Page

Q. A particle is in motion along a line between $x=0$ and $x=a$, the P.E. is infinite. The wave function for the particle is in the n^{th} state is given by $\psi_n = A \sin\left(\frac{n\pi x}{a}\right)$. Find the expression for normalised wave function.

Sol:- The normalised condition is determined as,

$$\int_0^a |\psi(x)|^2 dx = 1$$

$$\Rightarrow \int_0^a A^2 \sin^2 \frac{n\pi x}{a} \cdot dx = 1$$

$$\Rightarrow \frac{A^2}{2} \int_0^a 2 \sin^2 \frac{n\pi x}{a} \cdot dx = 1.$$

$$\Rightarrow A^2/2 \int_0^a \left[1 - \cos 2 \frac{n\pi x}{a}\right] dx = 1 \quad (\because 1-\cos 2\theta = 2\sin^2\theta)$$

$$\Rightarrow A^2/2 \left[\int_0^a dx - \int_0^a \cos 2 \frac{n\pi x}{a} dx\right] = 1.$$

$$\Rightarrow A^2/2 \left[x\Big|_0^a - \left(\frac{\sin 2\frac{n\pi x}{a}}{\frac{2n\pi}{a}}\right)\Big|_0^a\right] = 1$$

$$\Rightarrow \frac{A^2}{2}\left[(a-0) - \left(\frac{\sin 2n\pi x}{\frac{2n\pi}{a}}\right)\Big|_0^a = 1\right.$$

$$\Rightarrow \frac{A^2}{2}\left[a - \frac{a}{2n\pi}(\sin 2n\pi - \sin 0)\right] = 1$$

$$\Rightarrow \frac{A^2}{2}\left[a - 0\right] = 1.$$

$$\therefore A = \sqrt{\frac{2}{a}}$$

$\therefore$ Normalised wave function,

$$\boxed{\psi_n(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}}$$