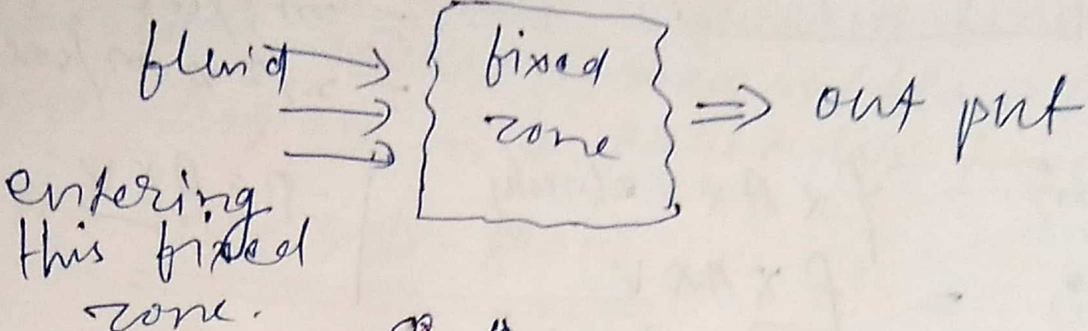
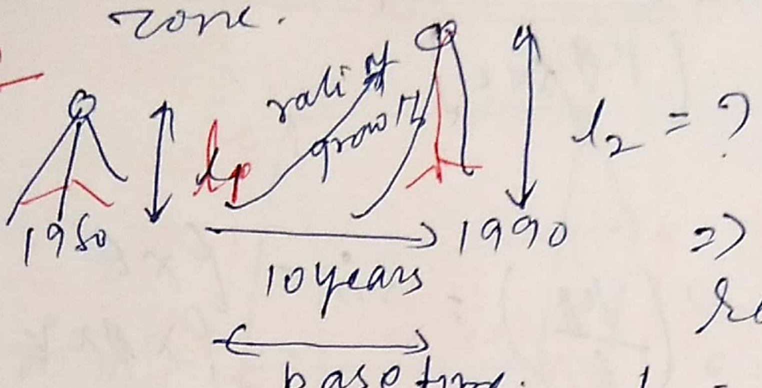
* continuity equation *

* Conservation of mass *



eg :-



$\Rightarrow l_1 + (\Delta l)$

rate = (x) cm / year.

$l_2 = l_1 +$ rate of growth × base time

(x) cm / year ↳ 10 years

differential eqn - limits

$\Rightarrow l_1 + \frac{\partial l}{\partial y} \times (dy)$ → base year change in year.

↳ rate of growth. partiall differential.

⇒ Continuity equation →

↳ Conservation of mass *

mass = kg.

→ in fluid mechanics mass rate is observed instead of only mass.

* mass rate → mass per unit time.

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$\dot{m} = \rho \times \text{discharge.}$

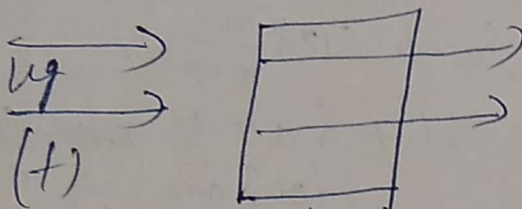
$$\boxed{\dot{m} = \rho \times Q}$$

$Q = \text{Area} \times \text{velocity}$
$= m^3/sec$
$= m^2 \times m/sec$

$\dot{m} = \rho \times A \times \text{velocity}$ | $Q = A \times \underline{V}$

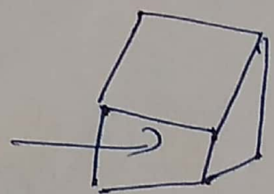
$\dot{m} = \rho \times A \times V$

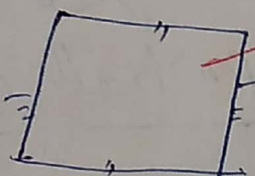
$$\boxed{\dot{m} = \rho \times Q}$$ (kg/sec.)



$\left(\frac{kg}{t}\right) = \dot{m} = \rho \times Q$
$= \rho \times A \times \underline{V}.$

* parallelopiped →



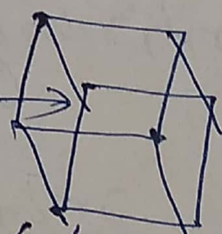


face

→ parallelogram.

* parallelopiped (3D)

(3D) →



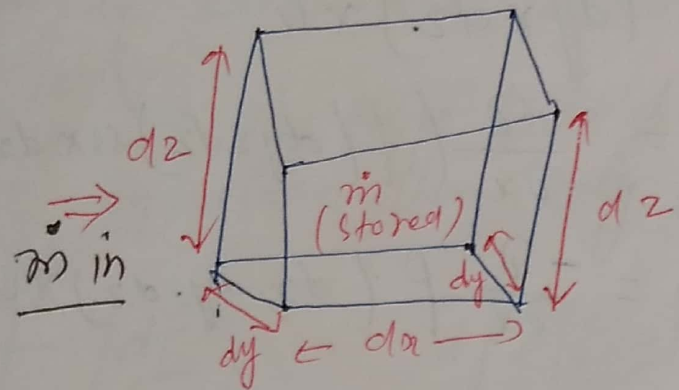
6-face

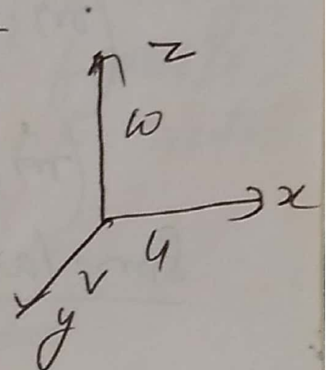
→ using-6-parallelogram

* Continuity equation *

→ It is based on Principle of conservation of mass i.e mass inflow in fixed region should be equal to mass outflow from that fixed region in a particular time.

* in 3-D-flow

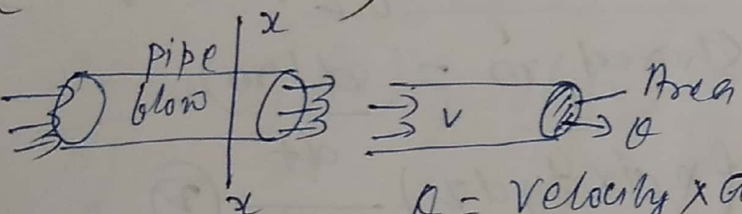




$V = f$ (space, x, y, z, t)

$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$ where $\hat{i}$ = unit vector in x-direction.

$(\dot{m}_{in} - \dot{m}_{out}) = \dot{m}_{stored}$.



Q = velocity × area
↳ cross sectional area of flow.

* in x-direction

$$(\dot{m}_{in})_x - (\dot{m}_{out})_x = (\dot{m})_{stored_x}$$

$$(\dot{m}_{in})_x = \rho \times A \times V = \rho \times Q$$

$$= \rho \times (dz \times dy) \times u$$

$(\dot{m}\,out) = \dot{m}_x\,in + \frac{\partial \dot{m}x}{\partial x} \times dx$

$\dot{m}$stored in x-direction $= (\dot{m}in)_x - (\dot{m}out)_x$

$= (\dot{m}in)_x - \{ (\dot{m}in)_x + \frac{\partial \dot{m}x}{\partial x} \times dx$

$(\dot{m}stored)_x = -\frac{\partial \dot{m}x}{\partial x} dx.$

$\dot{m}x = \rho \times (dy \times dz) \times u.$

$(\dot{m}stored)_x = \frac{\partial}{\partial x}(\rho(dy \times dz)u \times dx$

$(\dot{m}stored)_x = \frac{-\partial}{\partial x}\rho(dx \cdot dy \cdot dz) \times u.$

Similarly

$\dot{m}$stored along, y & z Co-ordinate axis

$(\dot{m}stored)_y = \frac{-\partial}{\partial y}\rho(dx\,dy\,dz) \times v$

$(\dot{m}stored)_z = -\frac{\partial \rho}{\partial z}(dx \cdot dy \cdot dz) \times w.$

Total mass stored $\dot{m} = \frac{d(m)}{dt}$

$\Rightarrow \frac{d}{dt}(\rho \times dx\,dy\,dz) \longrightarrow (2)$

Σ total mass stored in (x, y, z)
= total mass stored.

$\Rightarrow -\frac{\partial}{\partial x}\rho(dx\,dy\,dz)u - \frac{\partial}{\partial y}\rho(dx\,dy\,dz)v$
$- \frac{\partial}{\partial z}\rho(dx \cdot dy\,dz)w$
$= \frac{d}{dt}(\rho dx\,dy\,dz$

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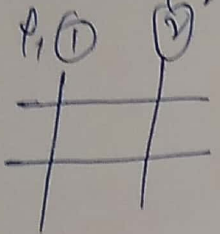
$$\Rightarrow -\frac{\partial}{\partial x}(\rho u)-\frac{\partial}{\partial y}(\rho v)-\frac{\partial}{\partial z}(\rho w)=\frac{d\rho}{dt}$$

Where ρ = mass density.

$$\therefore \Rightarrow \boxed{\frac{d\rho}{dt}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0}$$

↳ General continuity equation

↳ if flow field satisfy continuity equation then only flow is feasible else not.



$\left(\frac{d\rho}{dt}\right)$. $d\rho=(\rho_2-\rho_1)\neq 0$

↳ ρ ⇒ Not constant.
⇒ changing.
⇒ compressible.

* if $d\rho=0$ $(\rho_1=\rho_2)=0$
↳ incompressible

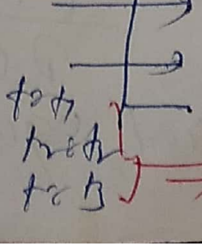
$$\Rightarrow \frac{d\rho}{dt}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$

$$\Rightarrow \frac{d\rho}{dt}+\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)+u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+w\frac{\partial \rho}{\partial z}=0$$

* for Steady flow

$\left\{\frac{d\rho}{dt}=0\right\}$

$$\left(\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0\right)$$



at any fixed Section ⇒ flow properties = constant.

* For Incompressible flow

ρ = constant i.e it is not changing

$$\Rightarrow \rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=0$$

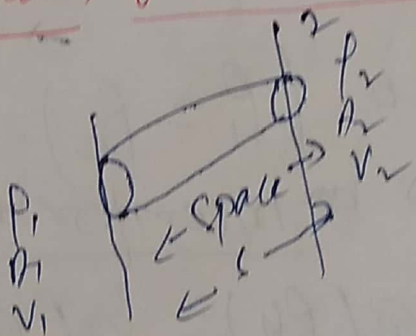
$$\Rightarrow \underset{\text{constant not zero.}}{\rho \neq 0} \text{ or } \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \checkmark$$

$\Rightarrow$. for steady & incompressible flow

$$\boxed{\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0} \quad (3-D)$$

$$\Rightarrow (2-D) \quad \left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0$$

* Continuity equation for one dimensional flow



$$\Rightarrow \frac{d\rho}{dt}+\frac{\partial(\rho A v)}{\partial s}=0$$

$$\Rightarrow \frac{d\rho}{dt}+\frac{\partial(\dot{m})}{\partial s}=0$$

$\rho A v = \dot{m}$ → Rate of change of mass (kg/sec)

$$\Rightarrow \frac{d\rho}{dt}+\frac{\partial(\rho \times A \times v)}{\partial s}=0$$

→ General equation for one dimensional flow.

$\Rightarrow$ Steady flow

$$\frac{d\rho}{dt} = 0 \qquad \frac{\partial(\rho A v)}{\partial s} = 0$$

i.e $\rho A v = \text{constant.}$

$$\boxed{\rho_1 A_1 v_1 = \rho_2 A_2 v_2} \rightarrow \text{for Steady flow only.}$$

* For Steady and Incompressible flow.

$\Rightarrow$ $\rho = \text{constant} = \underbrace{\rho_1 = \rho_2}_{\text{Incompressible}}$ | $\frac{d\rho}{dt} = 0.$ $\rightarrow$ at any particular section.

u.v.g

$$\therefore \boxed{A_1 v_1 = A_2 v_2}.$$

Q for Stead and Incompressible flow.

Vvali

$\vec{V} = (\lambda x y^3 - x^2 y)\hat{i} + (x y^2 - \frac{3}{4} y^4)\hat{j}$

find λ.

soln $\vec{V} = u\hat{i} + v\hat{j}$ $\Big\}$ $u = \lambda x y^3 - x^2 y$

$v = (x y^2 - \frac{3}{4} y^4)$

$\Rightarrow$ the given flow field must satisfy continuity equation.

for 2-D $\Rightarrow$ $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$\frac{\partial}{\partial x}(\lambda x y^3 - x^2 y) = (\lambda y^3 - 2xy)$$

Similarly $\frac{\partial v}{\partial y} = \partial(x y^2 - \frac{3}{4} y^4)$

$$= (x \cdot 2y - \frac{3}{\cancel{4}} \cancel{4} y^3)$$

$$\frac{\partial v}{\partial y} = (2xy - 3y^3)$$

$$\Rightarrow (\lambda y^3 - \cancel{2xy}) + (\cancel{2xy} - 3y^3) = 0$$

$$\lambda \cancel{y^3} = 3\cancel{y^3}.$$

$$\boxed{\lambda = 3}$$

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