Application of Schrodinger's wave equation

One Dimensional Linear Harmonic Oscillator :-

A particle undergoing S.H.M. in one dimension is called one-dimensional harmonic oscillator.

∵ in S.H.M., the restoring force is proportional to displacement i.e.,

$$F = -Kx \quad \text{--- (1)}$$

where K is a +ive constant called force constant.

According to Newton's second law, we have,

$$F = m\,d^2x/dt^2, \text{ where } m \text{ is the mass}$$

Therefore, by equation (1), we have,

$$m\,d^2x/dt^2 = -Kx$$

$$\Rightarrow m\,d^2x/dt^2 + Kx = 0$$

$$\Rightarrow d^2x/dt^2 + K/m\,x = 0 \quad \text{--- (2)}$$

$$\text{Let } \omega = \sqrt{K/m} \;\therefore\; \nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \quad \text{--- (3)}$$

The potential energy of oscillator is,

$$V = -\int^{x} Kdx = \frac{1}{2}Kx^2 \quad \text{(4)} \quad (\text{at } x=0, \text{ P.E.}=0 \text{ assum})$$

The one dimensional Schrodinger time independent equation is,

$$\partial^2\psi/dx^2 + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

$$\Rightarrow \partial^2\psi/dx^2 + \frac{2m}{\hbar^2}\left[E - \frac{1}{2}Kx^2\right]\psi = 0$$

Solving this equation under the condition that $\psi\psi^* \to 0$ at $x \to \infty$, we get the energy values of oscillator

$$\boxed{2E/\omega\hbar = 2n+1}$$

A few of these wave functions are :-