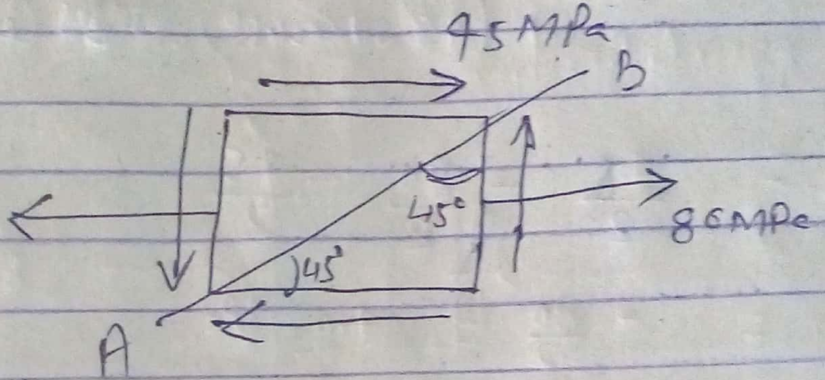
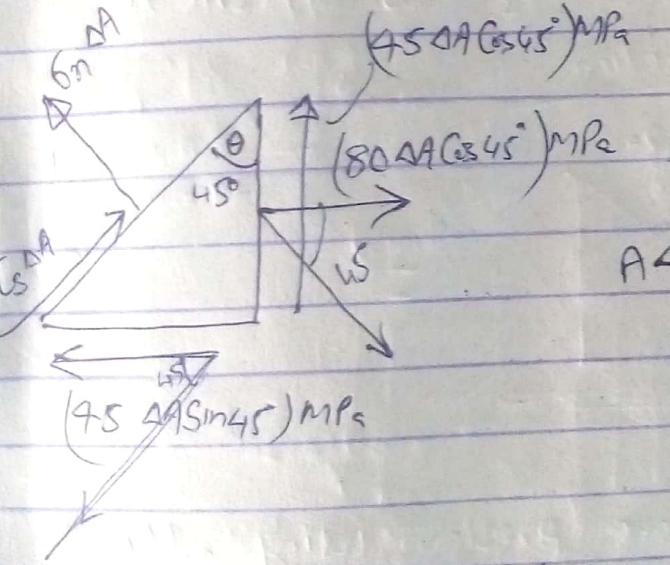
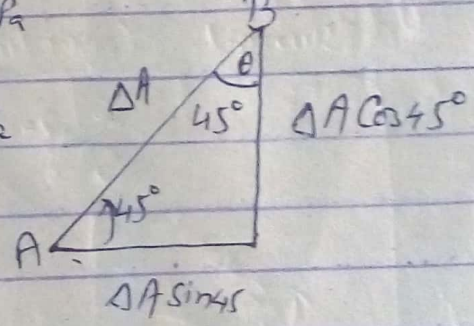
Question → Determine the normal stress and shear stress acting on the inclined plane AB. Solve the problem using the method of equilibrium







Consider the equilibrium

$$\sigma_n \Delta A = (80\,\Delta A \cos45°)\cos45° - (45\,\Delta A \sin45°)\cos45° + (45\,\Delta A \cos45°)\cos(135°)$$

$$\sigma_n = \frac{80}{2} - \frac{45}{2} - \frac{45}{2} = \frac{-90 \pm 80}{2}$$

$$= \frac{-10}{2} = -5\text{ MPa}$$

$$\boxed{\sigma_n = -5\text{ MPa}}$$

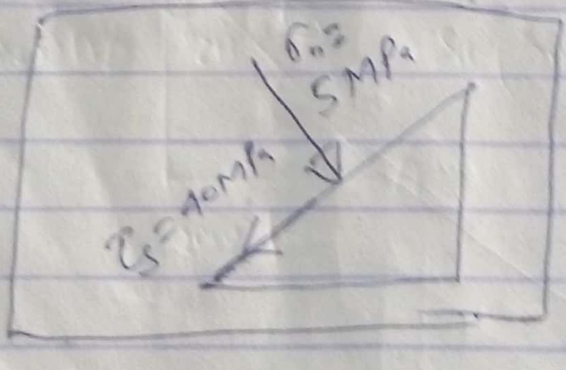
-ve sign indicate that σ_n is a Compressive stress

$$\tau_s \Delta A = (45\,\Delta A \sin 45)\cos 45^\circ + (80\,\Delta A \cos 45^\circ)\cos 45^\circ + (45\,\Delta A \cos 45^\circ)\cos(135^\circ)$$

$$\tau_s = \frac{45}{2} - \frac{80}{2} - \frac{45}{2}$$

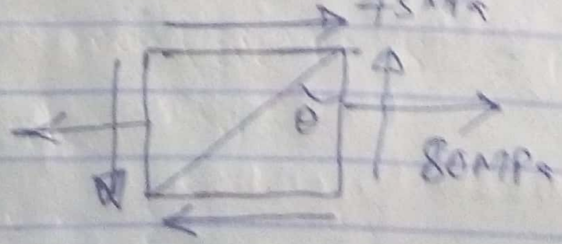
$$= -\frac{80}{2} = -40\ MPa$$

-ve sign indicate that τ_s has opposite to the assumed direction ie



Ans

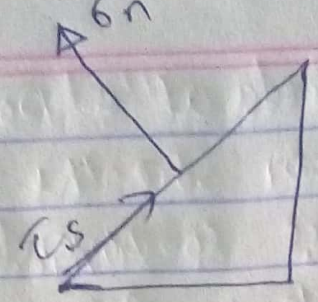
Q → Repeat the above problem using the stress transformation equation



Here $\theta = 45^\circ$ $\qquad \tau_{xy} = -45\ MPa$

$\sigma_x = 80\ MPa$

$\sigma_y = 0$



$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{80+0}{2} + \frac{80-0}{2} \cdot \cos 90^\circ + (-45) \sin 90^\circ$$

$$= 40 + 0 - 45$$

$$= -5 \text{ MPa}$$

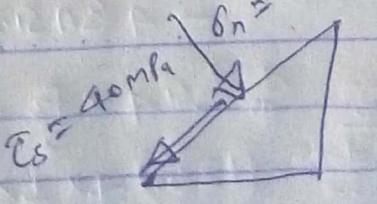
-ve sign indicate σ_n normal stress has opposite to assumed direction

$$\tau_s = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

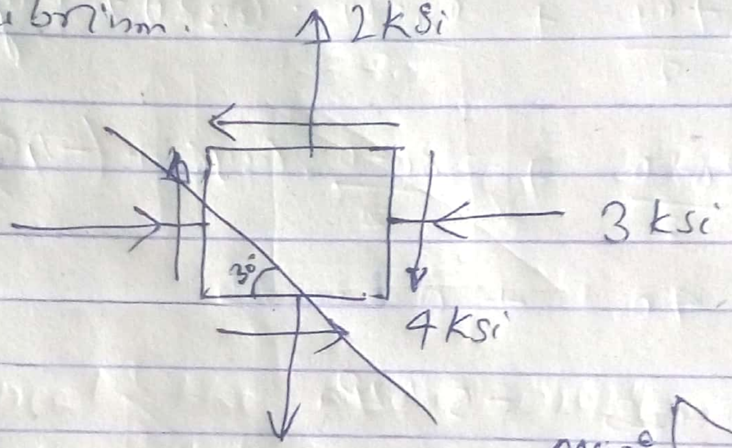
$$= -\left(\frac{80-0}{2}\right) \cdot \sin 90^\circ + (-45) \cos 90^\circ$$

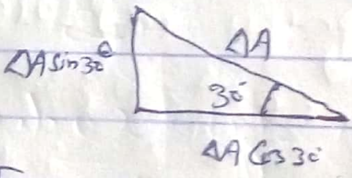
$$= -40 \text{ MPa}$$

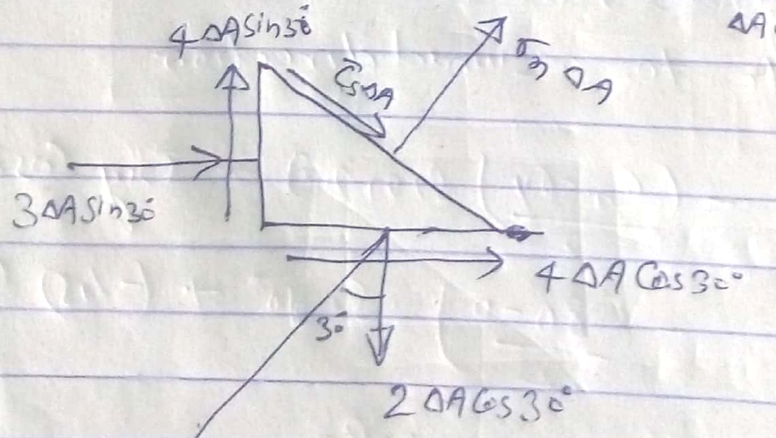
In this case also τ_s has opposite to assumed direction



Q → The state of stress at a point in a member is shown on the element. Determine the stress component acting on the inclined plane AB. Solve the problem using the method of equilibrium.







$$\sigma_n \Delta A = (2\,\Delta A \cos 30^\circ)\cos 30^\circ + (4\,\Delta A \cos 30^\circ)\cos(120^\circ)$$
$$+ (3\,\Delta A \sin 30^\circ)\cos(120^\circ)$$
$$- 4(\Delta A \sin 30^\circ)\cdot\cos 30^\circ$$

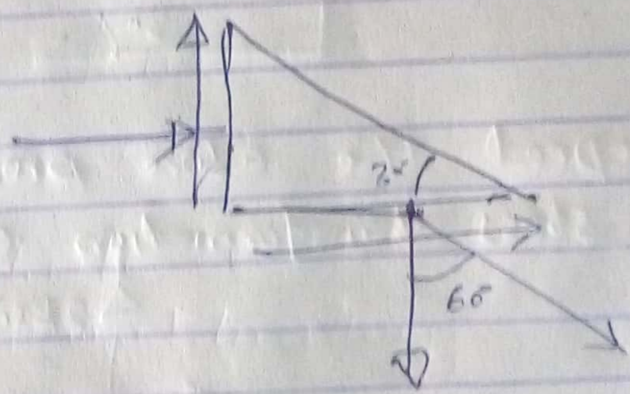
$$\sigma_n = 2 \times \frac{3}{4} - 4 \times \frac{\sqrt{3}}{2}\cdot\frac{1}{2} - 3 \times \frac{1}{2} \times \frac{1}{2}$$
$$- 4 \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$\sigma_\theta = \frac{3}{2} - \sqrt{3} - \frac{3}{4} - \sqrt{3}$$

$$\sigma_\theta = \frac{3}{2} - 2\sqrt{3} - \frac{3}{4}$$

$$\boxed{\sigma_\theta = -2.71 \text{ ksi}}$$

τ_s



$$\tau_s \Delta A = -(2\Delta A \cos 30^\circ)\cos 60^\circ - (4\Delta A \cos 30^\circ)\cos 30^\circ$$
$$- (3\Delta A \sin 30^\circ)\cos 30^\circ + (4\Delta A \sin 30^\circ)\cos 60^\circ$$

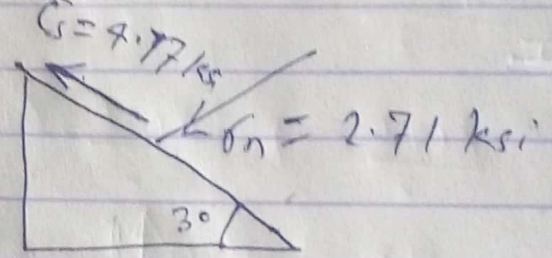
$$\tau_s = -2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} - 4 \times \frac{3}{4} - 3 \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$$
$$+ 4 \times \frac{1}{2} \times \frac{1}{2}$$

$$= -\frac{\sqrt{3}}{2} - 3 - \frac{3\sqrt{3}}{4} + 1$$

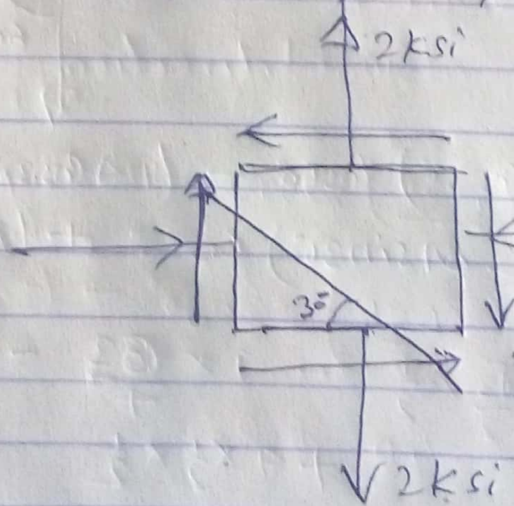
$$= -0.866 - 3 - 1.299 + 1$$

$$= -4.17 \text{ ksi}$$

-ve sign indicate that both normal stress & shear stress is opposite to the assumed direction



Q→ Repeat the above problem using the stress transformation equation.



Here $\sigma_x = -3$ ksi

$\sigma_y = 2$ ksi

$\tau_{xy} = 4$ ksi

$\theta = (120°)$

Now $\sigma_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2} \cdot \cos 2\theta + \tau_{xy} \sin 2\theta$

$\sigma_n = \left(\frac{-3+2}{2}\right) + \left(\frac{-3-2}{2}\right) \cos 240°$
$+ 4 \sin 240$

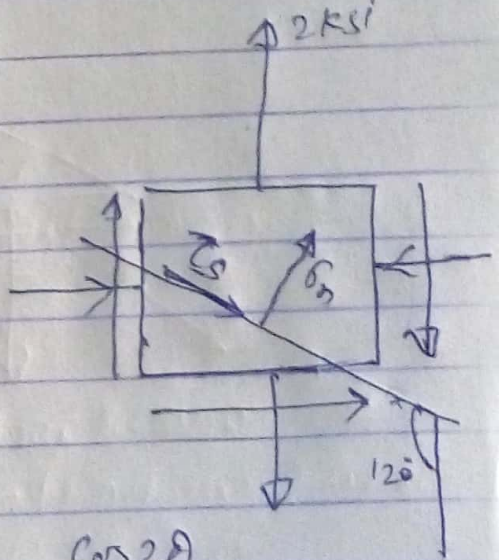
$= -\frac{1}{2} + -\frac{5}{2} \cdot (-\cos 60°) + 4(-\sin 60°)$

$= -\frac{1}{2} + \frac{5}{2} \times \frac{1}{2} - 4 \times \frac{\sqrt{3}}{2}$

$= -\frac{1}{2} + \frac{5}{4} - \frac{4\sqrt{3}}{2}$

$= -0.5 + 1.25 - 3.46$

$= \underline{-2.71 \text{ ksi}}$



$\tau_s = -\left(\frac{\sigma_x-\sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$

$= -\left(\frac{-3-2}{2}\right) \cdot \sin 240° + 4 \cos 240°$

$= +\frac{5}{2}\left(-\frac{\sqrt{3}}{2}\right) + 4 \times -\frac{1}{2}$

$= -\frac{5\sqrt{3}}{4} - 2$

$= -4.17 \text{ ksi}$

-ve sign indicate σ_n & τ_s is opposite to assumed direction.