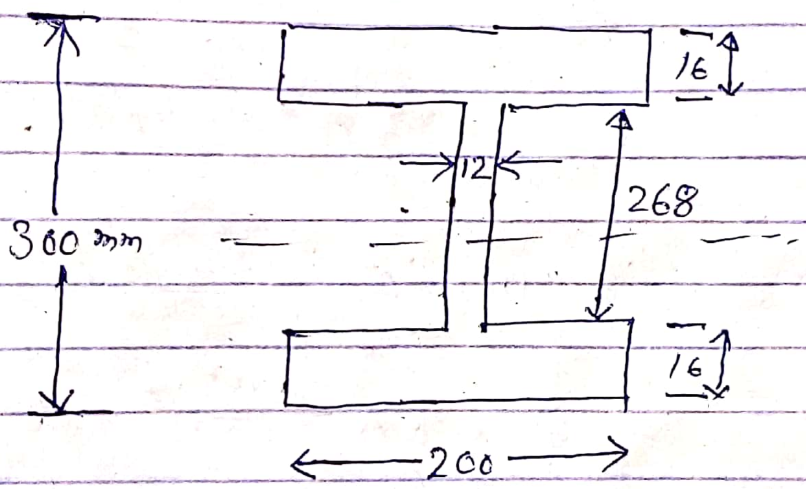
Q → A rolled steel joist 300 mm deep and 200 mm wide has flange 16 mm thick and web 12 mm thick. Calculate the proportion in which the flange and web resist (a) BM (b) S.F



$$I = \frac{1}{12} \times 200 \times (300)^3 - \frac{1}{12} \times 188 \times 268^3$$

$$= 148439465.3$$

$$\text{or } I = 2\left[\left(\frac{1}{12} \times 200 \times (16)^3\right) + (200 \times 16)(142)^2\right] + \frac{1}{12} \times 12 \times (268)^3$$

$$= 148439465.3 \text{ mm}^4$$

Case (a) Resistance to BM

consider a section δy thickness in the web at a distance y from NA

Stress $\sigma_b = \frac{M}{I} y$

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Elementary force offered by the strip $= \sigma_b A = \frac{M}{I} y \cdot (12 \delta y)$

Moment of elementary force about NA $= \delta M$

$\delta M = \frac{M}{I} y \cdot (12 \delta y) \cdot y$

$= \frac{M}{I} y^2 \cdot (12 \delta y)$

$\therefore M_r$ offered by the web

$\boxed{\frac{268}{2} = 134}$

$= M_{rw} = \int_{-134}^{134} 12 \frac{M}{I} y^2 \delta y$

$= \int_0^{134} 24 \frac{M}{I} y^2 dy$

$\frac{24 \times M \times (134)^3}{3 \times 14839465.3}$ $= \frac{24M}{I} \left[\frac{y^3}{3}\right]_0^{134}$

$\boxed{\text{Moment of resistance offered by the web} = 0.1297 M}$

Hence moment of resistance offered by the flange $= M - 0.1297 M$
$= 0.8703 M$

Hence flange takes up 87.03% of BM, while the web takes up only 12.97%.

Case (b) Resistance to shear force

Consider an elementory strip of dy thickness in the flange at a y distance from NA

The elementary area $da = 200dy$

Shear stress intensity

$$\tau = \frac{V}{2I}\left(\frac{D^2}{4} - y^2\right)$$

$$= \frac{V}{2I}\left[\frac{(300)^2}{4} - y^2\right]$$

$$= \frac{V}{2I}\left[22500 - y^2\right]$$

Shear resistance offered by the strip $= \tau\, da = \frac{V}{2I}(22500 - y^2) \times (200dy)$

$\therefore$ Shear resistence offered by one flange

$$= \frac{100V}{I}\int_{134}^{150}(22500 - y^2)\,dy$$

Now shear resistance offered by both flange

$$= \frac{200V}{I} \int_{134}^{150} (22500 - y^2)\,dy$$

$$= \frac{200V}{I} \left[22500y - \frac{y^3}{3}\right]_{134}^{150}$$

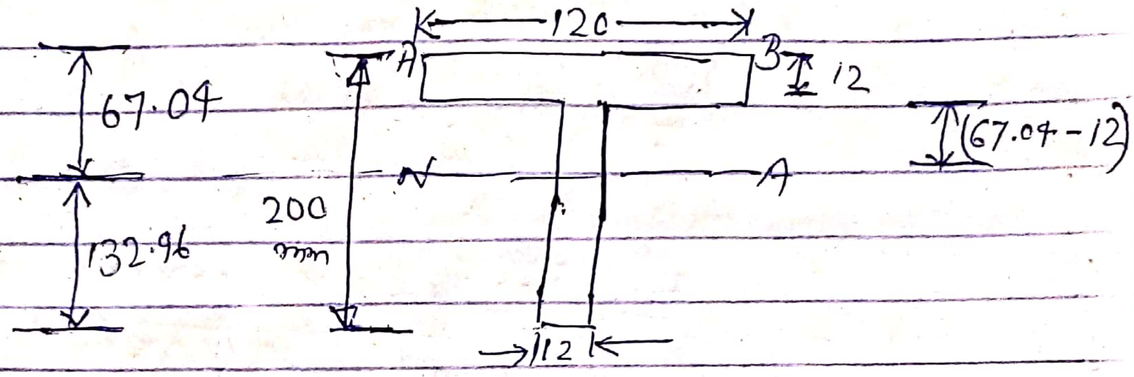
$$= \frac{7406933.334\,V}{148434965.3}$$

$$\simeq 0.05V$$

$\therefore$ Shear resistance offered by the web $= V - 0.05V$

$$= 0.95V$$

Q→ The cross section of a joist is a T-section 120 mm × 200 mm × 12 mm with 120 mm side horizontal. Sketch the shear stress distribution and hence find the maximum shear stress of it has to resist a shear force of 200kN.



$$\bar{y} = \frac{188 \times 12 \times 94 + 120 \times 12 \times 194}{188 \times 12 + 120 \times 12}$$

$= 132.96$ mm from bottom

$$I_{AB} = \frac{1}{3} \times 12 \times (200)^3 + \frac{1}{3}(108) \times (12)^3$$

$$= 3206.2208 \times 10^4 \text{ mm}^4$$

$$I_{NA} = 3206.2208 \times 10^4 - 3696\,(67.04)^2$$

(using parallel axis theorem)

$$= 15451047.53$$

$$= 1545.1047 \times 10^4 \text{ mm}^4$$

Area
$= 200 \times 12 + 108 \times 12$
$= 2400 + 1296$
$= 3696$

Now τ_{flange} (at the junction of the web and flange)

$$= \frac{V}{IB} A\bar{y}$$

$$= \frac{V}{I \times 120} 120 \times 12 \times (67.04 - 6)$$

$$= \frac{732.48\, V}{I}$$

$$= \frac{732.48 \times 200 \times 10^3}{1545.1047 \times 10^4} \quad \boxed{\text{as } V = 200\text{kN}}$$

$$= 9.48\ N/mm^2$$

Now τ_{web} at the junction with flange

$$= \tau_{flange} \times \frac{B}{b}$$

$$= 9.48 \times \frac{120}{12}$$

$$= 94.8\ N/mm^2$$

$(\tau_{web})_{max}$ at the centroid ie at NA (at 132.96 mm distance from bottom)

$$= \frac{V}{Ib} \left[(A\bar{y})_{flange} + (A\bar{y})_{web} \right]$$

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$$= \frac{V}{Ib}\left[120\times 12\,(67.04-6) + \left\{12\times(67.04-12)\times\left(\frac{67.04-12}{2}\right)\right\}\right]$$

$$= \frac{V}{I\times 12}\left[1440\times 61.04 + 12\times 55.04\times\frac{55.04}{2}\right]$$

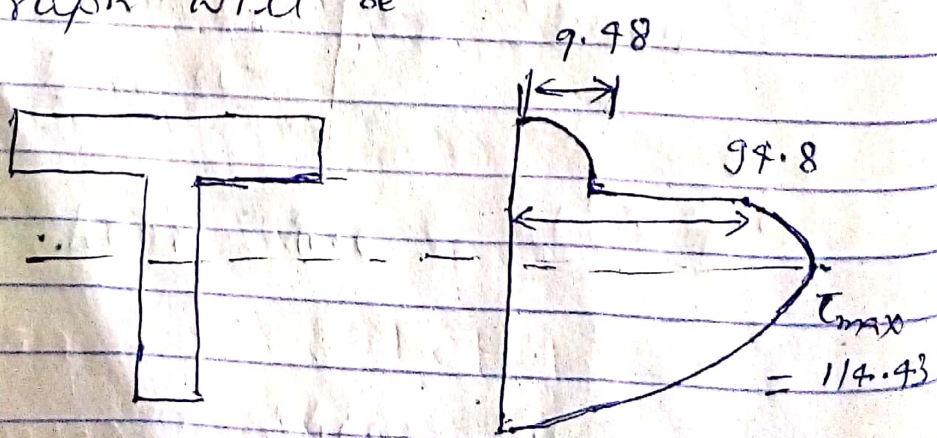
$$\simeq \frac{8840\,V}{I} \quad (8839.5008)$$

$\therefore (\tau_{web})_{maximum}$ at the centroid

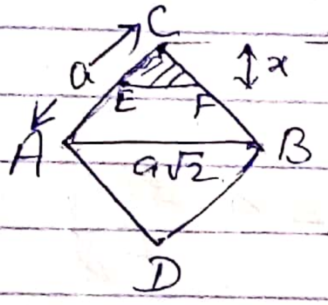
$$= \frac{200\times 10^3\times 8840}{1545\times 10^4}$$

$$= 114.43\ N/mm^2$$

Now graph will be

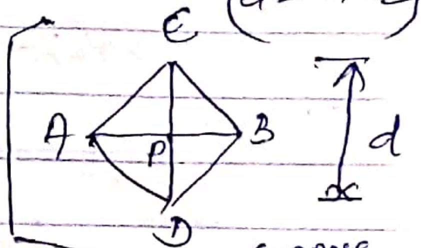


Q → A beam of square section is used as a beam with one diagonal horizontal. Find the magnitude and location of maximum shear stress in the beam. Also sketch the shear stress distribution across section.



$a/\sqrt{2} = \frac{AB}{2} = \frac{CD}{2}$

$(d = a\sqrt{2})$



Suppose
$CD = d$
$\therefore CP = \frac{d}{2}$
$\frac{d}{2} \to d$
$1 \to \frac{d}{d/2} = 2$
$x \to 2x$

Now $I_{AB} = 2 \times \frac{1}{12}(a\sqrt{2})\left(\frac{a}{\sqrt{2}}\right)^3$

$= \frac{1}{6} \times \frac{a^4}{2} = \frac{a^4}{12}$

At any distance x from apex C
width $EF = 2x$

$\boxed{\text{width } EF = 2x}$ Imp

$\therefore \tau = \frac{V}{Ib}$ (moment of the shaded area about NA ie about AB)

$= \frac{V}{I(2x)}\left[\frac{1}{2} \times 2x \cdot x\left(\frac{d}{2} - \frac{2}{3}x\right)\right]$

$= \frac{V}{\left(\frac{a^4}{12}\right) \cdot 2x}\left[\frac{x^2(3d - 4x)}{6}\right]$

$$\tau = \frac{V}{a^4} \cdot x(3d - 4x)$$

$$\boxed{\tau = \frac{V}{a^4}(3dx - 4x^2)} \rightarrow \text{parabola}$$

Now at $x = 0$, $\tau = 0$

at $x = \frac{d}{2}$

$$\tau = \frac{V}{a^4}\left[\frac{3d^2}{2} - 4 \cdot \frac{d^2}{4}\right]$$

$$= \frac{V}{a^4} \cdot \frac{d^2}{2} = \frac{V}{a^4} \cdot \frac{(a\sqrt{2})^2}{2}$$

$$= \frac{V \cdot a^2 \cdot 2}{2a^4}$$

$$\boxed{\tau_{at\, x = d/2} = \frac{V}{a^2}} = \tau_{mean}$$

Thus shear stress at NA is the τ_{mean}

For max^m shear stress

$$\frac{d\tau}{dx} = 0$$

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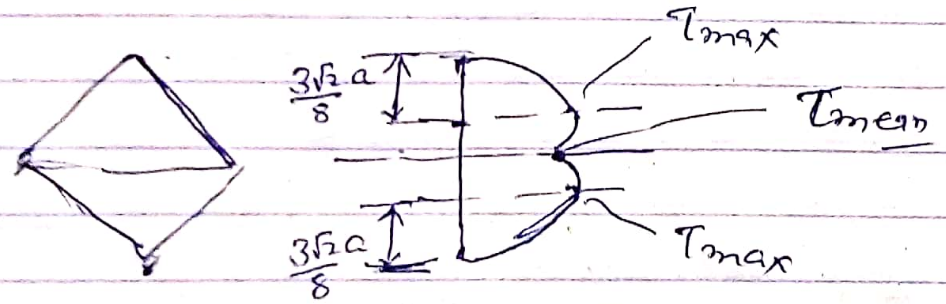
$$\therefore \frac{d}{dx}\left[\frac{V}{a^4}\left(3dx - 4x^2\right)\right] = 0$$

$$\Rightarrow \quad 3d - 8x = 0$$

$$\therefore x = \frac{3}{8}d = \frac{3}{8}a\sqrt{2}$$

$$x = \frac{3\sqrt{2}}{8}a$$



$$\tau_{max} = \frac{V}{a^4}\left[3 \times a\sqrt{2} \times \frac{3\sqrt{2}a}{8} - 4 \times \left(\frac{3\sqrt{2}}{8}a\right)^2\right]$$

$$= \frac{V}{a^2}\left[\frac{18}{8} - 4 \times \frac{18}{64}\right]$$

$$= \frac{V}{a^2}\left[\frac{4 \times 18}{64}\right] = \frac{V}{a^2} \cdot \frac{9}{8}$$

$$\boxed{\tau_{max} = \frac{9}{8}\frac{V}{a^2}}$$

$$\boxed{\therefore \frac{\tau_{max}}{\tau_{mean}} = \frac{9}{8} = 1.125}$$