Example 3.2. A wattmeter reads 25.34 watts. The absolute error in the measurement is – 0.11 watt. Determine the true value of power.

Solution:

Measured value, $A_m = 25.34$ watts

Absolute error, $\delta A = -0.11$ watt

True value, A = Measured value – absolute error

$= A_m - \delta A = 25.34 - (-0.11) = 25.45$ watts **Ans.**

Example 3.3. The measured value of a capacitor is 205.3 µF, whereas its true value is 201.4 µF. Determine the relative error.

[B.P. Univ. of Technology Orissa Electronics Measurements and Measuring Instruments, 2007]

Solution:

Measured value, $A_m = 205.3 \times 10^{-6}$ F

True value, $A = 201.4 \times 10^{-6}$ F

Absolute error, $\varepsilon_0 = A_m - A = 205.3 \times 10^{-6} - 201.4 \times 10^{-6} = 3.9 \times 10^{-6}$ F

Relative error, $\varepsilon_r = \dfrac{\varepsilon_0}{A} = \dfrac{3.9 \times 10^{-6}}{201.4 \times 10^{-6}} = 0.0194$ or 1.94% **Ans.**

Example 3.4. The expected value of the voltage across a resistor is 80 V. However, the measurement gives a value of 79 V. Calculate:

(i) absolute error (ii) % error (iii) relative accuracy (iv) % of accuracy.

[M.D. Univ. Electrical Measurement and Measuring Instruments, December-2010]

Solution: Measured value of voltage, $A_m = 79$ V

Expected value of voltage, A = 80 V

(*i*) Absolute error, $\varepsilon_o = A_m - A = 79 - 80 = -1$ V **Ans.**

(*ii*) % error $= \dfrac{A_m - A}{A} \times 100 = \dfrac{79 - 80}{80} \times 100 = -1.25\%$ **Ans.**

(*iii*) Relative accuracy $= 1 - \left|\dfrac{\varepsilon_o}{A}\right| = 1 - \dfrac{1}{80} = 0.9875$ **Ans.**

(*iv*) % of accuracy = 100 × relative accuracy = 100 × 0.9875 = 98.75% **Ans.**

Example 3.5. A resistor of value 4.7 kΩ is read as 4.65 kΩ in a measurement. Calculate (i) absolute error, (ii) % error and (iii) accuracy.

[U.P.S.C. I.E.S. Electronics and Telecommunication Engineering-I, 2009]

Solution: Measured value of voltage, $A_m = 4.65$ kΩ

True value of resistor, A = 4.7 kΩ

(*i*) Absolute error, $\varepsilon_o = A_m - A = 4.65 - 4.7 = -0.05$ kΩ or – 50 Ω **Ans.**

(*ii*) % error $= \dfrac{A_m - A}{A} \times 100 = \dfrac{4.65 - 4.7}{4.7} \times 100 = -1.064\%$ **Ans.**

(*iii*) Accuracy = 100 – |% error|% = 100 – 1.064 = 98.936% **Ans.**

Example 3.6. Define limiting errors.

A 0–10 A ammeter has a guaranteed accuracy of 1.5 per cent of full-scale reading. The current measured by the instrument is 2.5 A. Calculate the limiting values of current and the percentage limiting error. *[U.P. Technical Univ. Elec. Measurements and Measuring Instruments 2005-06]*

Solution: The magnitude of limiting error of the instrument,

$$\delta A = \varepsilon_r \times A = \frac{1.5}{100} \times 10 = 0.15 \text{ A}$$

The magnitude of current being measured is 2.5 A. The relative error at this current is

$$\varepsilon_r = \frac{\delta A}{A} = \frac{0.15}{2.5} = 0.06$$

Hence, the current under measurement is between the limits of

$$A = A_m(1 \pm \varepsilon_r) = 2.5\,(1 \pm 0.06) = (2.5 \pm 0.15)\text{ A}$$

So limiting values of current under measurement are 2.35 A and 2.65 A **Ans.**

$$\text{Limiting error} = \pm\frac{0.15}{2.5} \times 100 = \pm 6\% \quad \textbf{Ans.}$$

Example 3.7. A wattmeter having a range of 1,000 W has a error ± 1% of full-scale deflection. If the true power is 100 W, what would be range of the reading? Suppose the error is specified as percentage of true value, what would be the range of the readings?

[U.P. Technical Univ. Electrical Measurements and Measuring Instruments, 2006-07]

Solution: The magnitude of limiting error of the instrument,

$$\delta P = \varepsilon_r \times P = 0.01 \times 1{,}000 = 10\text{ W}$$

The magnitude of power being measured is 100 W

Range of the readings = P ± δP = (100 ± 10) W i.e. 90 to 110 watts **Ans.**

When the error is specified as a percentage of true value

$$\delta P = 0.01 \times 100 = 1\text{ W}$$

and Range of the readings = (100 ± 1) W i.e. 99 to 101 W **Ans.**

From the above example it is noteworthy that meters are guaranteed for better accuracies for full-scale reading but when the meters are used for lower readings the limiting error increases (say 1.5% to 6% in case of Example 3.6). If the quantity (current or voltage or any other one) under measurement is further reduced, the limiting error will further increase because the magnitude of the limiting error is a fixed quantity based on the full-scale deflection of the meter. The above examples also show the importance of taking measurements as close to full scale as possible.

Measurements or computations, combining guarantee errors, are usually made. Such a computation is illustrated in the following examples.

Example 3.8. In a Wheatstone bridge three decade resistance boxes are used which are guaranteed for ± 0.2%. An unknown resistor of R ohms is measured with this bridge. Determine the limits on resistor R imposed by the decade boxes.

Solution: Under balanced condition of Wheatstone bridge, unknown resistance R can be determined in terms of the resistances of three decade boxes, i.e. $R = \frac{R_1 R_2}{R_3}$. Here R_1, R_2 and R_3 are the resistances of the decade boxes and each is guaranteed to ± 0.2%. In worst case, the two terms in numerator may both be +ve to the full limit of 0.2% and the denominator may be –ve to the full limit of 0.2% giving a resultant error of 0.6%.

So the guarantee error is obtained by taking the direct sum of all the possible errors, adopting the algebraic signs giving the worst possible combination.

So limiting error = ±(0.2 + 0.2 + 0.2) = ±0.6% **Ans.**

Example 3.9. The current passing through a resistor of 100 ± 0.2 Ω is 2.00 ± 0.01 A. Using the relationship, $P = I^2R$, calculate the limiting error in the computed value of power dissipation.

[R.G. Technical Univ. Bhopal Electronic Instrumentation, Nov./Dec.-2007]

Solution: Percentage limiting error to resistance $= \pm\frac{0.2}{100} \times 100 = \pm 0.2\%$

Percentage limiting error to current $= \pm\frac{0.01}{2} \times 100 = \pm 0.5\%$

to assume that the individual errors connected may all be of such a sense as to affect the result in the same direction.

Provided that the errors are small, their effect on the final result is readily obtained from the simple rules outlined below:

Determination of maxm systematic Error:-

(i) Sum of Two or More Quantities. Let the final result y be the sum of measured quantities, u, v, and z, each of which is subjected to possible systematic error $\pm\delta u, \pm\delta v, \pm\delta z$.

Then the nominal result is

$$y = u + v + z$$

The relative increment of the function is given by

$$\frac{dy}{y} = \frac{d(u+v+z)}{y} = \frac{du}{y} + \frac{dv}{y} + \frac{dz}{y}$$

Expressing the result in terms of relative increment of the component quantities

$$\frac{dy}{y} = \frac{u}{y}\cdot\frac{du}{u} + \frac{v}{y}\cdot\frac{dv}{v} + \frac{z}{y}\cdot\frac{dz}{z}$$

Since the errors in the component quantities are represented by $\pm\delta u$, $\pm\delta v$ and $\pm\delta z$ then corresponding limiting error δy in y is given by

$$\frac{\delta y}{y} = \pm\left(\frac{u}{y}\cdot\frac{\delta u}{u} + \frac{v}{y}\cdot\frac{\delta v}{v} + \frac{z}{y}\cdot\frac{\delta z}{z}\right) \quad ...(3.6)$$

The above expression shows that the resultant systematic error is equal to the sum of the products formed by multiplying the individual systematic errors by the ratio of each term to the function.

Since no approximation has been made in working out this particular result, it is true for all values of the errors and is not restricted to the case of small errors.

(ii) Difference of Two Quantities.

Let $y = u - v$

$$\frac{dy}{y} = \frac{du}{y} - \frac{dv}{y}$$

Expressing the result in terms of relative increments of component quantities

$$\frac{dy}{y} = \frac{u}{y}\cdot\frac{du}{u} - \frac{v}{y}\cdot\frac{dv}{v}$$

If the errors in u and v are $\pm\delta u$ and $\pm\delta v$ respectively, the signs may be interpreted to give the worst possible discrepancy i.e., when the error in u is $+\delta u$ and the error in v is $-\delta v$ and vice versa, then the corresponding relative limiting error δy in y is given by

$$\frac{\delta y}{y} = \pm\left(\frac{u}{y}\cdot\frac{\delta u}{u} + \frac{v}{y}\cdot\frac{\delta v}{v}\right) \quad ...(3.7)$$

This expression is the same as obtained in first case. It may, however, be mentioned that in this case when u and v are almost equal in magnitude then the relative error in y would be very large.

(iii) Product of Two or More Quantities.

Let $y = uvz$

$$\log_e y = \log_e u + \log_e v + \log_e z$$

Differentiating with respect to y, we get

$$\frac{1}{y} = \frac{1}{u}\cdot\frac{du}{dy} + \frac{1}{v}\cdot\frac{dv}{dy} + \frac{1}{z}\cdot\frac{dz}{dy}$$

$$\text{or} \quad \frac{dy}{y} = \frac{du}{u} + \frac{dv}{v} + \frac{dz}{z}$$

Representing the errors in u, v, and z as $\pm\delta u$, $\pm\delta v$ and $\pm\delta z$ respectively, the error δy in y is given by

$$\frac{\delta y}{y} = \pm\left(\frac{\delta u}{u} + \frac{\delta v}{v} + \frac{\delta z}{z}\right) \quad ...(3.8)$$

From the above expression we conclude that the relative limiting error of the product of the terms is equal to the sum of the relative errors of the terms.

(iv) *Quotient of Two Quantities.*

$$\text{Let } y = \frac{u}{v}$$

$$\log_e y = \log_e u - \log_e v$$

Differentiating with respect to y, we have

$$\frac{1}{y} = \frac{1}{u}\cdot\frac{du}{dy} - \frac{1}{v}\cdot\frac{dv}{dy}$$

$$\text{or} \quad \frac{dy}{y} = \frac{du}{u} - \frac{dv}{v}$$

Representing the errors in u and v as $\pm\,\delta u$ and $\pm\delta v$ respectively, the relative error in y is given by

$$\frac{\delta y}{y} = \pm\frac{\delta u}{u} \mp \frac{\delta v}{v}$$

The maximum possible error occurs when $\delta u/u$ is +ve and $\delta v/v$ is –ve or vice versa.

∴ Relative limiting error in y is given by the expression

$$\frac{\delta y}{y} = \pm\left(\frac{\delta u}{u} + \frac{\delta v}{v}\right) \quad ...(3.9)$$

(v) *Power of a Factor*

$$\text{Let } y = u^n$$

where n may be +ve or –ve, integral or fractional

$$\log_e y = n \log_e u$$

Differentiating with respect to y, we get

$$\frac{1}{y} = n\cdot\frac{1}{u}\cdot\frac{du}{dy}$$

$$\text{or} \quad \frac{dy}{y} = n\frac{du}{u}$$

Hence the relative limiting error of y is

$$\frac{\delta y}{y} = \pm\, n\,\frac{\delta u}{u} \quad ...(3.10)$$

(vi) *Composite Factors*

$$\text{Let } y = u^n v^m$$

$$\log_e y = n \log_e u + m \log_e v$$

Differentiating with respect to y we get

$$\frac{1}{y} = \frac{n}{u}\frac{du}{dy} + \frac{m}{v}\frac{dv}{dy}$$

$$\text{or} \quad \frac{dy}{y} = n\frac{du}{u} + m\frac{dv}{v}$$

∴ Relative limiting error of y is

$$\frac{\delta y}{y} = \pm\left(n\frac{\delta u}{u} + m\frac{\delta v}{v}\right) \quad ...(3.11)$$

Example 3.11. Explain the limiting error in terms of true value. Two capacitors $C_1 = 150 \pm 2.4\ \mu F$, $C_2 = 120 \pm 1.5\ \mu F$ connected in parallel, what is the limiting error of the resultant capacitance C?
[*G.B. Technical Univ. Electronic Instrumentation and Measurements, 2011-12*]

Solution: We have $u = 150 \pm 2.4\ \mu F$
$v = 120 \pm 1.5\ \mu F$

When the two capacitors are connected in parallel, the resultant capacitance is

$$y = u + v = (150 \pm 2.4) + (120 \pm 1.5) = (270 \pm 3.9)\ \mu F$$

Therefore, the limiting error is $\pm$ 3.9 μF **Ans.**

Relative limiting error is

$$\frac{\delta y}{y} = \pm\frac{3.9}{270} = \pm 0.0144 \text{ or } \pm 1.44\% \text{ **Ans.**}$$

Example 3.12. The limiting errors for a four dial resistance box are:

Units	:	**± 0.2%**
Tens	:	**± 0.1%**
Hundreds	:	**± 0.05%**
Thousands	:	**± 0.02%**

If the resistance value is set at 4,325 Ω, calculate the limiting error for this value.
[*U.P.S.C. I.E.S. Electrical Engineering-I, 2007*]

Solution:

$$\text{Error in thousands} = \pm\frac{0.02}{100}\times 4{,}000 = \pm 0.8\ \Omega$$

$$\text{Error in hundreds} = \pm\frac{0.05}{100}\times 300 = \pm 0.15\ \Omega$$

$$\text{Error in tens} = \pm\frac{0.1}{100}\times 20 = \pm 0.02\ \Omega$$

$$\text{Error in units} = \pm\frac{0.2}{100}\times 5 = \pm 0.01\ \Omega$$

$$\text{Total error} = \pm(0.8 + 0.15 + 0.02 + 0.01) = \pm 0.98\ \Omega$$

$$\text{Limiting error} = \pm\frac{0.98}{4{,}325}\times 100 = \pm 0.02266\% \quad \textbf{Ans.}$$

Example 3.13. Two resistors having the following ratings: $R_1 = 200\ \Omega \pm 10\%$ and $R_2 = 500 \pm 5\%$. Calculate:

(i) the magnitude of error in each resistor.
(ii) the limiting error in ohms when the resistors are connected in series.
(iii) the limiting error in ohms when the resistors are connected in parallel.
[*J.N. Technological Univ. Hyderabad Electronic Measurements and Instrumentation, February/March 2012*]

Solution: (i) Magnitude of error in resistor R_1, $\delta R_1 = \pm\frac{10}{100}\times 200 = \pm 20\ \Omega$ **Ans.**

Magnitude of error in resistor R_2, $\delta R_2 = \pm\frac{5}{100}\times 500 = \pm 25\ \Omega$ **Ans.**

(*ii*) When the two resistors are connected in series

Equivalent resistance,

$$R_{se} = R_1 + R_2 = 200 + 500 = 700\ \Omega$$

Limiting error, $\delta R = \delta R_1 + \delta R_2 = \pm 20\ \Omega \pm 25\ \Omega = \pm 45\ \Omega$ **Ans.**

(*iii*) When the two resistors are connected in parallel

Equivalent resistance,

$$R_P = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{200 \times 500}{200 + 500} = \frac{1{,}000}{7}\ \Omega$$

Let $R_P = \dfrac{X}{Y}$

Then $X = R_1 \times R_2 = 200 \times 500 = 100{,}000$

$Y = R_1 + R_2 = 200 + 500 = 700$

$$\text{Error in } X = \frac{\delta R_1}{R_1} + \frac{\delta R_2}{R_2} = \pm 10 \pm 5 = \pm 15\%$$

$$\text{Error in } Y = \frac{\delta R_1}{Y} + \frac{\delta R_2}{Y} = \frac{R_1}{Y} \times \frac{\delta R_1}{R_1} + \frac{R_2}{Y} \times \frac{\delta R_2}{R_2} = \pm \frac{200}{700} \times 10 \pm \frac{500}{700} \times 5 = \pm \frac{45}{7}\%$$

So percentage error (maximum possible) in equivalent parallel resistance

= Error in X + Error in Y

$$= \pm 15\% \pm \frac{45}{7}\% = \pm \frac{150}{7}\%$$

$$\text{Limiting error in ohms} = \pm \frac{150}{700} \times \frac{1{,}000}{7} = \pm 30.6122\ \Omega \ \textbf{Ans.}$$

Example 3.14. Three resistors have the following ratings:

$$R_1 = 200\ \Omega \pm 5\%,\ R_2 = 100\ \Omega \pm 5\%,\ R_3 = 50\ \Omega \pm 5\%$$

Determine the magnitude of resultant resistance and limiting errors in percentage and ohms, if the above resistances are connected in (*a*) series and (*b*) parallel.

[*U.P. Technical Univ. Electrical and Electronics Measurements and Instruments, 2013-14*]

Solution: (*a*) When the resistances are connected in series

Equivalent resistance,

$$R_{se} = R_1 + R_2 + R_3 = 200 + 100 + 50 = 350\ \Omega \ \textbf{Ans.}$$

Relative limiting error of series resistances in percentage

$$= \frac{R_1}{R_{se}} \cdot \frac{\delta R_1}{R_1} + \frac{R_2}{R_{se}} \cdot \frac{\delta R_2}{R_2} + \frac{R_3}{R_{se}} \cdot \frac{\delta R_3}{R_3} = \pm\left(\frac{200}{350} \times 5 + \frac{100}{350} \times 5 + \frac{50}{350} \times 5\right) = \pm 5\% \ \textbf{Ans.}$$

Relative limiting error of series equivalent resistance in ohms

$$= \pm 350 \times \frac{5}{100} = \pm 17.5\ \Omega \ \textbf{Ans.}$$

(*b*) When the resistances are connected in parallel

The equivalent resistance is given by the expression

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_2 R_3 + R_1 R_3 + R_1 R_2}{R_1 R_2 R_3}$$

$$R_p = \frac{R_1 R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2} = \frac{200 \times 100 \times 50}{100 \times 50 + 200 \times 50 + 200 \times 100} = 28.57\ \Omega$$

Let $R_p = \dfrac{X}{Y}$

then $X = R_1 R_2 R_3 = 200 \times 100 \times 50 = 10{,}00{,}000$

and $Y = R_1 R_2 + R_2 R_3 + R_3 R_1 = y_1 + y_2 + y_3 = 200 \times 100 + 100 \times 50 + 50 \times 200 = 35{,}000$

$$\text{Error in } X = \frac{\delta R_1}{R_1} + \frac{\delta R_2}{R_2} + \frac{\delta R_3}{R_3} = \pm(5 + 5 + 5) = \pm 15\%$$

$$\text{Error in } y_1 = \frac{\delta R_1}{R_1} + \frac{\delta R_2}{R_2} = \pm(5 + 5) = 10\%$$

$$\text{Error in } y_2 = \frac{\delta R_2}{R_2} + \frac{\delta R_3}{R_3} = \pm(5 + 5) = 10\%$$

$$\text{Error in } y_3 = \frac{\delta R_3}{R_3} + \frac{\delta R_1}{R_1} = \pm(5 + 5) = \pm 10\%$$

Percentage error in Y

$$= \left(\frac{y_1}{Y}\frac{\delta y_1}{y_1} + \frac{y_2}{Y}\frac{\delta y_2}{y_2} + \frac{y_3}{Y}\frac{\delta y_3}{y_3}\right) \times 100 = \pm\left(\frac{20{,}000}{35{,}000} \times 10 + \frac{5{,}000}{35{,}000} \times 10 + \frac{10{,}000}{35{,}000} \times 10\right)$$

$$= \pm 10\%$$

∴ Percentage error (maximum possible) in equivalent parallel resistance

$= 15 + 10 = \pm 25\%$ **Ans.**

Error (maximum possible in equivalent parallel resistance in ohms)

$$= 28.57 \times \frac{25}{100} = 7.1425\ \Omega \ \textbf{Ans.}$$

Example 3.15. A resistor is measured by the voltmeter-ammeter method. The voltmeter reading is 123.4 V on the 250 V scale and the ammeter reading is 283.5 mA on the 500 mA scale. Both meters are guaranteed to be accurate within ±1 per cent of full-scale reading. Calculate (a) the indicated value of the resistance (b) the limits within which the result can be guaranteed.

[*R.G.P.V. Electronic Instrumentation, December 2005, G.B. Technical Univ. Electrical Measurements and Measuring Instruments, 2012-13*]

Solution: (*a*) Indicated value of resistance, $R = \dfrac{V}{I} = \dfrac{123.4}{283.5 \times 10^{-3}} = 435.27\ \Omega$ **Ans.**

The magnitude of limiting error of the voltmeter, $\delta V = \varepsilon_r V = \pm \dfrac{1}{100} \times 250 = \pm 2.5\ V$

The magnitude of voltage under measurement, $V = 123.4\ V$

The percentage limiting error at this voltage $= \pm \dfrac{2.5}{123.4} \times 100 = \pm 2.0259\%$

The magnitude of limiting error of the ammeter, $\delta I = \pm \dfrac{1}{100} \times 500 = \pm 5\ mA$

The magnitude of current under measurement, $I = 283.5\ mA$

The percentage limiting error at this current $= \pm \dfrac{5\ mA}{283.5\ mA} \times 100 = \pm 1.7637\%$

(*b*) Relative limiting error in resistance measurement,

$$\frac{\delta R}{R} \times 100 = \pm\left[\frac{\delta V}{V} + \frac{\delta I}{I}\right] \times 100$$

$$= \pm(2.0259 + 1.7637) = \pm 3.7896\% \ \textbf{Ans.}$$

Let $v = r(Q + S) = ru = 200 \times (2{,}000 + 2{,}000) = 0.8 \times 10^6$

Percentage error in $v = \frac{\delta r}{r} + \frac{\delta u}{u} = \pm 0.5 \pm 0.75 = \pm 1.25\%$

Let $x = QS = 2{,}000 \times 2{,}000 = 4 \times 10^6$

Percentage error in $x = \frac{\delta Q}{Q} + \frac{\delta S}{S} = \pm 1.0 \pm 0.5 = \pm 1.5\%$

Let $y = r(Q + S) + QS = v + x = 0.8 \times 10^6 + 4 \times 10^6 = 4.8 \times 10^6$

Percentage error in $y = \left[\frac{v}{y} \cdot \frac{\delta v}{v} + \frac{x}{y} \cdot \frac{\delta x}{x}\right] = \pm\left[\frac{0.8 \times 10^6}{4.8 \times 10^6} \times 1.25 + \frac{4 \times 10^6}{4.8 \times 10^6} \times 1.5\right] = \pm 1.458\%$

Percentage error in inductance

$$L_x = \frac{\delta C}{C} + \frac{\delta P}{P} + \frac{\delta S}{S} + \frac{\delta y}{y} = \pm 1.0 \pm 0.4 \pm 0.5 \pm 1.458 = \pm 3.358\% \text{ Ans.}$$

3.7 STATISTICAL ANALYSIS

No measurement is made with 100 per cent accuracy and, therefore, there is always some error, which varies from one determination to another, and gets introduced in the value of the quantity under measurement. It is a function of statistics to separate, as far as possible, the truth from error by narrowing and defining the region of doubt. But statistical study is mainly concerned with precision of measurement and so it cannot remove systematic errors from set of data. So systematic errors should be small as compared with residual or random errors.

To make statistical methods and interpretations meaningful, a large number of measurements is usually required.

Sometimes simple approach is required for describing and summarizing the results of the measurements. Some of these methods are described below.

(i) Arithmetic Mean. The most probable value of a measured variable is the arithmetic mean of the number of readings taken. Theoretically the best approximate value will be obtained when number of observations of the quantity under measurement is infinite but in practice, only a finite number of observations can be made. The arithmetic mean is given by the following expression

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots x_n}{n} = \frac{\Sigma x}{n} \qquad \ldots(3.12)$$

where $\bar{x}$ is arithmetic mean and $x_1, x_2, x_3, \ldots x_n$ are the readings taken and n is the number of readings taken.

(ii) Deviation from the Mean. The deviation of a reading is the amount by which it differs from the mean. If we have a set of readings $x_1, x_2, x_3 \ldots$ with mean $\bar{x}$, the deviations of the individual readings are

$$\text{Deviation of } x_1 = d_1 = x_1 - \bar{x}$$
$$\text{Deviation of } x_2 = d_2 = x_2 - \bar{x} \qquad \ldots(3.13)$$

Deviation from the mean may have a + ve or – ve value but the algebraic sum of all the deviations is always zero.

(iii) Average Deviation. Average deviation is the sum of the scaler values (without sign) of the deviations divided by the number of readings. Average deviation may be expressed as

$$D = \frac{|d_1| + |d_2| + |d_3| + \ldots |d_n|}{n} = \frac{\Sigma|d|}{n} \qquad \ldots(3.14)$$

Average deviation gives an indication of the precision of the instruments used in carrying out measurements. Low average deviation between readings shows that instruments used for measurements are highly precise.

(iv) Standard Deviation. The standard deviation of an infinite number of data is the square root of the sum of all the individual deviations squared, divided by the number of readings.

$$\text{Standard deviation, } \sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \ldots d_n^2}{n}} = \sqrt{\frac{\Sigma d^2}{n}} \qquad \ldots(3.15)$$

The standard deviation is also known as *root mean square deviation*, and is the most important factor in the statistical analysis of measurement data. Reduction in this quantity effectively means improvement in measurement.

In practice, the possible number of observations is finite. When the number of readings exceeds 20, the standard deviation is denoted by σ but if it is less than 20 the symbol 's' is used to denote the same. The standard deviation of a finite number of observations is given as

$$s = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \ldots d_n^2}{n-1}} = \sqrt{\frac{\Sigma d^2}{n-1}} \qquad \ldots(3.16)$$

Another expression for essentially the same quantity is the *variance* or *mean square deviation*, which is same as the standard deviation except that the square root is not extracted.

$$\text{So, variance, } V = \text{Mean square deviation} = \sigma^2 \qquad \ldots(3.17)$$

Variance is a convenient quantity for use in many computations because variances are additive. The standard deviation, however, has the advantage of being of the same units as the variable, making easy to compare magnitudes. Nowadays most scientific results are expressed in terms of standard deviation.

(v) Standard Deviation of Mean. When we have a multiple sample data, it is evident that the mean of various sets of data can be analyzed by statistical means. This may be accomplished by taking standard deviation of the mean given as

$$\sigma_m = \frac{\sigma}{\sqrt{n}} \qquad \ldots(3.18)$$

(vi) Standard Deviation of Standard Deviation. For a multiple sample data, the standard deviation of the standard deviation is given as

$$\sigma_\sigma = \frac{\sigma}{\sqrt{2n}} = \frac{\sigma_m}{\sqrt{2}} \qquad \ldots(3.19)$$

3.8 CHARACTERISTICS OF EXPERIMENTAL DATA

During measurement of any quantity, scattered data is obtained and this variation can be controlled by taking all care in all manipulations and by holding conditions as steady as possible during the period of measurement. But even with maximum care an unavoidable uncertainty remains. So no measurement can be carried out with absolute definiteness and as the measurements are made closer to limits, presence of smaller disturbances becomes more evident.

All known errors from data such as known systematic effects, calibration etc., should be removed first, before applying statistical methods as they are based on laws of chance, and not on consistent factors. Statistical analysis allows us to determine the best value possible from the given data and set the limits of uncertainty inherent in the scatter of the data.

The distribution of data in a set of readings may be presented in several ways, one of which is a block diagram or *histogram*. Table 3.1 shows a set of 60 current readings, that were taken at small intervals and recorded to the nearest of hundredth of an ampere. The nominal value of measured current is 10.00 A.

In Fig. 3.2, a histogram for Table 3.1 is shown, in which the number of readings are plotted against each observed current reading.

From the figure it is obvious that the largest number of readings (23) occurs at the central value of 10.00 A while the other readings are placed more or less symmetrically on either side of central value. If more readings are taken at smaller increments of 0.005 A intervals (200 readings), then the distribution of observations will remain approximately symmetrical about the central value and the shape of the histogram will be approximately the same as before. With more and more data taken at smaller and smaller increments, the contour of the histogram will finally become a smooth curve, as shown in Fig. 3.2 by the dashed line. The bell shaped curve is known as a *Gaussian curve*. The sharper and narrower the curve, the more definitely an observer may state that the most probable value of the true reading is the central value or mean value.

TABLE 3.1 *Tabulation of Current Readings*

Current Reading in Amperes	*Number of Readings*
9.97	1
9.98	3
9.99	13
10.00	23
10.01	15
10.02	4
10.03	1
Total	60

The normal, or Gaussian, law of errors is the basis for the major part of study of random effects.

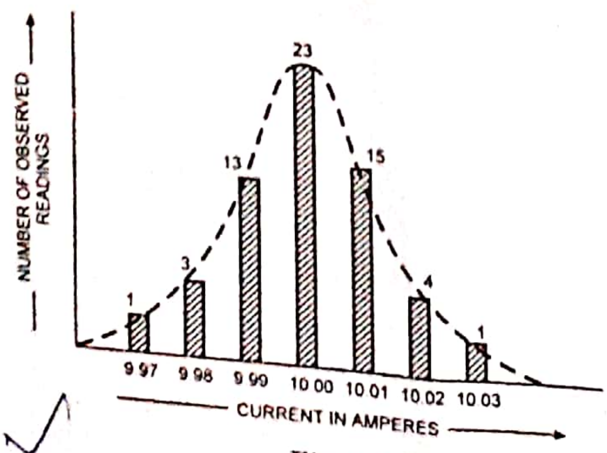

Fig. 3.2

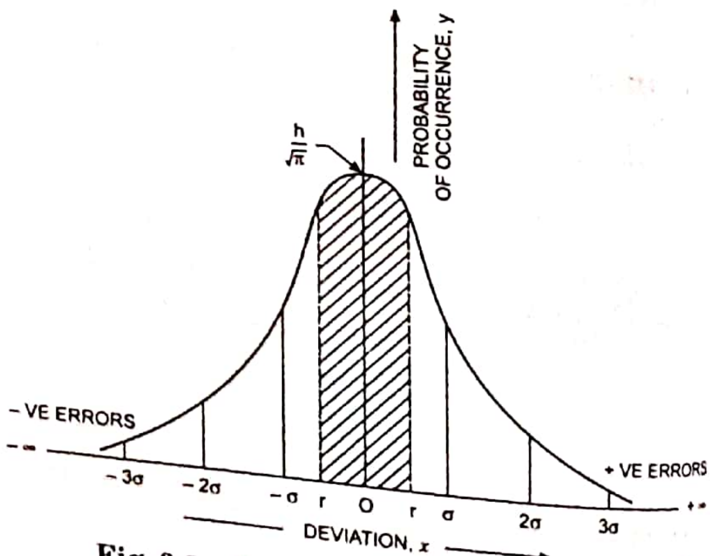

Fig. 3.3 *Curve For Normal Law of Error*

3.9 NORMAL LAW OF ERROR

When measurement of a quantity is carried out, the determinations are always finite and limited in number. Random effects cancel each other completely in an infinite set of measurements but it is not true for a small set of measurements. Hence in a limited set of measurements, mean of sample is not necessarily the mean of the larger set, and the standard deviation as obtained from the l[illegible] of measurements may not be the standard deviation of a larger sample or of the un[illegible] ally precision increases with the size of the sampl[illegible] ious samples have the propert[illegible]

The value of h is given as

$$h = \frac{1}{\sigma\sqrt{2}} \qquad ...(3.21)$$

If we substitute $h = \frac{1}{\sigma\sqrt{2}}$ in Eq. (3.20),

we have

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2} \qquad ...(3.22)$$

This form of equation is particularly useful, as σ is the quantity we ordinarily know and are interested in. σ is a quantity of the same units as the observed quantity and its mean. This makes it easy to visualize the dispersion in comparison with the mean.

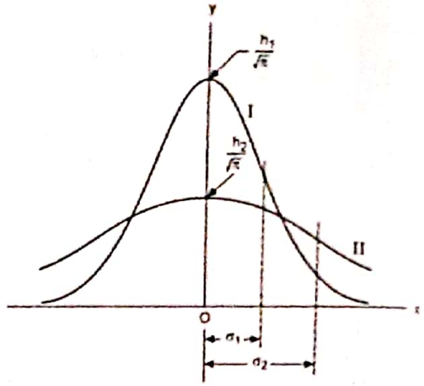

Fig. 3.4 *Comparison of Two Frequency Distribution Curves With Different Degrees of Dispersion*

3.10 PROBABLE ERROR

The area under the Gaussian probability curve shown in Fig. 3.3 within certain limits represents the number of cases among the observations within those deviation limits, expressed as a fraction of the total cases. The area between the limits $-\infty$ to $+\infty$ represents the entire number of observations and taken as unity.

The area under the curve between $-\sigma$ and $+\sigma$ limits represents the number of observations that differ from the mean by no more than the standard deviation. This area can be determined by integration in series, and for normally dispersed data, following the Gaussian distribution, the value is found to be 0.68, so 68 per cent of the cases for the normally dispersed data lie between the limits of $\pm\sigma$. Corresponding values of other deviations, expressed in terms of σ, are given in Table 3.2.

TABLE 3.2

Deviation (±)	*Fraction of Total Area Included*
0.6745σ	0.5000
1.0000σ	0.6828
2.0000σ	0.9546
3.000σ	0.9972
1.96σ	0.9500

If, for example, a large number of capacitors having nominal value of 10 μF are measured and the mean value is found to be 10.00 μF with a standard deviation of 0.02 μF, then we know that on an average 68% of all the capacitors have values lying between limits of ± 0.02 μF of the mean. There is then approximately a two to one chance that any capacitor, selected from the lot at random, will lie within these limits. If larger odds are required, then deviation may be extended to a limit of $\pm 2\sigma = \pm 0.04$ F. According to Table 3.3, this now includes 95% of all the cases, giving 21 to 1 odds and any capacitor selected at random lies within ± 0.04 μF of the mean value of 10.00 μF.

If ordinates are erected at deviations of $\pm r = 0.6745\,\sigma$, as shown in Fig. 3.3, half the area under the curve is enclosed between these limits. The quantity r is called the *probable error (PE)*. This value is probable, as shown, in the sense that there is an even chance that any one observation will have a random error no greater than $\pm r$.

The *probability of occurrence* can be stated in terms of *odds* which is the number of chances that a particular reading will occur when the error limit is specified. The odds can be determined as follows:

$$\text{Probability of occurrence} = \frac{\text{Odds}}{\text{Odds} + 1} \qquad ...(3.23)$$

The odds that the observation lies between $\pm\sigma$ limits are

$$\frac{\text{Odds}}{\text{Odds} + 1} = 0.6828$$

$$\text{or Odds} = 2.15 : 1$$

Probable error has been used in experimental work to some extent but standard deviation is more convenient in statistical work and is preferred.

3.11 MEASUREMENT DATA SPECIFICATION

After making statistical analysis of multi-sample data, the results of the measurements are to be specified. The results are expressed as deviations about a mean value. The deviations may be expressed as follows:

1. ***Standard Deviation.*** The result is expressed as $\bar{x} \pm \sigma$. The error limit in this case is the standard deviation which means that 68.28% (or about two-thirds) of all the readings have values which lie between limits of $\pm\sigma$ and the odds are 2.15 to 1. Thus there is approximately a two to one chance (or possibility) that a new observation will be within these limits.

2. ***Probable Error.*** The result is expressed as $\bar{x} \pm r$ or $\bar{x} \pm 0.6745\sigma$. It means that 50% or half of all the readings lie within these limits and odds are 1 to 1. There is an even chance that any one observation will be within these limits.

3. ***± 2σ Limits.*** The result is expressed as $\bar{x} \pm 2\sigma$. In this case, probability range is increased, approximately 95 per cent of all the readings fall within these limits and odds are 21 to 1.

4. ***± 3σ Limits.*** The result is expressed as $\bar{x} \pm 3\sigma$. The probability in this case is 0.9972 which means that 99.72% of all the readings fall within these limits i.e. practically all the readings are included in these limits. The odds of any observation falling within these limits are 356 to 1.

Example 3.22. The expected value of current through a resistor is 20 mA. However, the measurement yields a current value of 18 mA. Calculate (*i*) absolute error (*ii*) percentage error (*iii*) relative accuracy (*iv*) percentage accuracy (*v*) precision for 6th measurement if the set of 10 measurements are:

16, 19, 20, 17, 21, 18, 15, 16, 18 and 17 mA.

[*U.P.S.C. I.E.S. Electronics and Telecommunication Engineering I, 2014*]

Solution: Expected value of current, $A = 20$ mA

Measured value of current, $A_m = 18$ mA

(*i*) Absolute error, $\varepsilon_0 = A_m - A = 18 - 20 = -2$ mA **Ans.**

(*ii*) Percentage error $= \frac{A_m - A}{A} \times 100 = \frac{18-20}{20} \times 100 = -10\%$ **Ans.**

(*iii*) Relative accuracy $= 1 - \left|\frac{\varepsilon_0}{A}\right| = 1 - \frac{2}{20} = 0.9$ **Ans.**

(*iv*) Percentage accuracy = Relative accuracy × 100 = 0.9 × 100 = 90% **Ans.**

(*v*) Arithmetic mean of the set of 10 measurements,

$$\bar{x} = \frac{16+19+20+17+21+18+15+16+18+17}{10} = 17.7$$

$$\text{Precision for 6th measurement} = 1 - \left|\frac{x_6 - \bar{x}}{\bar{x}}\right| = 1 - \left|\frac{18 - 17.7}{17.7}\right| = 0.983 \text{ Ans.}$$

Example 3.23. The following set of 10 measurements was recorded during an experiment. Calculate the precision of fourth measurement.

Measurement No.	1	2	3	4	5	6	7	8	9	10
Quantity	98	102	101	97	100	103	98	106	107	99

[*U.P. Technical Univ. Electrical Measurements and Measuring Instruments, 2006-07*]

Solution: Arithmetic mean of the set of 10 measurements

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}}{10}$$

$$= \frac{98 + 102 + 101 + 97 + 100 + 103 + 98 + 106 + 107 + 99}{10} = \frac{1011}{10} = 101.1$$

Precision for fourth measurement

$$= 1 - \left|\frac{x_4 - \bar{x}}{\bar{x}}\right| = 1 - \left|\frac{97 - 101.1}{101.1}\right| = 1 - 0.04055 = 0.95945 \quad \textbf{Ans.}$$

Example 3.24. A circuit was tuned for resonance by eight different students, and the values of resonant frequency in kHz were recorded as 532, 548, 543, 535, 546, 531, 543 and 536. Calculate (i) the arithmetic mean (ii) deviations from mean (iii) the average deviation (iv) the standard deviation and (v) variance.

[*Rajasthan Technical Univ. Electronic Measurements and Instrumentation, 2006-2007, U.P.S.C. I.E.S. Electrical Engineering-I, 2013*]

Solution: (*i*) Arithmetic mean,

$$\bar{x} = \frac{532 + 548 + 543 + 535 + 546 + 531 + 543 + 536}{8} = \frac{4,314}{8} = 539.25 \text{ kHz} \quad \textbf{Ans.}$$

(*ii*) Deviations from mean,

$$\left.\begin{aligned} d_1 &= 532 - 539.25 = -7.25 \text{ kHz} \\ d_2 &= 548 - 539.25 = +8.75 \text{ kHz} \\ d_3 &= 543 - 539.25 = +3.75 \text{ kHz} \\ d_4 &= 535 - 539.25 = -4.25 \text{ kHz} \\ d_5 &= 546 - 539.25 = +6.75 \text{ kHz} \\ d_6 &= 531 - 539.25 = -8.25 \text{ kHz} \\ d_7 &= 543 - 539.25 = +3.75 \text{ kHz} \\ d_8 &= 536 - 539.25 = -3.25 \text{ kHz} \end{aligned}\right\} \quad \textbf{Ans.}$$

(*iii*) Average deviation,

$$D = \frac{\Sigma |d|}{n} = \frac{7.25 + 8.75 + 3.75 + 4.25 + 6.75 + 8.25 + 3.75 + 3.25}{8} = \frac{46.00}{8} = 5.75 \text{ kHz} \quad \textbf{Ans.}$$

(*iv*) Standard variation,

$$s = \sqrt{\frac{\Sigma d^2}{n-1}} \quad \text{because number of readings is 8, which is less than 20}$$

$$= \sqrt{\frac{(-7.25)^2 + (+8.75)^2 + (3.75)^2 + (-4.25)^2 + (+6.75)^2 + (-8.25)^2 + (+3.75)^2 + (-3.25)^2}{8-1}}$$

$= 6.54$ kHz **Ans.**

(*v*) Variance, $V = s^2 = (6.54)^2 = 42.772 \text{ (kHz)}^2$ **Ans.**

Example 3.25. The following 10 observations were recorded when measuring a voltage:

1	2	3	4	5	6	7	8	9	10
41.7	42	41.8	42	42.1	41.9	42.5	42	41.9	41.8

Find (*i*) mean (*ii*) standard deviation (*iii*) probable error of one reading.

[*U.P.S.C. I.E.S. Elec. Engineering-I, 2003*]

Solution: (*i*) Arithmetic mean,

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}}{10}$$

$$= \frac{41.7 + 42 + 41.8 + 42 + 42.1 + 41.9 + 42.5 + 42 + 41.9 + 41.8}{10} = \frac{419.7}{10} = 41.97 \quad \textbf{Ans.}$$

Deviations from the mean

$$\begin{aligned} d_1 &= 41.7 - 41.97 = -0.27 \\ d_2 &= 42 \;\; - 41.97 = +0.03 \\ d_3 &= 41.8 - 41.97 = -0.17 \\ d_4 &= 42 \;\; - 41.97 = +0.03 \\ d_5 &= 42.1 - 41.97 = +0.13 \\ d_6 &= 41.9 - 41.97 = -0.07 \\ d_7 &= 42.5 - 41.97 = +0.53 \\ d_8 &= 42 \;\; - 41.97 = +0.03 \\ d_9 &= 41.9 - 41.97 = -0.07 \\ d_{10} &= 41.8 - 41.97 = -0.17 \end{aligned}$$

(*ii*) Since the number of reading is 10, which is less than 20, the standard deviation is calculated from the equation

$$s = \sqrt{\frac{\Sigma d^2}{n-1}}$$

Standard deviation,

$$s = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2 + d_6^2 + d_7^2 + d_8^2 + d_9^2 + d_{10}^2}{10-1}}$$

$$= \sqrt{\frac{(-0.27)^2 + (+0.03)^2 + (-0.17)^2 + (+0.03)^2 + (+0.13)^2 + (-0.07)^2 + (+0.53)^2 + (+0.03)^2 + (-0.07)^2 + (-0.17)^2}{9}}$$

$$= \sqrt{\frac{0.4410}{9}} = 0.221 \quad \textbf{Ans.}$$

(*iii*) Probable error of one reading, $r = 0.6745 \times s = 0.6745 \times 0.221 = 0.149$ **Ans.**

Example 3.26. The following 10 observations were recorded when measuring a voltage:

41.7, 42.0, 41.8, 42.0, 42.1, 41.9, 42.0, 41.9, 42.5, 41.8.

Find: (*i*) the mean, (*ii*) the standard deviation, (*iii*) the probable error of one reading, (*iv*) the probable error of mean and (*v*) range.

Solution:

$$\left.\begin{aligned} &(i) \text{ Arithmetic mean, } \bar{x} = 41.97 \text{ V } \textbf{Ans.} \\ &(ii) \text{ Standard deviation, } s = 0.221 \text{ V } \textbf{Ans.} \end{aligned}\right\} \text{ As already worked out in Example 3.25}$$

(*iii*) Probable error of one reading, $r = 0.6745\, s = 0.6745 \times 0.221 = 0.149$ V **Ans.**

(*iv*) Probable error of mean, $r_m = \frac{r}{\sqrt{n-1}} = \frac{0.149}{\sqrt{10-1}} = 0.0497$ V **Ans.**

(*v*) Range $= 42.5 - 41.7 = 0.8$ V **Ans.**