* Dynamic Fluids

→ study of motion of fluid flow along with the force causing the flow.

→ Let us consider fluid is flowing in X- direction.

$$F = m \cdot a$$

or $$\boxed{F_x = m \cdot a_x}$$

① F_g → (gravity force)
② F_p → (pressure force)
(3) F_v → (viscous force)
(4) F_T → (Force due to turbulance)
(5) F_c → (Force due to compressiblity)

$$\therefore \boxed{F_x = (F_x)_g + (F_x)_p + (F_x)_v + (F_x)_T + (F_x)_c}$$

① If we neglected force due to compressibi-lity.

$$\therefore \boxed{(F_x) = (F_x)_g + (F_x)_p + (F_x)_v + (F_x)_T}$$

↳ Reynold's equation

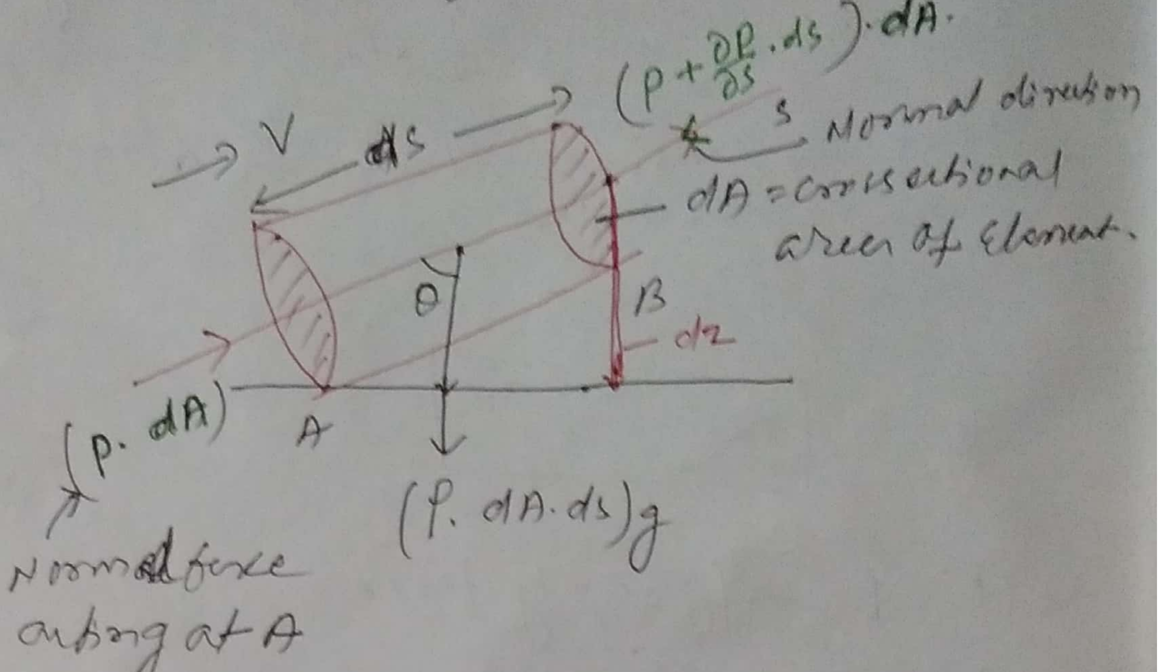
② If $(F_x)_c$ & $(F_x)_T$ are neglected.

i.e $\boxed{F_x = (F_x)_g + (F_x)_p + (F_x)_v}$ → Navier stocks equation

(3) If $\boxed{F_x = (F_x)_g + (F_x)_p}$ → Euler's equation.

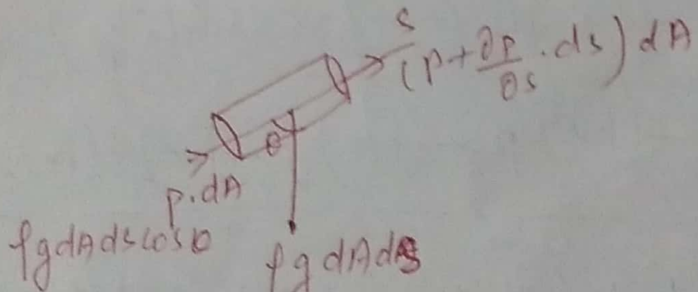
* Euler's equation of motion

→ The Resultant force will be equal to the sum of gravity force & pressure force



→ $$\boxed{P_s + ds = P + \frac{\partial P}{\partial s} \cdot ds}$$ → Taylor's equation.

→ Let us consider density of fluid = ρ.



(1) elements is moving along stream line

→ using Newton's 2nd law

$$F_s = m \cdot a_s$$

→ $$P \cdot dA - \left(P + \frac{\partial P}{\partial s} ds\right) dA - \rho g \, dA \, ds \cos\theta = (\rho \, dA \, ds) \times a_s$$

→ $$\cancel{P dA} - \cancel{P \cdot dA} + \frac{\partial P}{\partial s} ds \cdot dA - \rho g \, dA ds \cos\theta = (\rho \, dA \cdot ds) \times a_s$$

③

OR $-\frac{\partial P}{\partial s} dA \cdot ds - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \left\{ \frac{\partial v}{\partial t} + v \cdot \frac{\partial v}{\partial s} \right\}$

$\frac{\partial v}{\partial t}$ → local acceleration

$v \cdot \frac{\partial v}{\partial s}$ → convection acceleration

$$\rightarrow -\frac{\partial P}{\partial s} dA \cdot ds - \rho g\, dA\, ds \cos\theta = \rho\, dA \cdot ds \left\{ \frac{\partial v}{\partial t} + v \cdot \frac{\partial v}{\partial s} \right\} \quad (1)$$

(ii) If flow is steady

→ local acceleration will becomes zero.

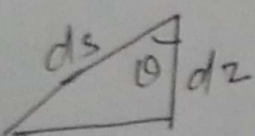
$$\boxed{\frac{\partial v}{\partial t} = 0}$$

→ Dividing equation (1) with $\rho dA \cdot ds$

$$\rightarrow -\frac{\partial P}{\rho \cdot \partial s} - g\cos\theta = v \cdot \frac{\partial v}{\partial s}$$

$$\rightarrow -\frac{\partial P}{\rho \cdot \partial s} - g\cos\theta - v \cdot \frac{\partial v}{\partial s} = 0$$

$$\rightarrow \boxed{\frac{\partial P}{\rho \partial s} + g\cos\theta + v \cdot \frac{\partial v}{\partial s} = 0}$$



$$\cos\theta = \frac{\partial z}{ds}$$

$$\therefore \frac{\partial P}{\rho \partial s} + g\frac{dz}{ds} + v \cdot \frac{\partial v}{\partial s} = 0$$

$$\rightarrow \frac{dP}{\rho \cdot ds} + g \cdot \frac{dz}{ds} + v \cdot \frac{dv}{ds} = 0$$

OR → $\boxed{\frac{dP}{\rho} + g \cdot dz + v\,dv = 0}$

↳ Euler's Equation of motion

* Important point for Euler's equations

(i) F_v = force due to viscosity is neglected,
F_c is also negligible.

→ So, this equation is valid for Non-viscous or Inviscid fluid flows.
↓
Ideal fluid flow.

(ii) $\boxed{\frac{\partial v}{\partial t} = 0}$ → for steady flow.

→ It is valid for steady flows.

(iii) F_T is Neglected.

→ flow is laminar or stream-line flow.

—·—·—

* Bernoulli's equation

→ $\boxed{\frac{dP}{\rho} + g\,dz + V\cdot dV = 0}$ → Euler's Equation

Integrating both side

→ $\int \frac{dP}{\rho} + \int g\,dz + \int V\cdot dV = \int(0)$

→ $\boxed{\rho = \text{constant}}$ → fluid is incompressible

→ $\boxed{\frac{P}{\rho} + gz + \frac{V^2}{2} = \text{constant}}$

→ $\frac{P}{\rho g} + z + \frac{V^2}{2g} = \frac{\text{const.}}{g} = \text{constant.}$

→ $\boxed{\frac{P}{\rho g} + z + \frac{V^2}{2g} = C}$ → Bernoullis equations.

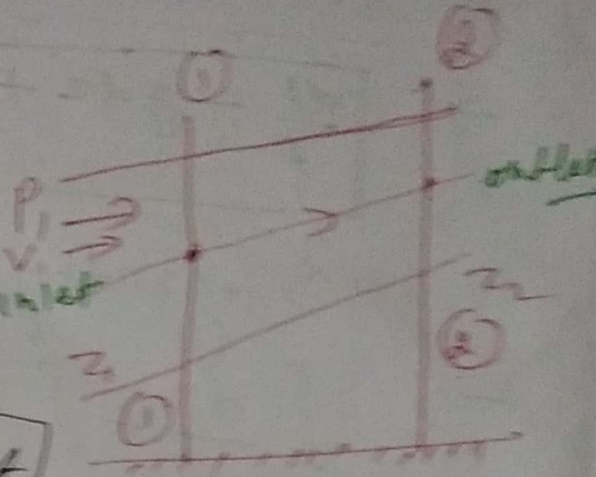
→ If flow is rotational

* Bernoullis Equation

(i) Flow is inviscid
Incompressible (ideal fluid)

(ii) steady flow.

(iii) Irrotational flow.



→ $\boxed{\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}}$

→ $\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2}$

→ Bernoullis equation for an Ideal fluid.

→ $\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$

1 2 3

terms 3 potential $\boxed{\text{Energy per unit weight}}$

↓

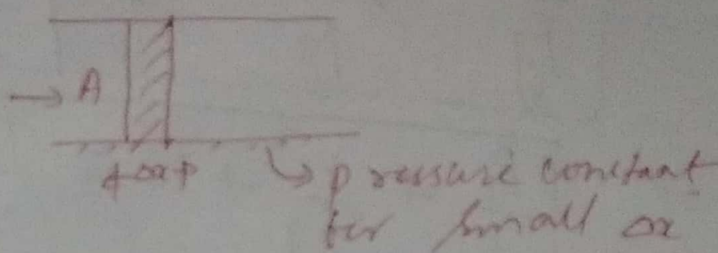
head.

or

potential head

$= \frac{mg \cdot z}{mg} = z = \text{head}$

terms 2 $\frac{V^2}{2g}$ = kinetic Energy per unit weight terms (3) $\frac{P}{\rho g}$



→ force required to maintain the flow = $P \cdot A$

→ work done by pressure to maintain the flow.

$$\boxed{W = P \cdot A \cdot \Delta x}.$$

→ Weight of Elements = $\rho . A \, \Delta x \cdot g$

$$= \frac{W}{\rho A \Delta x \cdot g} = \frac{PA \cdot \Delta x}{\rho A \cdot \Delta x \cdot g}$$

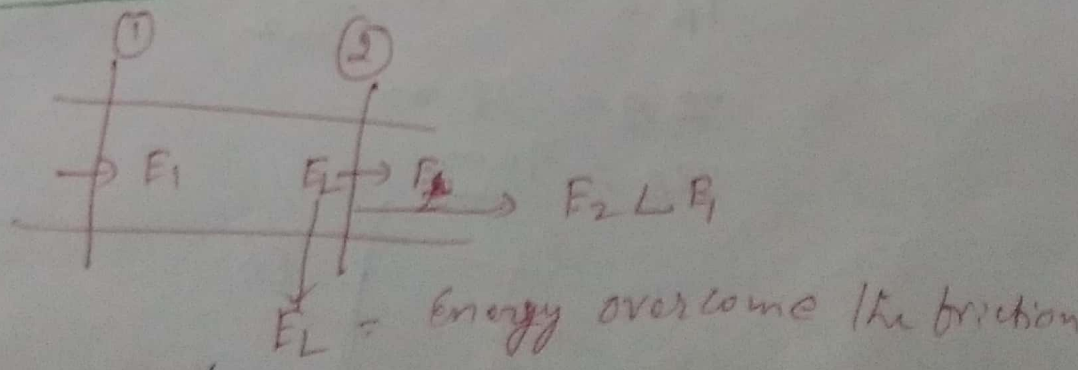
$$= \frac{P}{\rho g}$$

→ $\frac{P}{\rho g}$ = Work required to maintain the flow per unit weight

→ $\frac{P}{\rho g}$ = flow Energy.

→ Bernoullis of equation is ~~constant~~ conservation of Energy.

* Bernoulli's equation for Real fluids



→ Energy lost per unit Weight = Head loss

$$\rightarrow \boxed{\frac{E_1}{\rho g} = \frac{E_2}{\rho g} + h_L}$$

$$\rightarrow \boxed{\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L}$$

$\boxed{(E)_{inlet} > (E)_{outlet}}$

$$\rightarrow \boxed{\frac{E_{inlet} - E_{out}}{\rho g} = h_L}$$

Q A pipe of diameter 400 mm carries water at a velocity of 25 m/s, pressure at point A & B is 29.43 $\frac{N}{cm^2}$ and 22.563 $\frac{N}{cm^2}$ respectively. While datum head at A & B are 28 m & 30 m find Head loss between A & B

Soln Given

$D = 400 mm = 0.4 m$.

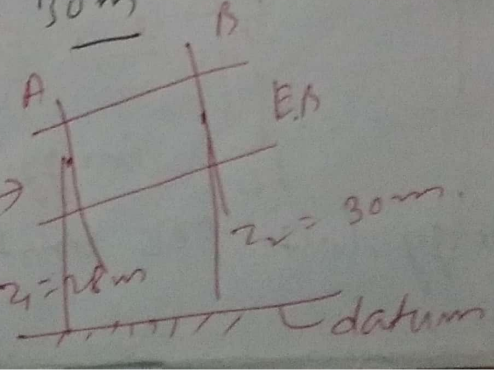
$V = 25 m/s$

$P_A = 29.43 \frac{N}{cm^2} = 29.43 \times 10^4 \frac{N}{m^2}$

$P_B = 22.563 \frac{N}{cm^2} = 22.563 \times 10^4 \frac{N}{m^2}$

$Z_A = 28 m$

$Z_B = 30 m$



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→ for incompressible fluid

$A_1 V_1 = A_2 V_2$ } continuity equation

$A_1 = A_2$ $\therefore V_1 = V_2 = 25$ m/sec.

→ $$\boxed{\left(\frac{V^2}{2g}\right)_A = \left(\frac{V^2}{2g}\right)_B}$$

→ $$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B + h_L$$

→ $$\frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 28 = \frac{22.563}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 30 + h_L$$

$$\therefore \boxed{h_L = 5\text{m}}$$