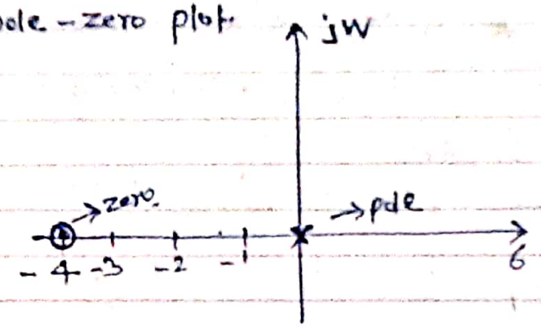
Network function.

Q:- A function is given by $Z(s) = \frac{s+4}{s}$. find the pole-zero plot.



For zero $s+4 = 0$, $s = -4$

For pole $s = 0$

Q:- A function is given by $Z(s) = \frac{2s}{s^2+6}$. Draw its pole zero plot.

Q:- Routh-Hurwitz criterion of stability of network function.

process

Let the polynomial be

$B(s) = b_0 s^m + b_1 s^{m-1} + b_2 s^{m-2} + \dots + b_m$.

First row and second row coefficient give.

$b_0 \quad b_2 \quad b_4 \quad \dots$

$b_1 \quad b_3 \quad b_5 \quad \dots$

Let $m=5$, the array will contain $(m+1)$ i.e 6 rows

Array

s^5	b_0	b_2	b_4
s^4	b_1	b_3	b_5
s^3	c_1	c_2	
s^2	d_1	d_2	
s^1	e_1		
s^0	f_1		

$$c_1 = \frac{\begin{vmatrix} b_0 & b_2 \\ b_1 & b_3 \end{vmatrix}}{b_1} = \frac{b_1 b_2 - b_0 b_3}{b_1}$$

$$c_2 = \frac{\begin{vmatrix} b_0 & b_4 \\ b_1 & b_5 \end{vmatrix}}{b_1} = \frac{b_1 b_4 - b_0 b_5}{b_1}$$

$$d_1 = \frac{\begin{vmatrix} b_1 & b_3 \\ c_1 & c_2 \end{vmatrix}}{c_1} = \frac{c_1 b_3 - b_1 c_2}{c_1}$$

$$d_2 = \frac{\begin{vmatrix} b_1 & b_5 \\ c_1 & 0 \end{vmatrix}}{c_1} = \frac{b_5 \cdot c_1 - 0}{c_1}$$

$$e_1 = \frac{\begin{vmatrix} c_1 & c_2 \\ d_1 & d_2 \end{vmatrix}}{d_1} = \frac{c_2 d_1 - c_1 d_2}{d_1}$$

$$f_1 = \frac{\begin{vmatrix} d_1 & d_2 \\ e_1 & 0 \end{vmatrix}}{e_1} = \frac{d_2 e_1}{e_1} = d_2$$

According to Routh Hurwitz criterion, the system is said to be stable, if and only if there are no changes in signs of the first column of the array. This gives the roots with negative real parts and hence gives the condition of stability.

Q:- check the given polynomials are Hurwitz

$P(s) = s^4 + s^3 + 2s^2 + 3s + 2$

$M(s) = s^4 + 2s^2 + 2$, $N(s) = s^3 + 3s$

$\therefore Z(s) = \frac{M(s)}{N(s)} = \frac{s^4+2s^2+2}{s^3+3s}$

For check

$s^3+3s)\ s^4+2s^2+2\ (s$

s^4+3s^2

$-s^2+2)\ s^3+3s\ (-s$

s^3-2s

$5s)\ -s^2+2\ (-\frac{s}{5}$

$-s^2$

$2)\ 5s\ (\frac{5s}{2}$

$5s$

$\times$

Since the continued fraction contains negative quotients hence the given polynomial is not Hurwitz.

Q:- check Hurwitz

i). $P(s) = s^4 + 11s^3 + 39s^2 + 51s + 20$ — Hurwitz

ii). $P(s) = s^4 + 7s^3 + 4s^2 + 18s + 6$ — not Hurwitz

Q.- Check the stability of the following polynomial by applying Routh - Hurwitz criterion.

$P(s) = s^4 + 2s^3 + 4s^2 + 12s + 10s^0$

Soln. In this

$b_0 = 1$, $b_2 = 4$, $b_4 = 10$, $b_1 = 2$, $b_3 = 12$

$c_1 = \frac{b_1 b_2 - b_0 b_3}{b_1} = \frac{8-12}{2} = -2$

$c_2 = \frac{b_1 b_4 - b_0 b_5}{b_1} = \frac{20 - 1\times 0}{2} = 10$

$d_1 = \frac{c_1 b_3 - b_1 c_2}{c_1} = \frac{-2\times 12 - 2\times 10}{-2} = 22$

$d_2 = b_5 = 0$

$e_1 = \frac{c_2 d_1 - c_1 d_2}{d_1} = \frac{10\times 22 - (-2)\times 0}{22} = 10$

$f_1 = d_2 = 0$

This gives

s^4	1	4	10
s^3	2	12	0
s^2	-2	10	0
s^1	22	0	
s^0	10		

There is a sign change in the 1st column of the array. Hence the system is not stable.

Positive real function :

Statement :- All real parts of the driving point immitances (Impedance or admittance) of RLC networks (passive networks) must be PR function.

$$Re\,[Z_{in}]\,(j\omega)] \geq 0$$

Q:- Check whether $F(s) = \frac{s+2}{s+1}$ is a PR function

Soln:

(I) Since all the quotient terms of F(s) are real hence F(s) is real if s is real.

(II) pole & zero plot are in s-plane. Hence it is real.

(III)

$$Re\,F(j\omega) = Re\left[\frac{j\omega+2}{j\omega+1}\right]$$

$$= Re\left[\frac{j\omega+2}{j\omega+1} \times \frac{-j\omega+1}{-j\omega+1}\right]$$

$$= Re\left[\frac{\omega^2+j\omega-2j\omega+2}{\omega^2+1}\right]$$

$$= \frac{\omega^2+2}{\omega^2+1}$$

For all values of ω, $Re[F(j\omega)] \geq 0$

The above tests certify that the given function is a PR function.

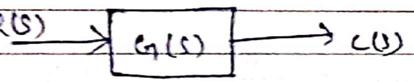
Transfer function :-

It is defined as the ratio of Laplace transform of the output and the Laplace transform of input is called transfer function

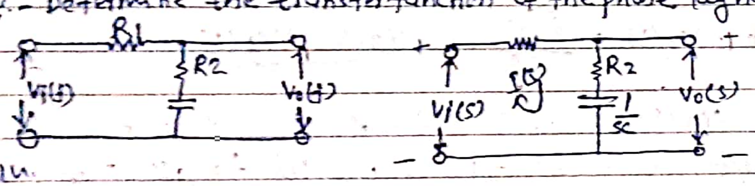
$$T.F = \frac{LT\text{ of output}}{LT\text{ of input}} = G(s)$$

$$\Rightarrow G(s) \triangleq \left.\frac{C(s)}{R(s)}\right|_{\text{all initial conditions are zero.}}$$

The transfer function can be represented as



Q:- Determine the transfer function of the phase lag network



Soln.

The fig. shows the network in s-domain

By KVL in the left-hand path

$$V_i(s) = R_1 I(s) + R_2 I(s) + \frac{1}{sc} I(s)$$

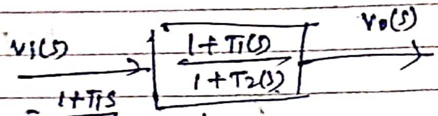
$$\Rightarrow V_i(s) = \left[\frac{(R_1+R_2)sc+1}{sc}\right] I(s)$$

By KVL in the right-hand mesh

$$V_o(s) = R_2 I(s) + \frac{1}{sc} I(s)$$

$$V_o(s) = \left[\frac{R_2 cs+1}{sc}\right] I(s)$$

$$\therefore G(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2(s+1)}{(R_1+R_2)sc+1} = \frac{1+T_1 s}{1+T_2 s}$$



Block diagram.

(7)

Q:- The ~~driving~~ point impedance of a network is given by

$$Z(s) = \frac{s^3+4s}{s^2+2}$$. Realise the network. or ~~synth.~~
or synthesis

Soln. steps.

$$s^2+2 \;)\; s^3+4s \;(\; s$$
$$\underline{-\; s^3+2s}$$
$$2s$$

$$Z(s) = s + \frac{2s}{s^2+2} = Z_1(s) + Z_2(s)$$

Thus $Z_1(s) = 1 \cdot s$ indicates that the series inductance would ~~not~~ have value of 1H.

step (ii).
$$Z_2(s) = \frac{2s}{s^2+2}, \quad Y_2(s) = \frac{s^2+2}{2s}$$

step (iii) $$Y_2(s) = \frac{s^2+2}{2s} = \frac{1}{2}s + \frac{1}{s} = Y_3(s) + Y_4(s).$$

$Y_3(s) = \frac{1}{2}s$ indicate the value of capacitance to be $\frac{1}{2}$F in parallel to $Y_4(s) = \frac{1}{s}$ which is evidently an inductor of $L = 1$ H.

The complete realisation is shown in fig.