Solution ①

We have, the Schrodinger equation as,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{—— (1)}$$

Given function is,

$$\psi = x\, e^{-m\omega x^2/2\hbar} \quad \text{—— (2)}$$

Differentiating the above funⁿ w.r.t. x, we get

$$\frac{d\psi}{dx} = e^{-m\omega x^2/2\hbar} - x \cdot \frac{m\omega 2x}{2\hbar} e^{-m\omega x^2/2\hbar}$$

$$= e^{-m\omega x^2/2\hbar}\left[1 - \frac{m\omega x^2}{\hbar}\right]$$

and

$$\frac{d^2\psi}{dx^2} = e^{-m\omega x^2/2\hbar}\left(0 - \frac{2m\omega x}{\hbar}\right) + \left(1 - \frac{m\omega x^2}{\hbar}\right)\left(-\frac{m\omega 2x}{2\hbar}\right)e^{-m\omega x^2/2\hbar}$$

$$= e^{-m\omega x^2/2\hbar}\left[-\frac{3m\omega x}{\hbar} + \frac{m^2\omega^2 x^3}{\hbar^2}\right]$$

$$= e^{-m\omega x^2/2\hbar} \cdot \frac{m\omega x}{\hbar}\left[-3 + \frac{m\omega x^2}{\hbar}\right]$$

$$\frac{d^2\psi}{dx^2} = \frac{m\omega x}{\hbar} e^{-m\omega x^2/2\hbar}\left[-3 + \frac{m\omega x^2}{\hbar}\right] \quad \text{—— (3)}$$

$$V = \frac{1}{2} m\omega^2 x^2 \quad \text{—— (4) (Given)}$$

On putting the value of eqⁿ ②, ③ and ④ in eqⁿ ①, we get,

$$\frac{m\omega x}{\hbar} e^{-m\omega x^2/2\hbar}\left[-3 + \frac{m\omega x^2}{\hbar}\right] + \frac{2m}{\hbar^2}\left[E - \frac{1}{2}m\omega^2 x^2\right] x \cdot e^{-m\omega x^2/2\hbar} = 0$$

$$\Rightarrow \frac{2m}{\hbar^2} \cdot x \cdot e^{-m\omega x^2/2\hbar}\left[-\frac{3\omega\hbar}{2} + \frac{m\omega^2 x^2}{2} + E - m\omega^2 x^2\right] = 0$$

$$\Rightarrow E - \frac{3\omega\hbar}{2} = 0$$

$$\therefore E = -\frac{3\omega\hbar}{2} \text{ proved}$$

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