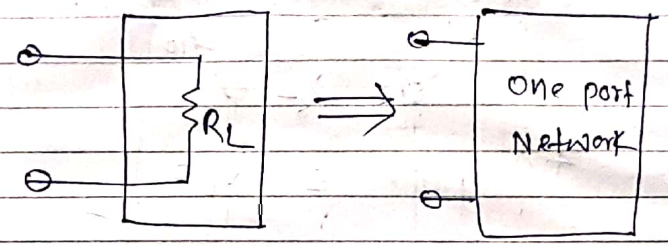
Two part Network

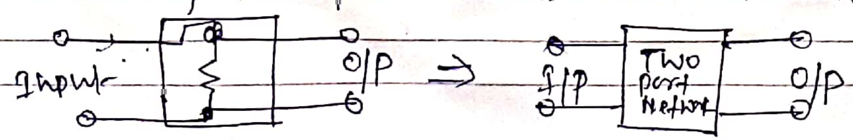
One port



Any active or passive network having only two terminals can be represented by an one port network. It is represent by a rectangular box. It must have two terminal. This rectangular box then represents one port or a two terminal network.

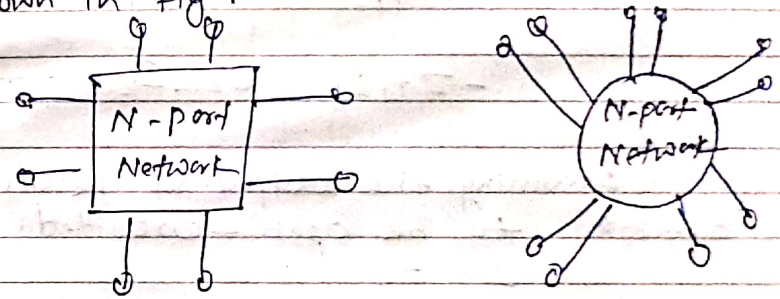
Two port :-

If the network box that represents a network consists of two pairs of terminal, where one pair of ~~terminal~~ terminals can be designated as input, the other pair being output it is called a two port network. Fig represents the diagramatic representation of a two port network. In such a network the driving source may be connected at the pair of input terminals while the other pair of terminals serves as output.

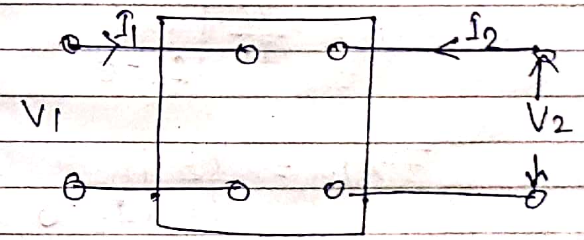


n-port network

If the network representation contains of 'n' - number of pairs of terminals, it is called a ~~n~~ n - port network. In a port network, the driving source may be connected in more than one number of pairs of terminals. The block diagram of an ~~network~~ n-port network has been shown in fig.



Z-parameters (open ckt Impedance parameters)



For the two port network In this port V_1 & I_1 are the inputs and I_2 & V_2 are outputs. It can be expressed in terms of

$$V = [Z][I] \quad \text{—— ①}$$

Where [Z] is the impedance matrix.
Equation ① can be expressed as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{—— (2)}$$

By equation ② gives

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{—— (3)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{—— (4)}$$

Assuming the output of the two-port network to be open-circuited.

$I_2 = 0$

Thus from eqn ③ with

$I_2 = 0,$

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

Again, assuming the input port of the same two port network to be open ckt

$I_1 = 0$

From eqn ③

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

and from equation

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

Then

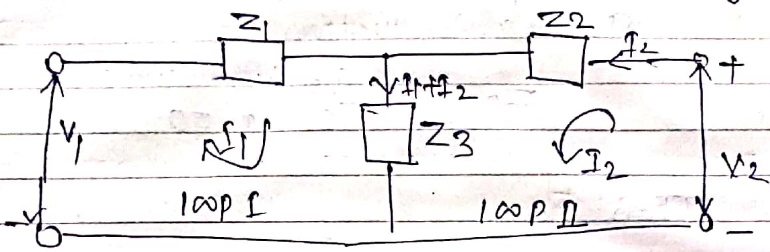
$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} \quad \& \quad Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

These are called input & o/p driving point impedance respectively,

and $Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$ & $Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$

are called & forwork transfer & impedance
Z_{11}, Z_{12}, Z_{21} & Z_{22} are called impedance parameters or open-circuit parameters of the two port network.

1. Find the Z-parameters for the following network



Soln: By KVL in the above fig, the loop eqns ~~be~~ can be written as

$$V_1 = I_1 Z_1 + (I_1+I_2) Z_3$$

$$\Rightarrow V_1 = (Z_1+Z_3) I_1 + Z_3 I_2 \quad \text{--- (I)}$$

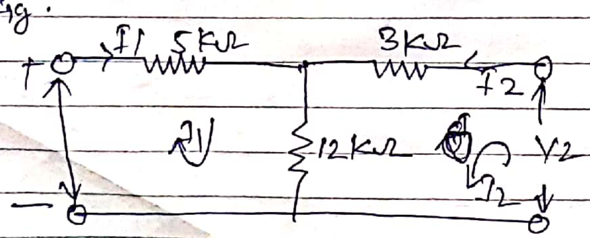
and $$V_2 = I_2 Z_2 + (I_1+I_2) Z_3$$

$$\Rightarrow V_2 = Z_3 I_1 + (Z_2+Z_3) I_2 \quad \text{--- (II)}$$

Comparing eqns (I) & (II) with the standard Z-parameter equations

$$Z_{11} = Z_1+Z_3 \; ; \; Z_{12} = Z_3$$
$$Z_{21} = Z_3 \; ; \; Z_{22} = Z_2+Z_3$$

Q:- Find Z-parameters for the network shown in fig.



Using by KVL for loops I & II.

$$V_1 = 5\times10^3 I_1 + 12\times10^3 (I_1+I_2)$$

$$\Rightarrow V_1 = 17\times10^3 I_1 + 12\times10^3 I_2 \quad \text{--- (I)}$$

and

$$V_2 = 3\times10^3 I_2 + 12\times10^3 (I_1+I_2)$$

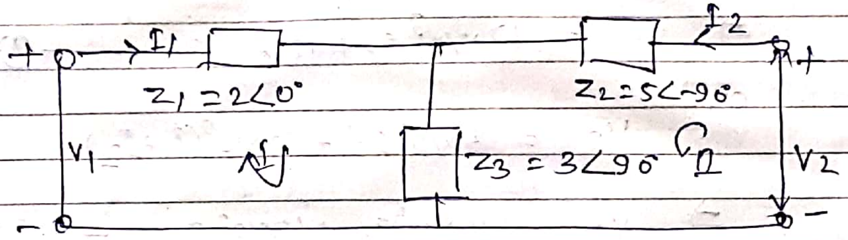
$$\Rightarrow V_2 = 12\times10^3 I_1 + 15\times10^3 I_2 \quad \text{--- (II)}$$

Comparing these two eqns (I) & (II) with Z-parameters.

$$Z_{11} = 17\times10^3 \Omega$$
$$Z_{12} = 12\times10^3 \Omega$$
$$Z_{21} = 12\times10^3 \Omega$$
$$Z_{22} = 15\times10^3 \Omega$$

7

Q:- In a T network of fig. $Z_1 = 2\angle 0°$, $Z_2 = 5\angle -90°$, $Z_3 = 3\angle 90°$, find the Z-parameters.



Soln

By KVL, the loop equations are

$V_1 = 2\angle 0° I_1 + 3\angle 90° (I_1 + I_2)$

$\Rightarrow V_1 = [2\angle 0° + 3\angle 90°] I_1 + 3\angle 90° I_2$. —(I)

and $V_2 = 5\angle -90° I_2 + 3\angle 90° (I_1 + I_2)$

$\Rightarrow V_2 = 3\angle 90° I_1 + [5\angle -90° + 3\angle 90°] I_2$ —(II)

From (I) & (II)

$Z_{11} = 2\angle 0° + 3\angle 90° = 2+3j$

$= 3.605\angle +56° \Omega$

$Z_{12} = 3\angle 90° \Omega$

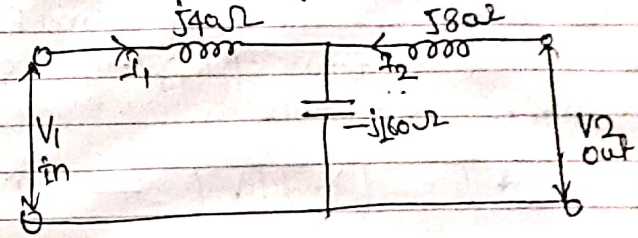
$Z_{21} = 3\angle 90° \Omega$

$Z_{22} = 5\angle -90° + 3\angle 90°$

$= -5j + 3j = -2j = 2\angle -90° \Omega$

8

Q:- Find the open ckt parameter of the two port network shown in fig.



Soln: Assuming open ckt at the output terminals

$V_1 = I_1 (j40 - 160j)$ V

$= -I_1 j120 \Omega$

Then $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = -j120 \Omega$

and

since $V_2 = I_1 (-j160$

$\therefore Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = -j160 \Omega$

Similarly,

$V_2 = I_2 j(80 - 160) = -jI_2 80$V

that gives

$Z_{22}|_{I_1=0} = \frac{V_2}{I_2} = -j80$V

and $Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{-j160 I_2}{I_2} = -j160\Omega$

$[\because V_1 = I_2(-j160\Omega)]$

Thus in this network

$Z_{11} = -j120\Omega$

$Z_{12} = -j160\Omega$ / $Z_{22} = -j80\Omega$

$Z_{21} = -j160\Omega$

9

Y-Parameters (short-ckt admittance parameters)

In a two port network, the input current I_1 and I_2 can be expressed in terms of input and output voltages V_1 and V_2 respectively as

$$[I] = [Y][V] \quad \text{—— (I)}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{—— (II)}$$

Equation (II) gives

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{—— (III)}$$

& $$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{—— (IV)}$$

Assuming output voltage $V_2 = 0$

From eqn (III) & (IV)

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$

Similarly, assuming the input port to be short ckt, $V_1 = 0$ then eqn (III) & (IV) gives

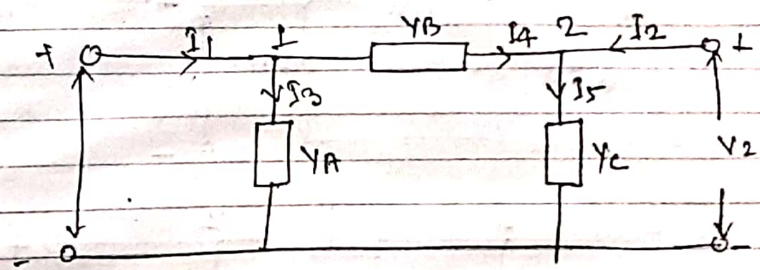
$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} \quad \&$$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

10

~~Y_{11} & Y_{22} are~~

Q:- Find the Y-parameters of the following π-ckt ~~and draw~~



By using KCL at node 1

$$I_1 = I_4 + I_3$$

$$\Rightarrow I_1 = V_1Y_A + (V_1 - V_2)Y_B$$

$$\Rightarrow I_1 = V_1(Y_A + Y_B) + (-Y_B)V_2 \quad \text{—— (I)}$$

Now, using KCL at node 2

$$I_2 = I_5 - I_4$$

$$\Rightarrow I_2 = V_2Y_C - (V_1 - V_2)Y_B$$

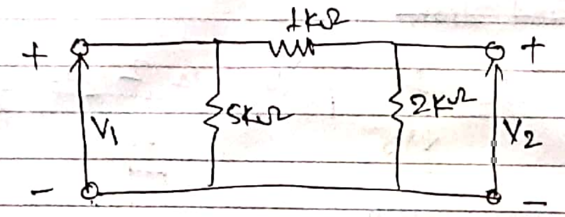
$$\Rightarrow I_2 = (-Y_B)V_1 + (Y_C + Y_B)V_2 \quad \text{—— (II)}$$

comparing eqn (I) & (II)

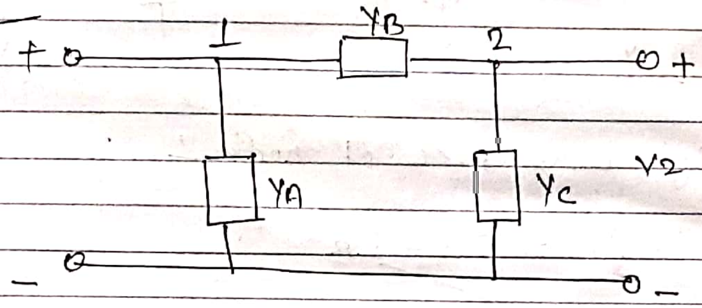
$Y_{11} = Y_A + Y_B$, $Y_{12} = -Y_B$

$Y_{21} = -Y_B$, $Y_{22} = Y_C + Y_B$

Q:- An π- attenuator has been shown in fig. Find the Y - parameters. ~~and~~



Soln.



$$\because Y_A = \frac{1}{5k\Omega} = 0.2\times10^{-3}\ \mho$$

$$Y_B = \frac{1}{1k\Omega} = 10^{-3}\ \mho$$

$$Y_C = \frac{1}{2k\Omega} = 0.5\times10^{-3}\ \mho$$

The Y-parameters are

$$Y_{11} = Y_A + Y_B = 1.2\times10^{-3}\ \mho$$

$$Y_{12} = -10^{-3}\ \mho$$

$$Y_{21} = -10^{-3}\ \mho$$

$$Y_{22} = Y_B + Y_C = 1.5\times10^{-3}\ \mho$$

Hybrid parameter

As both short ckt and open ckt terminals conditions are utilized, hence this parameter representation is known as hybrid parameter representation. In this form of representation, the voltage of the input port and the current of the ~~put~~ output port are expressed in terms of the current of the input port and the voltage of the output port.

Here,
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad \text{---} (1)$$

$$\Rightarrow V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{---} (2)$$

and
$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{---} (3)$$

Assuming short ckt conditions at the output $V_2 = 0$. This gives from eqn (2) & (3)

$$h_{11} = \frac{V_1}{I_1}$$

$$h_{21} = \frac{I_2}{I_1}$$

Here $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ & $h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$

This are called impedance and forward current gain. h_{11} is expressed in ohms while h_{21} is a unitless quantity.

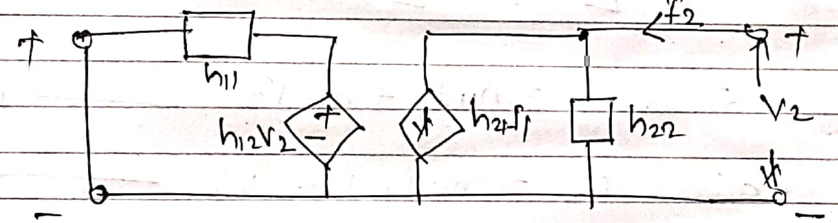
In similar way, from eqn (2) & (B) for, input ckt ~~condition~~ open-ckt condition with $I_1 = 0$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

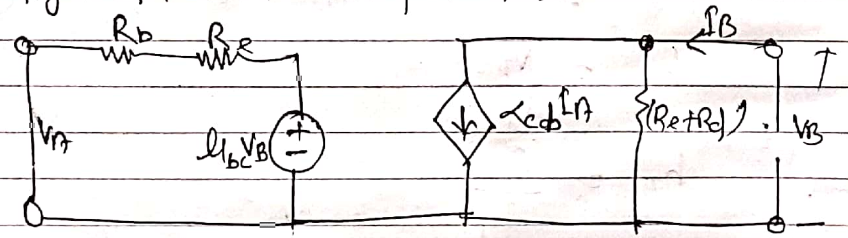
$$\& \; h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

These are called voltage gain and output admittance. h_{12} has got no unit while h_{22} has unit of mho.

The equivalent ckt of hybrid param



'Q':- CE transistor equivalent ckt has been shown in fig. Find the h-parameters



Apply KVL at the input port.

$$V_A = (R_b+R_e)I_A + \mu_{bc}V_B \quad \text{——— (1)}$$

Apply KCL at output port loop

$$I_B = \alpha_{cb}I_A + \frac{V_B}{R_e+R_d} \quad \text{——— (2)}$$

Write (1) & (2) in matrix form.

$$\begin{bmatrix} V_A \\ I_B \end{bmatrix} = \begin{bmatrix} R_b+R_e & \mu_{bc} \\ \alpha_{cb} & \frac{1}{R_e+R_d} \end{bmatrix} \begin{bmatrix} I_A \\ V_B \end{bmatrix}$$

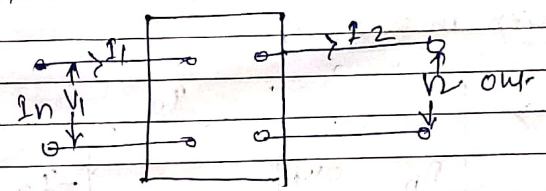
i.e. $h_{11} = R_b+R_e$; $h_{12} = \mu_{bc}$

$h_{21} = \alpha_{cb}$; $h_{22} = \frac{1}{R_e+R_d}$.

ABCD parameters

ABCD parameters are widely used in analysis of power transmission engineering where they are termed as circuit parameters. ABCD parameters are also called as "Transmission parameters".

It is conventional to designate the input port as sending end and output port as receiving end while representing ABCD parameters. Moreover, the output current direction, assumed in fig. is taken reverse.



Here, the ABCD parameter equations are given as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{ —— (I)}$$

Such that

$$V_1 = AV_2 + B(-I_2) \text{ —— (II)}$$

$$I_1 = CV_2 + D(-I_2) \text{ —— (III)}$$

Assuming receiving end to be open ckt, $I_2 = 0$

$$\therefore A = \frac{V_1}{V_2} \quad \& \quad C = \frac{I_1}{V_2}$$

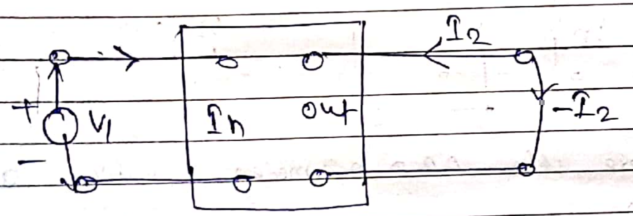
Being a ratio of voltages 'A' is called reverse ~~ratt~~ voltage ratio. ~~do~~ not have any unit. 'C' is known as transfer admittance and has the unit mho.

Let us assume that the receiving end short-ckt from eqn (I) & (II) with $V_2 = 0$

$$B = \frac{V_1}{-I_2} \quad \& \quad D = \frac{I_1}{-I_2}$$

D being a ratio of two currents, it is call reverse current ratio. It is unitless and B is expressed in ohm and is termed as transfer impedance.

Reciprocity in Z-parameter



Let the Z-parameter based two port network be represented as shown in fig. Here from the governing Z-parameter network.

$$V_1 = Z_{11}I_1 - Z_{12}I_2 \quad \text{—— (I)}$$

$$0 = Z_{21}I_1 - Z_{22}I_2 \quad \text{—— (II)}$$

From eqn (I) & (II)

$$I_2 = \frac{V_1 Z_{21}}{Z_{11}Z_{22} - Z_{21}Z_{12}} \quad \text{—— (III)}$$

Let us now interchange the input and output voltage & currents

Here from the governing equations of Z parameter network.

$$0 = -Z_{11}I_1 + Z_{12}I_2 \quad \text{—— (4)}$$

$$V_2 = -Z_{21}I_1 + Z_{22}I_2 \quad \text{—— (5)}$$

From eqn (4) & (5).

$$I_1 = \frac{V_2 Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}} \quad \text{—— (6)}$$

Assuming $V_1 = V_2$ comparision between (3) & (6)

$$Z_{12} = Z_{21}$$

Eqn This represents the condition of reciprocity.

Symmetry in Z-parameter.

Applying voltage V at input port and keeping the output port open.

$$V = Z_{11}I_1 \quad \text{i.e} \quad Z_{11} = \frac{V}{I_1}\bigg|_{I_2=0}$$

Again applying voltage V at output port and keeping open ckt at input port.

$$V = I_2 Z_{22} \quad \text{i.e.} \quad Z_{22} = \frac{V}{I_2}\bigg|_{I_1=0}$$

condition of symmetry is observed when $V/I_1 = V/I_2$ leading to the condition

$$Z_{11} = Z_{22}.$$

This is condition for symetry.

Condition of Reciprocity and symmetry.

Parameter	Condition for reciprocity	Condition for symmetry
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{21}$
h	$h_{12} = -h_{21}$	$\Delta h = 1$
ABCD	$AD - BC = 1$	$A = D$

Inter-Relationships between parameters of two port network :-

1) Z-parameters in terms of Y-parameters.

$$[Z] = [Y]^{-1}$$

$$\Rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

Thus $Z_{11} = \frac{Y_{22}}{\Delta Y}$, $Z_{12} = -\frac{Y_{12}}{\Delta Y}$

$Z_{21} = -\frac{Y_{21}}{\Delta Y}$ & $Z_{22} = \frac{Y_{11}}{\Delta Y}$

Here $\Delta y = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix} = Y_{11}Y_{22} - Y_{12}Y_{21}$.

2) Z parameters in terms of ABCD parameters

From the governing equation

$I_1 = CV_2 - DI_2$

$\Rightarrow V_2 = \frac{1}{C}I_1 + \frac{D}{C}I_2$ —(I)

Similarly

$V_1 = AV_2 - BI_2$

$\Rightarrow V_1 = \left[\frac{1}{C}I_1 + \frac{D}{C}I_2\right]A - BI_2$

$\Rightarrow V_1 = \frac{A}{C}I_1 + \frac{AD-BC}{C}I_2$ —(II)

compare eqn (I) & (II) with standard eq of Z-parameter

$\therefore Z_{11} = \frac{A}{C}$, $Z_{12} = \frac{AD-BC}{C}$

$Z_{21} = \frac{1}{C}$, $Z_{22} = \frac{D}{C}$.

3) Z-parameters in terms of hybrid parameters.

From h-parameter equations

$V_1 = h_{11}I_1 + h_{12}V_2$ —(I)

$I_2 = h_{21}I_1 + h_{22}V_2$ —(II)

From eqn (II)

$V_2 = -\frac{h_{21}}{h_{22}} + \frac{1}{h_{22}}I_2$ —(III)

From eqn (I)

$V_1 = h_{11}I_1 + h_{12}V_2$

$= h_{11}I_1 + h_{12}\left[-\frac{h_{21}}{h_{22}}I_1 + \frac{1}{h_{22}}I_2\right]$ —(IV)

$= h_{11}$

Comparing eqn (I) & (II) with standard form of Z-parameter,

$$Z_{11} = \frac{\Delta h}{h_{22}}, \quad Z_{12} = \frac{h_{12}}{h_{22}}$$

$$Z_{21} = -\frac{h_{21}}{h_{22}}, \quad Z_{22} = \frac{1}{h_{22}}.$$

Here $\Delta h = h_{11}h_{22} - h_{21}h_{12}$.

(4) Y-parameters in terms of Z-parameter

We know that $[I] = [Y][V] = [Y][Z][I]$ $\quad \because [V]=[Z][I]$

$$[Y] = [Z]^{-1}$$

$$\Rightarrow \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}.$$

Thus $Y_{11} = \frac{Z_{22}}{\Delta Z}, \quad Y_{12} = -\frac{Z_{12}}{\Delta Z}$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z}, \quad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

Here, $\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$

(5) Y-parameters in terms of Transmission parameter

From ABCD parameters

$$V_1 = AV_2 - BI_2$$

$$\Rightarrow I_2 = -\frac{1}{B}V_1 + \frac{A}{B}V_2 \quad \text{--- (I)}$$

$$\& \; I_1 = CV_2 - DI_2$$

$$= \left(\frac{D}{B}\right)V_1 - \frac{AD-BC}{B}\,V_2 \quad \text{--- (II)}$$

Comparing equations (I) & (II) with standard form of Y-parameter

$$Y_{11} = \frac{D}{B}, \quad Y_{12} = -\frac{AD-BC}{B}$$

$$Y_{21} = -\frac{1}{B}, \quad Y_{22} = \frac{A}{B}.$$

(6) Y-parameters in terms of h-parameter.

From h-parameter equation,

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$\Rightarrow I_1 = \left(\frac{1}{h_{11}}\right)V_1 + \left(-\frac{h_{12}}{h_{11}}\right)V_2 \quad \text{--- (I)}$$

$$\& \; I_2 = h_{21}I_1 + h_{22}V_2$$

$$\Rightarrow I_2 = \frac{h_{21}}{h_{11}}V_1 + \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}V_2 \quad \text{--- (II)}$$

Comparing these two equations with the standard form of Y-parameter network

$$Y_{11} = \frac{1}{h_{11}}, \quad Y_{12} = -\frac{h_{12}}{h_{11}}$$

$$Y_{21} = \frac{h_{21}}{h_{11}}, \quad Y_{22} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}$$

(7) ABCD parameter in terms of Z-parameter.

The Z-parameter equations are given by

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$\therefore I_1 = \left(\frac{1}{Z_{21}}\right) V_2 + \left(\frac{Z_{22}}{Z_{21}}\right)(-I_2)$ —— (I)

& $V_1 = Z_{11} I_1 + Z_{12} I_2$

$\therefore V_1 = \left\{ Z_{11} \left\{ \left(\frac{1}{Z_{21}}\right) V_2 + \left(-\frac{Z_{22}}{Z_{21}}\right) I_2 \right\} + Z_{12} I_2 \right.$

$\Rightarrow V_1 = \left(\frac{Z_{11}}{Z_{21}}\right) V_2 + (-I_2) \left[\frac{Z_{22} Z_{11} - Z_{12} Z_{21}}{Z_{21}}\right]$ —— (II)

Comparing eqn (I) & (II) with ABCD parameter equation.

$A = \frac{Z_{11}}{Z_{21}}, \quad B = \frac{Z_{22} Z_{11} - Z_{12} Z_{21}}{Z_{21}}$

$C = \frac{1}{Z_{21}}; \quad D = \frac{Z_{22}}{Z_{21}}.$

(8) ABCD parameters in terms of Y-parameters

From Y-parameters.

$I_2 = Y_{21} V_1 + Y_{22} V_2$

$\Rightarrow V_1 = \left(-\frac{Y_{22}}{Y_{21}}\right) V_2 + \left(-\frac{1}{Y_{21}}\right)(-I_2)$ —— (I)

and $I_1 = Y_{11} V_1 + Y_{12} V_2$ —— (II)

put the value of V_1 in eqn (II)

$I_1 = Y_{11} \left[\left(-\frac{Y_{22}}{Y_{21}}\right) V_2 + \left(\frac{1}{Y_{21}}\right) I_2 \right] + Y_{12} V_2$

$= \left[-\frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{21}} \right] V_2 - \left(-\frac{Y_{11}}{Y_{21}}\right) I_2$ —— (III)

Comparing eqn (I) & (III) with standard form of ABCD parameters.

$A = \left(-\frac{Y_{22}}{Y_{21}}\right), \quad B = -\frac{1}{Y_{21}}$

$C = \left[-\frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{21}} \right]; \quad D = \left(-\frac{Y_{11}}{Y_{21}}\right).$

(9) ABCD parameters in terms of h-parameters.

From h-parameter equations.

$I_2 = h_{21} I_1 + h_{22} V_2$

$\Rightarrow I_1 = \left(-\frac{h_{22}}{h_{21}}\right) V_2 + \left(-\frac{1}{h_{21}}\right)(-I_2)$ —— (I)

& $V_1 = h_{11} I_1 + h_{12} V_2$ —— (II)

put the value of I_1 in eqn (II)

$I_1 = \left(+\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{21}}\right) V_2 - \left(-\frac{h_{11}}{h_{21}}\right)(I_2)$ —— (III)

Compare eqn (I) & (III) with standard form of ABCD parameters.

$A = \left(-\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{21}}\right), \quad B = -\left(\frac{h_{11}}{h_{21}}\right)$

$C = \left(-\frac{h_{22}}{h_{21}}\right), \quad D = -\left(\frac{1}{h_{21}}\right)$

(10) h-parameters in terms of Z-parameters

From Z-parameters eqn.

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\Rightarrow I_2 = \left(-\frac{Z_{21}}{Z_{22}}\right) I_1 + \left(\frac{1}{Z_{22}}\right) V_2 \quad \text{—— (I)}$$

and $V_1 = Z_{11} I_1 + Z_{12} I_2$ —— (II)

put the value of I_2 in eqn (II)

$$V_1 = Z_{11} I_1 + Z_{12}\left[\left(-\frac{Z_{21}}{Z_{22}}\right) I_1 + \left(\frac{1}{Z_{22}}\right) V_2\right] \quad \text{—— (III)}$$

Comparing eqn (I) & (III) with standard form of h-parameters.

$$h_{11} = \frac{Z_{11} Z_{22} - Z_{21} Z_{12}}{Z_{22}}$$

$$h_{12} = \frac{Z_{12}}{Z_{22}}$$

$$h_{21} = -\frac{Z_{21}}{Z_{22}}$$

and $h_{22} = \dfrac{1}{Z_{22}}$

(11) h-parameters in terms of the y-parameters.

From y-parameters eqn

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{—— (I)}$$

& $I_2 = Y_{21} V_1 + Y_{22} V_2$ —— (II)

$$\Rightarrow V_1 = \left(\frac{1}{Y_{11}}\right) I_1 + \left(-\frac{Y_{12}}{Y_{11}}\right) V_2 \quad \text{—— (III)}$$

put V_1 in eqn (II)

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$= \left(\frac{Y_{21}}{Y_{11}}\right) I_1 + \left(\frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{11}}\right) V_2 \quad \text{—— (IV)}$$

comparing eqn (III) & (IV) with standard form of h-parameters.

$$h_{11} = \frac{1}{Y_{11}}$$

$$h_{12} = -\frac{Y_{12}}{Y_{11}}$$

$$h_{21} = \frac{Y_{21}}{Y_{11}}$$

$$h_{22} = \frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{11}}$$

(12) h-parameters in terms of ABCD-parameters.

From ABCD-parameters

$$I_1 = CV_2 - DI_2$$

$$\Rightarrow I_2 = \left(-\frac{1}{D}\right)I_1 + \left(\frac{C}{D}\right)V_2 \quad \text{—— (I)}$$

& $$V_1 = AV_2 - BI_2 \quad \text{—— (II)}$$

put value of I_2 in eqn (II)

$$V_1 = AV_2 - B\left[\left(-\frac{1}{D}\right)I_1 + \left(\frac{C}{D}\right)V_2\right]$$

$$= \left(\frac{B}{D}\right)I_1 + \left(\frac{AD-BC}{D}\right)V_2 \quad \text{—— (III)}$$

Comparing (II) & (III) with standard form of h-parameters.

$$h_{11} = \left(\frac{B}{D}\right)$$

$$h_{12} = \left(\frac{AD-BC}{D}\right)$$

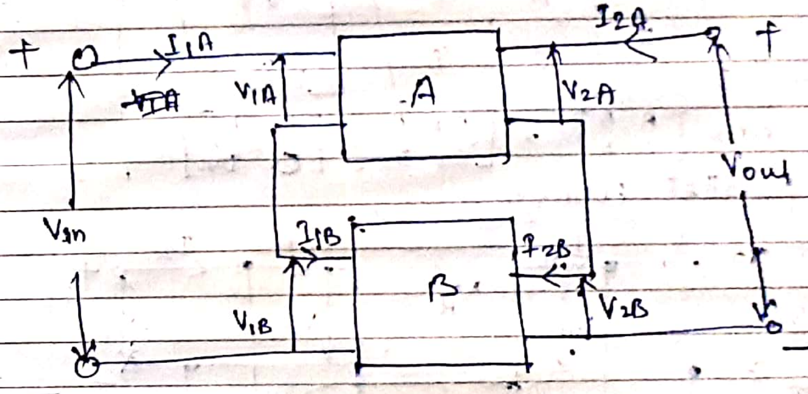
$$h_{21} = \left(-\frac{1}{D}\right)$$

$$h_{22} = \left(\frac{C}{D}\right)$$

Different types of interconnections of two port networks.

(1) series connection :-

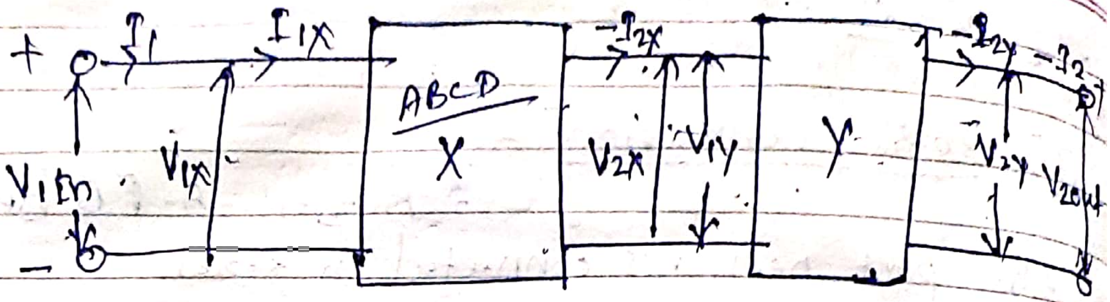
Let network A & B be the two port networks connected in series. All input and output currents and voltages shown in fig.



In this form connection.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11A}+Z_{11B} & Z_{12A}+Z_{12B} \\ Z_{21A}+Z_{21B} & Z_{22A}+Z_{22B} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

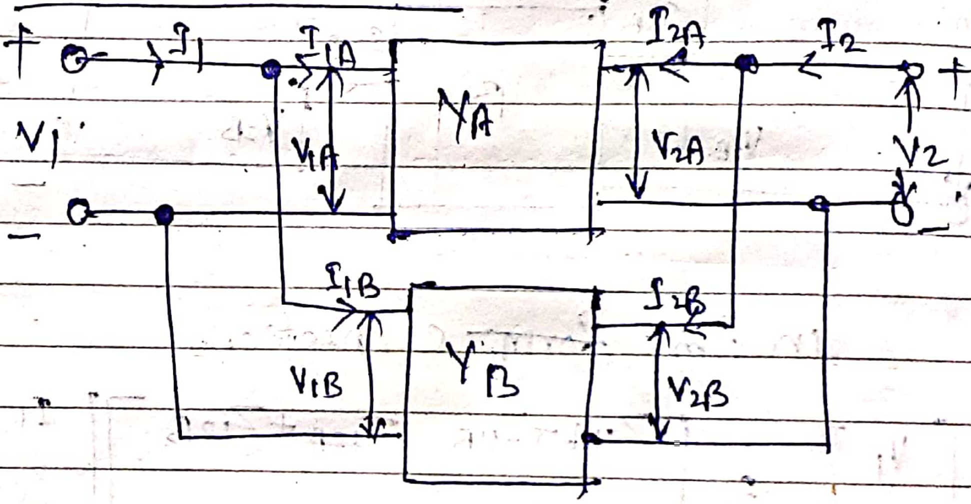
(II) Cascade connection:



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_Y \\ -I_Y \end{bmatrix}$$

Here $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} A_Y & B_Y \\ C_Y & D_Y \end{bmatrix}$

(III) parallel connection



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11A} + Y_{11B} & Y_{12A} + Y_{12B} \\ Y_{21A} + Y_{21B} & Y_{22A} + Y_{22B} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$