$$\tau_s' = -\frac{1}{2}[\sigma_x - \sigma_y]\sin(2(90+\theta)) + \tau_{xy}\cos 2(90+\theta)$$

$$= -\left\{-\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta\right\}$$

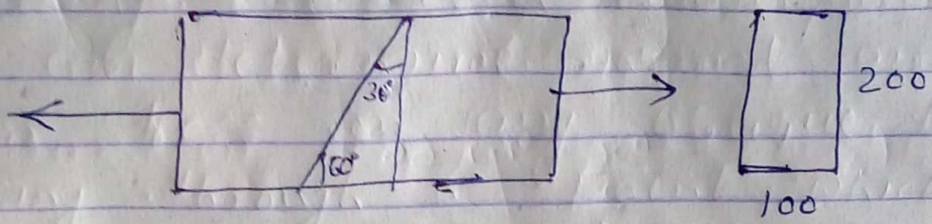
$$= -\tau_s$$

$$\therefore \tau_s' = -\tau_s$$

$$\therefore \boxed{\tau_s + \tau_s' = 0}$$

i.e complementary shear stress equal in magnitude but opposite in nature.

Q→ A tension member is formed by connecting with glue two wooden scantlings each 100mm x 200mm at their ends which are cut at an angle of 60° as shown in figure. The members is subjected to a pull P. Calculate the safe value of P if the permissible normal and shear in the glue are 2 N/mm² and 1 N/mm² respectively



Area = A = 100 × 200 = 20000 mm^2

Angle of cut with horizontal = 60°

Angle of cut (θ) with vertical

$= 90° - 60° = 30°$

$\sigma_n = 2 \, N/mm^2$

$\tau_s = 1 \, N/mm^2$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_n = \frac{\sigma_x}{2} + \frac{\sigma_x}{2} \cos 2\theta$$

$$= \frac{\sigma_x}{2} [1 + \cos 2\theta]$$

$$= \frac{\sigma_x}{2} [1 + \cos 2(30°)]$$

$$= \frac{\sigma_x}{2} [1 + \cos 60°]$$

$$= \frac{\sigma_x}{2} \times \frac{3}{2}$$

$$\therefore \sigma_x = \frac{4}{3} \sigma_n = \frac{4}{3} \times 2 = \frac{8}{3}$$

$$\boxed{\sigma_x = 2667 \, N/mm^2}$$

$$\tau_s = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\tau_s = -\frac{\sigma_x}{2} \sin 60^\circ$$

or $$\sigma_x = -\frac{2\tau_s}{\sin 60} = -\frac{2 \times 1}{\frac{\sqrt{3}}{2}}$$

$$= -\frac{4}{\sqrt{3}} = -2.309 \ N/mm^2$$

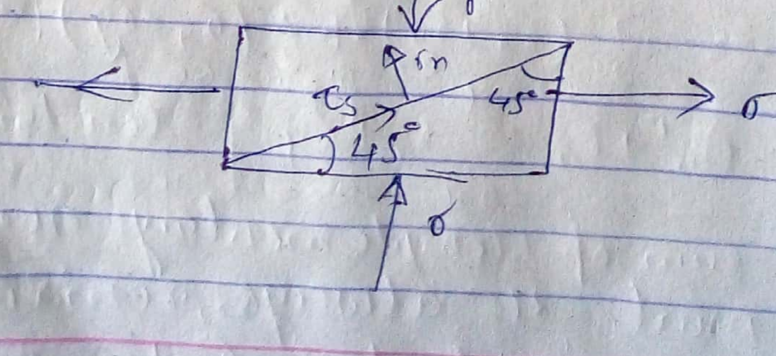
The safe stress p will be lesser of the above 2 value

ie 2.309 N/mm^2

$\therefore$ $P = 2.309 \times 20000 \times$ ~~10~~

$= 46.18$ kN

Q→ For a state of stress at a point as shown in figure, determine determine normal and shear stress acting on oblique plane



$$\sigma_n = \frac{1}{2}[\sigma_x + \sigma_y] + \frac{1}{2}[\sigma_x - \sigma_y]\cos 2\theta$$
$$+ \tau_{xy} \sin 2\theta$$

$$= \frac{1}{2}[\sigma - \sigma] + \frac{1}{2}[2\sigma]\cos 90^\circ$$
$$+ 0$$

$$= 0 + 0 + 0 = 0$$

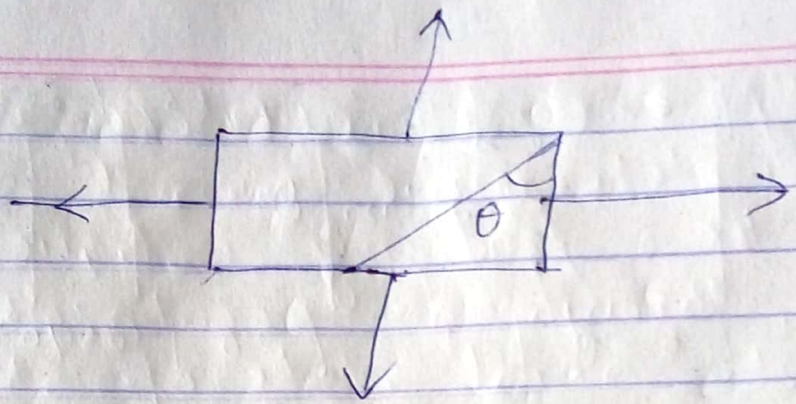
$$\cancel{\tau_s = \frac{1}{2}[\sigma + \sigma]}$$

$$\tau_s = -\left[\frac{\sigma_n - \sigma_y}{2}\right]\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{1}{2}[\sigma + \sigma]\sin 90^\circ + 0$$

$$\boxed{\tau_s = -\sigma}$$

Q Determine normal and shear stress on a oblique plane which is inclined at an angle θ, when a point is subjected to normal stress of same nature and equal in magnitude on two mutually ⊥r plane passing through that point.



$$\sigma_n = \frac{1}{2}[\sigma_x + \sigma_y] + \frac{1}{2}[\sigma_x - \sigma_y]\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{1}{2}[\sigma + \sigma] + \frac{1}{2}[\sigma - \sigma]\cos 2\theta + 0 \times \sin 2\theta$$

$$= \sigma$$

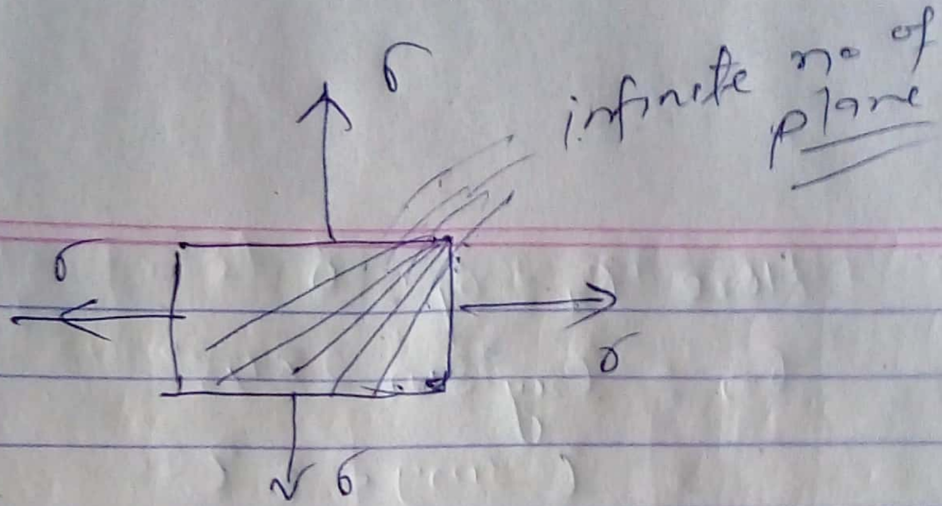
$$\tau_s = -\frac{1}{2}\{\sigma_x - \sigma_y\}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{1}{2}[\sigma - \sigma]\sin 2\theta + 0 \times \cos 2\theta$$

$$= 0$$

Observation → we see that In both case (σ_n & τ_s) is independent of θ

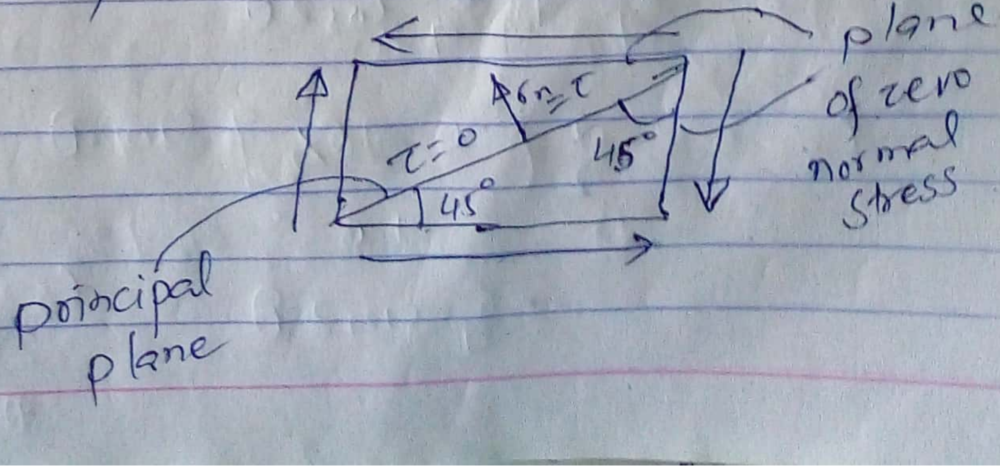
that means infinite no of such planes exists.



~~PRINCIPAL PLANE~~

PRINCIPAL PLANE :→ Principal plane is an oblique plane in which normal stress is non-zero (either max or min) but shear stress is zero, hence principal plane is also known as planes of zero shear stress or planes of maximum or minimum normal stress

Q→ For the state of stress at a point as shown in the figure, determine normal and shear stress on the oblique plane as shown in figure



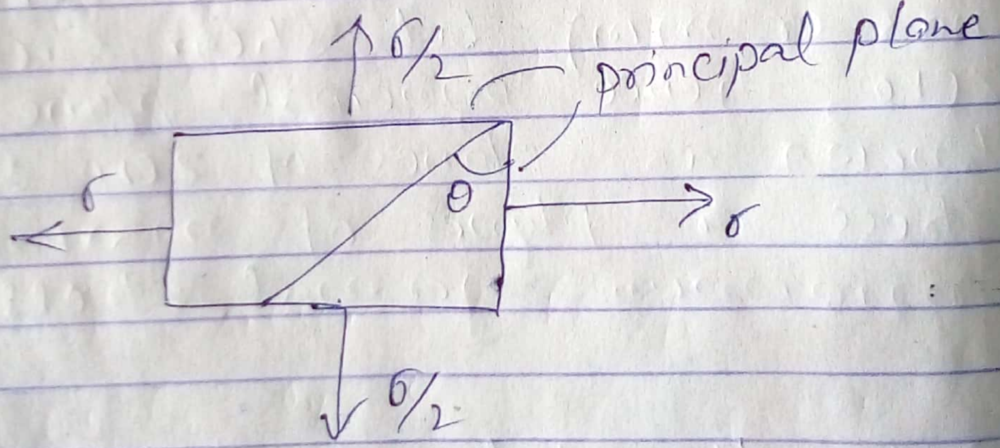
Here $\sigma_x = \sigma_y = 0$

$\sigma_n = \tau_{xy} \sin 2\times 45^\circ$

$= \tau \sin 90^\circ = \tau$

$\tau_s = \tau_{xy} \cos 2\times 45^\circ = 0$

Q→ For the state of stress shown in the figure, determine normal and shear stress on the oblique plane.



$\sigma_n = \frac{1}{2}\left[\sigma + \frac{\sigma}{2}\right] + \frac{1}{2}\left[\sigma - \frac{\sigma}{2}\right]\cos 2\theta$
$+ 0$

$= \frac{3\sigma}{4} + \frac{\sigma}{4}\cos 2\theta$

$\tau_s = -\frac{1}{2}\left[\sigma - \sigma/2\right]\sin 2\theta$
$+ 0$

$= -\frac{\sigma}{4}\sin 2\theta$

$\tau_s = 0$ when
$2\theta = 0, \text{ or } 180^\circ$
$\theta = \underline{0, 90^\circ}$

Location of Principal plane & magnitude of principal stress

$\downarrow$

$(\sigma_n)_{max}$ or $(\sigma_n)_{min}$

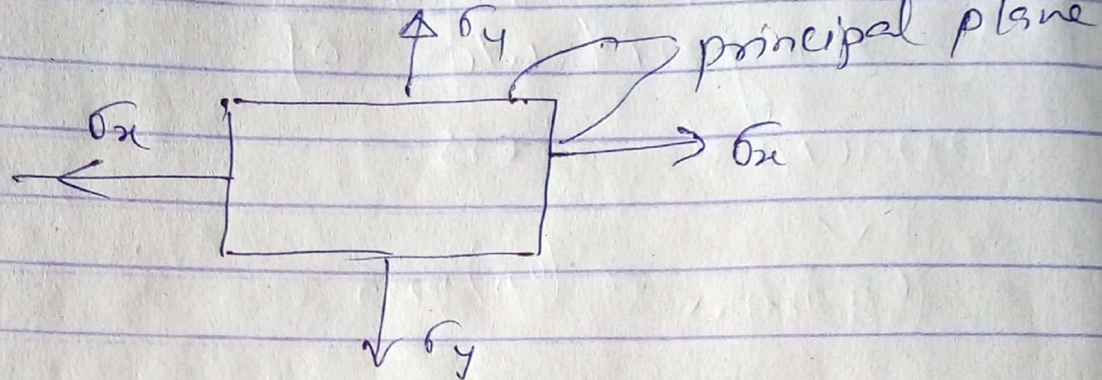
$$\boxed{\begin{array}{c}\text{Condition of Principal plane} \\ \tau_s = 0, \text{ or } \frac{d}{d\theta}(\sigma_n) = 0\end{array}}$$

or $\tau_s = -\frac{1}{2}[\sigma_x - \sigma_y]\sin 2\theta + \tau_{xy}\cos 2\theta = 0$

$$\Rightarrow \boxed{\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$$

Scanned with CamScanner

(1) pure normal state of stress



Here $\tau_{xy} = 0$

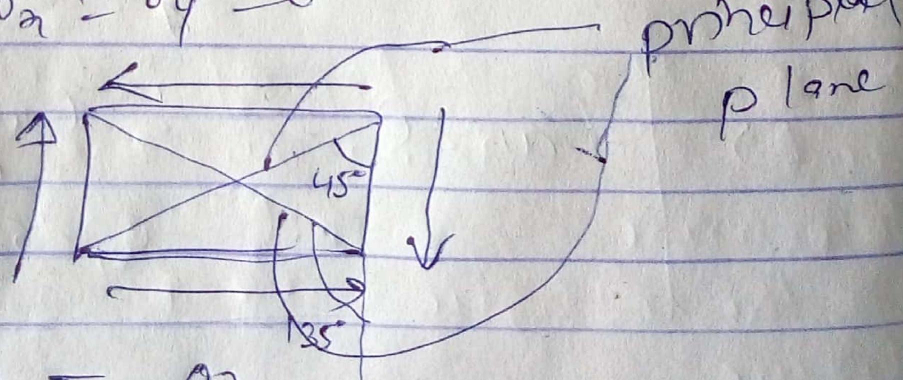
$\Rightarrow \tan 2\theta = 0$

$\Rightarrow 2\theta = 0$ or 180°

$$\boxed{\theta = 0 \text{ \& } 90^\circ}$$

(2) pure shear state of stress

$\sigma_x = \sigma_y = 0$



$\tan 2\theta = \infty$

$2\theta = 90^\circ$ or 270°

$\theta = 45^\circ$ or 135

principal plane are always mutually perpendicular to each other

as $\tan 2\theta$ has a period of $n\pi$

As $\tan 2\theta$ is period of π hence θ is $\pi/2$ hence principal plane are mutually $\perp r$ always

Conclusion → Under the biaxial state of stress there are two principal plane will exist

$$\boxed{\theta_2 = \theta_1 \pm 90^\circ}$$

In this case $\sigma_n = \tau_{xy} \sin 2\theta$ $(\theta = 45^\circ)$

$$\sigma_n = \tau_{xy} = \tau$$

$$\sigma_n' = \tau_{xy} \sin(2 \times 135)$$

$$= -\tau$$

Also $\sigma_n + \sigma_n' = 0$

$$= \sigma_x + \sigma_y = 0 \qquad \boxed{\sigma_n' = -\sigma_n}$$