Electrical & Electronics Measurements

Syllabus

Basics of Measuring system

Error analysis

Analog instruments
- PMMC
- EMMC
- MI
- ESV
- Thermal instruments.
- Rectifier type

Measurement of Resistance
- VA, AV methods
- * DC Bridges
- * AC Bridges (L, C, M).

Measurement of power
- DC power
- AC power

Reactive power measurement.

Measurement of Energy [1-ϕ EM, 3ϕ EM].

Potentiometers

Q-meter

power factor meter

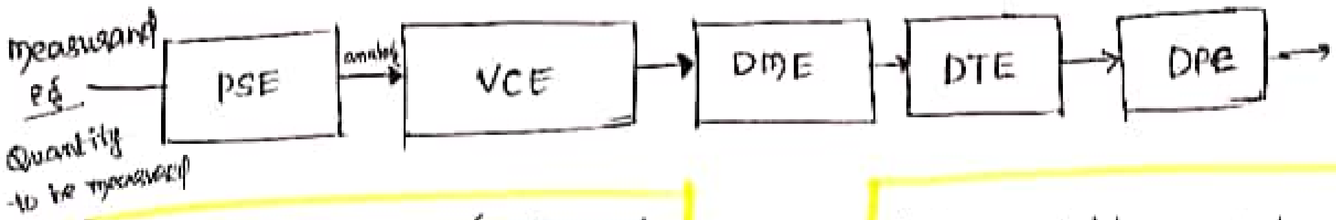
Flux meter

CRO

DVM

TRANSDUCERS.

Generalized building blocks of Measuring System :-



PSE – primary Sensing Element. (sensor)

Eg: Thermocouple (temperature)

i.e. Emf $\propto (\Delta T)$.

(Seebeck effect)

VCE – variable conversion element

Eg:- ADC, DAC converters

V to I, I to V, V to f. convert

Data manipulation element (DME)

Eg: Amplifier, modulator, attenuator

DTE – Data transmission Element.

Eg: Any transm. channel (optical fibre)

two-wire co-axial channel.

DPE – Data present elements

Eg: CRO, digital displays.

Analog pointer-scale indicator.

xy-recorder, LCD, LED display.

The purpose of measurement system is to present an observer with a numerical value. So that the observer can understand in easy way. Usually, the type of display preferred is BCD seven segment display.

In a gen. meas. system the following building blocks are given as.

1) VCE
2) DPE
3) DTE
4) DME
5) PSE

correct order is .. PSE, VCE, DME, DTE, DPE for the

Signal conditioning Elements : VCE, DME, DTE.

Types of measuring methods

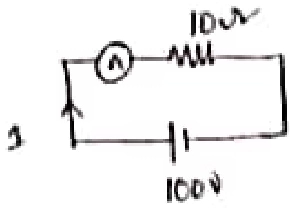
1. Direct methods. ⇒ Unknown quantity compared with known standards (mass, length, time.
2. Indirect Methods.

$\frac{\text{neutrons (n)}}{\text{protons (p)}} > 1.54 \Rightarrow$ unstable atoms ; (α) atomic number > 84.

1 meter =	1,650,763,73 no. of times of wavelength of radiation emitted by "KRYPTON-86" element
Time (1 sec) =	9,19,26,31,770 times of no. of periods of radiation corresponding to hyper-transmission levels of "cecium-136" element.
mass (1 kg) =	prototype unit of mass 1 kg = 2.2 pounds.

2. Indirect method of measurement (Eg: Amp, volts b/c we dont have the standard current b/c we cannot see (or) hold).

Eg:-



(i) $I_{true} = \frac{100}{10} = 10A$.

$I_{meas} = 9.5$ (say)

$\boxed{\text{Error} = A_{meas} - A_{true}}$

error = $9.5 - 10 = -0.5A$

(ii) $I_{meas} = 10.5$ (say)

error = $10.5 - 10 = +0.5A$.

Correction Factor :- (C.F)

The value that your are added (or) subtracted from the measure-d value is known as correction factor.

The error may be either positive (or) negative.

$$\boxed{C.F. = -(\text{Error})}$$

Static error: If error is independent of time is known as static error.

Dynamic error :- If error is changing w.r.t. time is known as dynamic error.

absolute error $= \delta A = A_m - A_t$

$$\% RSE = \left\{\begin{matrix} \% \text{ relative static error} \\ \text{(or)} \\ \% \text{ Limiting error} \end{matrix}\right\} = \frac{A_m - A_T}{A_T} \times 100 = \frac{\delta A}{A_T} \times 100.$$

Eg 1:

	A	B
	$\delta A = 1A$	$\delta B = 10A$

Good instrument.

a) only A
b) only B
c) A & B
d) None

Eg 2:

	A	B
	$\delta A = 1A$	$\delta B = 10A$
	$A_T = 2A$	$A_T = 1000A$

$\%\text{error} = \frac{1}{2} \times 100 > \%\text{error} = \frac{10}{1000} \times 100$

a)
b) only B
c)
d)

The quality of the instrument is given by the % relative static error, the error is always expressed in terms of true value of the instrument. If the accuracy of the instrument is mentioned by manufactures known as guarenteed accuracy error (GAE).

GAE (Guarenteed Accuracy Error) is always calculated w.r.t. Full Scale deflection

Eg :- (0–10)A meter ; $GAE = \pm 1\ FSD = \pm \frac{1}{100} \times 10 = \pm 0.1\ A.$

(is irrespective of load). ∴ constant error.

(Load or DC m/c brak test)

(I)

(Initial value)

1 kg	$1A \pm 0.1A$	⇒	$1 \times \frac{1}{100} = 0.1$	⇒	$(1 \pm 10\%)$
2 kg	$1A \pm 0.1A$	⇒	$2 \times \frac{1}{100} = 0.1$	⇒	$(1 \pm 5\%)$
3 kg	$3A \pm 0.1A$	⇒	$3 \times \frac{1}{100} = 0.1$	⇒	$(1 \pm 3.3\%)$
⋮					
10 kg	$10A \pm 0.1A$	⇒	$10 \times \frac{1}{100} = 0.1$	⇒	$(1 \pm 1\%)$

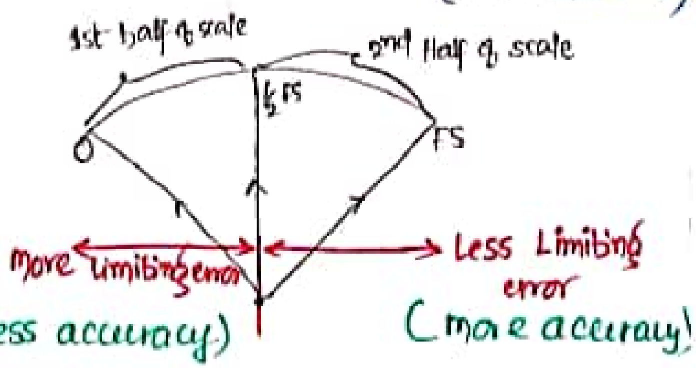
(Full scale Deflection)

GAE is constant error.

% Limiting error (variable error)

$x \rightarrow$ percentage Limiting error.

$$\boxed{\left(\text{True value}\right)\left(\frac{x}{100}\right) = \left(\text{GAE value}\right)}$$



GAE is always (measured) mentioned by manufacturer, it is always calculated w.r.t. full-scale value, It is also known as Constant error, whereas the % limiting error always calculated w.r.t true value, It decreases in magnitude when the pointer is moving from initial position to full scale value. So it is also called as variable error.

Q:- A (0-200V) is having a GAE of ± 1% of full scale reading. The voltage measured by this instruments is 50V. Find the corresponding limiting error.

Sol:-
$$GAE = \pm \frac{1}{100} \times 200 = \pm 2 \text{ volts}.$$

$$A_m = 50\text{volts} \cdot \pm 2$$

$$\% RSE = \frac{2}{50} \times 100 = 4\%.$$

Q:- $A_m = 205.5\ \mu F$
$A_T = 202.4\ \mu F$.

Sol:- $\% RSE = \frac{A_m - A_T}{A_T} \times 100 = \frac{3.1}{202.4} \times 100$
$= +1.53\%$.

Error Analysis

1. $y = x^n \Rightarrow \frac{dy}{dx} = ? \Rightarrow \frac{\delta y}{y} = \pm n \frac{\delta x}{x}$

$\log y = n \log x \Rightarrow \frac{1}{y} \cdot dy = \pm n \cdot \frac{dx}{x}$.

2. composite factors ; $y = x_1^m \cdot x_2^n$

$$\frac{\delta y}{y} = \pm m \frac{\delta x_1}{x_1} \pm n \frac{\delta x_2}{x_2};$$

$\frac{\delta x_1}{x_1}, \frac{\delta x_2}{x_2} \Rightarrow$ % limiting error is x_1 & x_2 respectively.

3. Addition (or) Subtraction of 3 variables.

$x = (x_1 + x_2 + x_3)$ (or) $x = x_1 - x_2 - x_3$

δx ... $\left[\frac{\delta x_1}{x} \ldots\right] \cdot \left(\frac{x_2}{x}\right) \frac{\delta x_2}{x_2} + \frac{x_3}{x} \frac{\delta x_3}{x_3}\right]$

proof :- $X = x_1 + x_2 + x_3$; (or) $X = x_1 + x_2 + x_3$.

diff. partially on both sides.

$$\delta X = \delta x_1 + \delta x_2 + \delta x_3$$

$$\frac{\delta X}{X} = \left(\frac{x_1}{x_1}\right)\frac{\delta x_1}{X} + \left(\frac{x_2}{x_2}\right)\frac{\delta x_2}{X} + \frac{\delta x_3}{X}\left(\frac{x_3}{x_3}\right)$$

$$\frac{\delta X}{X} = \pm\left[\frac{x_1}{X}\left(\frac{\delta x_1}{x_1}\right) + \frac{x_2}{X}\left(\frac{\delta x_2}{x_2}\right) + \frac{x_3}{X}\left(\frac{\delta x_3}{x_3}\right)\right]$$

4. Multiplication/division of variables.

$$X = x_1 x_2 x_3 \quad (\text{or}) \quad \frac{1}{x_1 x_2 x_3} \quad (\text{or}) \quad \frac{x_1}{x_2 x_3}$$

$$\frac{\delta X}{X} = \pm\left[\frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2} + \frac{\delta x_3}{x_3}\right];$$

(proof: Apply log on both sides, diff... you will get...).

5. $$y = \frac{x_1^m \cdot x_2^n}{x_3^p}$$

$$\frac{\delta y}{y} = \pm\left[m\frac{\delta x_1}{x_1} + n\frac{\delta x_2}{x_2} + p\frac{\delta x_3}{x_3}\right].$$

If any error is lying within the limits that means min. value and max. value, it is known as unknown error It is denoted by "±" symbol. Eg:- $(100 \pm 5)\Omega \Rightarrow (95\Omega$ to $100\Omega)$

Question Two resistors are given as $R_1 = 100 \pm 4\%$ $(100 \pm 4\Omega)$

$R_2 = 50 \pm 2\%$ $(50 \pm 4\Omega)$

(i) when they are connected in series, Find the equivalent resistance.
(ii) Find $(R_1 - R_2)$; (iii) Find $R_1 R_2$ (iv) $\frac{R_1}{R_2}$

Solution:-

(i) $(R_1+R_2) = (100 \pm 4\%) + (50 \pm 2\%)$

$= (100 \pm 4\Omega) + (50 \pm 1\Omega)$

$= 150 \pm 5\Omega$

$= 150 \pm \frac{5}{150} \times 100 = 150 \pm 3.33\%$

$$\frac{\delta x}{x} = \pm\left[\frac{x_1}{x}\frac{\delta x_1}{x_1} + \frac{x_2}{x}\frac{\delta x_2}{x_2}\right]$$

$$= \pm\left[\frac{100}{150}\times 4 + \frac{50}{150}\times 2\right]$$

$$= \pm 3.33\%$$

$\therefore$ $R_{eq} = \underset{\text{nominal value}}{150} \pm \underset{\text{\% Limiting error}}{3.33\%} = 150 \pm \underset{\text{error in value form}}{5\Omega}$. $(145\Omega \text{ to } 155\Omega)$

(ii) $(R_1-R_2) = (100 \pm 4\%) + (50 \pm 2\%)$

$x = x_1 - x_2 = 100 - 50 = 50\Omega$.

$$\frac{\delta x}{x} = \pm\left[\frac{x_1}{x}\frac{\delta x_1}{x_1} + \frac{x_2}{x}\frac{\delta x_2}{x_2}\right] = \pm\left[\frac{100}{50}\times 4 + \frac{50}{50}\times 2\right]$$

$$\frac{\delta x}{x} = \pm 10\%$$

$R_{eq} = 50 \pm 10\% = \underset{\text{nominal value}}{50} \pm \underset{\text{error in value form}}{5\Omega} \Rightarrow (45 \text{ to } 55)\Omega$

(iii) $R_1 R_2 \Rightarrow x = x_1 x_2 \rightarrow \frac{\delta x}{x} = \frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2}$

$\therefore$ $x = 100 \times 50 = 5000\Omega$; $\frac{\delta x}{x} = 4 + 2 = \pm 6\%$.

(iv) $\frac{R_1}{R_2} = \left(\frac{100 \pm 4\%}{50 \pm 2\%}\right) = \frac{100}{50} \pm (4+2)\% = 2 \pm 6\% = (2 \pm 0.12)\Omega$

(5)

In case of multiplication (or) division the % limiting errors are simply added but don't add the error in value form.

Q2 :- Two resistors are given as $R_1 = 100 \pm 6\Omega = (100 \pm 6\%)$

$R_2 = (50 \pm 2\Omega) = (50 \pm 4\%)$

(i) $R_1 + R_2$ (ii) $R_1 - R_2$ (iii) $R_1 R_2$ (iv) $\frac{R_1}{R_2}$

Sol :-

(i) $R_{eq} = R_1 + R_2 = 100 + 50 = 150\,\Omega$

$$\therefore \frac{\delta x}{x} = \pm\left(\frac{x_1}{x}\frac{\delta x_1}{x_1} + \frac{x_2}{x}\frac{\delta x_2}{x_2}\right) = \pm\left[\frac{100}{150}\times(6) + \frac{50}{150}(4)\right]$$

$$= \cancel{4.667\%}\ 5.33\%$$

$\therefore R_{eq} = 150 \pm \cancel{5.33} = 150 \pm 8\,\Omega . = (100 \pm 6\Omega) + (50 \pm 2\Omega)$

$= (150 \pm 8\Omega)$

$$\boxed{\left(\text{Nominal value}\right)\left(\frac{x}{100}\right) = \left(\text{Error in value}\right)}$$

(ii) $R_{eq} = R_1 - R_2 = 100 - 50 = 50\,\Omega . = (100 \pm 6) - (50 \pm 2)$

$= 50 \pm 8\,\Omega .$

$$\therefore \frac{\delta x}{x} = \pm\left(\frac{100}{50}(6) + \frac{50}{50}(2)\right) = \pm 16\%$$

$\therefore R_{eq} = 50 \pm 16\% = (50 \pm 8)\Omega$

(iii) $R_{eq} = R_1 R_2 = (100 \pm 6\%)(50 \pm 4\%) = 5000 \pm (10\%)$

$= (5000 \pm 500)\Omega$

(iv) $R_{eq} = \frac{R_1}{R_2} = \frac{100}{50} \pm (6 \pm 4) = 2 \pm 10\% = (2 \pm 0.2)\Omega$

(v) $R_{eq} = \frac{R_1 R_2}{\quad}$

In case of addition (or) subtraction the error in value form is simply added but don't add % limiting errors.

$$* \quad R_1 R_2 = (100 \pm 6\Omega)(50 \pm 2\Omega) = (100 \times 50 \pm 8\Omega) \times$$

$$= (100 \pm 6\%)(50 \pm 4\%) = (100 \times 50 \pm 10\%) \checkmark$$

$$* \quad R_{eq} = \frac{R_1 R_2}{(R_1 + R_2)} = \frac{(100 \pm 6\Omega)(50 \pm 2\Omega)}{(100 \pm 6\Omega) + (50 \pm 2\Omega)} = \frac{(100 \pm 6\%)(50 \pm 4\%)}{(100 + 50) \pm 8\Omega}$$

$$= \frac{(5000 \pm 10\%)}{(150 \pm 8\Omega)} = \frac{(5000 \pm 10\%)}{(150 \pm 5.33\%)}$$

$$= \frac{5000}{150} \pm (10 + 5.33)\%$$

$$R_{eq} = 33.33 \pm 15.33\%$$

$$R_{eq} = 33.33 \pm 5.1099\,\Omega$$

$$* \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

diff. partially on both sides ...

$$\frac{-1}{R_{eq}^2}\,\delta R_{eq} = \left(\frac{-1}{R_1^2}\,\delta R_1\right) + \left(\frac{-1}{R_2^2}\,\delta R_2\right)$$

$$\frac{1}{R_{eq}}\left(\frac{\delta R_{eq}}{R_{eq}}\right) = \frac{1}{R_1}\left(\frac{\delta R_1}{R_1}\right) + \frac{1}{R_2}\left(\frac{\delta R_2}{R_2}\right)$$

$$\frac{\delta R_{eq}}{R_{eq}} = \frac{R_{eq}}{R_1}\left(\frac{\delta R_1}{R_1}\right) + \frac{R_{eq}}{R_2}\left(\frac{\delta R_2}{R_2}\right)$$

$$\frac{\delta R_{eq}}{R_{eq}} = \pm\left[\left(\frac{R_2}{R_1 + R_2}\right)\left(\frac{\delta R_1}{R_1}\right) + \left(\frac{R_1}{R_1 + R_2}\right)\left(\frac{\delta R_2}{R_2}\right)\right]$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = 33.33\,\Omega.$$

$$\therefore \frac{\delta R_{eq}}{R_{eq}} = \pm\left[\frac{33.33}{100}\times(6) + \frac{33.33}{50}\times(4)\right] = \pm\frac{33.33}{5000}(3000+4000)$$

$$= \pm 4.667\,\%$$

$$\therefore R_{eq} = 33.33 \pm 4.667\,\% = 33.33 \pm 1.5556\ \Omega.$$

Short cut $\Rightarrow$ $$R_{eq} = \frac{(5000 \pm 10\%)}{(150 \pm 5.33\%)} = \frac{5000}{150} \pm (10 - 5.33)\%$$

Q:– The input of a electrical m/c is given as $6500 \pm 3\%$ output is given as $5000 \pm 2\%$. Find the loss of the m/c.
(ii) Find the efficiency.

Sol:– (i) Losses = Input − Output = $(6500 \pm 3\%) - (5000 \pm 2\%)$

$$= 1500 \pm (195 + 100) = 1500 \pm (295)$$

$$\text{Losses} = 1500 \pm 19.6667\,\%$$

(ii) $$\eta = \text{efficiency} = \frac{\text{output}}{\text{Input}} = \frac{5000 \pm 2\%}{6500 \pm 3\%}$$

$$= 76.923 \pm 5\%$$

$$= 76.923 \pm 3.846\%$$

Q:– 3 resistors are given as $R_1 = 50 \pm 2\%$; $R_2 = 37 \pm 2\%$; $R_3 = 75 \pm 2\%$; (i) when they are connected in series find the equivalent resistance

Sol:- (i) $R_{eq} = R_1 + R_2 + R_3 = 50 + 37 + 75 = 162 \pm$

$$= (50 \pm 1\,\Omega) \pm (37 \pm 0.74) \pm (75 \pm 1.5)$$

$$= 162 \pm 3.24\,\Omega$$

$$= 162 \pm 2\%$$

(or)

$$= (50 \pm 2\%) + (37 \pm 2\%) \pm (75 \pm 2\%)$$

$$= (50 + 37 + 75) \pm 2\%$$

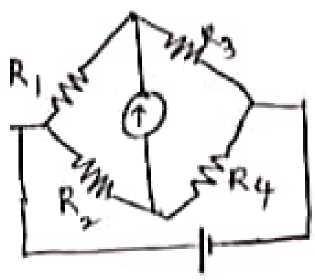
$$= 162 \pm 2\%$$

Note:- If 'n' different resistors are having same % limiting error (x), when they are connected in series.... The % limiting error in the equivalent resistor is also $x\%$.

$$R_1 + x\%,\ R_2 + x\%\ \ldots,\ R_n + x\%$$

$$R_{eq} = (R_1 + R_2 + \cdots + R_n) \pm x\%$$

Q:- In a following wheatstone bridge, the arms of the resistances are given as $R_1 = 500 \pm 5\%$, $R_2 = 1000 \pm 5\%$, $R_3 = 200 \pm 5\%$ find R_4



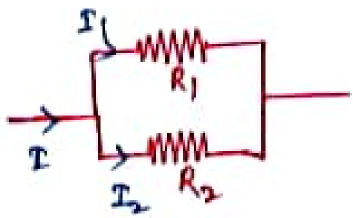
Sol:-

$$R_1 R_4 = R_2 R_3.$$

$$R_4 = \frac{R_2 R_3}{R_1}$$

$$= \frac{200 \times 1000}{500} \pm (5 + 5 + 5) = 400 \pm 15\%$$

Q:- Find the total current from the following parallel ckt. ⑦



$I_1 = (150 \pm 1)\, A$

$I_2 = (250 \pm 2)\, A$

Sol:- $I = I_1 + I_2 = (150 + 250) \pm (1+2) = (400 \pm 3)\, A$.

Q:- In the above problem if the errors are given in the form of standard deviation. Find the total current and also the resultant standard deviation

a) (400 ± 2)
b) (400 ± 2.24)
c) (400 ± 3)
d) (400 ± 3.24)

Sol:-

$I_1 = (150 \pm \underbrace{1}_{\sigma_1})\, A$

$I_2 = (250 \pm \underbrace{2}_{\sigma_2})\, A$

$\sigma_1 = 1, \quad \sigma_2 = 2.$

$I = I_1 + I_2 . \Rightarrow \frac{\partial I}{\partial I_1} = 1 \; ; \; \frac{\partial I}{\partial I_2} = 1$

$$\frac{\partial I}{I} = \sqrt{\left(\frac{\partial I}{\partial I_1}\right)^2 \cdot \sigma_1^2 + \left(\frac{\partial I}{\partial I_2}\right)^2 \sigma_2} = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{1^2 + 2^2} = \sqrt{5} = 2.24$$

$$\therefore I = (I_1 + I_2) \pm \frac{\partial I}{I} = 400 \pm 2.24$$

$$\boxed{\sigma_{resultant} = \frac{\partial x}{x} = \sqrt{\left(\frac{\partial x}{\partial x_1}\right)^2 \sigma_1^2 + \left(\frac{\partial x}{\partial x_2}\right)^2 \sigma_2^2 + \dots + \left(\frac{\partial x}{\partial x_n}\right)^2 \sigma_n^2}}$$

(*) If the data given in variance form. $\therefore V_{resultant}$.

$$V_{result} = \left(\sqrt{(1)^2 \times 1 + (1)^2 \times 2}\right)^2 = \left(\sqrt{3}\right)^2 = (1.732)^2 = 3$$

a) 400 ± 2
b) 400 ± 1.732
c) 400 ± 2.24
d) None. (400 ± 3).

Scanned with CamScanner

Errors

Systematic Errors

(1, 2, 3)

Constructional (or) Instrumental error

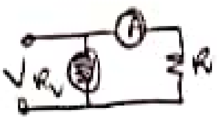
Ex: (i) Inherent shortcomings of instruments (or) defective problems (or) defective parts (or) worn parts.

Eg:- weak magnet, weak spring

(ii) misuse of instruments.

Some of the Instruments which are gravity control can be suitable for vertical placements only. If they are placing horizontally it may not give proper reading. The customer always expect same results

(iii) Loading effect. (Ideally, $R_a = 0$, $R_v = \infty$; practically not possible)



$R_{Th} = \frac{V}{A} = \frac{R_a + R}{BI}$

Random Errors

We don't know the reason for

Random Errors

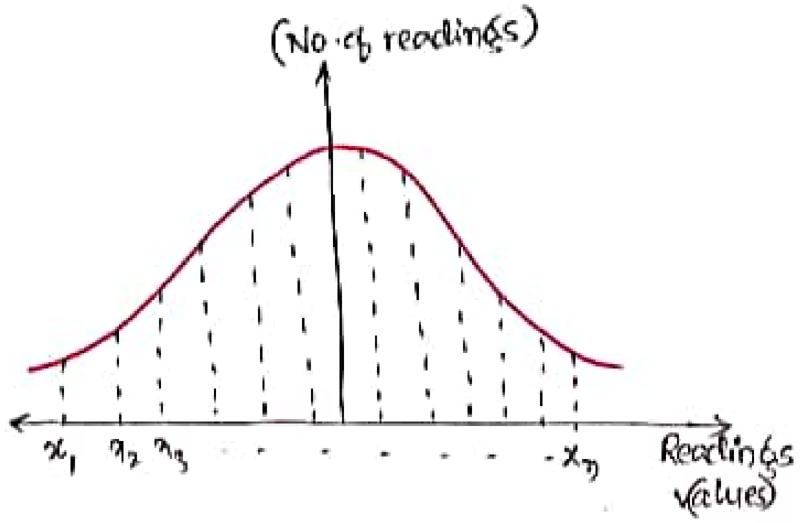
(or) probable errors

(or) Accidental errors.

These are the errors occurs randomly, These are also called as residual errors. These errors are related confidence interval. These errors can be minimized by using Gausian statistical analysis. It has been consistently found that experimental results show variation from one reading to another, even after all systematic errors are accounted for. These errors are due to multitude of small factors. The quantity being measured is affected by many happenings in the universe. Some of them we are aware rest are unaware ... are lumped together called random (or) residual errors. These errors are remain even after systematic errors have been taken care of, we call these are residual errors. Probability can be applied to study the ra... These is no other way as the random errors are known & only statistical study can lead us to the best approximation of the true value of the quantity.

Gausian statistical Analysis :-

(8)



$$\text{Arithmatic mean (AM)} = \bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} = \boxed{\frac{\sum_{i=1}^{n} x_i}{n} = \bar{x}}$$

Deviations (di) ; $d_1 = x_1 - \bar{x}$; $d_2 = x_2 - \bar{x}$ . . . , $d_n = x_n - \bar{x}$

$$\boxed{d_i = x_i - \bar{x}}$$

$$\text{mean deviation} = \bar{D} = \frac{|d_1| + |d_2| + \cdots + |d_n|}{n}$$

$$\text{standard deviation} (\sigma) = \sqrt{\frac{d_1^2 + d_2^2 + \cdots + d_n^2}{n}}$$

$$\therefore \boxed{\sigma = \sqrt{\frac{\sum_{i=1}^{n} |d_i|^2}{n}}}$$ for infinite no. of observation. i.e. $n > 20$.

$$\boxed{\sigma = \sqrt{\frac{\sum_{i=1}^{n} |d_i|^2}{(n-1)}}}$$ for finite no. of observations (or) $n \leq 20$.

$$\text{Variance (V)} = \sigma^2 = (\text{standar deviation})^2$$

probable error $(\gamma) = 0.6745\sigma$; $\gamma \propto \sigma$

$$\boxed{\gamma = 0.6745\sigma}$$

$r = 0.8453\,\bar{D}$

$\therefore\ r = 0.6745\sigma = \frac{0.4765}{h} = 0.8453\,\bar{D}$

$h = \frac{0.4765}{(0.6745)\sigma}$

$h = \frac{0.706}{\sigma}$

$h = \frac{1}{\sigma\sqrt{2}}$

$\boxed{h\sigma = \frac{1}{\sqrt{2}}}$

mean probable error $= r_m$

$r_m = \frac{0.6745\sigma}{\sqrt{(n-1)}} = \frac{r}{\sqrt{(n-1)}}$ for $(n \le 20)$

$\boxed{r_m = \frac{r}{\sqrt{n}}}$; $n > 20$.

standard deviation of mean $\boxed{\sigma_m = \frac{\sigma}{\sqrt{n}}}$

standard deviation of standard deviation

$\Rightarrow \boxed{\sigma_\sigma = \frac{\sigma}{\sqrt{2n}}}$

$\sigma_\sigma = \frac{\sigma}{\sqrt{n}} \times \frac{1}{\sqrt{2}} = \frac{\sigma_m}{\sqrt{2}}$

$\therefore\ \boxed{\sigma_\sigma = \frac{\sigma_m}{\sqrt{2}}}$

minimum range of error $= x_{avg} - x_{min}$

maximum range of error $= x_{max} - x_{avg}$.

Average range of error $= \frac{(x_{avg} - x_{min}) + (x_{max} - x_{avg})}{2}$

$\boxed{\text{Avg. Range of error} = \frac{x_{max} - x_{min}}{2}}$

Find the (i) AM

(ii) deviations(d_i);(iii) mean deviations ($\bar{D}$)

(iv) std. deviation (σ) ; (V) variance = σ^2 ;(vi) probable e

(v) mean probable error ; (vi) σ_m (std deviation of

(vii) σ_σ (std. deviation of std. deviation).

(viii) min. range of error, max range of error.

Sol:-

$$AM = \bar{x} = 10 + 0.065 \simeq 10.08$$

$$d_1 = 10.03 - 10.08 = -0.05$$

$$d_2 = 10.11 - 10.08 = +0.03$$

$$d_3 = 10.12 - 10.08 = +0.04$$

$$d_4 = 10.08 - 10.08 = 0.00$$

$$\bar{D} = \frac{|0.05| + |0.03| + 0.04 + 0}{4}$$

$$\bar{D} = 0.03$$

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + d_4^2}{(n-1)}}$$

$$= \sqrt{\frac{0.05^2 + 0.03^2 + 0.04^2 +}{(4-1)}}$$

$$= \sqrt{\frac{25 + 9 + 16}{3 \times 10^4}}$$

$$\sigma = 0.0408.$$

$$\text{variance} = \sigma^2 = 0.001684$$

mean probable error $(r) = 0.6745\sigma = 0.6745 \times 0.0408 = 0.0$

stand. deviation of mean $(\sigma_m) = \frac{\sigma}{\sqrt{n}} = \frac{0.0408}{\sqrt{4}} = 0.0204$

st. de. of st. de. $(\sigma_\sigma) = \frac{1}{\sqrt{2}}\sigma_m = 0.0144$

mean probable error $(r_m) = \frac{r}{\sqrt{n-1}} = \frac{0.0275}{\sqrt{3}} = 0.015$

min range of error = ~~10.07~~ $x_{avg} - x_{min} = 10.08 - 10.03 = 0.05$

max. " " = $x_{max} - x_{avg} = 10.12 - 10.08 = 0.04$

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case(i) If Errors are given in the form of standard deviations (σ)

Let $x = f(x_1, x_2, x_3, \dots, x_n)$.

$= f(\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_n)$.

$$\sigma_{resultant} = \frac{\partial x}{x} = \sqrt{\left(\frac{\partial x}{\partial x_1}\right)^2 \sigma_1^2 + \left(\frac{\partial x}{\partial x_2}\right)^2 \sigma_2^2 + \cdots + \left(\frac{\partial x}{\partial x_n}\right)^2 \cdot \sigma_n^2}$$

case(ii) If errors are given in the form of variance ($v = \sigma^2$).

let $x = f(x_1, x_2, x_3, \dots, x_n)$

$x = f(v_1, v_2, v_3, \dots, v_n)$

$$V_{resultant} = \frac{\partial x}{x} = \left(\frac{\partial x}{\partial x_1}\right)^2 v_1 + \left(\frac{\partial x}{\partial x_2}\right)^2 v_2 + \cdots + \left(\frac{\partial x}{\partial x_n}\right)^2 v_n.$$

case(iii) If errors are given in the form of probable error (r)

$\because (r \propto \sigma)$

$x = f(r_1, r_2, \dots, r_n)$.

$$r_{resultant} = \frac{\partial x}{x} = \sqrt{\left(\frac{\partial x}{\partial x_1}\right)^2 r_1^2 + \left(\frac{\partial x}{\partial x_2}\right)^2 r_2^2 + \cdots + \left(\frac{\partial x}{\partial x_n}\right)^2 r_n^2}$$

case(iv) If the errors are given in the form of uncertainity (ω).

(doubtness of the ~~error~~ measurement)

Uncertainity refers to the doubtness of the measurement and % limiting errors are not at all same.

$x = f(\omega_1, \omega_2, \dots, \omega_n)$

$$\omega_{resul} = \frac{\partial x}{x} = \sqrt{\left(\frac{\partial x}{\partial x_1}\right)^2 \omega_1^2 + \left(\frac{\partial x}{\partial x_2}\right)^2 \omega_2^2 + \cdots + \left(\frac{\partial x}{\partial x_n}\right)^2 \omega_n^2}$$

Q. (11) Two identical resistors $\Rightarrow R_1 = R_2 = 100\Omega \pm 1\%$.

case (i) series connection. $R_1 = R_2 = 100 \pm 1\,\Omega$ $\hookrightarrow \sigma_1 = \sigma_2 = 1$.

$$R_{eq} = R_1 + R_2$$

$$\frac{\partial R_{eq}}{\partial R_1} = 1 \quad ; \quad \frac{\partial R_{eq}}{\partial R_2} = 1 \,;$$

$$\frac{\partial R_{eq}}{R_{eq}} = \sqrt{\left(\frac{\partial R_{eq}}{\partial R_1}\right)^2 \sigma_1^2 + \left(\frac{\partial R_{eq}}{\partial R_2}\right)^2 \sigma_2^2} = \sqrt{1^2\times 1^2 + 1^2 \times 1^2} = \sqrt{2}$$

$$\therefore R_{eq} = (100+100) \pm \sqrt{2} = 200 \pm \underbrace{1.414}_{\sigma} = 200 \pm \frac{1}{\sqrt{2}}\,\%$$

$$\% = \frac{\sqrt{2}}{200} \times 100 = \frac{1}{\sqrt{2}}\,\%$$

case (ii) parallel connection.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad ; \quad R_{eq} = 50\Omega$$

$$\left(-\frac{1}{R_{eq}^2}\right)\frac{\partial R_{eq}}{\partial R_1} = \left(-\frac{1}{R_1^2}\right) \quad ; \quad \left(-\frac{1}{R_{eq}^2}\right)\frac{\partial R_{eq}}{\partial R_2} = -\frac{1}{R_2^2}$$

$$\sigma_{result} = \frac{\partial R_{eq}}{R_{eq}} = \sqrt{\left(\frac{R_{eq}}{R_1}\right)^4 \cdot \sigma_1^2 + \left(\frac{R_{eq}}{R_2}\right)^4 \sigma_2^2} = \sqrt{2} \times \left(\frac{R_{eq}}{R_1 = R_2}\right)^2$$

$$= \sqrt{2} \times \left(\frac{50}{100}\right)^2 = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}}$$

$$\therefore R_{eq} = 50 \pm \frac{1}{2\sqrt{2}} = 50 \pm \frac{100}{50 \times 2\sqrt{2}} = 50 \pm \frac{1}{\sqrt{2}}\,\%$$

Note :- "n" no. of resistors are connected in series.

$$R_{eq} = R_1 + R_2 + \cdots + R_n$$

$$\frac{\partial R_{eq}}{\partial R_1} = \frac{\partial R_{eq}}{\partial R_2} = \cdots = \frac{\partial R_{eq}}{\partial R_n} = 1.$$

$$\therefore \frac{\partial R_{eq}}{R_{eq}} \Rightarrow \boxed{\sigma_{resultant} = \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2}}$$

Q:- (WB) chap1 (conv 1)

$R_1 = 20\,K\Omega \pm 5\% = 20K \pm \underbrace{1\,K\Omega}_{\sigma_1}$

$R_2 = 30\,K\Omega \pm 10\% = 30K \pm \underbrace{3\,K\Omega}_{\sigma_2}$

case(i) :- parallel. $R_{eq} = 12\,K\Omega$ i.e $= \frac{R_1 R_2}{R_1 R_2} = 12K.$

$$\therefore \frac{\partial R_{eq}}{\partial R_1} = \left(\frac{R_{eq}}{R_1}\right)^2 = \left(\frac{12}{20}\right)^2; \quad \frac{\partial R_{eq}}{\partial R_2} = \left(\frac{R_{eq}}{R_2}\right)^2 = \left(\frac{12}{30}\right)^2$$

$$\therefore \sigma_{result} = \frac{\partial R_{eq}}{R_{eq}} = \sqrt{\left(\frac{\partial R_{eq}}{\partial R_1}\right)^2 \sigma_1^2 + \left(\frac{\partial R_{eq}}{\partial R_2}\right)^2 \sigma_2^2}$$

$$= \sqrt{1^2 \left(\frac{12}{20}\right)^4 + 3^2\left(\frac{12}{30}\right)^4} = \frac{(12^2)}{(10)^2}\sqrt{\frac{1}{16} + \frac{9}{81}}$$

$$= 1.44\sqrt{\frac{81+144}{16 \times 81}} = 1.44 \times \frac{15}{4 \times 9} = 9\,K\Omega.$$

case(ii) :-

Characteristics of the Instruments:-

1. Accuracy.
2. precision.
3. Linearity.
4. Sensitivity
5. Dead time
6. Dead zone.
7. Drift.
8. Threshold
9. Resolution
10. fedility.

1. accuracy ⇒ degree of closeness. ⇒ GAE. (constant error).

Accuracy refers to the degree of closeness. It is an indication of the measured value. How much close to the true value. If the accuracy is mentioned by manufacturer is known as GAE i.e. guarenteed accuracy error.

CLASS-1 instruments ⇒ if accuracy = ±1 FSD i.e. 1% GAE.

2. precision (repeatable (or) consistency).

Precision refers to repeatability (or) consistancy, The most repeated value from the given set of recordings.

Eg:- $I_T = 2A$.

A	B
1.9A	1.8A
1.8 A	1.5A
1.6 A	1.5A
1.7 A	1.9 A
1.5 A	1.5A
1.4 A	1.6A
1.3 A	1.5A
1.8 A	1.5A
1.2A	1.4A

precised value (1.8A)

precised value. (1.5)A

⇒ instrument 'B' is more precise but not accurate.

⇒ instrument 'A' is less precise but more accurate.

∴ precision ~~nev~~ may not be the accurate indication.

Highly precise instrument doesn't mean that highly accurate. Because the most precise instruments will give the wrong reding (so that precision never conforms accuracy.

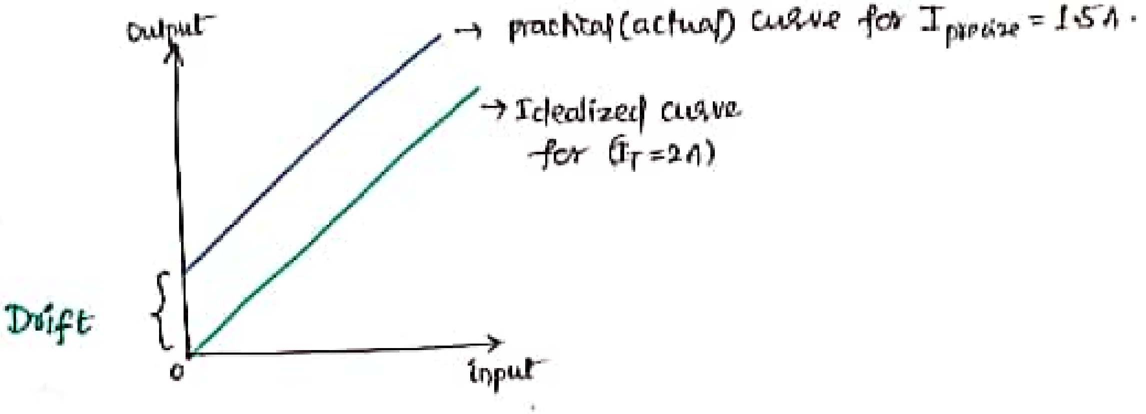
⇒ $I_{T(\text{val})} = 2A$; (1.5A) reading is Reproducability.

out of 10 readings ⇒ repeated 5 times ⇒ 50%

⇒ " 8 " ⇒ 8%

⇒ " 10 " ⇒ 100%

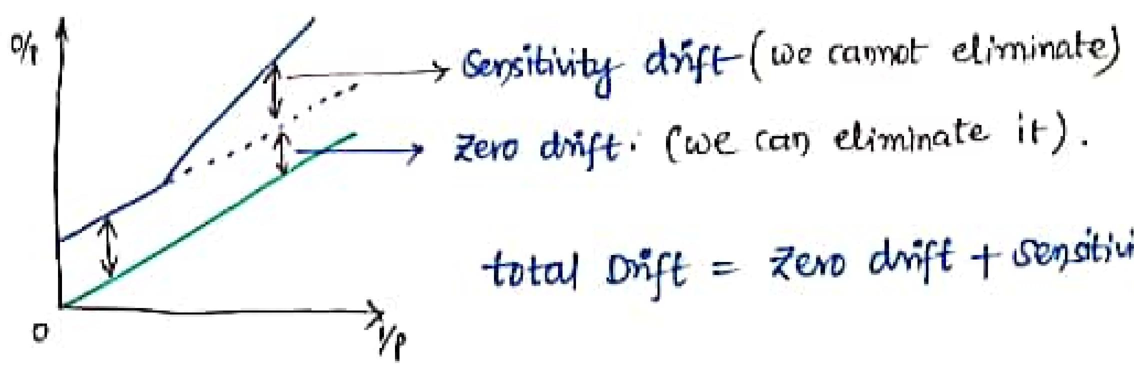
Reproducability refers to the degree of repeatability.



*. Reproducability :- Refers to the degree of repeatability.

A perfectly reproducable instrument is having zero drift.

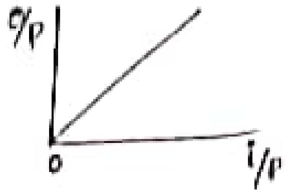
"Zero Drift can be eliminated by re calibrating the instrument"



total Drift = Zero drift + sensitivity drift.

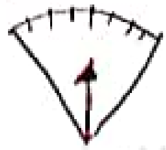
Linearity :- slope of curve (o/p vs i/p) = constant.

i.e. (output) $\propto$ (input).



proportional output ; slopes = constant.

i.e. Uniform scale reading.



non-Linearity :-

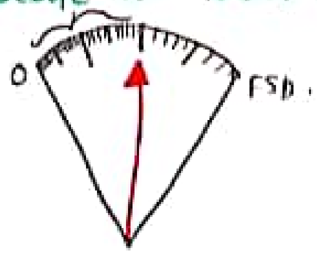
slopes $\neq$ constant

(i.e. variable slope).

(Eg:) o/p: (output) $\propto$ (input)^2.

cramped scale at lower end.

Eg: $\theta \propto I^2$



$I = 1A \Rightarrow \theta = 1^2$

$I = 2A \Rightarrow \theta = 2^2$

$I = 5A \Rightarrow \theta = 5^2$

$\vdots$

$I = 10\,Amp \Rightarrow \theta = 10^2$

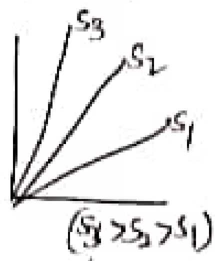
$I = 20\,A \Rightarrow \theta = 20^2$

$I = 30\,A \Rightarrow \theta = 30^2$

If the output follows the input with a proportional relationship then the instrument is said to be linear. Otherwise, if the output follows the input with a square law relationship then the instrument is said to be non-linear. For a non-linear instruments scale is cramped at lower end.

Sensitivity :- Slope of curve $\equiv$ sensitivity $= \dfrac{d(o/p)}{d(i/p)} = \dfrac{dy}{dx}$;

It is defined as the ratio of infinitismall change in output to the change in input is known as sensitivity.



For linear instruments — constant sensitivity
non-lear instrument — variable sensitivity.

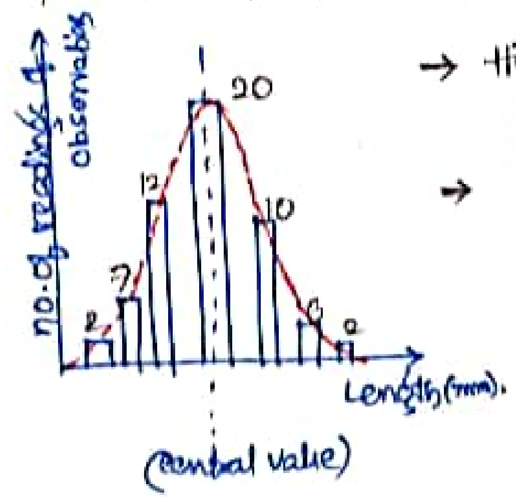
High sensitive (elements) instruments will respond for ~~high~~ small inputes.

units → mm/volt ; Degree/volt ; Radian/volt

(AM) also

Central value :- If we make a large no. of measurements and if the plus effects are equal to the minus effects, they would cancel each other and we would obtain the scatter round a central value. This condition is frequently met in practice.

Histogram :- When a no. of multisample observations are taken experimentally there is a scatter of data about some central value. One method of presenting test results in the form of a histogram.



→ Histogram is also called as a frequency distribution curve.

→ With more and more data taken at smaller and smaller increments the histogram would finally change into a smooth curve.

The most probable value of measured variable (variate) is the arithmatic mean of the no. of readings taken. Theoretically, an infinite no. of readings would give the best result, although in practice, only finite no. of measurements can be made.

Measure of Dispersion from the mean :-

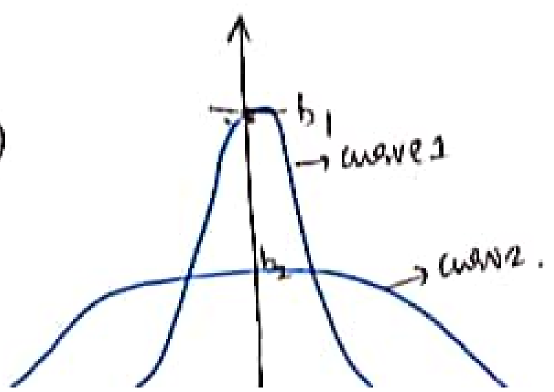
Dispersion :- The property which denotes the extent to which the values are dispersed about the central value called dispersion.

dispersion = spread = scatter

Curve 1 :- Greater precision $(h_1 > h_2)$

Curve 2 :- Lower precision

$x \Rightarrow$ deviation from the central value.



Dispersion is more for curve 2. that means ....

A large dispersion indicates that some factors involved in the measurement process are not under close control and ∴ it becomes more difficult to estimate the measured quantity with confidence and definiteness.

Range ⇒ $(x_2 - x_1)$; $(x_4 - x_3)$;

Deviation :- Deviation is the departure of the observed reading from the arithmatic mean of the group of readings.

$$d_1 = x_1 - \bar{x}$$
$$d_2 = x_2 - \bar{x}$$
$$\cdots\cdots\cdots$$
$$d_n = x_n - \bar{x}$$

$$\Sigma d_i = d_1 + d_2 + \cdots + d_n$$
$$= (x_1 - \bar{x}) + (x_2 - \bar{x}) + \cdots + (x_n - \bar{x})$$
$$= (x_1 + x_2 + x_3 + \cdots + x_n) - n\bar{x} \qquad \because \bar{x} = \frac{\sum^{n} x_i}{n}$$
$$= n\bar{x} - n\bar{x}$$
$$\Sigma d_i = 0.$$

∴ Algebraic sum of deviations is alway zero.

Highly precice instruments yield a low value of average deviation between the readings. The average deviation is an indication of the precision of the instuments used in making the measurements.

i.e. $$\bar{D} = \frac{|d_1| + |d_2| + \cdots + |d_n|}{n} = \text{avg. deviation.}$$

$$(\text{precision}) \propto \frac{1}{(\text{avg. deviation})}$$

Q:- There are 3 sets of data having average deviations of values 0.6, 0.3, and 0.5 for A, B, C set of datas respectively. Then what is the correct order of precision of measurements of A, B, C set of data.

(a) $A > B > C$

(b) $A > C > B$

(c) $B > C > A$ ✓

(d) we cannot say until we get information about standard deviation.

Sol:- $$\text{precision} \propto \frac{1}{(\text{avg. deviation})} \propto \frac{1}{(\text{dispersion} = \text{spread} = \text{scatter})}$$

Normal (or) Gaussian Distribution Curve of Errors :-

This is the basis for the major part of study of random errors. This type of distribution is most frequently met in normal practices.

The law of probability states the normal occurance of deviations from average value of an infinite no. of measurements (or) observations can... be expressed as..

$$y = \frac{h}{\sqrt{\pi}} \exp(-h^2x^2) \; ; \; y = \frac{1}{\sigma\sqrt{2\pi}} \exp(-x^2/2\sigma^2).$$

x = magnitude of deviation from mean.

y = no. of readings at any deviation x.
(The probability of occurence of deviation x)

h = a constant called precision index

σ = Standard deviation.

σ = usually known as quantity of interest.

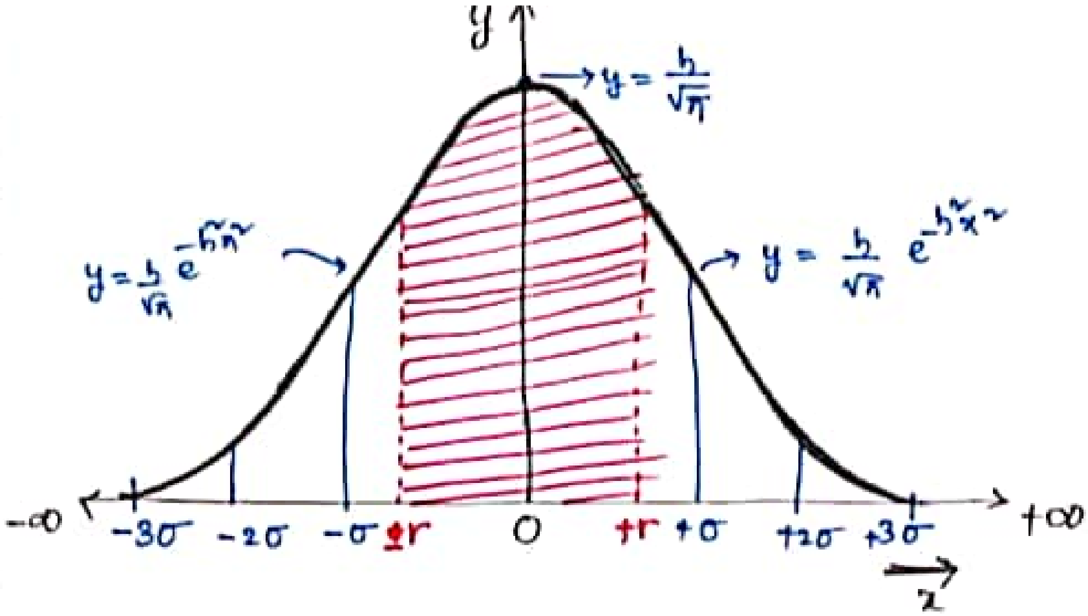


Fig:- Normal probability curve (or) Gaussian curve.

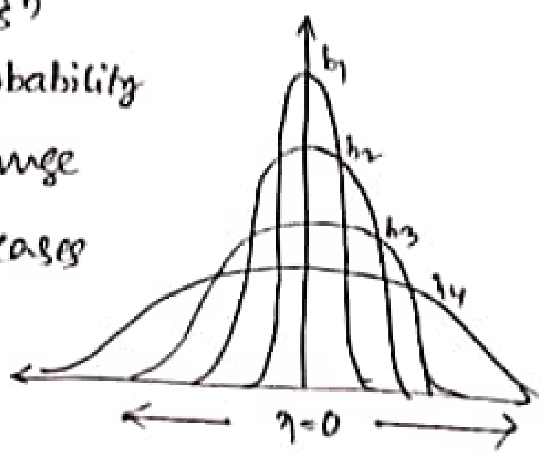
→ The value of h determines the sharpness of the curve since the curve drops sharply owing to the term $(-h^2)$ being in the exponent. The sharp curve evidently indicates that the deviations are more closely grouped together around deviation $x=0$.

⇒ It is clear that the probability that a variate lies in a given range becomes less as the deviation of the range becomes greater.

If greater the value of h ... for x more probability less.

∴ Thus the name ... precision index for h is reasonable.

➡ A large value of h represents high precision of the data because the probability of occurrence of variates in a given range falls off rapidity as the deviation increases because the variantes tends to cluster (becomes closer) into a narrow range.



Probable Error :-

The confidence in the best value (most probable value) is connected with the sharpness of the distribution curve.

Total area of the Guassian curve = 1.

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-3\sigma}^{3\sigma} \exp(-x^2/2\sigma^2)\,dx = 1.$$

A convenient measure of precision is the quantity (r), called probable error.

$$\frac{h}{\sqrt{\pi}} \int_{-r}^{r} \exp(-h^2x^2)\,dx = \frac{1}{2}$$

$$\therefore \quad r = 0.6745\sigma = \frac{0.4769}{h}$$

average deviation for normal curve :

$$\bar{D} = \int_{-\infty}^{+\infty} |x|\,y\,dx$$

$$h = \frac{1}{\sqrt{\pi}\,\bar{D}} \Rightarrow \bar{D} = \frac{r}{0.4769\sqrt{\pi}} = \frac{r}{0.8453}$$

$$r = 0.8453\bar{D}$$

$$PE = r = 0.6745\sigma = \frac{0.4769}{h} = 0.8453\bar{D}.$$

for finite readings (n) ;- probable error $(r_n) = 0.6745\frac{\sigma}{\sqrt{n}}$

standard deviation of mean $\sigma_m = \frac{\sigma}{\sqrt{n}}$

standard deviation of standard deviation $\sigma_\sigma = \frac{\sigma}{\sqrt{2n}} = \frac{\sigma_m}{\sqrt{2}}$

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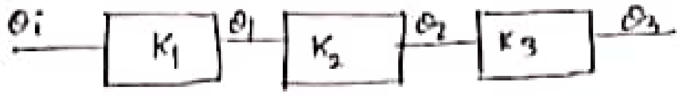
$$S_V = \frac{R_m}{V_{FSD}} \left(\frac{\Omega}{\text{Volt}}\right) = \frac{1}{I_{FSD}} \; {}^{1}/_{mA}$$

$R_m \rightarrow$ meter resistance;
$V_{FSD} \rightarrow$ Full scale deflection;



$$S = K = \frac{\Delta o/p}{\Delta i/p} = \frac{\theta_o}{\theta_i}$$

$$S_\phi = \frac{\theta_3}{\theta_i} = \frac{\theta_o}{\theta_i} = \frac{\theta_o}{\theta_2} \cdot \frac{\theta_2}{\theta_1} \cdot \frac{\theta_1}{\theta_i} = K_1 K_2 K_3 .$$

$$(S_v)_{overall} = (S_{v_1}) \cdot (S_{v_2}) \cdot (S_{v_3}).$$

The overall sensitivity of a cascaded system is the multiplication of their individual sensities.

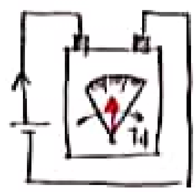
Q:- two meters having full sale current of 5UA, 100A respectively Find their sensitivities in Ω/volts, state which meater has greater sensitivity.

Sol:- $S_{V_A} = \frac{1}{50} \; \Omega/V = 0.02 \; \Omega/V$; $S_{V_B} = \frac{1}{100} \; \Omega/V$

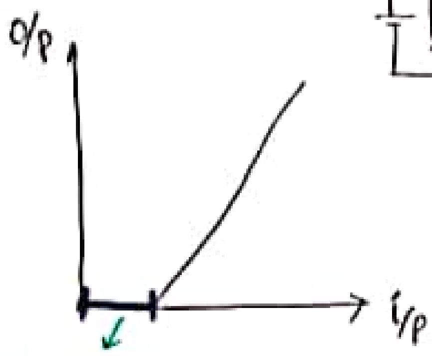
$S_A = 20 \; m\Omega/V$; $S_B = 10 \; m\Omega/V$.

$\therefore S_A > S_B$.

Dead Time :-



$\Rightarrow$ no-electrical inertia
(∵ mass of electron is 9.31×10^{-31} Kg)
$\Rightarrow$ mechanical (i.e. mass) inertia.



Dead time.

$$T_{electrical} = \frac{L \,(mH)}{R \,(k\Omega)} = \mu sec.$$

$$T_{elect.} = RC = msec/\mu sec.$$

$$T_{mech} = \frac{m}{B} \text{ seconds}$$

(m, B, K systems) $\quad m \rightarrow$ mass

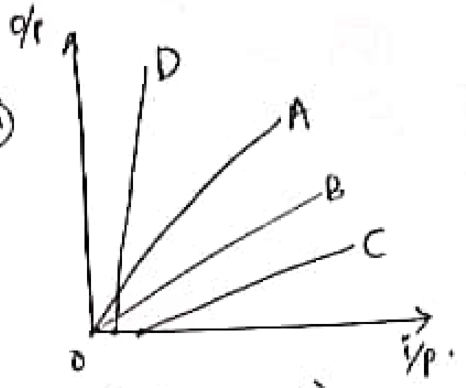
$\therefore$ $(\text{Time-const})_{mech.} > (\text{Time-constant})_{electrical}$.

The time taken by the instrument inorder to give the response is known as dead-time.

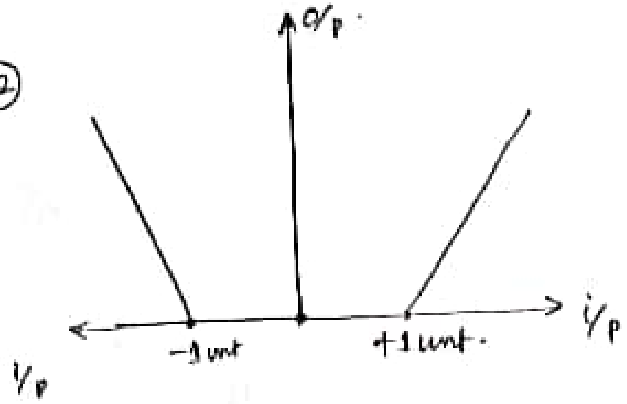
There is no electrical ~~mat~~ inertia b/c the mass of electron is very very small. Where as every mechanical body will offer some inertia so that it takes considerable amount of time inorder to give the response. B/c mechanical time constants are always greater than that of electrical time constants.

Dead zone :- For the largest value of input. The response of the instrument is zero. Beyond this input value the instrument gives the response. The corresponding portion of input where the output is known as dead zone.

Threshold :- At what particular input value, the instrument will give the response is known as threshold (or) pick-up

① 

(A>B>D>C)

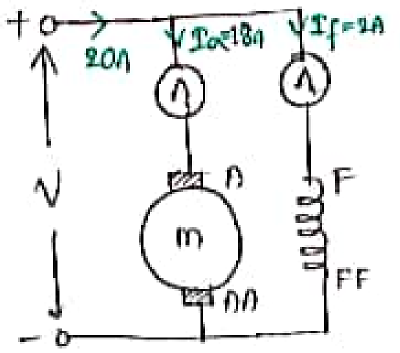
② 

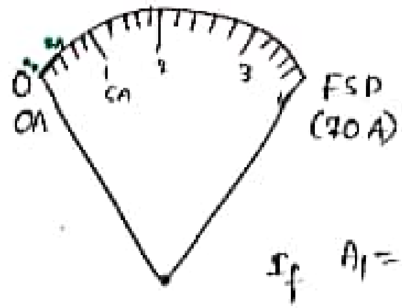
① Identify the from the above plot which instrument is the best inst : A

② Find the dead zone , threshold ... = ±1 unit

deadzone = 2 units

Resolution :- refers to clarityness (or) certainity

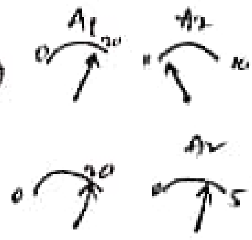




$\Rightarrow$ 1 unit $= \frac{20}{4\times5} = 1$ A.

I_f $A_1 = A_2$ (0-20A) $\Rightarrow$

I_f A_1(0-20)
A_2(0-5)



more no. of divisions $\Rightarrow$ more clarity, more resolutions.

The smallest value of change in input that we can detect with more clariliness (or) more certainity is known as resolution.

Fedelity :- The readings obtained how much faithfully represented in the record book is known as fedelity.

Significant figures :-
When we are representing the significant figures in a resultant variable always we will select the min. significant figure of the given variable.

Types of standards.

1. International stds $\Rightarrow$ IEEE, ISO

   Eg:. std. voltage $\Rightarrow$ western company.

   western std. cell $\Rightarrow$ E = 1.0183 = 1.0183 volt

   (Highest accuracy). $\Rightarrow$ not available for common man (patents require).

2. National standards (primary standards). ISI.

   (much more accurate). , not available

3. secondary standards (Industrial standards).

   Every industry has its own standards.... , not available

4. working standards → least accurate.
available to common man.

Secondary instruments: In all these secondary instruments either current quantity (or) voltage quantity is converted into mechanical deflecting torque and by means of scale and pointer we can obtaine the reading.

1. Indicating type instruments.
2. Recording type ".
3. Integrating type ".

1. These are the instruments which display the reading only at the time of measurement, It doesn't have any storage facility.
   Eg: All analog instrument, PMMC V,A, EMMC V,A, MI V,A, ohmmeter, wattmeter, megger...

2. Eg: Substation recording type, magnetic tape recorder, XY-recorder, cock-pit voice recorder, ECG (medical research patient monitors).
   These are the instruments, which record the continuous variations of an electrical quantity. Substation....
   Eg:- seismograph, speedometer.

3. 1-φ Energy meters (domestic purpose); 3φ energy (industrial).

   These are the instruments, which will give the electrical energy supplied to a consumer over a specific period of time.

There are (3) essential components in all indicating instruments. (17)

1. Deflecting torque (T_d)
2. Controlling " (T_c) → (i) o/p ∝ i/p ; (ii) if i/p = 0 ; pointer should bring back to '0' po
3. damping " (T_{damp}).

The torque which is required to move the pointer from its initial position, due to continuous current is flowing through the instrument, continous deflecting torque is exerted on the pointer (so that allways the pointer will be reaching full-scale reading, which is undesirable.

At steady-state both deflecting & controlling torques are equal in magnetude and acting opposite in direction so that the moving system produces oscillations, which is undesirable. To reduce the no. of oscillations (or) to damp out the no. of oscillatio we are going to apply a torque is called damping torque.

If the damping torque is absent, nothing will happen, but it (pointer) will take more time to settle at final steady-state process value.

Effects :-

1. Magnetic field effect. → PMMC (A),(V), EMMC (A),(V),(W),
2. Electro-static field effect → ESV-(V)
3. Electromagnetic field of attraction / Repulsion. → M.I (A),(V)
4. Electromagnetic induction effect. → All Induction type meters (A),(V),(W) Energy meter ✓
5. Thermal effect / heating → Bolometer (temperature), RTD (resistance-temp-detector), Thermistor, Thermocouple, hot-wire
6. Chemical effect → DC ampere-hour meter (V.I.t). Eg: vehicles, inverter.
7. Hall effect → (magnetic field measurement), flux/Gauss meter, pointing vector type wattmeter.
8. piezoelectric effect → piezo-electric transducers.

rectifier alo

pointing vector type instruments → It is used to measure the power loss density in a magnetic measurement.

PMMC

$T_d \propto I$

$I = +ve \Rightarrow T_d \Rightarrow +ve$

$I = -ve \Rightarrow T_d \Rightarrow -ve$

$T_{d\,net} = 0$

* cannot work for AC only DC.

EMMC

$T_d \propto I^2$

$I = +ve \Rightarrow T_d = +ve$

$I = -ve \Rightarrow T_d = +ve$

$T_{d\,net} \neq 0$

* can work for AC & DC.

ESV

$T_d \propto V^2$.

(AC & DC)

Induction

(T/F)

∴ only for AC

attraction/Repulsion

$T_d \propto I^2$ (AC & DC)

Thermal

$T_d \propto$ (heat)

$T_d \propto (I^2 r)(time)$

(AC & DC).

$\theta \propto I^2$

Note :- 1. Except PMMC for DC, induction type for AC remaining all other can be working for both AC as well as DC.

2. One and only one instrument is called PMMC always reads average value and remaining any other instrument always reads rms value including rectifier type instruments.

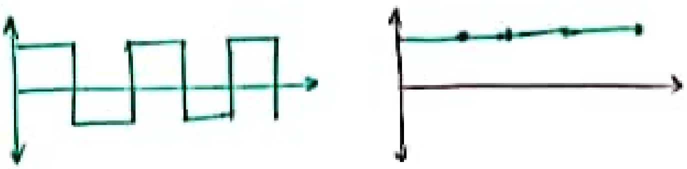
3. The rectifier type instruments are calibrated in such a manner by multiplying its form factor inorder to read RMS value.

$\boxed{\text{Reading of rectifier} = (FF) \times \text{reading of PMMC}}$

HWR ⇒ FF = 1.57 ⇒ (2.22X).
FWR ⇒ FF = 1.11

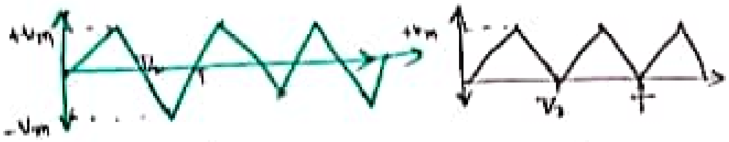
***
4. If the scale is multiplied by a factor of 1.11, it will give correct reading only for AC sinusoidal input full-wave rectifier. It gives wrong readings for AC sinusoidal input halfwave rectifier, squarewave input signal, triangular wave i/p signal and also for saw-tooth wave i/p signal.

Eg:-

1. 

$V_{RMS} = V_m$, $V_{avg} = V_m$

$K_f = \frac{V_{rms}}{V_{avg}} = 1$.

2. 

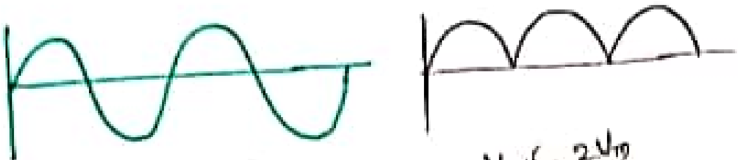
$K_f = \frac{V_{rms}}{V_{avg}} = \frac{2}{\sqrt{3}} = 1.1$

$V_{rms} = \frac{V_m}{\sqrt{3}}$ $V_{avg} = \frac{V_m}{2}$

3. 

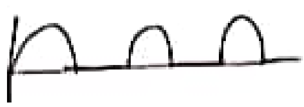
$K_f = \frac{V_{rms}}{V_{avg}} = \frac{2}{\sqrt{3}} = 1.15$

$V_{rms} = \frac{V_m}{\sqrt{3}}$ $V_{avg} = \frac{V_m}{2}$

4. 

$K_f = \frac{V_{rms}}{V_{avg}} = \frac{\pi}{2\sqrt{2}} = (1.11$

$V_{rms} = \frac{V_m}{\sqrt{2}}$ $V_{avg} = \frac{2V_m}{\pi}$

5. 

$K_f = \frac{V_{rms}}{V_{avg}} = \frac{\pi}{2} = 1.57$

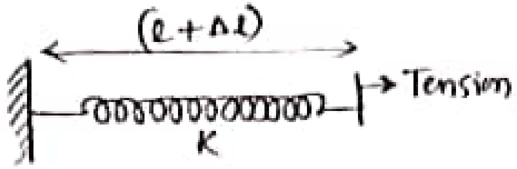
Control (Spring) Torque :— (T_c).

1. Spring control technique. → Helical spring. ⇒ ESV of parallel plate type. / → Spiral spring.

2. Gravity control

Spring Control

(i) Helical spring



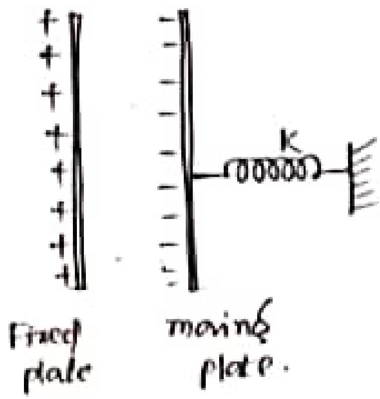
(restoring force) ∝ (displacement)

$$F_c \propto x$$

$$F_c = Kx$$

$$K = \frac{F_c}{x} \quad \frac{N}{m}$$

stiffness coefficient = Spring constant.



(ii) Spiral spring.

Torque ⇒ twisting moment

torque = Force × ⊥r distance ⇒ $\tau = Fx$ Nm.

$$T_c = K_c\theta \Rightarrow K_c = \frac{T_c}{\theta_c} \quad \frac{Nm}{radians} \Rightarrow \text{spring constant}.$$

common material to prepare spiral spring.

→ phosphor bronze ⇒ $K_c = 12 \times 10^9$ kg/m²

→ Beryllium Copper (costly)

Some of the Instruments, which has low resistance like MCoil MC milli-ammeter. In those instruments beryllium Copper is prefered to prepare the spring but it is very costly.

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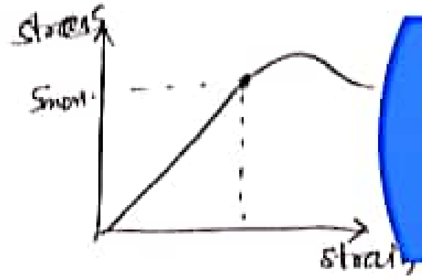
properties of spring. (Requirements).

(i) Non-magnetic material

(ii) $(\text{stress})_{max} <$ Elastic limit

(iii) $\frac{L}{t} = \frac{\text{length}}{\text{thickness}} \simeq 3000$; $\theta = 90°$.



$$\boxed{T_c = \frac{E b t^3}{12 L} \theta};$$

E = Young's modulus of elasticity (Kg/…

L = length of the spiral spring. (m…

t = thickness of spring (m)

b = width/depth of spring (m).

θ = deflection of pointer (radians)

K_c = Spring constant.

$$T_c = K_c \theta.$$

$$\boxed{K_c = \frac{E b t^3}{12 L}} \left(\frac{Nm}{radi}\right).$$

$E = 12 \times 10^9$ Kg/m². (for phosphor bronze).

$$\boxed{\frac{L}{t} = \frac{E\theta}{2 S_{max}}} \simeq 3000; \ \theta = 90°;$$

Every control spring must satisfy the above requirements... Some times spring is used as leads of the instruments that means it is a current carrying element. The area of cross-section of the spring must be sufficient inorder carry the rated current otherwise the spring may suffer from internal heating problem. Sothat spring may be broken.

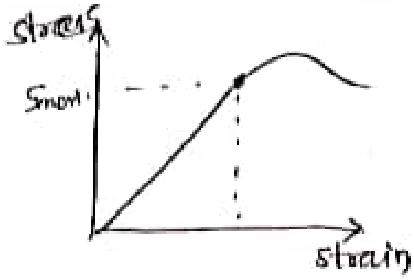
Spring problems

1. Ageing effect.

2. $l_t = l_0(1 + \alpha \Delta t)$.

$$K = \frac{F}{x} = \frac{F}{l + \Delta l} \Rightarrow l\uparrow \Rightarrow K\downarrow$$

properties of spring. (Requirements).



(i) Non-magnetic material

(ii) $(\text{stress})_{max} < \text{Elastic limit}$

(iii) $\frac{L}{t} = \frac{\text{length}}{\text{thickness}} \simeq 3000$; $\theta = 90°$.

$$\boxed{T_c = \frac{E b t^3}{12 L} \theta} ;$$

E = young's modulus of elasticity (Kg/m^2)

L = length of the spiral spring. (m)

t = thickness of spring (m)

b = width/depth of spring (m).

θ = deflection of pointer (radians).

K_c = Spring constant.

$T_c = K_c \theta$.

$$\boxed{K_c = \frac{E b t^3}{12 L}} \left(\frac{Nm}{radi}\right).$$

E = 12×10^9 Kg/m^2. (for phosphor bronze).

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Spring problems

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$$K = \frac{F}{x} = \frac{F}{l + \Delta l} \Rightarrow l\uparrow \Rightarrow K\downarrow$$

Because of error due to temperature the stiffness of the spring can be changed so that always the instrument gives the wrong readings. Due to ageing effect of the spring the stiffness const reduces so that the instrument always reads more. Spring control technique is costly compared to gravity control.

case(i) :- If $T_d \propto I \Rightarrow$ linear

We may obtain linear scale if the deflecting torque is proportional to the current flowing through the instrument.

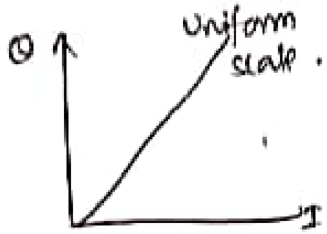
$$T_d = K_d I \rightarrow ①$$

$$T_c = K_c \theta \rightarrow ②$$

at Steady-state $\Rightarrow$ $|T_d| = |T_c|$

$$K_d I = K_c \theta$$

$$\theta = \left(\frac{K_d}{K_c}\right) I \Rightarrow \theta \propto I$$

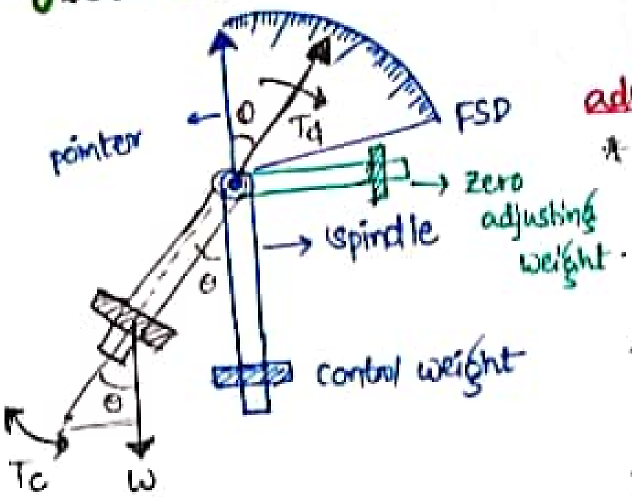


case(ii) :- If $T_d \propto I^2 \Rightarrow$ non-uniform scale.

$\theta \propto I^2$

Gravity Control :-

$$T_d = T_c = K(mg\sin\theta)$$



adv

* No ageing effect (b/c mg won't change).

$\therefore$ $\boxed{T_c \propto (\sin\theta)}$

* No current flow, no temperature effect.

* Cost of the arrangement is low.

disadvantage :- (Level zero error). present.

In a gravity control technique a small weight is attached to the moving system. So that because of acceleration due to gravity there is a gravitational pull is developed so that it will bring the pointer towards reading value. (20)

disadvantages

1. This technique is best suitable only for vertical placements. If these instruments are placed horizontally it may not give the proper reading.

2. The place where your are keeping the meter must be plaine surface, If it is inclined surface there is a level zero error will occur, it can be adjusted by using zero adjusting weight.

application: This type of techniquie is used in. Wall-mounted and panel boad type instruments.

$$T_c = K_g \sin\theta \; ; \; T_d = K_d I \quad (T_d \propto I)$$

$$K_d I = K_g \sin\theta$$

$$\boxed{I \propto (\sin\theta)} \Rightarrow \text{non-linear scaling.}$$

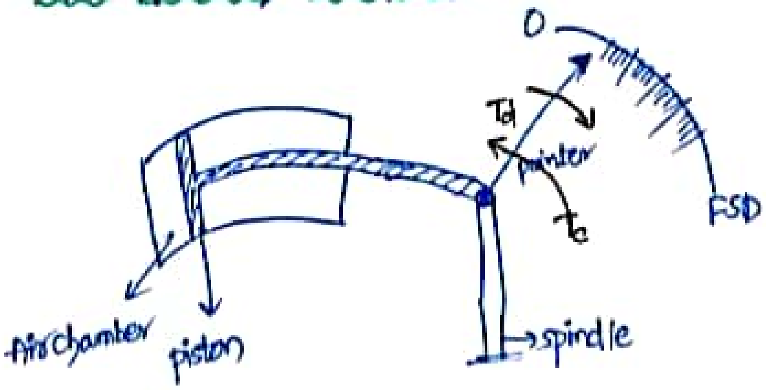
if $(T_d \propto I^2)$

$$\boxed{I^2 \propto (\sin\theta)}$$

Damping techniques :-

1. Air friction damping (MI, EMMC)
2. Fluid friction (Electrostatic voltmeter, Wall-mounted, panel board type).
3. Eddy current damping (PMMC, Induction type)
4. Electromagnetic damping technique. (Galvanometers)

Air friction damping :-



Effectiveness order ⇒ (eddy current) > (fluid) > (air friction)

priority order ⇒ (eddy current) > (air friction) > (fluid).

In this type of damping technique a light disc of aluminium, vane (or) piston, which has a considerable area inorder to develope the friction, which is placed inside the air-chamber, it is connected to the spindle so that the friction offered by air will oppose the motion of the pointer.

This arrangement requires less maintainances and cost is lesser.

Fluid Friction Damping technique :-

It is the more effective form of damping technique in this type of damping technique an aluminium vane is placed

The fluid must satisfy the following requirements. (2)

(i) Fluid shouldn't evaporate quickly.

(ii) fluid viscosity should not change with temperature.

(iii) The fluid should not have any corrosive action upon the metals of the instrument.

(iv) The fluid must be very good insulator.

Disadvantage

(i) Due to leakage of the fluid it is difficult to keep the instrument clean. So that it requires more maintanc

(ii) cost is also more.

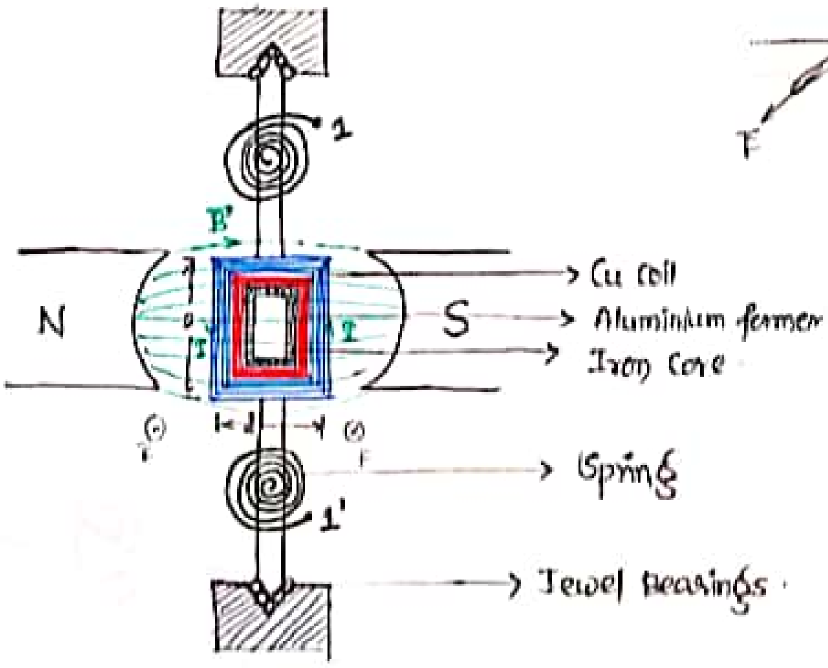
applications :- It is used in Electrost. V., panel board, wall-mount

Eddy Current Damping technique :-

PMMC Instrument :-

* U shaped - permanent magnet
    * ALNICO
    * Hard magnetic material
    * Very high cohesive forces.
    * Very strong internal magnetic field. ($B = 0.1\ Wb/m^2$ to $1\ Wb/m$

* Stationary iron-core :- to make the magnetic field radial field



$\vec{F} = (\vec{i} \times \vec{B})$

$F = BIl \sin\theta$.

Lorentz Force & Flemings Left Hand Rule

Here, $\theta = 90^\circ$. $\bar{B} \perp \bar{l}$

$\therefore F = BIl$

Torque = $F \times \perp d$

$T = F \times d$

$T = BILD$

$T = BIA$

(per conductor)

For N turns

$\therefore T = (BIA)N$

$\therefore T_d = BINA$

$T_d = (BNA)I$

$T_d = K_d I$

$\therefore \boxed{T_d \propto I}$

* Jewel-bearings → To reduce friction

made up of **SAPPHIRE**

* Aluminium Former :-
    (i) provide the base for copper coil
    (ii) To provide eddy currents path.

* Copper coil :-
    (i) To carry current
    (ii) To provide T_d.

* Type of control (PMMC) – Spring control.

* Spring (dual purpose)

(i) To provide controlling torque (T_c)

$$T_c \propto \theta \Rightarrow T_c = K_c \theta.$$

(ii) Used as Leads of instrument.
(current carrying element).

at steady-state $T_d = T_c$

$$K_d I = K_c \theta.$$

$$\theta = \frac{K_d}{K_c} I \Rightarrow \boxed{\theta \propto I.}$$

Uniform scale.

$$\text{Sensitivity} = \frac{d\theta}{dI} = \frac{K_d}{K_c} = \frac{BNA}{K_c} = \text{constant}.$$

$$\text{Sensitivity} = \frac{(\text{magn. Flux density})(\text{No. of turns})(\text{Area of Cross-section})}{(\text{stiffness of spring})}.$$

Due to aging effect of magnet

(i) magnetic field strength (B)↓ decreases.

$$\theta \propto B \Rightarrow \theta\downarrow \propto B\downarrow$$

(ii) (K_c) spring constant / stiffness will decreases.

$$\uparrow\theta_c \propto \frac{1}{K_c\downarrow}$$

$$\therefore \boxed{\theta \propto \frac{B}{K_c}}$$

$$\frac{\delta\theta}{\theta} = \frac{\delta B}{B} - \frac{\delta K_c}{K_c}$$

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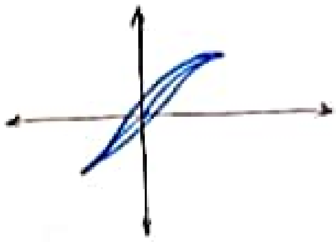
advantages of PMMC

* Linear scale.
* produced torque is very high due strong magnetic field. $(T \propto B)$
* Less weight of moving system ⇒ Less friction.
* High $\left(\frac{T}{W}\right) = \frac{\text{torque}}{\text{weight}}$ ration ⇒ high sensitivity.

$$S = 20{,}000\ \Omega/\text{volts to } 30{,}000\ \Omega/\text{volts}.$$

"In all analog instruments the torque to weight ratio will decide the frictional errors."

* Frictional errors are reduced by jewel-bearings also.
* Aluminium former will have very thin hysteresis loop area.



thin hysteresis curve for light weight aluminum former.

The word hysteresis means that the loading energy may n... equal to the unloading energy, reduced hysteresis error in a PMMC instrument b/c of aluminium former, It has very thin hysteresis loop.

* Low power consumption (25 μWatts – 200 μWatts)
  So that No internal heating/temperature rise problem. reduced temperature error, still any other temp. errors are ~~present~~ It can be reduced by swamping resistor.
* A long open 360° circular scale is available.

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* Error due to external magnetic field is known as stray magnetic field error, In a PMMC instrument reduced stray magnetic field error b/c of very strong internal magnetic field.

* More accuracy.

disadvantages of PMMC :-

* Cannot work for ac
* cost is more (PM, jewel-bearings) & delicate construction.

Application :-

* It is mainly used in air-craft systems, aero-space industry because of self-shielding property. → i.e. we shouldn't require any protecting cover b/c the operating field of meter is very strong.

Note :- All errors are lesser in PMMC instrument compared to any other instrument. So that this instrument has more accuracy & high sensibility.

Electrical Equivalent ckt/diagram :-

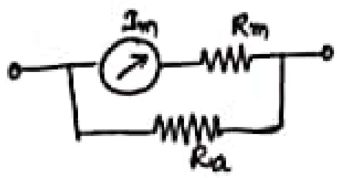
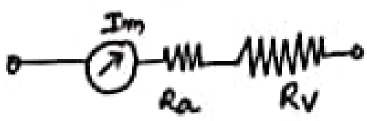


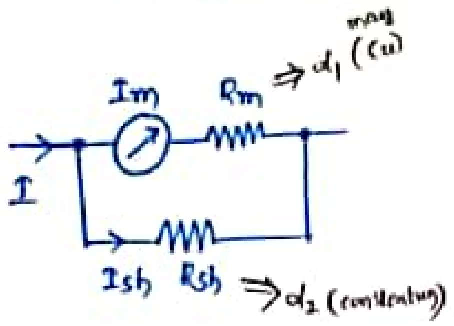
Fig. PMMC instrument
also called ammeter
i.e. of order (μA).



* without any shunt multiplier → (0-5 μA)
multiplier → (0-200 A)

* Without any series multiplier ⇒ (0 - 50 mV)
* With series multiplier ⇒ (0 - 20,000 V) (or) (0 - 30,000 V).

Extension Range of PMMC Ammeter :-



$I = I_{extension} = I_m + I_{sh}$.

$V_{sh} = V_m$

$\therefore\ I_{sh} R_{sh} = I_m R_m$

$$R_{sh} = \frac{R_m I_m}{I_{sh}} = \frac{R_m (I - I_{sh})}{I_{sh}}$$

$$= \frac{R_m I_m}{(I - I_m)} = \frac{R_m}{\left(\frac{I}{I_m} - 1\right)}$$

multiplication factor (m)

$$m = \frac{I}{I_m} = \frac{I_{exten}}{I_m} = \_\times 10^6$$

$$\boxed{R_{sh} = \frac{R_m}{(m-1)}}$$

$$\boxed{m = 1 + \frac{R_m}{R_{sh}}}$$

$\theta \propto I_m$

$\theta \propto (I_m \times m)$

$\theta \propto I_{extension}$

$\theta \propto I$

$R_{sh} \Rightarrow$ milli Ωs (very very low).

Maganin (or) Constanton.

properties of shunt resistors :-

(i) should carry large current without rise in temperature.

(ii) resistance should not vary with time & temperature.

(iii) Shunt should have very low temperature coefficient.

$\therefore$ constant resistance is possible.

(iv) Shunt should have very low thermal emf with cuppor.

AC instrument ⇒ Magn

DC instru ⇒ Constan

(cu)

All these above properties can be satisfied by maganin.

Particularly for DC instruments maganin is preferred b/c it should have very low thermal emf with copper.

Where as in the AC instruments always constanton is prefered because the produced thermal emf by Cu-constantum pair is always unidirectional. Which will not have any affect on AC instrument readings.

DC instruments ⇒ Manganin.

AC insbruments ⇒ Constanton

MANGANIN ⇒ (4% Ni + 84% Cu + 12% manganese)

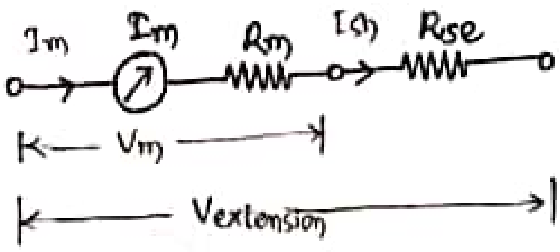
Extension Range of PMMC voltmeter :-

$\theta \propto V_m$

$\theta \propto (V_m)m$

$\theta \propto V_m \times \frac{V}{V_m}$

$\boxed{\theta \propto V}$



$$I_m = I_{sh} \Rightarrow \frac{V_m}{R_m} = \frac{(V - V_m)}{R_{se}}$$

$$\boxed{R_{se} = R_m(m-1)}$$

$$\boxed{m = 1 + \frac{R_{sh}}{R_m}}$$

m → multiplication factor

$m = \frac{V}{V_m} = 1 + \frac{R_{se}}{R_m}$

$m >>> 1$ (always).

Series resistances are preferred (prepared by Eureka material.

$R_{shunt} \longrightarrow$ Manganin

$R_{series} \longrightarrow$ Eureka

coil $\longrightarrow$ Cuppor

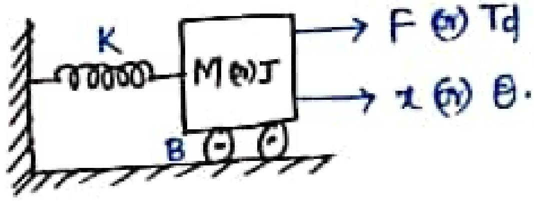
former $\longrightarrow$ Aluminium

Spring $\longrightarrow$ phosper-bronze.

Jewel-bearings $\longrightarrow$ Sapphire

PMMC ≡ 2nd order system of rotation :-

≡ All analog/measuring instruments.



$$F_d = m\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

(or)

$$T_d = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt^2} + K_c\theta.$$

$$F_d(s) = (ms^2 + Bs + K)\,\theta(s)$$

$$T/F = \frac{L(o/p)}{L(i/p)} = \frac{1}{ms^2+Bs+K} \Rightarrow \text{CEqn } ms^2+Bs+K=$$

$$s^2 + \left(\frac{B}{m}\right)s + \left(\frac{K}{m}\right) = 0$$ compared with 2nd order proto type system

$$s^2+2\xi\omega_n s+\omega_n^2=0.$$

$$2\xi\omega_n = \frac{B}{m}$$

$$\omega_n^2 = \frac{K}{m} \Rightarrow \omega_n = \sqrt{\frac{K}{m}} \Rightarrow \boxed{f_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\ }}$$

$$\xi = \frac{B}{2m\omega_n} \Rightarrow$$

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$$\therefore \quad \xi = \frac{B}{2\sqrt{mK}}$$

If $\xi = 0 \Rightarrow$ undamped

$\xi = 1 \Rightarrow$ critical damping

$\xi < 1 \Rightarrow$ Under damping (practical System).

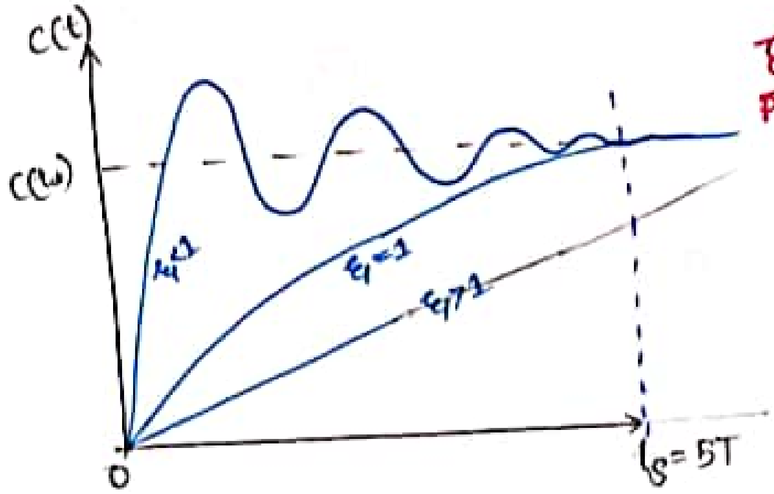
$\xi > 1 \Rightarrow$ over damping

$t_{settling} = \frac{3}{\xi\omega_n} \Rightarrow$ for 5% tolerance band

, $t_s = \frac{4}{\xi\omega_n} \Rightarrow$ for 2% tolerance band.

τ = time period = $\frac{1}{\xi\omega_n}$; if $\xi = 0$; τ = infinite.

* $Q = \frac{1}{2\xi}$; Underdamping $\Rightarrow Q > 1$ (quality factor).



$\xi_{practical}$ (0.6 to 0.8).

$\therefore$ All analog instruments are 2nd order systems, practically under-damped technique is prefered whose 'ξ' value is in the order of 0.6 to 0.8, In a under damped system always the pointer is reaching full-scale value before coming

In critically damped system without making oscillation directly it moves to steady-state. (20

Note :- (i) Potentiometer is zero order instrument.

$$T/F = \frac{V_o(s)}{X(s)} = K S^0 = \text{constant}.$$

(ii) Thermometer is 1st order instrument.

$$T/F = \frac{K}{(1+s\tau)}$$

$K \rightarrow$ static sensitivity ; τ = time response.

(iii) All analog instruments are 2nd order instruments.

$$T/F = \frac{1/m}{\left(s^2 + \frac{B}{m}s + \frac{K}{m}\right)}$$; m = mass, B = friction, K = spring constant.

$\left\{\begin{matrix} \text{PMMC} \\ \text{EMMC} \end{matrix}\right\} \Rightarrow$ Spring serves dual purpose (Tc & current carrying element).

$\Downarrow$

If one of spring breaks-down the produced deflecting torque will be zero. Then then the pointer always shows zero deflection

$\left\{\begin{matrix} \text{ESV} \\ \text{MI} \end{matrix}\right\} \Rightarrow$ Spring serves only control torque

$\Downarrow$

If spring breaks-down always the pointer will be swinging beyond the full-scale.

Moving Iron Instruments :-

→ minimum reluctance principle

→ variable self-inductance concept.

$$\phi \propto I$$

$$N\phi = LI$$

$$N\frac{d\phi}{dt} = L\frac{dI}{dt} + I\frac{dL}{dt}$$

energy stored in a magnetic field $= \frac{1}{2}LI^2$

$$e\,dt = L\,dI + I\,dL$$

$$eI\,dt = LI\,dI + I^2dL$$

incremental energy stored $= \frac{1}{2}(L+dL)(I+dI)^2$

energy supplied $= LI\,dI + I^2dL$ —(1)

change in energy stored $= \frac{1}{2}(L+dL)(I+dI)^2 - \frac{1}{2}LI^2$

$$\Delta E = \frac{1}{2}(L+dL)(I^2+dI^2+2I\,dI) - \frac{1}{2}LI^2$$

$$= \frac{1}{2}L\,dI^2 + \frac{1}{2}\times 2IL\,dI + I^2dL + dI^2\cdot dL + I\,dI\,dL$$

$$= \left(\frac{1}{2}L\,dI^2\right) + (IL\,dI + \tfrac{1}{2}I^2dL) + (dL\cdot dI^2) + (I\,dI\cdot dL)$$

$$\cong (IL)\,dI + \frac{1}{2}I^2dL \quad \text{—(2)}$$

$\left(\frac{1}{2}L\,dI^2,\ dL\cdot dI^2,\ dI,dL \text{ are neglible}\right)$

Mech work + change in energy stored $\cong$ Energy supplied

Mechanical work done $dW = T_d \cdot d\theta$.

According to law of conservation of energy...

$$dW + \frac{1}{2}I^2dL + (IL)\,dI = LI\,dI + I^2dL$$

$$dW = \frac{1}{2}I^2dL$$

Moving Iron Instruments :-

→ minimum reluctance principle

→ variable self-inductance concept.

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$$\Delta E = \frac{1}{2}(L+dL)(I^2+dI^2+2I\,dI) - \frac{1}{2}LI^2$$

$$= \frac{1}{2}L\,dI^2 + \frac{1}{2}\times 2IL\,dI + I^2 dL + dI^2 \cdot dL + I\,dI\,dL$$

$$= \left(\frac{1}{2}L\,dI^2\right) + (IL\,dI + \frac{1}{2}I^2 dL) + (dL \cdot dI^2) + (I\,dI \cdot dL)$$

$$\cong (IL)\,dI + \frac{1}{2}I^2 dL \quad \text{—(2)}$$

($\frac{1}{2}L\,dI^2$, $dL \cdot dI^2$, dI, dL are neglible)

Mech. work done + change in energy stored $\cong$ Energy supplied

Mechanical work done $dW = T_d \cdot d\theta$.

According to law of conservation of energy...

$$dW + \frac{1}{2}I^2 dL + (IL)\,dI = LI\,dI + I^2 dL$$

$$dW = \frac{1}{2}I^2 dL$$

$$T_d \cdot d\theta = \frac{1}{2} I^2 dL$$

$$\therefore \boxed{T_d = \frac{I^2}{2} \frac{dL}{d\theta}}$$

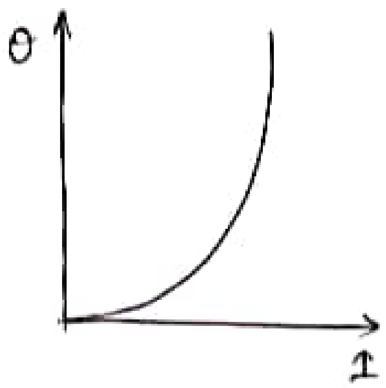
If $\frac{dL}{d\theta}$ = constant then $(T_d \propto I^2)$

$\therefore$ AC & DC can be used.

Type of control :- Spring control (only control, not current carrying cl

at steady-state $T_d = T_c$

$$K_c \theta = \frac{I^2}{2} \frac{dL}{d\theta}$$



$$\therefore \boxed{\theta = \frac{I^2}{2K_c}\left(\frac{dL}{d\theta}\right)}$$ ⇒ Attracted MI type ammeter.

$\therefore \boxed{\theta \propto I^2}$ non-linear (non-uniform) scale

$\therefore$ scale is cramped at lower end to obtain all readings.

$$I = \frac{V}{Z} = \frac{V}{|R+j\omega L|} = \frac{V}{\sqrt{R^2+\omega^2 L^2}}$$

$$\boxed{\theta = \frac{V^2}{(Z^2) 2K_c}\left(\frac{dL}{d\theta}\right)}$$ ⇒ Attracted MI type voltmeter

$$\Rightarrow \boxed{\theta = \frac{V^2}{2K_c(R^2+\omega^2 L^2)} \frac{dL}{d\theta}}$$

advantages

* works for both AC & DC.
* Reduced frictional errors because heavy part of the instrument is in stationary position that means coil.
* Robust construction
* Less cost

disadvantages :-

1. Non-linear scale ⇒ less accurate at lower ends of scale.
2. weak operating field b/c of electromagnet

$$B = 0.006\ wb/m^2 - 0.0075\ wb/m^2$$

$$B = (6m - 7.5m\ wb/m^2).$$

∴ Td produced lesser, $\omega \rightarrow$ moderate weight.

3. ∴ Low torque-weigh ration, Low sensitivity

$$S_v = (30\ \Omega/v \text{ to } 40\ \Omega/v).$$

4. Hysteresis error is more b/c of iron piece, it is a ferro magnetic material.
5. B/c of more hysteresis error the instrument may not follow the perfect square law. So that instrument accuracy is lesser.
6. More power consumption, more internal heating problem. more temperature error.
7. $$\theta \propto \frac{V^2}{Z^2} \propto \frac{V^2}{f^2} \propto \frac{(10^{-3})^2}{(10^3)^2} \propto 10^{-12}$$

This instrument severely suffers from frequency error particularly when we are measuring current & voltages in a low voltage & high frequency communication ckt.

Useful frequency range ⇒ (0 - 125) Hz.

∴ MI is best suitable for measurement of V & I at powers frequency. At f = 50Hz the instrument

In this instrument the operating field is weak so that stray-magnetic field error is more. So we required some external protection i.e. provided by wooden body/box.

$$\theta = \frac{I^2}{2K_c}\left(\frac{dL}{d\theta}\right) \qquad \boxed{\theta \propto I^2} \Rightarrow \text{non-linear}$$

$$\theta = \left(\frac{I}{2K_c}\right)\left(I\frac{dL}{d\theta}\right)$$

To have scale in linear form $\therefore$ $\boxed{\theta \propto I} \Rightarrow$ linear

K', $\theta = K'I$

$$\boxed{I\frac{dL}{d\theta} = (\text{constant})_1} \text{ (or)}$$

$$\boxed{\theta\frac{dL}{d\theta} = (\text{constant})_2} \quad \text{Linear scale.}$$

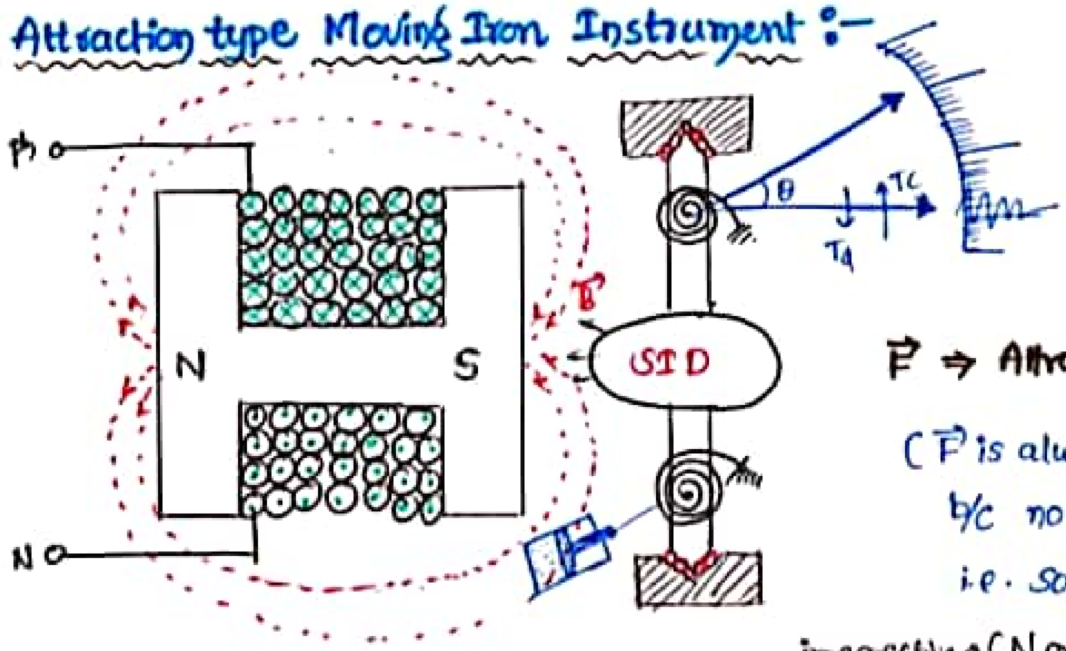
$\therefore$ Linear scale $\Rightarrow$ $\theta\frac{dL}{d\theta}$ = constant (or) $I\frac{dL}{d\theta}$ = constant

& multiply non-linear scale with K' to get linear scale.

Electrical Equivalent ckt/diagram :-

(Diagram)

Attraction type Moving Iron Instrument :-

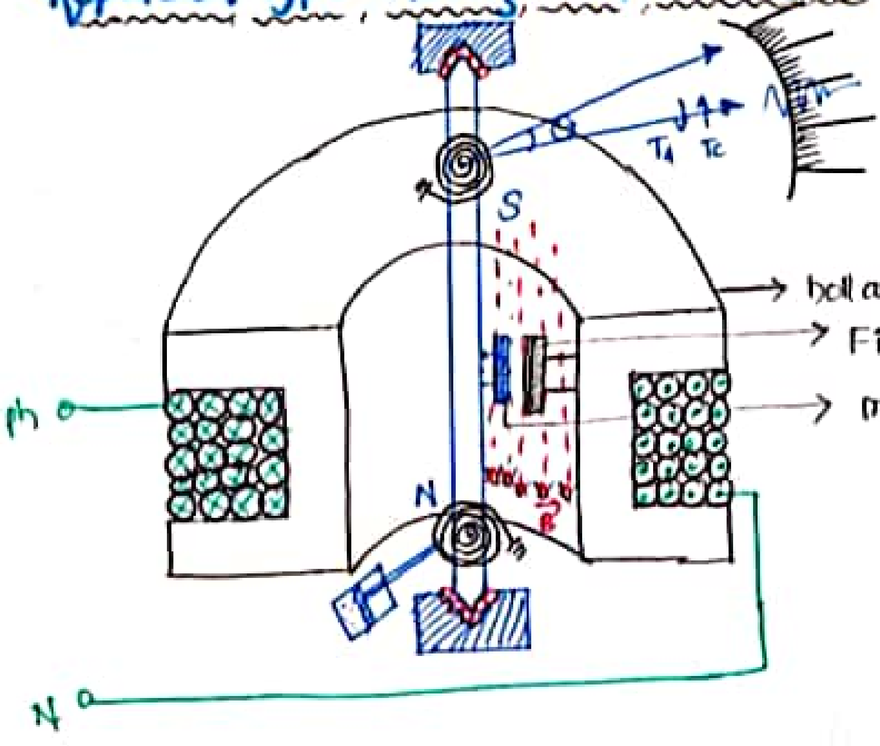


$\vec{F} \Rightarrow$ Attraction

($\vec{F}$ is always attraction b/c no pole on the SID i.e. Soft Iron disc).

irrespective q(N or S on SID side (created pole)

Repulsion Type Moving Iron Instrument :-



$\vec{F} \Rightarrow$ Repulsion

→ hollow cylinder (dumbell shape)

→ Fixed Iron Rod (same pole)

→ moving Iron Rod (same pole)

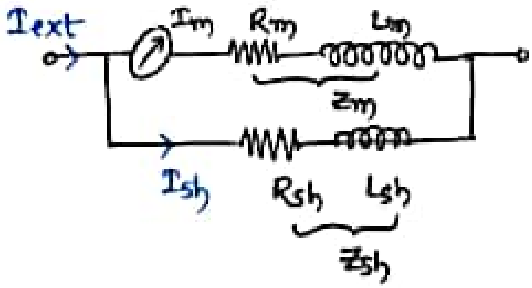
repulsion.

F, Td, θ.

(Inside hollow dumbel part .... spindel is placed)

Flux is South to North, two rods get magnetise with same pole on each.

Extension Range of MI ammeter :-



$$I_{extension} = I = I_m + I_{sh}$$

$$V_m = V_{sh}$$

$$I_m Z_m = I_{sh} Z_{sh}$$

$$\frac{I_m}{I_{sh}} = \frac{Z_{sh}}{Z_m}$$

$$\frac{I_m}{I_{sh}} = \frac{\sqrt{R_{sh}^2 + \omega^2 L_{sh}^2}}{\sqrt{R_m^2 + \omega^2 L_m^2}} \Rightarrow \frac{I_m}{I_{sh}} = \left(\frac{R_{sh}}{R_m}\right)\sqrt{\frac{1 + \omega^2\left(\frac{L_{sh}}{R_{sh}}\right)^2}{1 + \omega^2\left(\frac{L_m}{R_m}\right)^2}}$$

$$I_m = f(\text{frequency}).$$

$$\frac{I_m}{I_{sh}} = \left(\frac{R_{sh}}{R_m}\right)\sqrt{\frac{1 + \omega^2 \tau_{sh}^2}{1 + \omega^2 \tau_m^2}}$$

If $\tau_{sh} = \tau_m \Rightarrow \frac{L_{sh}}{R_{sh}} = \frac{L_m}{R_m}$

$$\therefore \boxed{\frac{I_m}{I_{sh}} = \frac{R_{sh}}{R_m}}$$ i.e. Independent of frequency.

$$m = \frac{I_{ext}}{I_m} = 1 + \frac{R_m}{R_{sh}} \Rightarrow 1 + \frac{Z_m}{Z_{sh}} = m.$$

$$R_{sh} = \frac{R_m}{(m-1)} \; ; \quad \boxed{Z_{sh} = \frac{Z_m}{(m-1)}}$$

$$\therefore m = 1 + \frac{Z_m}{Z_{sh}} = 1 + \sqrt{\frac{R_m^2 + \omega^2 L_m^2}{R_{sh}^2 + \omega^2 L_{sh}^2}}$$

To make the MI ammeter independent of frequency the shunt time-constant should be equal to meter time-constant.

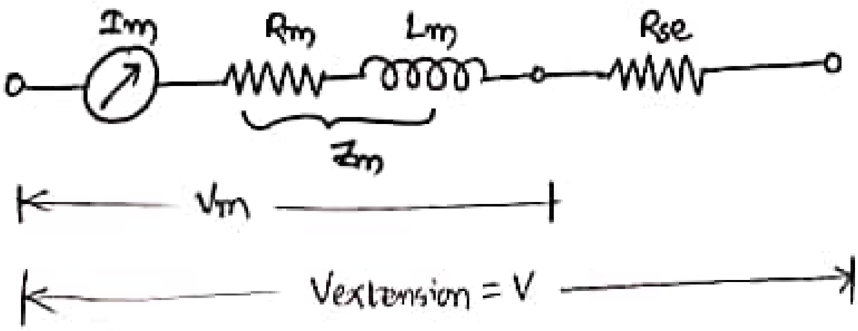
*** ⇒ Which one of the following error is **absent** in PMMC instrument

a) Hysteresis error
b) temperature error
c) stray-magnetic error

} very very small magnitude

d) reactance error (or) frequency error. → zero error.

Extension range of MI voltmeter :-



$$m = \frac{V_{ext}}{V_m} = \frac{V}{V_m} = \frac{I_m(Z_{total})}{I_m(Z_m)} = \frac{|(R_m+R_{se})+j\omega L_m|}{|R_m+j\omega L_m|}$$

$$m = \sqrt{\frac{(R_m+R_{se})^2+\omega^2L_m^2}{R_m^2+\omega^2L_m^2}}$$

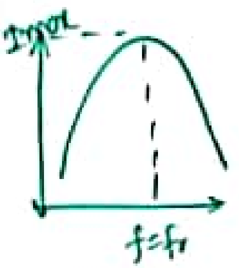
⇒ to nullify reactance of inductor can connect Capacitor.

(i) If C_{se} is connected

RLC series resonance,

$Z_{min} = R$

$I = I_{max} = I_{coil}$



i.e max saturate coil.

(ii) If C_{sh} is connected

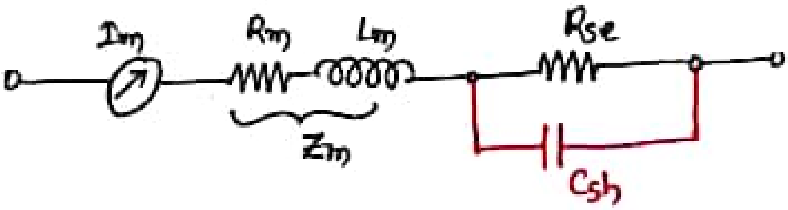
RLC parallel, resonance

$X_L = X_C \Rightarrow Z_{max} = R$

$\therefore I = I_{min} = I_{coil}$

no... saturation of coil.

Calculation of C_{sh} value :-



$$Z_{total} = (R_m + j\omega L_m) + \frac{R_{se} \frac{1}{j\omega C_{sh}}}{R_{se} + \frac{1}{j\omega C_{sh}}} \times 1$$

$$= (R_m + j\omega L_m) + \frac{R_{se}}{(1 + j\omega C_{sh} R_{se})} \times \frac{(1 - j\omega C_{sh} R_{se})}{(1 - j\omega C_{sh} R_{se})}$$

$$Z_{total} = R_m + j\omega L_m + \frac{R_{se}(1 - j\omega C_{sh} R_{se})}{1 + \omega^2 C_{sh}^2 R_{se}^2}$$

at resonance Z = resistive, $\mathrm{Im}(Z_{total}) = 0$.

$$\omega L_m - \frac{R_{se}^2 \omega C_{sh}}{(1 + \omega^2 C_{sh}^2 R_{se}^2)} = 0.$$

$$L_m = \frac{C_{sh} R_{se}^2}{(1 + \omega^2 C_{sh}^2 R_{se}^2)}$$

if $1 >>> (\omega^2 C_{sh}^2 R_{se}^2)$

$$\therefore \quad L_m = C_{sh} R_{se}^2 \Rightarrow \boxed{C_{sh} = \frac{L_m}{R_{se}^2}}$$ Theoretical

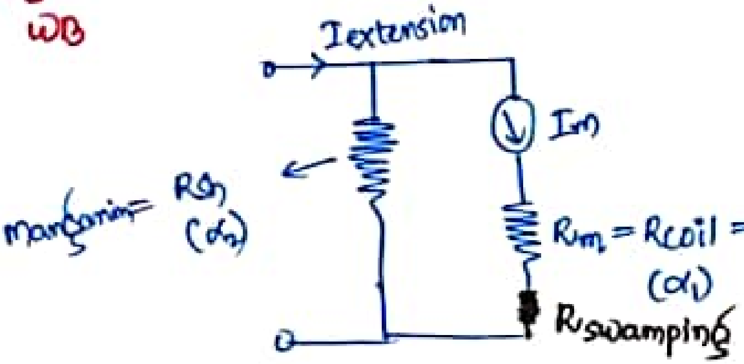
$$\boxed{C_{sh} = 0.41 \frac{L_m}{R_{se}^2}}$$ practical (including "f" effect).

To make the MI voltmeter independent of frequency a capacitor must be connected in parallel with R_{se}, whose value of capacitance is $0.41 \frac{L_m}{R_{se}^2}$.

Note:– MI instrument works on the principle of change in self-inductance (or) minimum reluctance property.

PMMC conv. ③ ⇒ Pg. 10 WB

Swamping resistance. in Series with coil of meter.



$\left(I_m = \frac{V_{FSD} = V_m}{R_{cuT}}\right)$

$R_{sh} \neq f(\text{temperature})$ (or) very low (α_T).

$R_{swamping} \neq f(\text{temperat})$ ⇒ $\alpha_T = \text{constant}$.
(manganin resistance). ∵ doesn't affected by temperature rise.

practically $\boxed{R_{swamp} = (20-30)\, R_{coil}}$

** $\therefore \quad I_m = \frac{V_m}{(R_{cuat_T} + R_{sw})}$

coil is prepared by Copper wire which has +ve temperature coefficient, shunt is prepared by manganin, which has constant temperature coefficient (or) negligible temperature coefficient. So that there is a temp. diff will exists b/w

To reduce the temperature difference both coil and shunt be made up of same material. For that a resistance (31) which is connected in series with the cupper coil whose chara-cteristics are similar to manganin (constant temperature coefficient). The value of resistance is around 20 to 30 times that of cupper coil resistance is known as swamping resistance

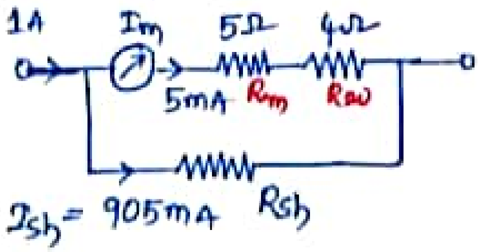
problem:- $R_m = R_{coil} = 5\Omega$.

$R_{sw} = 4\Omega$.

$I_m = 5mA$.

$I = I_{ext} = 1A$.

$R_{sh} = ?$



$$\therefore \quad R_{sh} = \frac{(R_m + R_{sw})}{(m-1)} \; ; \quad m = \frac{I}{I_m}$$

$$R_{sh} = \frac{5+4}{\left(\frac{1}{5\times10^{-3}} - 1\right)} = \frac{9}{\left(\frac{10\times10^{2}}{5} - 1\right)} = \frac{9}{199} = 0.0452261\,\Omega$$

$$R_{sh} = 45\,m\Omega$$

(P) which one of the following instrument works on the principle of change in mutual Inductance

sol:-

(a) PMMC

(b) MI

(c) EMMC ✓

(d) All

Electro magnetic moving coil (EMMC)
(or)
Electro dynamo type instrument (EDM) :-

1. Fixed coil ⇒ electromagnet
2. Moving coil

$T_d \propto i_1 i_2$

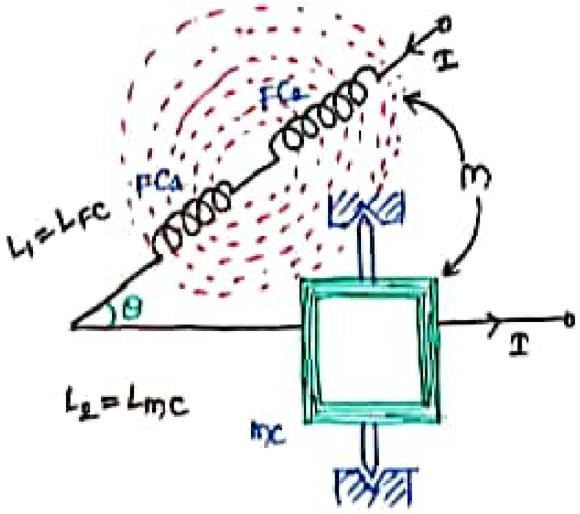
$T_d \propto \phi i_2$

$T_d \propto B i_2$

$$T_d = \frac{I^2}{2}\frac{dL}{d\theta} \Rightarrow MI.$$

$$T_d = \frac{I^2}{2}\frac{dL_{eq}}{d\theta} \Rightarrow EMMC.$$

$$L_{eq} = L_1 + L_2 + 2M.$$



$$\therefore T_d = \frac{I^2}{2}\frac{d(L_1+L_2+2M)}{d\theta}$$

$$\therefore \boxed{T_d = I^2 \frac{dM}{d\theta}}$$

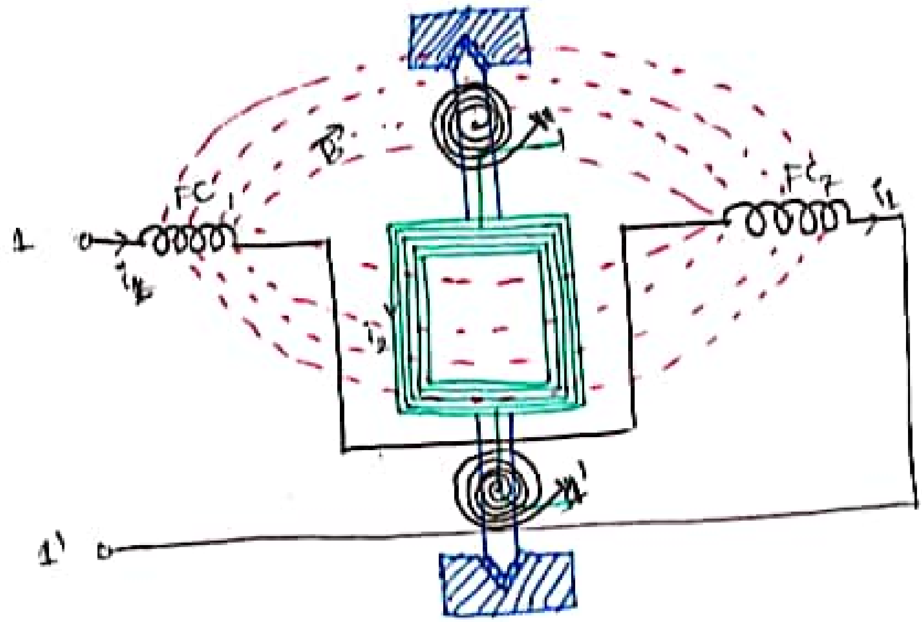
EMMC.

$L_1 = L_{FC} =$ constant

$L_2 = L_{MC} =$ constant

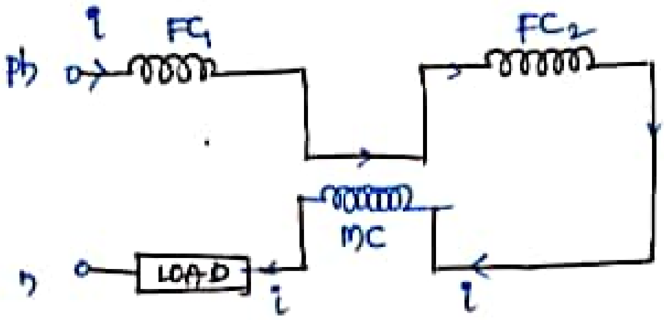
$M =$ variable

If $\frac{dM}{d\theta} =$ constant $\Rightarrow \boxed{T_d \propto I^2}$



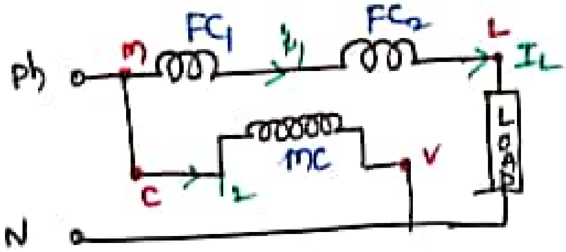
Incase of A, V ⇒ FC & M.C are in series ⇒ $I_1 = I_2 = I$

$i_{FC} = i_{MC} = i$



Incase of W ⇒ F.C & M.C are connected parallel

$i_1 \neq i_2$



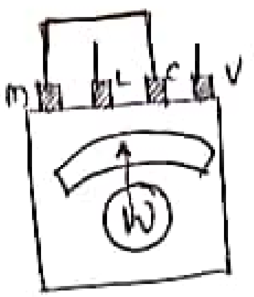
FC = current coil connected in series with Load.

Fixed coil ≡ current coil ≡ Series coil ≡ field coil

$i_L = i_1$ = large ∴ ⇒ thick-wire, weight of Fixed coil more

$i_2 = i_{PC}$ = small ∴ ⇒ thin-wire, weight of moving coil less

(pressure coil ≡ moving coil ≡ voltage coil).



⇒ air-core coil (no saturation & no hysteresis error).

The energy stored in the form of magnetic field....

mechanical workdone $dW = T_d \cdot d\theta$

$$\therefore \frac{dW}{d\theta} = T_d$$

stored energy change $\equiv$ mechanical work done on pointer.

$$\frac{dW}{d\theta} = \frac{dE}{d\theta}$$

$$\boxed{T_d = I_1 I_2 \frac{dm}{d\theta}}$$

$$\therefore \boxed{T_d \propto I_1 I_2} \quad \text{if } \frac{dm}{d\theta} = \text{constant.}$$

$$\boxed{T_d \propto \phi i_2}$$

type of control :- Spring control (dual purpose)

(i) controlling torque

(ii) current carrying element.

$$T_c = K_c \theta.$$

at steady-state ; $T_c = T_d$.

$$K_c \theta = I_1 I_2 \frac{dm}{d\theta}.$$

$$\therefore \boxed{\theta = \frac{I_1 I_2}{K_c} \frac{dm}{d\theta}} \qquad \frac{dm}{d\theta} = \text{constant}$$

$$\boxed{\theta \propto i_1 i_2} \rightarrow \text{EDM } (W).$$

In case of (A) & (V) $\Rightarrow i_1 = i_2 = I$.

$$\therefore \theta = \frac{I^2}{K_c} \frac{dm}{d\theta}$$

$\boxed{\theta \propto I^2} \longrightarrow$ EDW (A) (V).

$\theta \propto I^2 \Rightarrow \therefore$ instrument works for both AC & DC

$\Rightarrow$ non-linear (or) non-uniform scale. (33)

$$i = \frac{V}{Z} \Rightarrow \boxed{\theta = \frac{I^2}{K_c}\frac{dm}{d\theta} = \frac{V^2}{Z^2 K_c}\frac{dm}{d\theta}} \text{ EDM}-(V).$$

$$\therefore \boxed{\theta \propto \frac{V^2}{f^2}}$$

frequency error at < Low voltage / High frequ.

advantages :-

* works for AC & DC
* Hysteresis errors are completely absent.
  b/c air-core coils
* EMMC type instruments are known as transfer instruments

The instruments which are working for both AC as well as DC, the instrument 1st it used for DC then without making any changes if the same instrument is used for AC supply. If it reads/gives the correct instrument (reading), Those instruments are said to be transfer instrume. These transfer instruments are used in the process of calibration of other instruments (AC) disadvantages :-

* Non-uniform scale.
* weak operating field b/c of electromagnets

$B = 0.005\ wb/m^2$ to $0.006\ wb/m^2$

$B = 5m\ wb/m^2$ to $6m\ wb/m^2$

$B_{PMMC} > B_{MI} > B_{EMMC}$

$$\left(\frac{T_d}{\omega}\right)_{PMMC} > \left(\frac{T_d}{\omega}\right)_{MI} > \left(\frac{T_d}{\omega}\right)_{EMMC}$$

$(S)_{PMMC} > (S)_{MI} > (S)_{EMMC}$; sensitivity.

* Low torque to weight ratio.
* Low sensitivity.
* More powe consumption.
* More internal heating problem.
* Temperature error is more
* Frictional error is more.
* stray magnetic field error is more.
* $T_d \propto \frac{V^2}{f^2} \Rightarrow \theta \downarrow \propto \frac{V^2 \downarrow}{f^2 \uparrow}$

This instrument severely suffers from frequency error.

usefull frequency range : (0-125) Hz

* Incase low-grade instruments (accuracy is less around greater than ±5% of full-scale deflection). (31)

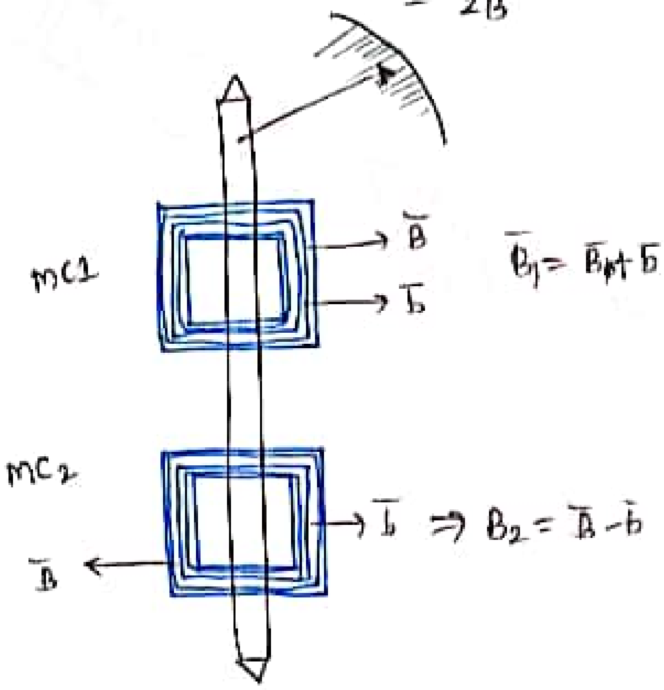
$f = 10\ Hz$ to $1000 Hz$

* Even we can use upto $f = 10 KHz$ by using.

ASTATIC arrangement.

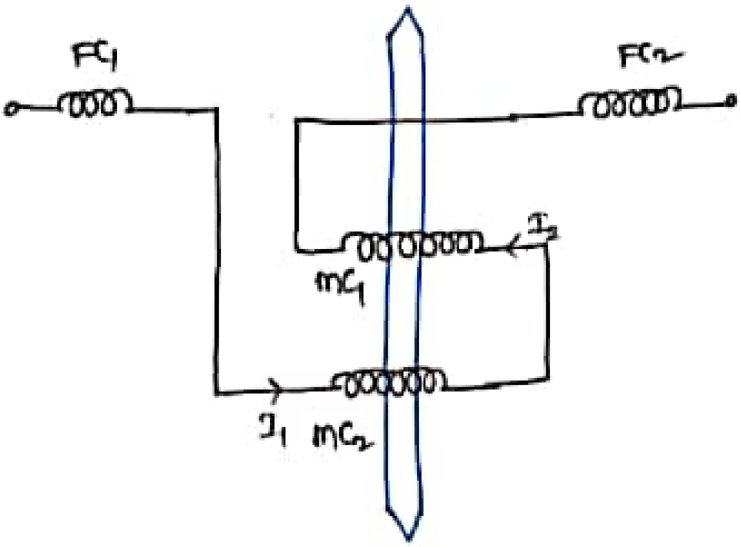
In this, there are two moving coils placed on same spindle, they are connected in such a manner. The flux produced by two coils equal in magnitude and acting opposite in direction. Let $\vec{B}$ is the operating field produced by coil, Let $\vec{b}$ is the stray-magnetic field.

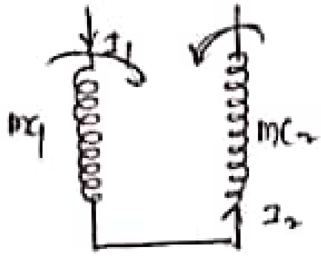
$$\therefore \quad \bar{B}_{net} = \bar{B}_1 + \bar{B}_2$$
$$= \bar{B} + \bar{b} + \bar{B} - \bar{b}$$
$$= 2\bar{B}$$



advantages of astatic arrangement

* $T_d \propto 2\bar{B}$
* Accuracy increased
* Stray magnetic field error is absent.
* Sensitivity increases.
* frequency range of instrument is extended.





(P) The EMMC instrument follows the square law

(a) over a entire operating range of instrument

(b) $\theta = (0-90^\circ)$

(c) $-90^\circ \leq \theta \leq 90^\circ$

(d) $\theta = (-45^\circ \text{ to } 45^\circ)$

$$\theta = \frac{I_1 I_2}{K_c}\frac{dm}{d\theta} = \frac{I^2}{K_c}\frac{dm}{d\theta}$$

$$T_d = I^2 \frac{dm}{d\theta}$$

$M = K\sqrt{L_{fc} L_{mc}}$; $K \rightarrow$ coefficient of coupling.

$K = 1 \Rightarrow$ for perfectly couple.

$K = 0 \Rightarrow$ No coupling (decoupled).

$0 \leq K \leq 1.$

$0 \leq \cos\theta \leq 1.$

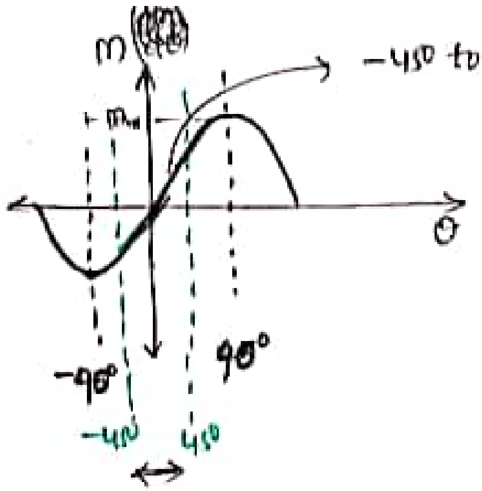
$M = \cos\theta \sqrt{L_1 L_2}$

$M_{max} = \sqrt{L_1 L_2}$

$M_{min} = 0.$

$\therefore \quad M = M_{max}\cos\theta.$

$$\frac{dM}{d\theta} = -M_{max}\sin\theta.$$



$-45°$ to $45° \Rightarrow$ linear $\Rightarrow (m \propto \theta) \Rightarrow \frac{dm}{d\theta} =$ constant.
$(\theta \propto I)$

(35)

$$T_d = I^2(-m_{max}\sin\theta)$$

$$T_d = -I^2 m_{max}\sin\theta$$

Incase of AC supply :- (EMMC)

$i_1 = i_{FC}$

$\phi = \alpha$

$i_2 = i_{MC}$

Let $i_1 = I_{m1}\sin\omega t$

$i_2 = I_{m2}\sin(\omega t - \phi)$

$\omega = 2\pi f = \frac{2\pi}{T}$

i.e. i_2 lags i_1 by $\phi = \alpha$.

$$T_{avg} = \frac{1}{T}\int_0^T \left(i_1 i_2 \frac{dm}{d\theta}\right) dt$$

$$= \frac{1}{T}\int_0^T I_{m1} I_{m2} \sin\omega t \cdot \sin(\omega t - \phi)\, dt$$

$$= \frac{I_{m1} I_{m2}}{2T}\int_0^T 2\sin\omega t \cdot \sin(\omega t - \phi)\, dt.$$

$$= \frac{I_{m1} I_{m2}}{2T}\left[\int_0^T \left[\cos\phi \cdot \cancel{dt} - \cos(2\omega t - \phi)\right] dt\right]$$

$$= \frac{I_{m1} I_{m2}}{2T}\left(\int_0^T \cos\phi\, dt - \int_0^T \cos(2\omega t - \phi)\, dt\right)$$

$$= \frac{I_{m1} I_{m2}}{2T}\left[(\cos\phi)T - \left[\frac{\sin(2\omega t - \phi)}{2\omega}\right]_0^T\right]$$

Scanned with CamScanner

$$\therefore \quad T_{avg} = \frac{I_{m1} I_{m2}}{2} \cos\phi$$

$$T_{avg} = (I_{rms1})(I_{rms2}) \cos\phi$$

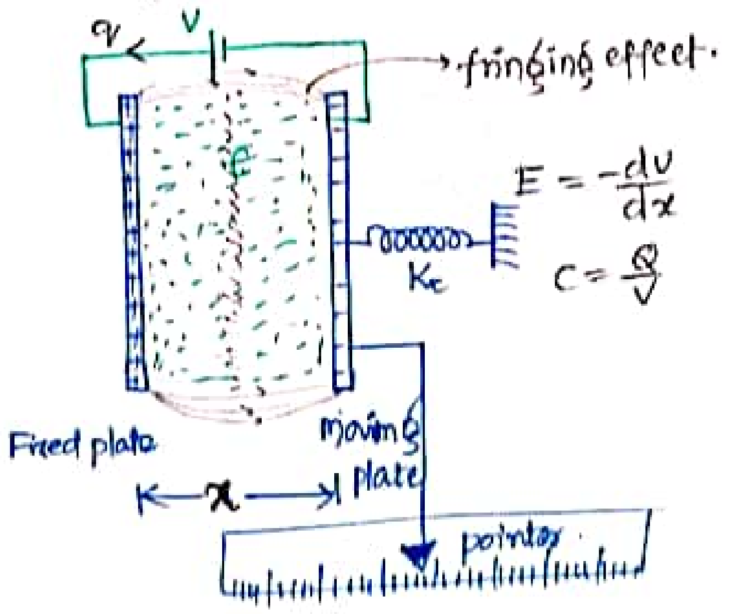
$$\boxed{(\text{Torque})_{avg} = (I_{rms1})(I_{rms2}) \cos(\phi - \alpha)}$$ EDM type meter.

Incase of AC supply, the average torque is calculated in terms of RMS value of fixed coil current and moving coil current.

Electrostatic Voltmeter :-

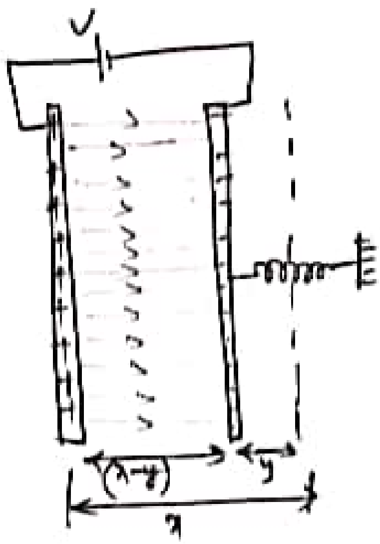
It works on the principle of change in capacitance, It reads only voltage b/c basically it is voltmeter.

(mobility of +ve charge) < (mobility of −ve charge)



Linear-moment of pointer on Non-linear scale.

$F = Eq$



$C = \frac{A\epsilon}{x}$ $\quad (\because C = \frac{A\epsilon}{d})$.

$C' = \frac{A\epsilon}{(x-y)}$

Pressure ↑ ⇒ Force ↑ ⇒ capacitance ↑
(or)
stress

$C' = C + \Delta C$

$V' = V + \Delta V$

∴ it is called aslo.. as...

Capacitive transeducer.

$Q = CV$

$dQ = C dV + (dC) V$

$V dQ = (CV) dV + V^2 (dC)$

⇒ No current flow through medes only displacement current not convensional current ∴ ESV ammeter won't exists.

Energy supplied $= CV dV + V^2 dC$

Energy ~~supplied~~ stored $= \frac{1}{2} CV^2$.

incremental energy stored $= \frac{1}{2} (C+dC)(V+dV)^2$

change in energy stored $= \frac{1}{2} (C+dC)(V+dV)^2 - \frac{1}{2} CV^2$

$= \frac{1}{2} (C+dC)(V^2+dV^2+2VdV) - \frac{1}{2} CV^2$

$= \frac{1}{2} CV^2 + \frac{1}{2} C dV^2 + CVdV + \frac{1}{2} dC V^2 + \frac{1}{2} dC dV^2 + \frac{1}{2} dC(2VdV) - \frac{1}{2} CV^2$

$= \left(\frac{1}{2} C dV^2 + \frac{1}{2} dC dV^2 + VdC dV\right) + CVdV + \frac{1}{2} V^2 dC$

$\cong CVdV + \frac{1}{2} V^2 dC$.

Energy supplied = Change in energy stored + Mechanical energy stored in spring. (work done)

$$\left(V^2 dC + CV\,dV\right) = \left(\frac{1}{2}V^2 dC + CV\,dV\right) + (\text{Force}) \times (\text{displacement})$$

$$\frac{1}{2}V^2 dC = F_d\,(dx) = Fy$$

$$F_d = \frac{1}{2}V^2\left(\frac{dC}{dx}\right)$$

$$\therefore \boxed{F_d = \frac{V^2}{2}\left(\frac{dC}{dx}\right)}$$

voltage responsible meter.

both AC & DC.

If $\frac{dC}{dx} = \text{constant}$; $F_d \propto V^2$

Type of control :- Helical spring.

$$\boxed{F_c = Kx}$$

K = spring constant.

At steady-state ; $F_d = F_c$

$$\frac{V^2}{2}\frac{dC}{dx} = Kx$$

$$\therefore \boxed{x = \frac{V^2}{2K}\frac{dC}{dx}}$$

$\frac{dC}{dx} = \text{constant}$, $\boxed{x \propto V^2}$ ⇒ scale is non-linear

⇒ used to measure both AC, DC.

scale is non-linear, Motion of the pointer is linear.

In a parallel plate electrostatic voltmeter, the motion of the pointer is linear but the scale is non-linear. (37)

advantages :-

* $F_d \propto V^2$
  ∴ both ac & dc can measure.
* No magnetic field ⇒ No hysteresis error
* No stray magnetic field error.
* No current flow, only ~~char~~ static charge,
  ∴ So that No power loss. No internal heating problem.
* So that there is no temperature error.
* It draws practically zero current from the supply.
* The waveform & frequency errors are un-important.

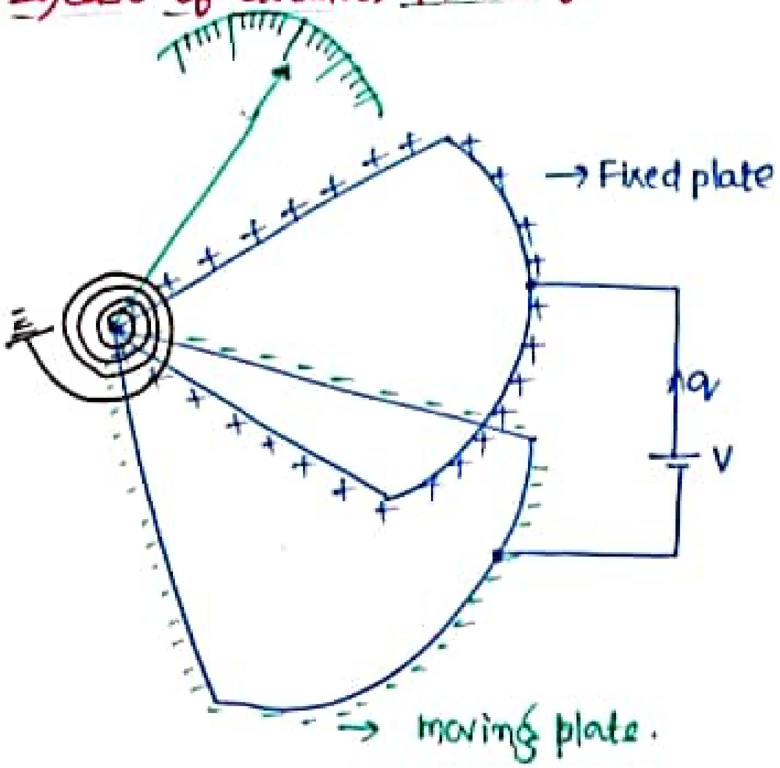
disadvantages :-

* Non uniform scale
* ($F_d \propto V^2$) only not for measurement of current.
* Only for high voltage measurements i.e. of the order (kV).
* when we are measuring low voltages, the produced deflecting force is very small, in the order of μNewtons.

$$F_d \propto (mV)^2 \Rightarrow \mu N \Rightarrow \text{no deflection}$$
$$F_d \propto (kV)^2 \Rightarrow MN$$

∴ this is not suitable for low voltage measurement but best suitable for high voltages like in power systems

Incase of circular plates :- (plates are rotating)



Mechanical work done $= dW = T_d \cdot d\theta$.

According to law of conservation of energy :

Energy supplied = change in energy + Mech. work done.

$$CV\,dV + (dC)V^2 = CV\,dV + \frac{1}{2}V^2 dC + (T_d\, d\theta)$$

$$\frac{1}{2}V^2 dC = T_d\, d\theta.$$

$$\therefore \boxed{T_d = \frac{V^2}{2}\left(\frac{dC}{d\theta}\right)}$$

A = circular plate areas

If $\frac{dC}{d\theta}$ = constant → $\boxed{T_d \propto V^2}$ ⇒ AC & DC.

Type of control :- Spring contro (spiral spring) $T_c = K_c\theta$.

at steady state $T_c = T_d$

$$K_c\theta = \frac{V^2}{2}\frac{dC}{d\theta}$$

$$\boxed{\theta = \frac{V^2}{2K_c}\frac{dC}{d\theta}}$$

Types of ESV :–

1. Repulsion type ⇒ ($< 10 KV$) ⇒ (Gold Leaf ESV)
2. Attractive type ⇒ ($> 20 KV$) → (Kelvin-absolute electrometer)
3. Attractive & repulsion type ⇒ (Symmetrical type ESV) ($10 KV - 20 KV$)

$$F_d = \frac{V^2}{2}\left(\frac{dc}{dx}\right) = \frac{V^2}{2}\left(\frac{c}{x}\right) \qquad ; c = \frac{A\varepsilon}{x} = \frac{A\varepsilon_0\varepsilon_r}{x}$$

$$= \frac{V^2}{2}\,\frac{A\varepsilon/x}{x}$$

$$F_d = \frac{V^2}{2}\,\frac{A\varepsilon}{x^2} \Rightarrow F_d = \frac{V^2}{2x^2}(A\varepsilon)$$

$$F_d = \left(\frac{A\varepsilon}{2}\right)\left(\frac{V^2}{x^2}\right)$$

$$V = \sqrt{\frac{2F_d x^2}{A\varepsilon}}$$

$$\boxed{V = x\sqrt{\frac{2F_d}{A\varepsilon}}}$$ Kelvin-absolute electrometer.

If any instruments gives the reading in terms of the physical parameters (or) constents of instrument... then it is called Absolute instrument.

Absolute .. { relay current ~~meter~~ balance meter – current
tangent galvanometer
lorentz force meter – resistance

Absolute instruments are used in the process of calibration of other instruments.

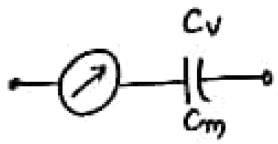
$F_d = \frac{V^2}{2}\frac{dC}{dx}$ (i) Kelvin - absolute electrometer. $V = x\sqrt{\frac{F_d(2)}{A\varepsilon}}$

(ii) rayleigh current balance meter $F_d = I^2 \cdot \frac{dm}{dx}$

(iii) Loretz force meter

(iv) Tangent galvonometer.

Extension Range of ESV :-

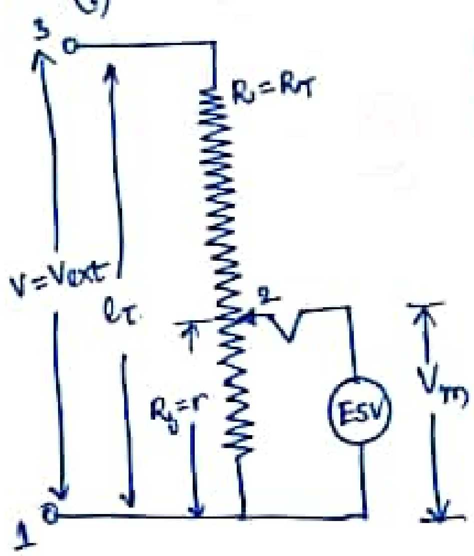


(i) It can be obtained by using potentiometer

(ii) By using external capacitor (or) series capacitor

(iii) Multiple series capacitors

(i)



{Length Ratio} = {Voltage Ratio} = {Resistance Ratio}

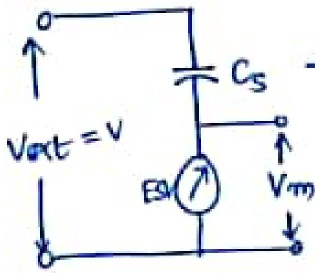
$$\frac{x}{l_t} = \frac{R_o}{R_t} = \frac{V_m}{V_{ext}}$$

$$\frac{x}{l_t} = \frac{r}{R} = \frac{V_m}{V} = \frac{1}{m}$$

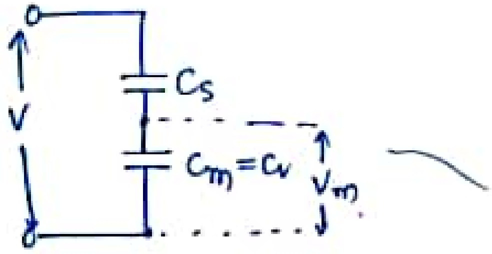
m = multiplication factor $= \frac{V_{ext}}{V_m} = \frac{V}{V_m}$

$\therefore\ m = \frac{R}{r}$ $\quad\therefore \Rightarrow\ V_{ext} = V = \left(\frac{R}{r}\right) V_m$.

(ii)



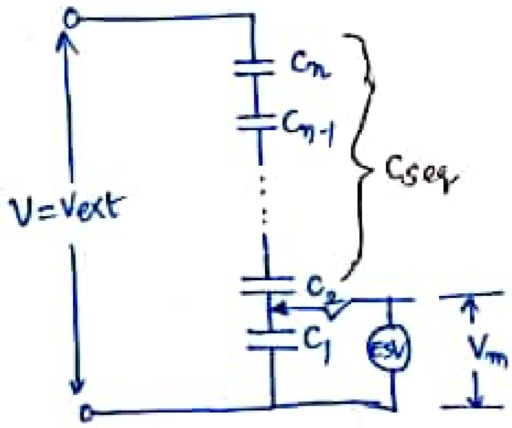
→ external capacitor



$$V_m = \left(\frac{C_S}{C_S + C_m}\right) V_{ext} \Rightarrow m = \frac{V_{ext}}{V_m} = 1 + \frac{C_m}{C_S} = \text{multiplication factor.}$$

$$\therefore \quad \boxed{C_S = \frac{C_m}{(m-1)}}$$

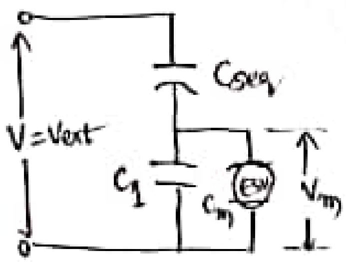
(iii) Multiple Series Capacitors



Let C_{seq} is the equivalent capacitance of series capacitor except the capacitance across ESV meter.

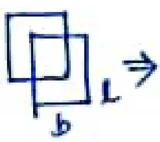
$$\frac{1}{C_{seq}} = \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}$$

$\Rightarrow$



$$\therefore \quad V_m = \left(\frac{C_{seq}}{C_{seq} + C_1 + C_m}\right) V_{ext}$$

$$\therefore m = \frac{V}{V_m} = 1 + \left(\frac{C_m + C_1}{C_{seq}}\right)$$



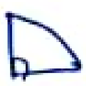
$\Rightarrow A = l \times b.$

sector

$\Rightarrow A = \frac{1}{2} r^2(\theta) \Rightarrow \frac{1}{2} r^2 (2\pi) = \pi r^2$

Full circle.

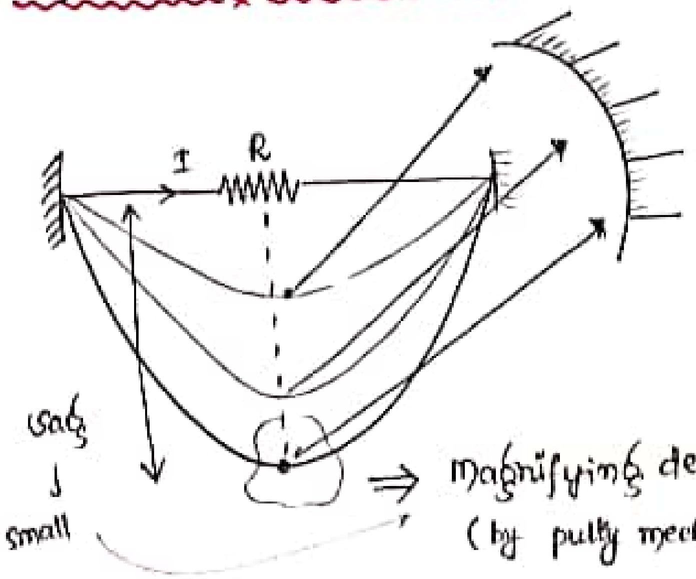
$\Rightarrow A = \frac{\pi r^2}{2}$



$A = \frac{\pi r^2}{4}$

which one of the following instrument works on the principle of heating effect ..... Thermal instrument.

Thermal Instrument :- (RF instruments) ; Radio frequency



$\Rightarrow$ magnifying devices (by pully mechanism).

$l_t = l_0(1+\alpha\Delta t)$

$l_t - l_0 = l_0 \alpha \Delta t$

$\Delta l \uparrow = l \alpha \Delta t \uparrow$

$\theta \propto (\text{sag}) \propto (\Delta l) \propto (\Delta t) \propto (\text{heat produced}) \propto (I^2 R)$.

Let $\Rightarrow$ $l \rightarrow$ length of wire in 'm'

$R \rightarrow$ resistance of wire in Ω

$I \rightarrow$ current flow.

$H \rightarrow$ Rate of heat generation in "wall"

$S_a \rightarrow$ Heat dissipating surface (area) - m^2

$\Delta t \rightarrow$ temperature rise in °C

$U \rightarrow$ coefficient of heat dissipation in watt/m²°c.

$\alpha \rightarrow$ temperature coefficient

$$\therefore U = \frac{H}{S_a(\Delta t)} \Rightarrow \Delta t = \frac{H}{U S_a} = \frac{\Delta l}{l\alpha}$$

$$\therefore \Delta l = \frac{l\alpha H}{U S_a}$$

$\boxed{\theta \propto [\Delta l\,(sag)] \propto I^2}$ ⇒ working for both AC & DC

it reads always RMS value.

⇒ scale is non-uniform.

$$\Delta l = \frac{l\alpha}{US_a}(I^2R) \Rightarrow \boxed{\Delta l = \frac{l\alpha}{US_a}\left(\frac{V^2}{R}\right)}$$ Hotwire voltmeter.

Advantages :-

* Works for both AC & DC
* No magnetic field ∴ there is no stray-magnetic field error
* No Hysteresis error.
* No reactance term in the principle, No frequency error.

  ∴ Thermal instruments can be used in a frequency range of several mega Hz. i.e. upto radio frequency range.

  So that thermal instruments popularly known as RF instruments

Disadvantages :-

* Non-uniform scale.
* Difficult construction.
* The ambient temperature will affect the actual readings of the instruments. So that the instrument accuracy is lesser.
* Thermal instruments are slow (sluggish) in response because the time taken by the wire to get heat up.
* Skin effect is more pronounced when we are used at high frequency.

$$\text{Skin effect} \propto \left(\frac{d^2 f\,\mu_r}{\rho}\right)$$

$$\therefore (\text{Skin effect}) \propto f$$

* To reduce the skin effect in this instruments tubular (or) hollow conductors are preffered $I > 3$ Amp.
* Thermal instruments has limited over-loading capability around 150%. otherwise wire gets damaged (breaks).

In produced sag is given by $\boxed{Sag = s = \sqrt{\frac{1}{2} l \Delta l}}$

Magnification of sag :-

It is defined as the ratio of produced sag to the change in length.

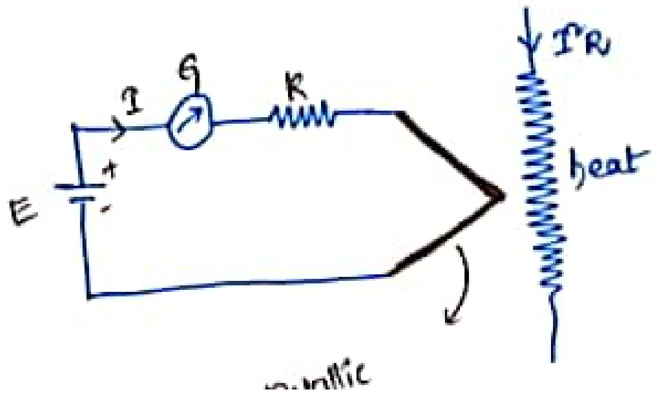
$$m = \frac{sag}{\Delta l} = \frac{\sqrt{\frac{1}{2} l \Delta l}}{\Delta l} = \sqrt{\frac{l}{2(\Delta l)}}$$

$$\boxed{m = \sqrt{\frac{l}{2(\Delta l)}}}$$

Resistance Temperature Detector :-

RTD is a temperature measuring device, which works on the principle of change in resistance of a metallic conductor due to heat produced by heater element, which has positive temperature coefficient. (PTC)

by
(i) conduction — contact
(ii) convection
(iii) Radiation. } non-contact type.



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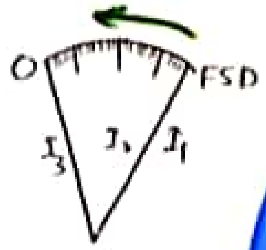
$$I_1 = \frac{E}{R} \qquad I_2 = \frac{E}{R'} = \frac{E}{(R+\Delta R)}$$ (4.1)

$$R_t = R_0(1+\alpha\,\Delta t)$$

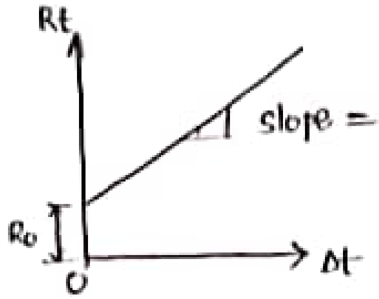
$$\therefore\ I_2 < I_1 .$$

$$\Delta R = (R_t - R_0) = R_0\alpha\,\Delta t .$$

$$I_3 < I_2 < I_1$$



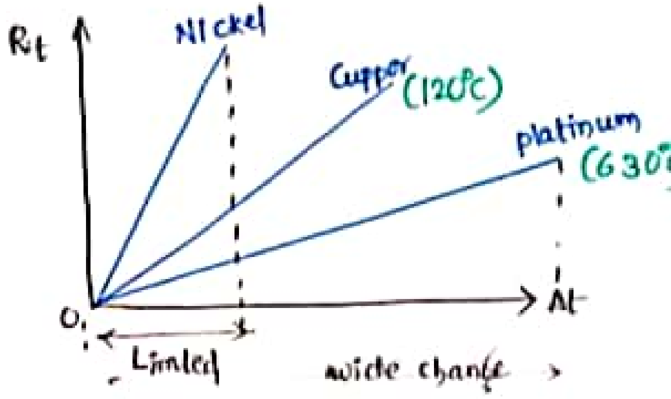
$\therefore$ Galvanometer should have $\boxed{\text{reverse scale}}$



$$\text{slope} = R_0\alpha\,\Delta t = \frac{\Delta R}{\Delta t}$$

(RTD) Most commonly used material for RTD ⇒ platinum.

platinum is preferable b/c it is highly stable at higher temperature. It can be used upto 630°C.



Cuppor is used for moderate temperature range only. Around (120°)

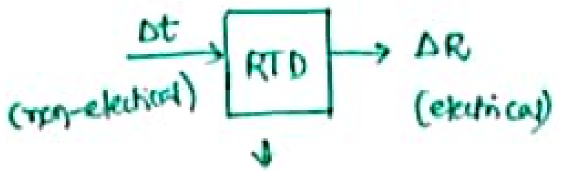
Silver & Gold are commonly used materials b/c of low resistivity (Ag & Au)

In general, the temperature range of RTD we can use upto 183°C

Sensitivity → $(S_{Ni} > S_{Cu} > S_{platinum})$.

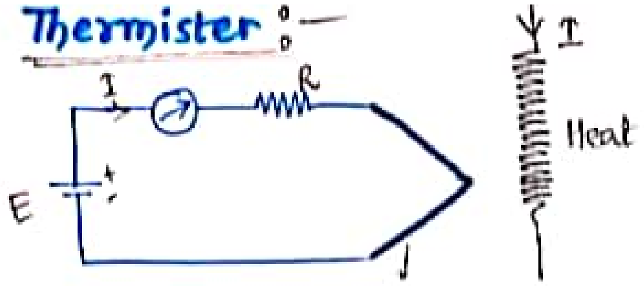
$$\Rightarrow S_{RTD} = \frac{\Delta R}{\Delta t}$$

$$\text{Sensitivity} \propto \frac{1}{\text{temp ra}}$$



is called transducer

Thermister :-



Semiconductor

$\alpha = NTC$
$= -ve$

$$R_t = R_0(1 - \alpha \Delta t).$$

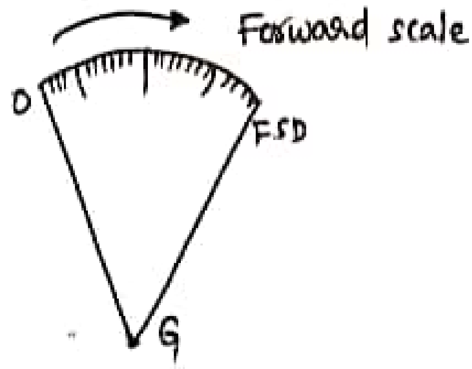
$$\therefore \Delta R_t = R_t - R_0$$

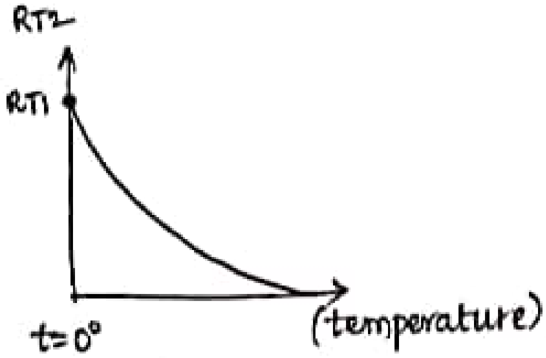
(Temp) ↑ ⇒ (Resistance) ↓

It decreases exponentially not linear

(-ve temp. coeff is ~~re~~placed inplace of PTC to over-come reverse -scale in RTD... but it is now called as Thermister)

$$R_{T2} = R_{T1} \cdot e^{-\beta\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$$





$t = \infty°c$ (All semiconductors will behave like conductors (or) Supercond)

$\beta \rightarrow$ coefficient of temperature. In °Kelvin.

$T_1, T_2 \rightarrow$ kelvin temperature.

$$\beta = (3000°K \text{ to } 4000°K).$$

Thermister has more sensitivity than RTD.

Temp. range ⇒ (−55°c to 15°c)
(or)

Semiconductor material → Sintered mixture of metallic oxides. (12)
(Fe, Co, Ni + oxides)

platinum, Iridium ⇒ zero Temp. coeff.

{d-block, semiconductor, Electrolytic solutions} → −ve temp. coeffient.

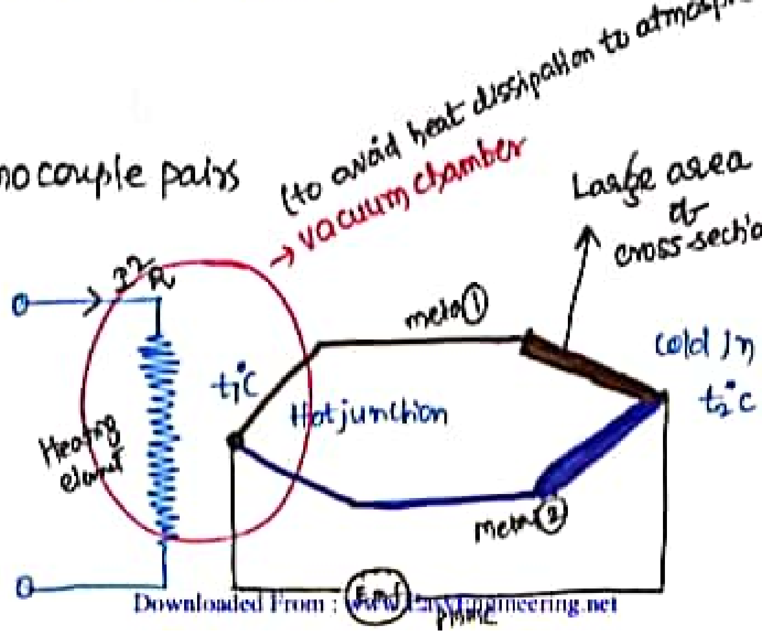
All metals .... ⇒ positive temp. coeff.

working principle → Thermister is a temperature measuring device which works on the principle of change in resistance of a me Semiconducting material due to heat produced by heater eleme which has negative temp. coeff.

Thermo-couple :-

It works on the principle of Seebeck effect. There are two dissimilar metals are maintained at two different tempera-tures namely hot junction and cold junction, there is some electro flow is observed from hot junction to cold junction is known as Seebeck effect. The produced electron flow is always unidirectio It can be measured by PMMC (∵ DC flow).

Cu-Con Cupper – Constonton
Fe-Con Iron – Constanton
Chromel – Alumel
} Thermo couple pairs



$\therefore$ produced emf = $\boxed{e = a\cdot(\Delta t) + b\,(\Delta t)^2}$

where, $\Delta t \rightarrow$ rise in temperature (or) temperature difference b/w junctions.

$a \rightarrow$ constante (or) V per °C

$$a = \frac{emf}{(\Delta t)} = \text{sensitivity of thermocouple.}$$

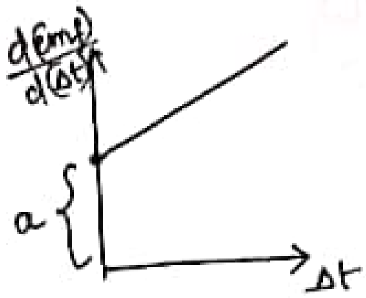
$$\boxed{a \Rightarrow 40\ \mu V/°c \text{ to } 50\ \mu V/°c}$$

$b \rightarrow$ constant (or) V per °c²

$$\boxed{b \Rightarrow (1\ \mu V/°c^2 \text{ to } 2\ \mu V/°c^2)}$$

Quadratic non-linearity ⇒ Thermocouple

exponential " ⇒ Thermister.



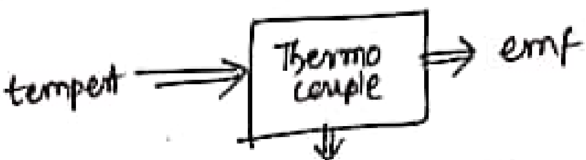
$$\frac{d(emf)}{d(\Delta t)} = a + 2b(\Delta t)$$

rate of production of electron flow (or) emf change due to temperature change

slope = 2b.

$$\boxed{emf \propto (\Delta t) \propto (heat) \propto (I^2R)}$$

$\therefore$



active transeducer

RTD, Thermister ⇒ passive transducer (source)

Thermocouple ⇒ Active transeducer (no source,

Note

→ The produced emf in thermocouple is in DC nature, it is in the order of several milli-volts, The produced emf is detected by PMMC but the scale is calibrated to read RMS value of AC current flowing through the heater element.

→ RTD, thermisters are passive transeducers, where as thermocouple is an active transeducer b/c It doesn't requir any external source.

Temperature range of Thermocouple upto 1100°C. Even we can use upto 2000°.

Q:- Arrange the above Instruments in the order of decreasing of sensitivity

1. RTD
2. Thermister
3. Thermocouple.

$$S_{Thermister} > S_{RTD} > S_{Thermocouple}$$

$\Downarrow$ (−55° to 150) $\quad\Downarrow$ 1803° $\quad\Downarrow$ 1100°C

Non-linearity ⇒ Thermister > Thermocouple > RTD

Note
Inspite of low accuracy and difficult construction, thermal instruments are popularly used for the measurement of current & voltages at low voltage & high frequency communication ckt and in wire-less networks

Accuracy Order :-

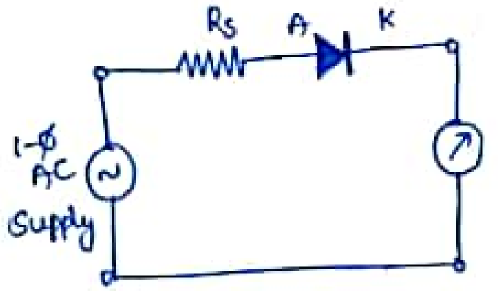
(PMMC) > (Rectifier) > (ESV) > (EMMC) > (MI) > (Thermal instrum) > (Induction)
⇓ Highest ⇓ Least.

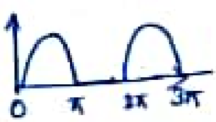
Rectifier Type of Instruments :-

Cu_2O, Mercury Arc rectifiers ⇒ Olden days
Diodes, SCRs .... ⇒ Modern days

- HWR
- FWR

1. Half Wave Rectifier :-





$$V_{AC} = V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$$V_{DC} = V_{avg} = \frac{V_m}{\pi} = \frac{\sqrt{2}\, V_{RMS}}{\pi}$$

$$\frac{V_{DC}}{V_{AC}} = \frac{V_{avg}}{V_{RMS}} = \frac{\sqrt{2}}{\pi} = 0.45$$

$$I_{AC} = \frac{V_{AC}}{(R_S + R_m)}$$

$$V_{DC} = 0.45\, V_{AC}$$

$$I_{DC} = 0.45\, I_{AC}$$

Scanned with CamScanner

(11)

$$\frac{1}{(I_{AC\,FSD})} = 0.45 \times \frac{1}{(I_{DC\,FSD})}$$

$$S_{AC(HWR)} = 0.45\,(S_{DC(HWR)})$$

$$(R_s + R_m) = 0.45 \times V_{AC} \times \frac{1}{I_{DC}} = 0.45\ V_{AC} \cdot S_{DC}$$

$$R_s = (0.45\ S_{DC})\,V_{AC} - R_m$$

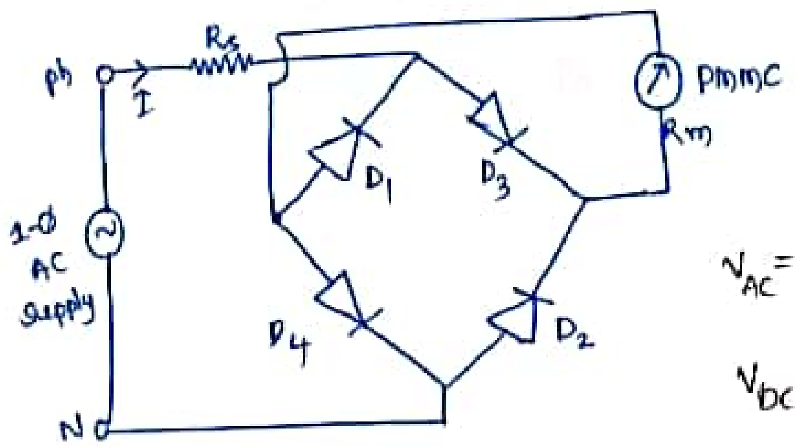
$$R_s = S_{AC} \cdot V_{AC} - R_m \Rightarrow \text{if } R_D = 0$$

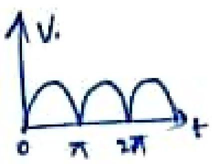
if $R_D \neq 0$ $\therefore$ $\boxed{R_s = S_{AC}\,V_{AC} - R_m - R_D}$

$$FF = K_f = \frac{V_m/\sqrt{2}}{V_m/\pi} = \frac{\pi}{\sqrt{2}} = 2.22$$

$\boxed{(\text{Reading of HWR type instrument}) = (2.22)(\text{PMMC meter reading})}$

Full wave Rectifier :-





$$V_{AC} = V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$$V_{DC} = V_{avg} = \frac{2V_m}{\pi}$$

$$V_{DC} = \frac{2}{\pi}(\sqrt{2}\ V_{RMS}) = 0.9\ V_{DC}$$

$$I_{AC} = \frac{V_{AC}}{R_s + R_m} \quad ; \quad I_{DC} = 0.9\ I_{AC}$$

$S_{AC} = 0.9\ S_{DC}$

$$(R_s + R_m) = (0.9)\, V_{AC}\, S_{DC} = S_{AC} V_{AC}$$

$$\boxed{R_s = S_{AC} V_{AC} - R_m} \Rightarrow \text{ideal diode } R_D = 0$$

$$\boxed{R_s = S_{DC} V_{DC} - R_m - 2R_D} \Rightarrow \text{practical identical diodes } R_D \neq 0.$$

$$FF = \frac{V_{rms}}{V_{avg}} = \frac{V_{AC}}{V_{DC}} = \frac{2.22}{2} = 1.11$$

$\therefore$ $\boxed{(\text{Reading of FWR reading}) = (1.11)(\text{pmmc reading})}$

Sensitivity of FWR = 2 (Sensity of HWR)
DC (or) AC

$$S_{AC\,FWR} = 2.S_{DC\,HWR}$$

advantages :—

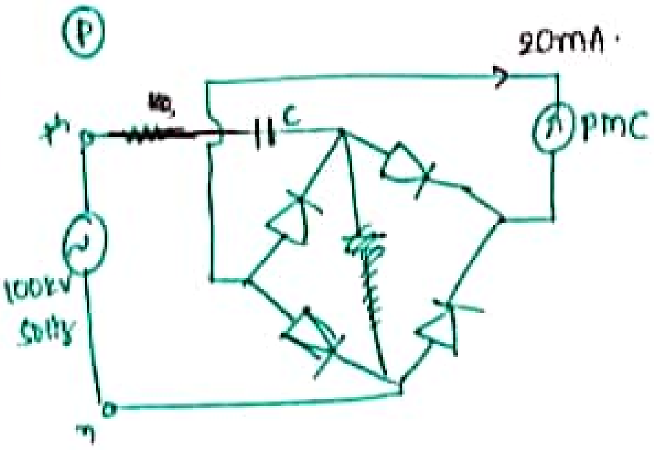
* Uniform scale
* working for both AC & DC
* Low power consumption
* High sensitivity in the order of several thousands Ω/V (1000 Ω/V to 2000 Ω/V)
* No frequency error, It can be used over a wide frequency range in the order of several KHz (or) MHz range.

Disadvantages :-

* It needs separate calibration for each input waveform.
* It has different sensitivity like AC aswell as DC sensitivity
* It consists of a diode which is a semi-conductor device,

* The diode will offer some capacitance known as stray capacit-ance of the diode. Due to this capacitance the instrument readings are affected. (4.5)

(P)



$C = ?$

$$I_{DC} = (0.9)\, I_{AC}$$

$$= 0.9 \times \frac{V_{AC}}{(R_s + R_m)}$$

$$R_s = \frac{1}{j\omega C} = X_C$$

$V_{AC} = V_{rms} = 100\,KV.$

$$I_{DC} = 0.9 \times \frac{V_{AC}}{(X_C + R_m)}$$

not mention; $R_m = 0$

$$\therefore I_{DC} = 0.9 \times \frac{V_{AC}}{X_C}$$

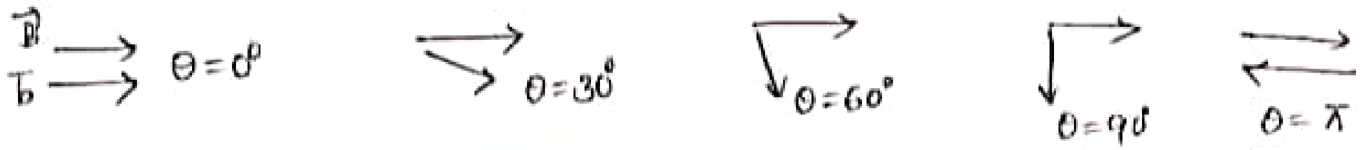
$$20\,m = \left| j\omega C \times 0.9 \times 100 \times 10^3 \right|$$

$$\therefore C = \frac{20 \times 10^{-6}}{100\pi \times 0.9 \times 100} = \frac{2}{9\pi} \times 10^{-8} = 7.07 \times 10^{-10}$$

$$C = 707\ pF.$$

Let B = operating field.
b = stray field.
$\theta(\vec{b}, \vec{B})$.

$$B_{Resultant} = \sqrt{B_1^2 + B_2^2 + 2B_1B_2\cos\theta}$$



$B_R = \sqrt{B_1^2 + b^2 + 2Bb}$ (parallel aiding)

$B_R = \sqrt{B_1^2 + b^2 + \sqrt{3}Bb}$

$B_R = \sqrt{B_1^2 + b^2 + Bb}$

$B_R = \sqrt{B_1^2 + b^2}$

$B_R = \sqrt{B_1^2 + b^2 - 2Bb}$ (parallel