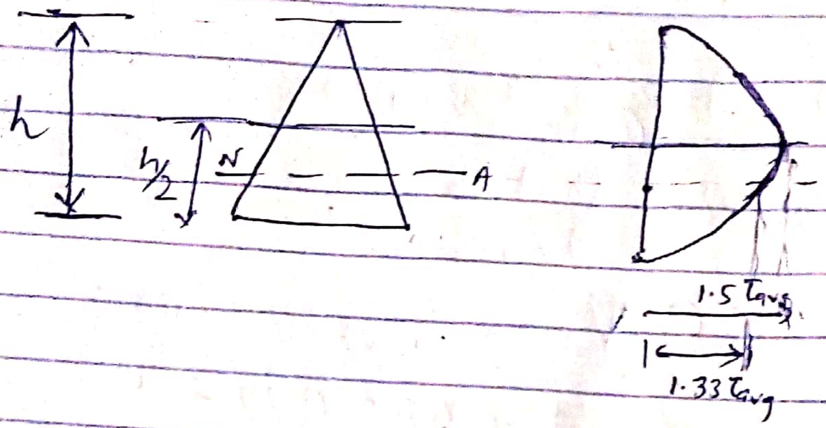
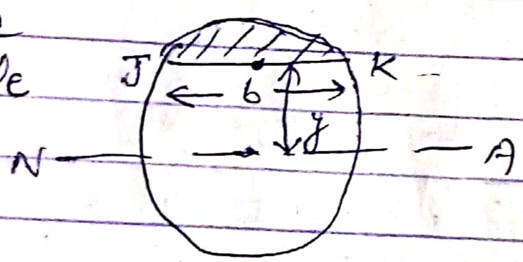
$\therefore$ graph is



Distribution of shearing stress over a circular section

~~Let r be the radius of circle~~



width of JK $= b = 2\sqrt{r^2-y^2}$

Area of the shaded strip

$$= 2\sqrt{r^2-y^2}\,dy$$

Moment of this area about NA

$$= \left(2\sqrt{r^2-y^2}\cdot dy\right)\cdot y$$

$$\therefore A\bar{y} = \int_y^r 2y\sqrt{r^2-y^2}\cdot dy$$

since $b = 2\sqrt{r^2 - y^2}$

$\Rightarrow b^2 = 4(r^2 - y^2)$

Diff with respect to y

$2b\,db = 4(-2y)\,dy$

$$\boxed{y\,dy = -\frac{1}{4} b\,db}$$

$$\therefore A\bar{y} = \int_b^0 b \cdot \left(-\frac{1}{4} b\,db\right)$$

$$= \int_b^0 -\frac{1}{4} b^2\,db$$

$$= \frac{1}{4}\int_0^b b^2\,db$$

$$= \frac{1}{4}\left[\frac{b^3}{3}\right]_0^b$$

$$= \frac{1}{12} b^3$$

$$\therefore \tau = \frac{V}{Ib} A\bar{y} = \frac{V}{Ib} \cdot \frac{1}{12} b^3$$

$$= \frac{V \times b^2}{12 I} = \frac{V}{12 I}\left(2\sqrt{r^2 - y^2}\right)^2$$

(as $b = 2\sqrt{r^2 - y^2}$)

$$\boxed{\therefore\ \tau = \frac{V}{3I}\left(r^2 - y^2\right)} \rightarrow \text{eq}^n \text{ of parabola}$$

at point $y = r$, $\tau = 0$

at point $y = 0$ τ is max

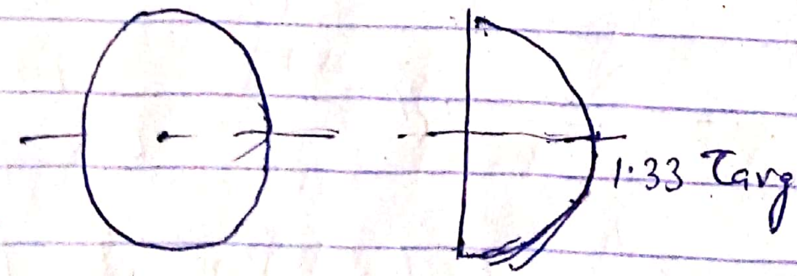
Now $\tau_{max} = \frac{V}{3I}\left(r^2\right)$

$$= \frac{V}{3}\frac{(d/2)^2}{\frac{\pi}{64}d^4}$$

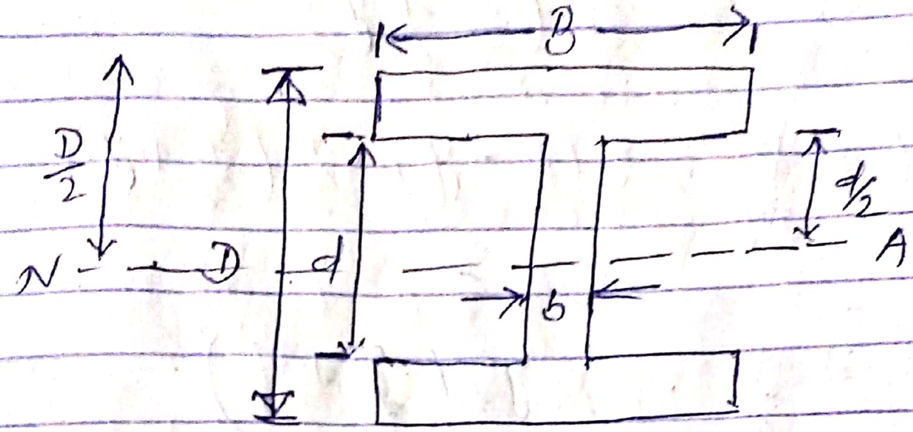
$$= \frac{16V}{3\cdot\pi d^2} = \frac{4V}{3\left(\frac{\pi}{4}d^2\right)}$$

$$= \frac{4}{3}\left(\frac{V}{\frac{\pi}{4}d^2}\right) = \frac{4}{3}\tau_{avg}$$

$$\boxed{\tau_{max} = 1.33\ \tau_{avg}}$$



Distribution of shearing stress over an I Section



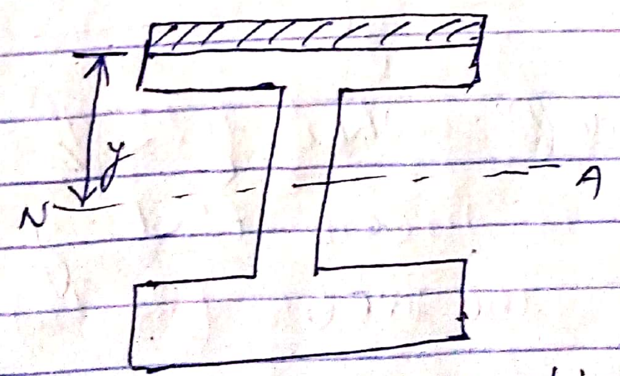
$$\tau = \frac{V \cdot A\bar{y}}{Ib}$$

Now, we shall discuss two important cases

(a) when $y > \frac{d}{2}$

(b) when $y < d/2$

First Case (a) when $y > d/2$



A = Area of the shaded strip = $B\left(\frac{D}{2} - y\right)$

$$\bar{y} = \frac{1}{2}\left(\frac{D}{2} - y\right) + y$$

$$= \frac{1}{2}\left(\frac{D}{2} + y\right)$$

$$\therefore \tau = \frac{V}{IB} \cdot B\left(\frac{D}{2} - y\right) \cdot \frac{1}{2}\left(\frac{D}{2} + y\right)$$

$$\boxed{\tau = \frac{V}{2I}\left(\frac{D^2}{4} - y^2\right)}$$

Now at $y = \frac{D}{2}$ $\tau = 0$

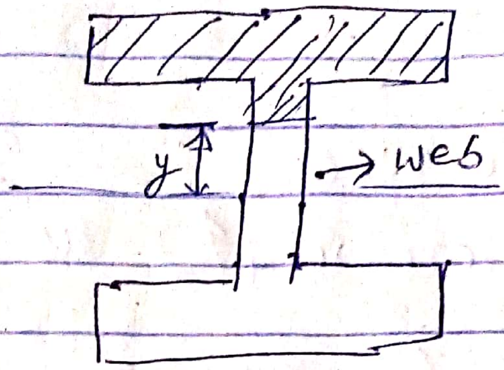
and at $y = \frac{d}{2}$ $\tau = \frac{V}{8I}(D^2 - d^2)$

$$\boxed{\tau = \frac{V}{8I}(D^2 - d^2)} \quad \text{— A}$$

2nd Case (b) when $y < d/2$

it means that y lies in the web

Scanned with CamScanner



In this case $A\bar{y}$ for the flange

$$A\bar{y} = B\left(\frac{D}{2}-\frac{d}{2}\right)\left(\frac{d}{2}+\frac{1}{2}\left(\frac{D}{2}-\frac{d}{2}\right)\right)$$

$$= \frac{B}{8}\left(D^2-d^2\right) \quad \text{--- (1)}$$

and value of $A\bar{y}$ for the web

$$= b\left(\frac{d}{2}-y\right)\cdot\left\{y+\left(\frac{d}{2}-y\right)\cdot\frac{1}{2}\right\}$$

$$= \frac{b}{2}\left(\frac{d^2}{4}-y^2\right) \quad \text{--- (2)}$$

$\therefore$ Total $A\bar{y} = \frac{B}{8}\left(D^2-d^2\right)+\frac{b}{2}\left(\frac{d^2}{4}-y^2\right)$

$$\boxed{\therefore \tau = \frac{V}{Ib}\left[\frac{B}{8}\left(D^2-d^2\right)+\frac{b}{2}\left(\frac{d^2}{4}-y^2\right)\right]}$$

Observation

(i) In the web

τ increases ($\uparrow$) as y decreases ($\downarrow$)

(ii) parabolic curve

(iii) at NA $y = 0$ τ is maximum

$$\therefore \tau_{max} = \frac{V}{Ib}\left[\frac{B}{8}(D^2 - d^2) + \frac{bd^2}{8}\right]$$

$$\boxed{\tau_{max} = \frac{V}{8I}\left[\frac{B}{b}(D^2 - d^2) + d^2\right]}$$

Now τ at $y = \frac{d}{2}$

i.e. at junction of the top of the web and bottom of flange.

$$\boxed{\tau = \frac{V}{Ib} \cdot \frac{B}{8}(D^2 - d^2)} \text{ at } y = \frac{d}{2}$$

$$= \frac{V}{8I} \cdot \frac{B}{b}(D^2 - d^2) = \frac{V}{8I}(D^2 - d^2) \times \frac{B}{b}$$

Thus we see that τ change abruptly from $\frac{V}{8I}(D^2-d^2)$ {as in eqn A} to

$$\frac{V}{8I}(D^2-d^2) \times \left(\frac{B}{b}\right)$$

i.e suddenly increases by $\frac{B}{b}$ time

Observation

$$\tau_{web} = \frac{B}{b} \tau_{flange}$$