³; $t_i = 65°C$; $t = 65 - 11 = 54°C$; $\tau = 1.15$ min = 69 s.

Heat transfer coefficient, *h*:

Biot number, $B_i = \frac{hL_c}{k} = \frac{h \cdot (R/2)}{k}$

Since *heat transfer coefficient has to be calculated, so assume that the internal resistance is negligible and B_i is less than 0.1.*

Using eqn. (4.4), we have

$$\frac{\theta}{\theta_i} = \frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA_s}{\rho Vc}\tau\right] = e^{-B_i F_0}$$

or, $ln\left[\frac{t - t_a}{t_i - t_a}\right] = -\frac{hA_s}{\rho Vc} \cdot \tau$

or, $h = \frac{\rho Vc}{A_s \tau} \, ln\left[\frac{t_i - t_a}{t - t_a}\right]$

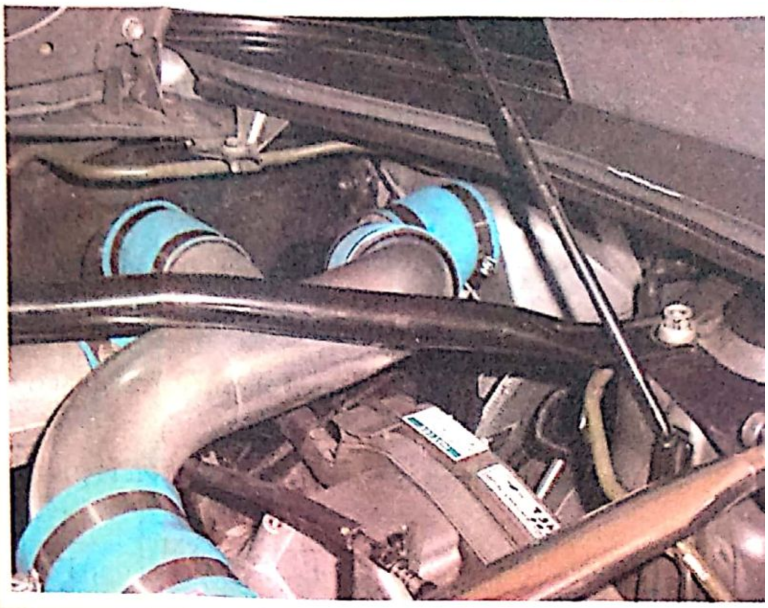
or, $h = \left(\frac{\frac{4}{3}\pi R^3}{4\pi R^2}\right)\frac{\rho c}{\tau} \, ln\left[\frac{t_i - t_a}{t - t_a}\right]$

or, $h = \frac{R}{3}\frac{\rho c}{\tau} \, ln\left[\frac{t_i - t_a}{t - t_a}\right]$

Substituting the proper values, we have

$$h = \frac{0.00625}{3} \times \frac{8850 \times (0.4 \times 1000)}{69}$$

$$ln\left[\frac{(65 - 28)}{(54 - 28)}\right] = \mathbf{37.71\ W/m^2K} \quad \textbf{(Ans.)}$$



Cooling of IC engine.

Example 4.6. *A steel ball 50 mm in diameter and at 900°C is placed in still atmosphere of 30°C. Calculate the initial rate of cooling of the ball in °C/min.*

Take: ρ = 7800 kg/m³, c = 2 kJ/kg°C (for steel); h = 30 W/m²°C.

Neglect internal thermal resistance. **(M.U.)**

Solution. *Given*: $R = \frac{50}{2} = 25$ mm = 0.025 m; $t_i = 900°C$; $t_a = 30°C$, $\rho = 7800$ kg/m³;

C = 2 kJ/kg°C; h = 30W/m²°C; τ = 1 min = 60 s.

The temperature variation in the ball (with respect to time), neglecting internal thermal resistance, is given by:

$$\frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA_s}{\rho Vc}\tau\right] \quad \text{...[Eqn. (4.4)]}$$

where, $\frac{hA_s}{\rho Vc} \cdot \tau = \frac{h \times 4\pi R^2}{\rho \times \frac{4}{3}\pi R^3 \times c}\tau = \frac{3h\tau}{\rho Rc} = \frac{3 \times 30 \times 60}{7800 \times 0.025 \times (2 \times 1000)} = 0.01385$

Substituting the values in the above equation, we get

$$\frac{t - 30}{900 - 30} = e^{-0.01385} = \frac{1}{e^{0.01385}} = 0.9862$$

or, $t = 30 + 0.9862\ (900 - 30) = 888°C$

$\therefore$ Rate of cooling = 900 – 888 = **12°C/min. (Ans.)**

Example 4.7. *A cylindrical ingot 10 cm diameter and 30 cm long passes through a heat treatment furnace which is 6 m in length. The ingot must reach a temperature of 800°C before it comes out of the furnace. The furnace gas is at 1250°C and ingot initial temperature is 90°C. What is the maximum speed with which the ingot should move in the furnace to attain the required temperature? The combined radiative and convective surface heat transfer coefficient is 100 W/m² °C. Take k (steel) = 40 W/m°C and α (thermal diffusivity of steel) = 1.16×10^{-5} m²/s.*

Solution. *Given* : D = 10 cm = 0.1 m; L = 30 cm = 0.3 m; t_i = 1250°C; t = 800°C; t_a = 90°C; k = 40 W/m°C; h = 100 W/m²°C; $\alpha = 1.16 \times 10^{-5}$ m²/s.

Characteristic length, $L_c = \frac{V \text{ (volume)}}{A_s \text{ (surface area)}} = \frac{\frac{\pi}{4} D^2 L}{\left[\pi DL + \frac{\pi}{4} D^2 \times 2\right]} = \frac{DL}{4L + 2D}$

$$= \frac{0.1 \times 0.3}{4 \times 0.3 + 2 \times 0.1} = 0.02143 \text{ m}$$

Biot number, $Bi = \frac{h L_c}{k} = \frac{100 \times 0.02143}{40} = 0.0536$

As $Bi < 0.1$, then internal thermal resistance of the ingot for conduction heat flow can be neglected.

∴ The time versus temperature relation is given as

$$\frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA_s}{\rho Vc}\tau\right]$$

Now,

$$\frac{hA_s}{\rho Vc} = \frac{k}{k} \cdot \frac{hA_s}{\rho Vc} = \left(\frac{k}{\rho c}\right)\left(\frac{h}{k}\right)\left(\frac{A_s}{V}\right) = \alpha \cdot \frac{h}{k} \cdot \frac{A_s}{V}$$

$$= 1.16 \times 10^{-5} \times \frac{100}{40} \times \frac{1}{0.02143} = 0.001353$$

Substituting the values in the above equation, we get

$$\frac{800 - 90}{12 - 90} = e^{-0.001353\tau} = \frac{1}{e^{0.001353\tau}}$$

or, $0.612 = \frac{1}{e^{0.001353\tau}}$ or $e^{0.001353\tau} = 1.634$

or, $0.001353\tau = ln\ (1.634) = 0.491$

∴ $\tau = \frac{0.491}{0.001353} = 362.9$ s

Velocity of ingot passing through the furnace,

$$v = \frac{\text{Furance length}}{\text{Time}} = \frac{6}{362.9} = \mathbf{0.01653\ m/s}$$ **(Ans.)**

Example 4.8. *A 15 mm diameter mild steel sphere (k = 42 W/m°C) is exposed to cooling airflow at 20°C resulting in the convective coefficient h = 120 W/m²°C.*

Determine the following:

(i) Time required to cool the sphere from 550°C to 90°C.

(ii) Instantaneous heat transfer rate 2 minutes after the start of cooling.

(iii) Total energy transferred from the sphere during the first 2 minutes.

For mild steel take: ρ = 7850 kg/m³, c = 475 J/kg°C and α = 0.045 m²/h.

Solution: *Given* : $R = \frac{15}{2} = 7.5$ mm $= 0.0075$ m; $k = 42$ W/m°C; $t_a = 20$°C, $t_i = 550$°C; $t = 90$°C; $h = 120$ W/m²°C.

(i) Time required to cool the sphere from 550°C to 90°C, τ :

The characteristic length L_c is given by,

$$L_c = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} = \frac{0.0075}{3} = 0.0025 \text{ m}$$

Biot number, $Bi = \frac{hl_c}{k} = \frac{120 \times 0.0025}{42} = 0.007143$

Fourier number, $F_0 = \frac{\alpha\tau}{l_c^2} = \frac{0.045 \times \tau}{(0.0025)^2} = 7200\ \tau$ (where τ is in hour)

since $Bi < 0.1$, so we can use lump theory to solve this problem.

The temperature variation with time is given by

$$\frac{t - t_a}{t_i - t_a} = e^{-BiF_o}$$

Substituting the values, we get

$$\frac{90 - 20}{550 - 20} = e^{-0.007143 \times 7200\,\tau} = e^{-51.43\,\tau}$$

$$0.132 = e^{-51.43\,\tau} \quad \text{or} \quad e^{51.43\,\tau} = \frac{1}{0.132} = 7.576$$

or $$51.43\,\tau = 2.025 \quad \text{or} \quad \tau = \frac{2.025}{51.43} = 0.03937\ h = 141.7\ s\ \textbf{(Ans.)}$$

(ii) Instantaneous heat transfer rate 2 minutes (0.0333 h) after the start of cooling, Q_i:

$$Qi = -hA_s\,(t_i - t_a)\ e^{-B_iF_o} \qquad \text{...[Eqn. 4.7(a)]}$$

Now $BiF_o = (0.007143)\ (7200 \times 0.0333) = 1.7126$

$\therefore$ $Q_i = -120 \times 4\,\pi \times (0.0075)^2\ (550 - 20)\ e^{-1.7126}$

$= -$ **8.1W (Ans.)**

The negative sign shows that *heat is given off by sphere.*

(iii) Total energy transferred from the sphere during first 2 minutes (0.0333 h), Q' :

$$Q' = \rho Vc\,(t_i - t_a)\ [e^{-BiF_o} - 1]$$

$$= 7850 \times \frac{4}{3}\pi \times (0.0075)^3\ (475)\ (550 - 20)\ [e^{-1.7126} - 1]$$

$= (-)$ **2862.3 J (Ans.)**

Example 4.9. *The decorative plastic film on copper sphere 10 mm is diameter is cured in an oven at 75°C. After removal from oven, the sphere is exposed to an air stream at 10 m/s and 23°C. Estimate the time taken to cool the sphere to 35°C using Lump theory.*

Use correlation :

$$Nu = 2 + \left[0.4\ (Re)^{0.5} + 0.06\ (Re)^{2/3}\right] (Pr)^{0.4} \left(\frac{\mu_a}{\mu_s}\right)^{0.25}$$

for determination of correlation co-efficient h, use following properties of air and copper :

For copper : $\rho = 8933\ kg/m^3$, $k = 400\ W/mK$, $c_p = 380\ J/kg°\ C$

For air at 23°C : $\mu = 18.16 \times 10^{-6}\ N\text{-}s/m^2$, $\nu = 15.36 \times 10^{-6}\ m^2/s$

$k = 0.0258\ W/m\ K$, $Pr = 0.709$, *and*

$\mu_s = 19.78 \times 10^{-6}\ N\text{-}s/m^2$, *at* 35°C. **(N.M.U., 1998)**

Solution. *Given* : $D = 10$ mm $= 0.01$ m; $t_1 = 75°C$, $V = 10$ m/s; $t_a = 23°C$; $t = 35°C$

Time taken to cool the sphere, τ :

$$Re = \frac{VD}{\nu} = \frac{10 \times 0.01}{15.36 \times 10^{-6}} = 6510$$

$$Nu = 2 + \left[0.4 \times (6510)^{0.5} + 0.06\ (6510)^{2/3}\right] \times (0.709)^{0.4} \times \left(\frac{18.16 \times 10^{-6}}{19.78 \times 10^{-6}}\right)^{0.25}$$

$$= 2 + [32.27 + 20.92] \times 0.87 \times 0.979 = 47.3$$

or, $$Nu = \frac{hD}{k} = 47.3$$

$\therefore$ $$h = \frac{k}{D} \times 47.3 = \frac{0.0258}{0.01} \times 47.3 = 122 \text{ W/m}^2\text{°C}$$

The time taken to cool from 75°C to 35°C may be found from the following relation :

$$\frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA_s}{\rho Vc} \cdot \tau\right]$$

$$\frac{35 - 23}{75 - 23} = \exp\left[-\frac{122 \times 4\pi R^2}{\rho \times \frac{4}{3}\pi R^3 \times c} \cdot \tau\right]$$

$$0.2308 = \exp\left(-\frac{122 \times 3}{8933 \times 0.005 \times 380} \cdot \tau\right) = e^{-0.02156\tau}$$

or, $$e^{0.02156\,\tau} = \frac{1}{0.2308} = 4.333$$

or, $$0.2156\,\tau = 1.466$$

or, $$\tau = \frac{1.466}{0.2156} = \mathbf{68\ s\ (Ans.)}$$

Example 4.10. *An egg with mean diameter of 40 mm and initially at 20°C is placed in a boiling water pan for 4 minutes and found to be boiled to the consumer's taste. For how long should a similar egg for same consumer be boiled when taken from a refrigerator at 5°C. Take the following properties for egg:*

$k = 10$ W/m°C, $\rho = 1200$ kg/m³, $c = 2$ kJ/kg°C and h (heat transfer coefficient) = 100 W/m²°C

Use lump theory.

(N.M.U.)

Solution. *Given* : $R = \frac{40}{2} = 20$ mm = 0.02 m; $t_i = 20$°C; $\tau = 4 \times 60 = 240$ s; $k = 10$ W/m°C; $\rho = 1200$ kg/m³; $c = 2$ kJ/kg°C; $h = 100$ W/m²°C.

For using the lump theory, the required condition is $B_i < 0.1$

$$B_i = \frac{hL_c}{k}$$ where L_c is the characteristic length which is given by,

$$L_c = \frac{V\ \text{(volume)}}{A_s\ \text{(surface area)}} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3}$$

$\therefore$ $$B_i = \frac{h}{k} \times \frac{R}{3} = \frac{100 \times 0.02}{10 \times 3} = 0.067$$

As $B_i < 0.1$, we can use lump theory.

The temperature variation with time is given by :

$$\frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA_s}{\rho Vc}\tau\right] \quad ...(1)$$

$$\frac{hA_s}{\rho Vc} = \left(\frac{h}{\rho c}\right)\left(\frac{A_s}{V}\right) = \left(\frac{100}{1200 \times 2000}\right)\left(\frac{3}{R}\right)$$

$$= \left(\frac{100}{1200 \times 2000}\right)\left(\frac{3}{0.02}\right) = 0.00625$$

Substituting the values in eqn. (1), we get

$$\frac{t - 100}{20 - 100} = e^{-0.00625 \times 240} = e^{-1.50} = \frac{1}{e^{1.50}} = \frac{1}{4.4817} = 0.223$$

or, $t = 100 + (20 - 100) \times 0.223 = 82.16°C$ say 82°C.

Now let us find 'τ' when the given data is : $t_i = 5°C$, $t_a = 100°C$ and $t = 82°C$

Again using eqn. (1), we get

$$\frac{82 - 100}{5 - 100} = e^{-0.00625\,\tau} = \frac{1}{e^{0.00625\,\tau}}$$

or, $0.1895 = \frac{1}{e^{0.00625\,\tau}}$ or $e^{0.00625\tau} = 5.277$

or, $0.00625\,\tau = 1.6633$ or $\tau = \frac{1.6633}{0.00625} = 266.13$ s = **4.435 minutes (Ans.)**