In a similar way, the *transfer admittance* is defined as the ratio of current transform at output port to the voltage tranform at the input port. It is given as

$$Y_{12}(s) = \frac{I_2(s)}{V_1(s)} \quad \text{...(14.7)}$$

14.4 VOLTAGE AND CURRENT TRANSFER RATIO

Voltage and current transfer ratios are basically transfer functions. *Voltage transfer ratio* is the ratio of voltage transform at output port to the voltage transform at the input port. It is usually denoted by $G(s)$. Thus,

$$G_{12}(s) = \frac{V_2(s)}{V_1(s)} \quad \text{...(14.8)}$$

In a similar way, the ratio of transform of current at output port to that at the input port of a two port network gives the *current transfer ratio*. It is represented by $\alpha(s)$. Thus,

$$\alpha_{12}(s) = \frac{I_2(s)}{I_1(s)} \quad \text{...(14.9)}$$

EXAMPLE 14.1 *A series R-L circuit is in parallel with a capacitance. Find the input impedance in Laplace domain.*

SOLUTION. Figure E14.1 represents the parallel combination of one series *R-L* circuit with a capacitance in Laplace domain. The input impedance is $Z(s)$ which is obviously

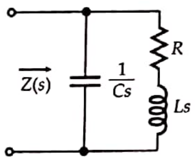

Fig. E14.1

$$Z(s) = \frac{(1/Cs)(R+Ls)}{\frac{1}{Cs} + (R+Ls)}$$

$$= \frac{R+Ls}{1+RCs+LCs^2}$$

$$= \frac{s+\frac{R}{L}}{C\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right)}$$

$\therefore$ Input impedance

$$Z(s) = \frac{s+(R/L)}{C\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right)}.$$

EXAMPLE 14.2 *Find $Z_{11}(s)$, $Z_{21}(s)$ in the following circuit. (Fig. E14.2)*

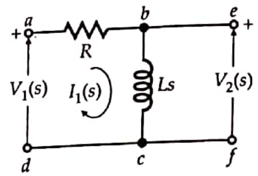

Fig. E14.2

SOLUTION. KVL gives for *loop abcd*,

$$RI_1(s) + I_1(s)Ls = V_1(s) \quad \text{...(i)}$$

or,
$$I_1(s)(R+Ls) = V_1(s)$$

$\therefore$
$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} = (R+Ls) \quad \text{...(ii)}$$

Again in *loop bcfe*, KVL yields

$$V_2(s) = I_1(s)Ls$$

which gives,
$$Z_{12}(s) = \frac{V_2(s)}{I_1(s)} = Ls. \quad \text{...(iii)}$$

Equations (*ii*) and (*iii*) give the required result.

EXAMPLE 14.3 *Obtain the transfer function $V_2(s)/V_1(s)$ in Fig. E14.3.*

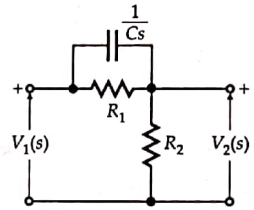

Fig. E14.3

SOLUTION. The equivalent impedance of $\frac{1}{Cs}$ and R_1 is given by $Z(s)$.

Obviously,

$$Z(s) = \frac{R_1 \times \frac{1}{Cs}}{R_1 + \frac{1}{Cs}} = \frac{R_1}{R_1Cs+1}.$$

Applying KVL in *left loop* of Fig. E14.3, we get

$$V_1(s) = I_1(s)[Z(s) + R_2]$$

$$V_2(s) = R_2\, I_1(s).$$

$$\therefore \quad V_2(s)/V_1(s) = \frac{R_2}{R_2 + Z(s)}$$

$$= \frac{R_2}{R_2 + \dfrac{R_1}{R_1 Cs + 1}}$$

$$= \frac{R_2(1 + R_1 Cs)}{R_2 + R_1 R_2 Cs + R_1}$$

$$= \frac{R_2 + R_1 R_2 Cs}{R_1 + R_2 + R_1 R_2 Cs}$$

$$= \frac{R_2 R_1 C\left(s + \dfrac{1}{R_1 C}\right)}{R_2 R_1 C\left(s + \dfrac{R_1 + R_2}{R_2 R_1 C}\right)}$$

i.e., Voltage transfer ratio

$$= \frac{R_2 R_1 C\left(s + \dfrac{1}{R_1 C}\right)}{R_2 R_1 C\left(s + \dfrac{R_1 + R_2}{R_1 R_2 C}\right)}.$$

EXAMPLE 14.4 *Obtain $Z_{12}(s)$ for a parallel R-C network where $R = 1\Omega$; $C = 1$ F.*

SOLUTION. In Fig. E14.4,

$$V_2(s) = I_1(s)\left[\frac{R \times \dfrac{1}{Cs}}{R + \dfrac{1}{Cs}}\right]$$

or, $$\frac{V_2(s)}{I_1(s)} = \frac{R}{RCs + 1} = \frac{1}{\left(s + \dfrac{1}{RC}\right)C}$$

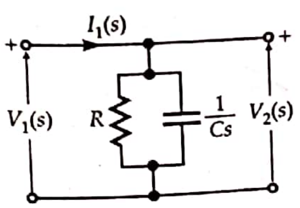

Fig. E14.4

$$\therefore \quad Z_{12}(s) = \frac{V_2(s)}{I_1(s)}$$

$$= \frac{1}{C\left(s + \dfrac{1}{RC}\right)} = \frac{1}{1(s+1)}$$

$$\therefore \quad Z_{12}(s) = \frac{1}{s+1}.$$

EXAMPLE 14.5 *Find (V_2/V_1) and (V_2/I_1) in Fig. E14.5.*

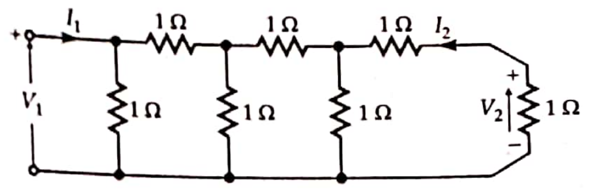

Fig. E14.5

SOLUTION. The figure is redrawn (Fig. E14.6) with marking of nodes and voltages at the node.

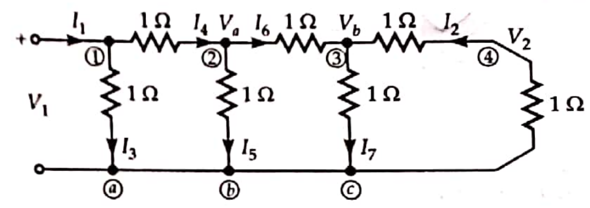

Fig. E14.6

At *node (1)*, application of KCL yields

$$I_1 = I_3 + I_4 = \frac{V_1}{1} + \frac{V_1 - V_a}{1} = 2V_1 - V_a \qquad \text{...(1)}$$

At *node (2)*,

$$I_4 = I_5 + I_6$$

or, $$\frac{V_1 - V_a}{1} = \frac{V_a}{1} + \frac{V_a - V_b}{1}$$

or, $$V_1 - V_a = V_a + V_a - V_b$$

or, $$V_1 - 3V_a + V_b = 0 \qquad \text{...(2)}$$

At *node (3)*,

$$I_6 = I_7 - I_2$$

or $$\frac{V_a - V_b}{1} = \frac{V_b}{1} - \frac{V_2 - V_b}{1}$$

or, $$V_a - V_b = V_b - V_2 + V_b$$

or $$V_2 + V_a - 3V_b = 0 \qquad \text{...(3)}$$

At *node (4)*, KCL yields,

Current through load of 1Ω = Current flows between nodes (3) and (4)

i.e., $\frac{V_2}{1} = \frac{V_b - V_2}{1}$

or $2V_2 - V_b = 0$ ...(4)

From (4), $V_b = 2V_2$ and from (3) we then get $V_a = 5V_2$

From (2), we get

$V_1 - 3(5V_2) + (2V_2) = 0$ or, $V_1 = 13\,V_2$

and from (1), we get

$I_1 = 2\,(13\,V_2) - (5V_2) = 21\,V_2$

Then finally, we get

$V_2 / V_1 = \frac{1}{13}$ and $V_2 / I_1 = \frac{1}{21}$.

Example 14.6 *Find Z(s) for the following network (Fig. E14.7).*

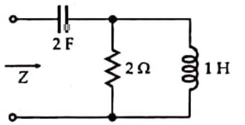

Fig. E14.7

Solution. The transform network is shown in Fig. E14.7(*a*).

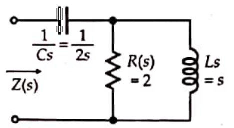

Fig. E14.7 (*a*)

$$Z(s) = [R(s) \,||\, Ls] + \frac{1}{Cs} = \frac{2 \times s}{2+s} + \frac{1}{2s},$$

$$= \frac{4s^2 + s + 2}{2s(s+2)} = \frac{4s^2 + s + 2}{2s^2 + 4s}$$

$$\therefore \quad Z(s) = \frac{4s^2 + s + 2}{2s^2 + 4s}.$$

Example 14.7 *Find the driving point impedance for the network shown in Fig. E14.8.*

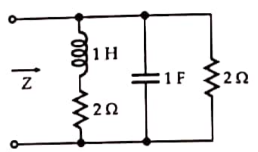

Fig. E14.8

Solution. The transfer network is shown in Fig. E14.8(*a*).

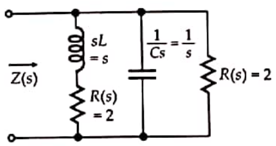

Fig. E14.8 (*a*)

Here $Z(s) = \left[R_2(s) \,||\, \frac{1}{Cs}\right] || [SL + R_1(s)]$

$$= \frac{2 \times \frac{1}{s}}{2 + \frac{1}{s}} || (2+s) = \frac{\frac{2}{s}}{\frac{2s+1}{s}} || (2+s)$$

$$= \frac{2}{(2s+1)} || (2+s) = \frac{\frac{2}{(2s+1)} \times (2+s)}{\frac{2}{2s+1} + 2 + s}$$

$$= \frac{2(s+2)}{2 + 4s + 2 + 2s^2 + s}$$

$$= \frac{2(s+2)}{2s^2 + 5s + 4}.$$

i.e., Z(s), the driving point impedance $= \frac{2(s+2)}{2s^2 + 5s + 4}$.

Example 14.8 *Find the driving point admittance of the network shown in Fig. E14.9.*

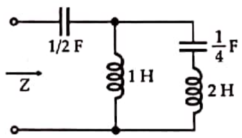

Fig. E14.9

Solution. The transform network is shown in Fig. E14.9(*a*)

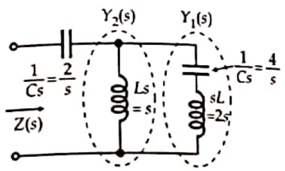

Fig. E14.9 (*a*)

Here $Y_1(s) = \frac{1}{2s + \frac{4}{s}} = \frac{1}{\frac{2s^2+4}{s}} = \frac{s}{2s^2+4}$

$$Y_2(s) = \frac{1}{s}$$

$$\therefore \quad Y_E(s) = Y_1(s) + Y_2(s) = \frac{s}{2s^2+4} + \frac{1}{s}$$

$$= \frac{3s^2 + 4}{2s^3 + 4s}$$

$$\therefore \quad Z_E(s) = \frac{1}{Y(s)} = \frac{2s^3 + 4s}{3s^2 + 4}$$

Thus $Z(s) = Z_E(s) + \frac{1}{Cs}$

$$= \frac{2s^3 + 4s}{3s^2 + 4} + \frac{2}{s}$$

$$= \frac{2s^4 + 10s^2 + 8}{3s^3 + 4s}$$

$$\because \quad Y(s) = \frac{1}{Z(s)},$$

here $Y(s) = \frac{3s^3 + 4s}{2s^4 + 10s^2 + 8}$.

Example 14.9 *Find the transfer function of the network shown in Fig. E14.10.*

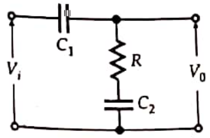

Fig. E14.10

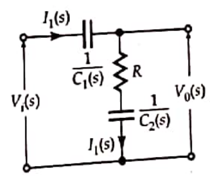

Fig. E14.10 (*a*)

Solution. Transforming the given network in the Laplace domain [Fig. E14.10(*a*)].

$$V_i(s) = I_1(s)\left[R + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right] \quad ...(1)$$

and $V_0(s) = I_1(s)\left[R + \frac{1}{C_2 s}\right]$

$$\therefore \quad \frac{V_0(s)}{V_i(s)} = \frac{R + \frac{1}{C_2 s}}{R + \frac{1}{C_1 s} + \frac{1}{C_2 s}}$$

$$= \frac{(RC_2 s + 1)C_1 s}{(RC_1 C_2 s^2 + C_1 s + C_2 s)}$$

$$= \frac{C_1(1 + RC_2 s)}{C_1 + C_2 + RC_1 C_2 s}$$

i.e., the transfer function (voltage transfer ratio) is given by

$$\left[\frac{C_1(1 + RC_2 s)}{C_1 + C_2 + RC_1 C_2 s}\right].$$

Example 14.10 *Find the expression of voltage transfer ratio for the network shown in Fig. E14.11.*

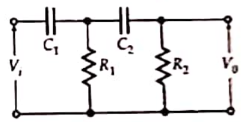

Fig. E14.11

Solution. The equivalent circuit of Fig. E14.11 in Laplace domain is shown in Fig. E14.11(*a*).

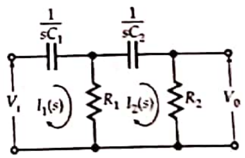

Fig. E14.11 (*a*)

Here, in the *leftmost loop*,

$$V_i(s) = I_1(s)\left[R_1 + \frac{1}{C_1 s}\right] - R_1 I_2(s) \quad ...(a)$$

in the *second loop*,

$$-R_1 I_1(s) + \left(R_1 + R_2 + \frac{1}{C_2 s}\right) I_2(s) = 0 \quad ...(b)$$

and in the *rightmost loop*,

$$R_2 I_2(s) = V_0(s) \quad ...(c)$$

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$\therefore M_{02}$ (distance between "zero" to "pole" at $-2) = 2$

and $\phi_{02} = 180°$

Also, $M_{32} = 1$ and $\phi_{32} = 0°$

[$\because$ the distance between pole at -3 and pole at -2 is 1 unit and ϕ_{32} is directed in opposite sense to ϕ_{02}]

$$\therefore \quad K_2 = H\frac{M_{02}\,e^{j\phi_{02}}}{M_{32}\,e^{j\phi_{32}}} = 10\frac{2e^{j180°}}{e^{j0°}}$$

$$= 20\,e^{j180°} = -20.$$

Next we consider the pole at -3.

$$\therefore \quad M_{03} = 3\,;\ \phi_{03} = 180°$$

Also, $M_{23} = 1\,;\ \phi_{23} = 180°$

$$\therefore \quad K_1 = H\frac{M_{03}\,e^{j\phi_{03}}}{M_{23}\,e^{j\phi_{23}}}$$

$$= 10\frac{3\,e^{j180°}}{e^{j180°}} = 30$$

This gives $v(t) = 30\,e^{-3t} - 20\,e^{-2t}$.

EXAMPLE 14.24 *Obtain the pole zero diagram of the given function and obtain the time domain response.*

$$I(s) = \frac{2s}{(s+1)(s^2+2s+4)}$$

SOLUTION. Using partial fraction,

$$I(s) = \frac{2s}{(s+1)(s^2+2s+4)}$$

$$= \frac{K_1}{(s+1)} + \frac{K_2}{(s+1-j\sqrt{3})} + \frac{K_3}{(s+1+j\sqrt{3})}$$

$$\therefore i(t) = K_1\,e^{-t} + K_2\,e^{-(1-j\sqrt{3})t} + K_3\,e^{-(1+j\sqrt{3})t}$$

The pole zero plot has been exhibited in the adjacent figure (Fig. E14.27).

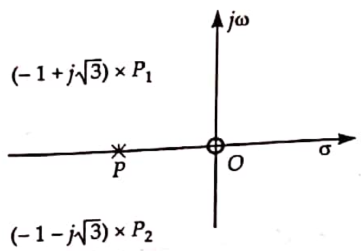

Fig. E14.27

$$K_1 = H\frac{M_{OP}\,e^{j\phi_{OP}}}{M_{P_1-P}\,M_{P_2-P}\,e^{j(\phi_{P_1-P}+\phi_{P_2-P})}}$$

$$= 2\frac{1.e^{j180}}{(\sqrt{3}\times\sqrt{3})\,e^{j(-90°+90°)}}$$

$$[\because M_{OP} = 1\,;\ M_{P_1-P} = M_{P_2-P} = \sqrt{3}$$
$$\phi_{OP} = 180°\,;\ \phi_{P_1-P} = -90°\,;\ \phi_{P_2-P} = 90°]$$

$$\therefore \quad K_1 = 0.67\,e^{j180°} = -0.67$$

and
$$K_2 = H\frac{M_{OP_1}\,e^{j\phi_{OP_1}}}{M_{P-P_1}\,M_{P_2-P_1}\,e^{j(\phi_{P-P_1}+\phi_{P_2-P_1})}}$$

(Please check with Fig. E14.28)

$$= 2\frac{2e^{j\left[\tan^{-1}\frac{OP}{PP_1}+90°\right]}}{(\sqrt{3}\times 2\sqrt{3})\,e^{j(90°+90°)}}$$

$$= 0.67\,e^{j\left[\tan^{-1}\frac{OP}{PP_1}+90°-180°\right]}$$

$$= 0.67\,e^{-j(60°)}$$

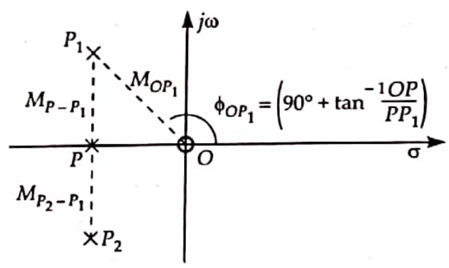

Fig. E14.28

$\therefore K_3$, being the conjugate of K_2 will be given by

$$K_3 = 0.67\,e^{j(60°)}\ [K_3 = K_2^*]$$

$$\therefore \quad i(t) = -0.67\,e^{-t} + 0.67\,e^{+j(60°)}\,e^{-(1+j\sqrt{3})t}$$
$$+ 0.67\,e^{-j(60°)}\,e^{-(1-j\sqrt{3})t}.$$

EXAMPLE 14.25(a) *Check the stability of the following polynomial by applying Routh-Hurwitz criterion :*

$$P(s) = s^4 + 2s^3 + 4s^2 + 12\,s + 10$$

SOLUTION. Routh array of the polynomial can be obtained from the following coefficients.

$$b_0 = 1;\ b_2 = 4;\ b_4 = 10;\ b_1 = 2;\ b_3 = 12$$

$$c_1 = \frac{b_1\,b_2 - b_0\,b_3}{b_1} = \frac{8-12}{2} = -2$$

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$$\therefore \quad c_2 = \frac{b_1 b_4 - b_0 b_5}{b_1} = \frac{20 - 1\times 0}{2} = 10$$

$$d_1 = \frac{c_1 b_3 - b_1 c_2}{c_1} = \frac{-2\times 12 - 2\times 10}{-2} = 22$$

$$d_2 = b_5 = 0$$

$$e_1 = \frac{c_2 d_1 - c_1 d_2}{d_1} = \frac{10\times 16 - (-2)\times 0}{16} = 10$$

$$f_1 = d_2 = 0$$

This gives

s^4	1	4	10
s^3	2	12	
s^2	−2	10	
s^1	22	0	
s^0	10		

There is a sign change in the 1st column of the array. Hence the system **is not stable.**

EXAMPLE 13.25(b) *For what value of 'K' the polynomial $s^4 + 10s^3 + 35s^2 + Ks + 24 = 0$ is said to be stable ? Solve by using Routh Hurwitz Array.*

SOLUTION. Given Polynomial is

$$s^4 + 10s^3 + 35s^2 + Ks + 24 = 0.$$

Routh Hurwitz Array is shown below

s^4	1	35	24
s^3	10	K	
s^2	$\frac{35\times 10 - K\times 1}{10} = \alpha_1$	$\frac{10\times 24 - 0\times 1}{10} = 24$	
s^1	$\frac{\alpha_1 \times K - 24\times 10}{\alpha_1}$		
s^0	24		

For stability of the network, $\alpha_1 > 0$

i.e., $\frac{350 - K}{10} > 0$ or $K < 350.$

Hence the range K is $(0 < K < 350)$.

EXAMPLE 14.26(a) *A polynomial is given by*

$$P(s) = s^3 + 2s^2 + 4s + M.$$

"M" is adjustable. Apply Routh Hurwitz criterion.

SOLUTION. Routh array gives

s^3	1	4
s^2	2	M
s^1	$\frac{8-M}{2}$	
s^0	M	

We see that if M is less than 8, the system is stable. There will be no change in the sign of 1st column. If $M = 8$, then also there is no change in sign in the first column and hence the system will also be stable.

EXAMPLE 13.26(b) *Check whether the following polynomial $P(s) = s^4 + s^3 + 2s^2 + 2s + 3$ is stable or not. Comment on your findings.*

SOLUTION. Given polynomial is

$$P(s) = s^4 + s^3 + 2s^2 + 25 + 3$$

where total number of rows are $(4 + 1) = 5$

Routh Hurwitz Array is shown below :

s^4	1	2	3
s^3	1	2	
s^2	$\frac{2-2=0}{1} = 0$	3	
s^1	∞		
s^0			

In above array, we found third row of first column has a zero value. Hence the subsequent value is infinite. Let us now assume a small value ε in place of the zero value. The modified array is given below :

s^4	1	2	3
s^3	1	2	
s^2	ε	3	
s^1	$\frac{2\varepsilon - 3}{\varepsilon} > 0$		
s^0	3		

For stability point of view, $\varepsilon > 0$ *i.e.,* $\frac{2\varepsilon - 3}{\varepsilon} > 0$.

This gives $\varepsilon > 3/2$

Hence to have the stability of the function, we should have any value of ε more than $(3/2)$.

EXAMPLE 14.27 *What should be the value of L such that $Z(s) = 1$ in the network of Fig. E14.29 ?*

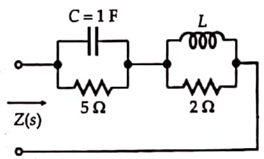

Fig. E14.29

$$= V_2(s)\left[2s + \frac{1}{s} + \frac{2}{s} + \frac{1}{s^3} - s\right]$$

$$= V_2(s)\left[\frac{2s^4 + s^2 + 2s^2 - s^4 + 1}{s^3}\right]$$

$$= V_2(s)\left[\frac{s^4 + 3s^2 + 1}{s^3}\right]$$

$$\therefore \quad \frac{V_2(s)}{V_1(s)} = \frac{s^4}{s^4 + 3s^2 + 1} = G_{21}(s)$$

i.e., $$G_{12}(s) = \frac{s^4}{s^4 + 3s^2 + 1}.$$

EXAMPLE 14.31 *Find $G_{21}(s)$ for the network shown in Fig. E14.36 when $V_1(s)$ is the applied voltage at the input terminals.*

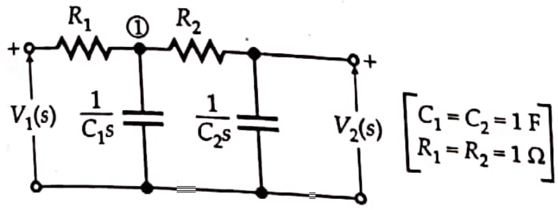

Fig. E14.36

SOLUTION. In the s-domain network of Fig. E14.36, let $V_3(s)$ be the potential at the junction of R_1, R_2, C_1 *i.e.,* at *node* (1).

Applying KCL at node (1),

$$\frac{V_1(s) - V_3(s)}{R_1} + \frac{V_2(s) - V_3(s)}{R_2} = \frac{V_3(s)}{\frac{1}{C_1 s}}$$

$$= V_3(s)\, C_1 s$$

or, $$\frac{V_1(s)}{R_1} = V_3(s)\left[sC_1 + \frac{1}{R_1} + \frac{1}{R_2}\right] - \frac{V_2(s)}{R_2} \qquad ...(1)$$

It may also be evident that the current passing through R_2 also passes through C_2

i.e., $$\frac{V_3(s) - V_2(s)}{R_2} = \frac{V_2(s)}{\frac{1}{sC_2}}$$

or, $$0 = \left[\frac{-V_3(s)}{R_2}\right] + V_2(s)\left[sC_2 + \frac{1}{R_2}\right]$$

i.e., $$V_3(s) = V_2(s)[sC_2 R_2 + 1] \qquad ...(2)$$

Utilising (2) in (1),

$$\frac{V_1(s)}{R_1} = V_2(s)[1 + sC_2 R_2]\left[sC_1 + \frac{1}{R_1} + \frac{1}{R_2}\right] - \frac{V_2(s)}{R_2}$$

or, $$V_1(s) = V_2(s)\left[\{R_1 + sC_2R_2R_1\}\left\{sC_1 + \frac{1}{R_1} + \frac{1}{R_2}\right\} - \frac{R_1}{R_2}\right]$$

or, $$\frac{V_2(s)}{V_1(s)} = G_{12}(s)$$

$$= \frac{1}{\left[\{R_1 + sC_2R_1R_2\}\left\{sC_1 + \frac{1}{R_1} + \frac{1}{R_2}\right\} - \frac{R_1}{R_2}\right]}$$

$$= \frac{1}{[\{1 + s\}\{s + 1 + 1\} - 1]} = \frac{1}{(s + 1)[(s + 2) - 1]}$$

i.e., $$G_{12(s)} = \frac{1}{(s+1)^2}.$$

EXAMPLE 14.32 *Obtain the voltage transfer function of the network shown in Fig. E14.37.*

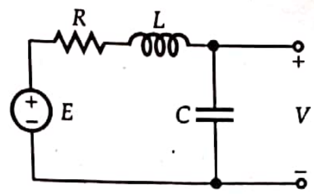

Fig. E14.37

SOLUTION. Let us first transform the network to frequency domain (Fig. E14.38).

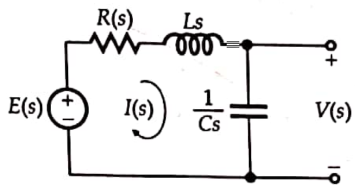

Fig. E14.38

Here, $$I(s) = \frac{E(s)}{R + Ls + \frac{1}{Cs}}$$

$$\therefore \quad V(s) = I(s) \times \frac{1}{Cs} = \frac{E(s)}{R + Ls + \frac{1}{Cs}} \cdot \frac{1}{Cs}$$

$$\therefore \quad \frac{V(s)}{E(s)} = \frac{\frac{1}{Cs}}{R + Ls + \frac{1}{Cs}} = \frac{1}{LCs^2 + RCs + 1}$$

Obviously, $\frac{V(s)}{E(s)}$ gives the voltage transfer ratio and

$$\frac{V(s)}{E(s)} = \frac{1}{LCs^2 + RCs + 1}.$$

This gives,

$$I_2(s) = I_0(s)\frac{\dfrac{C_2 s}{1+RC_2 s}}{C_1 s + \dfrac{C_2 s}{1+RC_2 s}}$$

$$= I_0(s)\frac{C_2 s}{C_1 s(1+RC_2 s)+C_2 s};$$

$$\frac{I_2(s)}{I_0(s)} = \text{Current transfer ratio}$$

$$= \frac{sC_2}{sC_2 + sC_1(1+RC_2 s)}$$

Thus, current transfer ratio

$$= \frac{C_2}{C_2 + C_1(1+RC_2 s)}.$$

EXAMPLE 14.37 *Obtain the driving point admittance of the network shown in Fig. E14.46.*

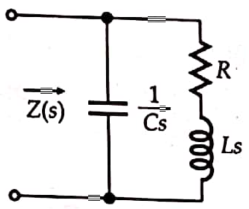

Fig. E14.46

SOLUTION.

$$Z(s) = \frac{(R+Ls)\dfrac{1}{Cs}}{R+Ls+\dfrac{1}{Cs}} = \frac{\dfrac{1}{Cs}(R+Ls)}{\dfrac{1+RCs+LCs^2}{Cs}}$$

$$= \frac{R+Ls}{1+RCs+LCs^2}$$

$\therefore$ $Y(s)$, the driving point admittance

$$= \frac{1}{Z(s)} = \frac{LCs^2+RCs+1}{R+Ls},$$

i.e.,

$$\mathbf{Y(s) = \frac{1+RCs+LCs^2}{R+Ls}}.$$

EXAMPLE 14.38 *Find the driving point impedance of the network shown in Fig. E14.47.*

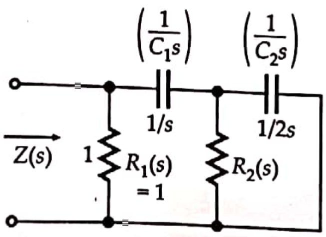

Fig. E14.47

SOLUTION. R_2 and C_2 are parallel

$$\therefore \quad Y_2(s) = Y_{R_2}(s) + Y_{C_2}(s)$$

$$= 1 + \frac{1}{Z_{C_2}(s)} = 1 + \frac{1}{\dfrac{1}{2s}} = 1+2s$$

i.e.,

$$Z_2(s) = \frac{1}{1+2s}$$

However, $Z_2(s)$ is in series with C_1

$$\therefore \quad Z_1(s) = Z_2(s) + \frac{1}{C_1 s} = \frac{1}{1+2s} + \frac{1}{s} = \frac{3s+1}{s(2s+1)}$$

$$\therefore \quad Y_1(s) = \frac{s(2s+1)}{3s+1}$$

Again $Z_1(s)$ and $R_1(s)$ are in parallel.

$$\therefore \quad Y(s) = \frac{1}{Z_1(s)} + \frac{1}{R_1(s)}$$

$$= \frac{s(2s+1)}{3s+1} + 1 = \frac{2s^2+4s+1}{3s+1}$$

$$\therefore \quad Z(s) = \frac{1}{Y(s)} = \frac{3s+1}{2s^2+4s+1}$$

i.e., Driving point impedance $= \mathbf{\dfrac{3s+1}{2s^2+4s+1}}$.

EXAMPLE 14.39 *In the network of Fig. E14.48, find pole-zero plot.*

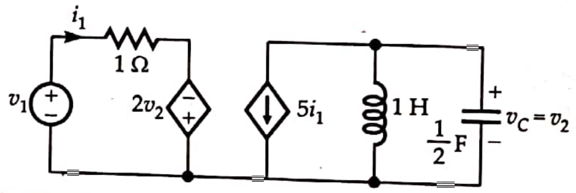

Fig. E14.48

SOLUTION. Let us first convert Fig. E14.48 to s-domain circuit (Fig. E14.49).

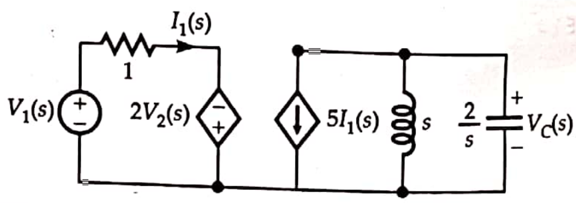

Fig. E14.49

Using KCL at *right hand loop,*

$$5I_1(s) + \frac{V_C(s)}{s} + \frac{V_C(s)}{2/s} = 0 \qquad \text{...(1)}$$