10.7. FLOW OF VISCOUS FLUID BETWEEN TWO PARALLEL PLATES

10.7.1. One Plate Moving and Other at Rest—Couette Flow

Let us consider laminar flow between two parallel flat plates located at a distance b apart such that the lower plate is at rest and the upper plate moves uniformly with a constant velocity U as shown in Fig. 10.20. A small rectangular element of fluid of length dx, thickness dy and unit width is considered as a free body (see Fig. 10.20). The forces acting on the fluid element are:

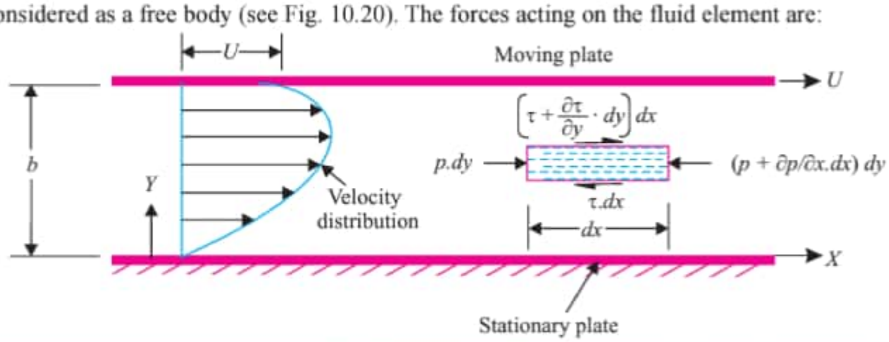

Fig. 10.20. Couette flow.

1. The pressure force, $p.dy \times 1$ on the left end,
2. The pressure force, $\left(p + \frac{\partial p}{\partial x} \cdot dx\right) dy \times 1$ on the right end,
3. The shear force, $\tau.dx \times 1$ on the lower surface, and
4. The shear force, $\left(\tau + \frac{\partial \tau}{\partial y} \cdot dy\right) dx \times 1$ on the upper surface.

For steady and uniform flow, there is no acceleration and hence the resultant force in the direction of flow is *zero*.

$$\therefore \quad p.dy - \left(p + \frac{\partial p}{\partial x} \cdot dx\right) dy - \tau\, dx + \left(\tau + \frac{\partial \tau}{\partial y} \cdot dy\right) dx = 0$$

or,
$$-\frac{\partial p}{\partial x} \cdot dx \cdot dy + \frac{\partial \tau}{\partial y} \cdot dy \cdot dx = 0$$

Dividing by the volume of the element $dx.dy$, we get:

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y} \qquad ...(10.19)$$

Eqn. (10.19) shows the interdependence of shear and pressure gradients and is *applicable for laminar as well as turbulent flow*. Accordingly the pressure gradient, in the direction of flow, is *equal* to the shear gradient across the flow.

According to Newton's law of viscosity for laminar flow the shear stress, $\tau = \mu \cdot \frac{du}{dy}$. Substituting for τ in eqn. (10.19), we get:

$$\frac{\partial p}{\partial x} = \mu \cdot \frac{\partial^2 u}{\partial y^2}$$

Since $\frac{\partial p}{\partial x}$ is independent of y, integrating the above equation twice w.r.t. y gives:

$$u = \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x}\, y^2 + C_1 y + C_2 \qquad ...(10.20)$$

where, C_1 and C_2 are the constants of integration to be evaluated from the known boundary conditions. In the present case the boundary conditions are:

At $\quad y = 0,\ u = 0$, and at $\ y = b,\ u = U$

$$\therefore \quad C_2 = 0, \quad \text{and} \quad C_1 = \frac{U}{b} - \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) b$$

Hence, substituting the values of C_1 and C_2 in eqn. (10.20), it yields the following equation for the *velocity distribution* for generalised *Couette flow:*

$$u = \frac{U}{b}\, y - \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x}\, (by - y^2) \qquad ...(10.21)$$

The eqn. (10.21) indicates that the velocity distribution in Couette flow depends on both U and $\left(\frac{\partial p}{\partial x}\right)$. However, the pressure gradient $\left(\frac{\partial p}{\partial x}\right)$ in this case may be either positive or negative. In a particular case when $\left(\frac{\partial p}{\partial x}\right)$ equals *zero*, there is no pressure gradient in the direction of flow, then, we have $u = U \cdot \frac{y}{b}$ which indicates that the velocity distribution is *linear*. This particular case is known as *simple (or plain) Couette flow or simple shear flow*.

The discharge per unit width (q) may be obtained as follows:

$$q = \int_0^b u \cdot dy = \int_0^b \left[\frac{U}{b} y - \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} (by - y^2)\right] dy$$

$$= U \cdot \frac{b}{2} - \frac{b^3}{12\mu} \cdot \frac{\partial p}{\partial x} \quad ...(10.22)$$

The *distribution of shear stress* across any section may be determined by using Newton's law of viscosity. Thus,

$$\tau = \mu \cdot \frac{\partial u}{\partial y} = \mu \left[\frac{U}{b} - \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} (b - 2y)\right]$$

$$= \mu \cdot \frac{U}{b} - \frac{1}{2} \cdot \frac{\partial p}{\partial x} (b - 2y) \quad ...(10.23)$$

The type of flow discussed above (*i.e.* flow of viscous fluid between two plates-one *stationary* and the other *moving*) is known as **generalised Couette flow.**

10.7.2. Both Plates at Rest

In this case the equations for velocity, discharge q and the shear stress can be obtained from similar equations for generalised Couette flow by putting $U = 0$. Thus for flow between two stationary parallel plates, shown in Fig. 10.21, we have:

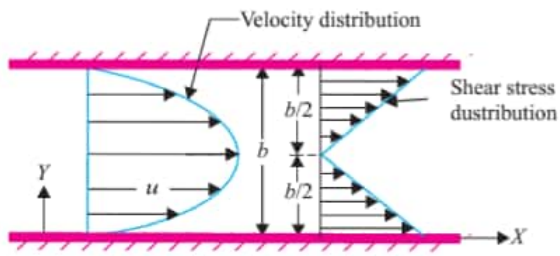

Fig. 10.21. Flow between stationary plates.

Velocity,

$$u = -\frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} (by - y^2) \quad ...(10.24)$$

[Eqn. (10.24) represents the plane *Poiseuille flow*]

Discharge per unit width,

$$q = -\frac{b^3}{12\,\mu} \cdot \frac{\partial p}{\partial x} \quad ...(10.25)$$

$$\text{Shear stress, } \tau = -\frac{1}{2} \cdot \frac{\partial p}{\partial x} (b - 2y) \quad ...(10.26)$$

10.7.3. Both Plates Moving in Opposite Directions

For flow between parallel plates, the velocity distribution is given by:

$$u = \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} \cdot y^2 + C_1 y + C_2 \quad ...\text{Eqn. } (10.20)$$

In the present case the boundary conditions are:

At, $y = 0,\ u = -V$, and

At, $y = b,\ u = U$

Substituting these boundary conditions in eqn. (10.20), we get:

$$-V = C_2 \quad i.e.\ C_2 = -V$$

and,

$$U = \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x}\, b^2 + C_1 b - V$$

or, $$U + V = \frac{1}{2u} \cdot \frac{\partial p}{\partial x} b^2 + C_1 b$$

$\therefore$ $$C_1 = (U + V)\frac{1}{b} - \frac{1}{2\mu}\frac{\partial p}{\partial x} \cdot b$$

Hence the eqn. (10.20) becomes:

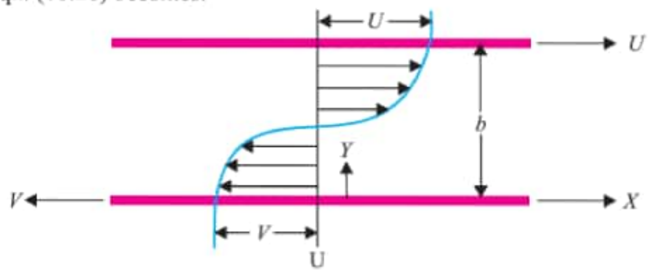

Fig. 10.22. Flow between parallel horizontal plates, both the plates moving in opposite directions.

$$u = \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} \cdot y^2 + \left[(U + V)\frac{1}{b} - \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} \cdot b\right] y - V$$

$\therefore$ $$u = (U + V)\frac{y}{b} - \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x}(by - y^2) - V \quad \text{...(10.27)}$$

The distance y at which the velocity u is zero may be determined as follows:

$$(U + V)\frac{y}{b} - \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x}(by - y^2) - V = 0$$

Rearranging the above equation, we have:

$$\frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} y^2 + \left(\frac{U + V}{b} - \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} \cdot b\right) y - V = 0$$

Solving this quadratic equation, we have:

$$y = \frac{-\left[\frac{U + V}{b} - \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} \cdot b\right] \pm \sqrt{\left(\frac{U + V}{b} - \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} \cdot b\right)^2 + 4 \cdot \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} \cdot V}}{2 \times \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x}}$$

$$= \frac{-\left[\frac{U + V}{b} - \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} \cdot b\right] \pm \sqrt{\left(\frac{U + V}{b}\right)^2 + \frac{1}{4\mu^2}\left(\frac{\partial p}{\partial x}\right)^2 b^2 - \frac{(U - V)}{\mu} \cdot \frac{\partial p}{\partial x}}}{\frac{1}{\mu} \cdot \frac{\partial p}{\partial x}}$$

The above equation will yield two values of y, one which is +*ve* and less than b will be accepted and the other one rejected.

The discharge per unit width of plates is given by,

$$q = \int_0^b u \, dy$$

$$= \int_0^b \left[(U + V)\frac{y}{b} - \frac{1}{2\mu}\frac{\partial p}{\partial x}(by - y^2) - V\right] dy$$

$$= (U+V)\left[\frac{y^2}{2b}\bigg|\right]_0^b - \frac{1}{2\mu}\cdot\frac{\partial p}{\partial x}\left[b\cdot\frac{y^2}{2} - \frac{y^3}{3}\bigg|\right] - V\,[y]_0^b$$

$$= (U+V)\frac{b}{2} - \frac{1}{2\mu}\cdot\frac{\partial p}{\partial x}\left(\frac{b^3}{2} - \frac{b^3}{3}\right) - Vb$$

$$= (U-V)\frac{b}{2} - \frac{1}{12\mu}\frac{\partial p}{\partial x}b^3 \qquad ...(10.28)$$

The distribution of shear stress across any section may be determined by using Newton's law of viscosity. Thus,

$$\tau = \mu\cdot\frac{du}{dy}$$

$$= \mu\frac{d}{dy}\left[(U+V)\frac{y}{b} - \frac{1}{2\mu}\cdot\frac{\partial p}{\partial x}(by - y^2) - V\bigg|\right]$$

$$= \mu\left[\frac{U+V}{b} - \frac{1}{2\mu}\cdot\frac{\partial p}{\partial x}(b-2y)\bigg|\right]$$

$$= \mu\left(\frac{U+V}{b}\right) - \frac{1}{2}\frac{\partial p}{\partial x}(b-2y)$$

$$= (U+V)\frac{\mu}{b} - \frac{\partial p}{\partial x}\left(\frac{b}{2} - y\right) \qquad ...(10.29)$$

The distance y at which the shear stress will be zero is obtained by putting eqn. (10.29) to zero. Thus,

$$(U+V)\frac{\mu}{b} - \frac{\partial p}{\partial x}\left(\frac{b}{2} - y\right) = 0$$

or,
$$\frac{\partial p}{\partial x}\left(\frac{b}{2} - y\right) = (U+V)\frac{\mu}{b}$$

or,
$$\frac{b}{2} - y = \frac{(U+V)\frac{\mu}{b}}{\frac{\partial p}{\partial x}}$$

∴
$$y = \frac{b}{2} - \frac{\mu}{b}\left(\frac{U+V}{\partial p/\partial x}\right) \qquad ...(10.30)$$

Example 10.24. *Determine the direction and amount of flow per metre width between two parallel plates when one is moving relative to the other with a velocity of 3 m/s in the negative direction, if $\frac{\partial p}{\partial x} = -100\times10^6$ N/m^3 and $\mu = 0.4$ poise and distance between the plates is 1 mm.*

(MGU, Kerala)

Solution. *Given:* $U = -3$ m/s; $\frac{dp}{dx} = -100\times10^6$ N/m^3, $\mu = 0.4$ poise $= 0.4\times\frac{1}{10} = 0.04$ Ns/m^2; $b = 1$ mm $= 0.001$ m.

We know that,
$$q = U\cdot\frac{b}{2} - \frac{b^3}{12\mu}\cdot\frac{\partial p}{\partial x} \qquad \text{[Eqn. (10.22)]}$$

Substituting the values, we have:

$$q = -3\times\frac{0.001}{2} - \frac{0.001^3}{12\times0.04}\times(-100\times10^6) = 0.2068\ \text{m}^3/\text{s}$$

Hence, *amount of flow per metre width* = **0.2068 m^3/s. (Ans.)**

Positive direction (*i.e.* in the direction opposite to that of the moving plate). **(Ans.)**

Example 10.25. *Two parallel plates kept 100 mm apart have laminar flow of oil between them with a maximum velocity of 1.5 m/s. Calculate:*

(*i*) *The discharge per metre width,*

(*ii*) *The shear stress at the plates,*

(*iii*) *The difference in pressure between two points 20 m apart,*

(*iv*) *The velocity gradient at the plates, and*

(*v*) *The velocity at 20 mm from the plate.*

Assume viscosity of oil to be 24.5 poise.

Solution. Distance between the parallel plates, $b = 100 \text{ mm} = 0.1 \text{ m}$

Maximum velocity of the oil, $u_{max} = 1.5 \text{ m/s}$

Viscosity of the oil, $\mu = 24.5 \text{ poise} = 2.45 \text{ Ns/m}^2$

(*i*) The discharge per metre width, q:

In this case the average velocity of flow,

$$\bar{u} = \frac{2}{3} u_{max} = \frac{2}{3} \times 1.5 = 1.0 \text{ m/s}$$

$$\therefore \quad q = \bar{u} \times b = 1.0 \times 0.1 = \mathbf{0.1 \ m^3/s \ per \ m} \ \textbf{(Ans.)}$$

(*ii*) The shear stress at the plates τ_0 :

We know,
$$q = \frac{b^3}{12\mu}\left(-\frac{\partial p}{\partial x}\right) \quad \text{...[Eqn. (10.25)]}$$

Substituting the values, we have:

$$0.1 = \frac{0.1^3}{12 \times 2.45}\left(-\frac{\partial p}{\partial x}\right)$$

or,
$$\left(-\frac{\partial p}{\partial x}\right) = \frac{0.1 \times 12 \times 2.45}{0.1^3} = 2940 \text{ N/m}^2\text{/m}$$

The shear stress across any section is given by:

$$\tau = \frac{1}{2}\left(-\frac{\partial p}{\partial x}\right)(b - 2y) \quad \text{...[Eqn. (10.26)]}$$

The shear stress at the plates is obtained by putting $y = 0$ in the above equation. Thus,

$$\tau_0 = \frac{1}{2}\left(-\frac{\partial p}{\partial x}\right)b$$

$$= \frac{1}{2} \times 2940 \times 0.1 = \mathbf{147 \ N/m^2} \ \textbf{(Ans.)}$$

(*iii*) Pressure difference between two points 20 m apart:

We know,
$$-\frac{\partial p}{\partial x} = 2940$$

or,
$$-\partial p = 2940 \ \partial x$$

Integrating w.r.t. x, we get:

$$\int_{p_1}^{p_2}(-\partial p) = \int_{x_1}^{x_2} 2940 \ (\partial x)$$

or,
$$p_1 - p_2 = 2940 \ (x_2 - x_1)$$

$$= 2940 \times 20 = 58800 \text{ N/m}^2 \text{ or } \mathbf{58.8 \ kN/m^2} \ \textbf{(Ans.)}$$

(iv) The velocity gradient at the plates, $\left(\frac{\partial u}{\partial y}\right)_{y=0}$:

$$\tau_0 = \mu \cdot \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

or,
$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\tau_0}{\mu} = \frac{147}{2.45} = \mathbf{60\ s^{-1}}\ \textbf{(Ans.)}$$

(v) The velocity at 20 mm from the plate:

$$u = \frac{1}{2\mu}\cdot\left(\frac{-\partial p}{\partial x}\right)(by - y^2) \qquad \text{...[Eqn. (10.24)]}$$

$$= \frac{1}{2\times 2\cdot 45} \times 2940\ (0.1 \times 0.02 - 0.02^2)$$

$(\because y = 20 \text{ mm} = 0.02 \text{ m})$

$$= \mathbf{0.96\ m/s}\ \textbf{(Ans.)}$$

Example 10.26. *A liquid of viscosity of 0.9 poise is filled between two horizontal plates 10 mm apart. If the upper plate is moving at 1 m/s with respect to the lower plate which is stationary and the pressure difference between two sections 60 m apart is 60 kN/m², determine:*

(i) The velocity distribution,

(ii) The discharge per unit width, and

(iii) The shear stress on the upper plate.

Solution. Viscosity of the liquid,

$\mu = 0.9$ poise $= 0.09$ Ns/m²

Distance between the plates,

$b = 10$ mm $= 0.01$ m

Velocity of the upper plate, $U = 1$ m/s

Pressure difference between the sections 60 m apart = 60 kN/m²

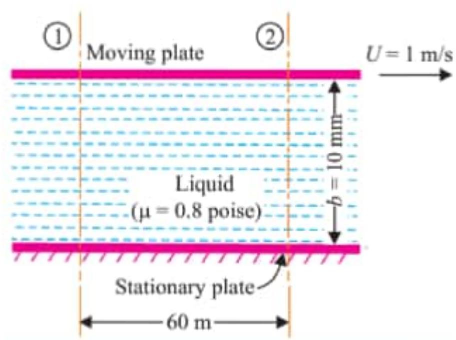

Fig. 10.23

$$\therefore \quad \left(-\frac{\partial p}{\partial x}\right) = \frac{60\times 10^3}{60} = 103\ \text{N/m}^2/\text{m}$$

(i) The velocity distribution:

The system corresponds to Couette flow for which the velocity distribution is given as:

$$u = \frac{U}{b}y + \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)(by - y^2) \qquad \text{...[Eqn. (10.21)]}$$

$$= \frac{1}{0.01}y + \frac{1}{2\times 0.09}\times 10^3(0.01\times y - y^2)$$

$$= y\,(100 + 55.55 - 5555.55\,y)$$

$$= y\,(155.55 - 5555.55\,y)$$

Hence, the *velocity distribution is*: $\boldsymbol{u = y\,(155.55 - 5555.55\,y)}$ **(Ans.)**

(ii) Discharge per unit width, q:

$$q = \int_0^b u\,dy$$

$$= \int_0^{0.01} (155.55\,y - 5555.55\,y^2)\,dy$$

$$= \left[155.55 \times \frac{y^2}{2} - 5555.55 \times \frac{y^3}{3}\right]_0^{0.01}$$

$$= \left[155.55 \times \frac{0.01^2}{2} - 5555.55 \times \frac{0.01^3}{3}\right]$$

$$= (0.007777 - 0.001852) = \mathbf{0.005925\ m^3/s}$$ **(Ans.)**

(iii) The shear stress on the upper plate, τ_0 :

Shear stress, $\tau = \mu \cdot \left(\frac{\partial u}{\partial y}\right) = \mu \frac{\partial}{\partial y}(155.55\,y - 5555.55\,y^2)$

$$= 0.09\,(155.55 - 11111.1\,y)$$

For the top plate, $y = 0.01$ m

$\therefore$ $\tau_0 = 0.09\,(155.55 - 11111.1 \times 0.01) = \mathbf{4\ N/m^2}$ **(Ans.)**

Example 10.27. *Fluid is in laminar motion between two parallel plates under the action of motion of one of the plates and also under the presence of a pressure gradient in such a way that the net forward discharge across any section is zero.*

(i) *Find out the point where minimum velocity occurs and its magnitude.*

(ii) *Draw the velocity distribution graph across any section.*

Solution. In the given case of flow, the velocity distribution is given by:

$$u = \frac{U}{b}y - \frac{1}{2\mu}\frac{\partial p}{\partial x}(by - y^2) \quad \text{...[Eqn. (10.21)]}$$

and, the discharge per unit width,

$$q = \frac{Ub}{2} - \frac{b^3}{12\mu}\cdot\frac{\partial p}{\partial x} \quad \text{...[Eqn. (10.22)]}$$

Net forward discharge,

$$q = 0 \quad \text{...(Given)}$$

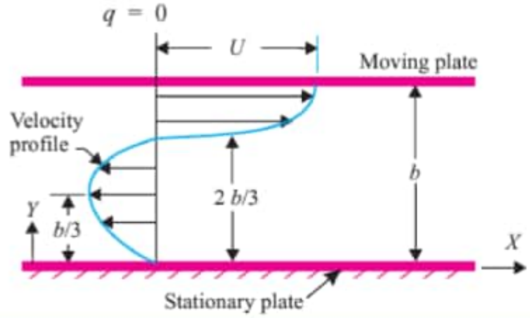

Fig. 10.24

$$\therefore \qquad 0 = \frac{Ub}{2} - \frac{b^3}{12\mu}\cdot\frac{\partial p}{\partial x}$$

or, $$\frac{\partial p}{\partial x} = \frac{Ub}{2}\times\frac{12\mu}{b^3} = \frac{6\mu U}{b^2}$$

Minimum velocity occurs where,

$$\frac{\partial u}{\partial y} = 0$$

Thus, $$\frac{\partial}{\partial y}\left[\frac{U}{b}\cdot y - \frac{1}{2\mu}\frac{\partial p}{\partial x}(by - y^2)\right] = 0$$

or, $$\frac{U}{b} - \frac{1}{2\mu}\frac{\partial p}{\partial x}(b - 2y) = 0$$

or, $$\frac{U}{b} = \frac{1}{2\mu}\frac{\partial p}{\partial x}(b - 2y)$$

or, $$(b - 2y) = \frac{(U/b)\times 2\mu}{(\partial p/\partial x)}$$

or, $$2y = b - \frac{(U/b)\times 2\mu}{(\partial p/\partial x)}$$

or, $$y = \frac{b}{2} - \frac{(U/b)\times\mu}{6\mu\, U/b^2} = \frac{b}{3} \qquad \left(\because \frac{\partial p}{\partial x} = \frac{6\mu U}{b^2}\right)$$

Hence, *minimum velocity occurs at a distance* $\boldsymbol{\frac{b}{3}}$ *from the fixed plate*. **(Ans.)**

The *magnitude of minimum velocity* is obtained by putting $y = \frac{b}{3}$ in the equation of velocity distribution. Thus,

$$u = \frac{U}{b}\cdot y - \frac{1}{2\mu}\frac{\partial p}{\partial x}(by - y^2)$$

or, $$u_{\min} = \frac{U}{b}\times\frac{b}{3} - \frac{1}{2\mu}\times\frac{6\mu U}{b^2}\left(b\times\frac{b}{3} - \frac{b^2}{9}\right)$$

$$= \frac{U}{3} - \frac{2U}{3} = -\frac{\boldsymbol{U}}{\boldsymbol{3}} \quad \textbf{(Ans.)}$$

(*ii*) Velocity distribution graph:

The velocity distribution graph may be drawn by substituting arbitrary values of y such as $0.1b$, $0.2b$, $0.3b$ etc. in the equation,

$$u = \frac{U}{b}y - \frac{3U}{b^2}(by - y^2),$$

and computing u in terms of U.

Also, when $u = 0, y = 0$ and $\frac{2}{3}b$

The velocity distribution graph is shown in Fig. 10.24.

Example 10.28. *Show that the discharge per unit width between two parallel plates distance b apart, when one plate is moving at velocity U while the other one is held stationary, for the condition of zero shear stress at the fixed plate is* $q = \frac{Ub}{3}$

Solution. The given case of flow corresponds to Couette flow for which the velocity distribution given by:

$$u = \frac{U}{b} \cdot y + \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)(by - y^2)$$

$\therefore$ Discharge per unit width,

$$q = \int_0^b u \cdot dy = \int_0^b \left[\frac{U}{b} \cdot y + \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)(by - y^2)\right] dy$$

$$= \left[\frac{U}{b} \cdot \frac{y^2}{2} + \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\left(b \cdot \frac{y^2}{2} - \frac{y^3}{3}\right)\right]_0^b$$

$$= \frac{U}{b} \cdot \frac{b^2}{2} + \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\left(\frac{b^3}{2} - \frac{b^3}{3}\right)$$

$$= \frac{Ub}{2} + \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\frac{b^3}{6} \quad \text{...(i)}$$

$$\text{Stress, } \tau = \mu \cdot \frac{\partial u}{\partial y}$$

$$= \mu \frac{d}{dy}\left[\frac{Uy}{b} + \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)(by - y^2)\right]$$

$$= \mu \left[\frac{U}{b} + \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)(b - 2y)\right]$$

But shear stress at the surface of fixed plate $(y = 0) = 0$ ... (*Given*)

$$\therefore \quad 0 = \mu\left[\frac{U}{b} + \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)b\right]$$

or,
$$\frac{U}{b} + \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)b = 0$$

or,
$$\frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right) = -\frac{U}{b^2}$$

Making substitution for this expression in (*i*), we get:

$$q = \frac{Ub}{2} - \frac{U}{b^2} \times \frac{b^3}{6} = \mathbf{\frac{Ub}{3}} \quad \text{...(Proved)}$$

Example 10.29. *Laminar flow of a fluid of viscosity 0.9 Ns/m² and specific gravity 1.26 occurs between a pair of parallel plates of extensive width, inclined at 45° to the horizontal, the plates being 10 mm apart. The upper plate moves with a velocity of 2.0 m/s relative to the lower plate and in a direction opposite to the fluid flow. Pressure gauges mounted at two points 1 m vertically apart on the upper plate record pressures of 250 kN/m² and 80 kN/m² respectively. Determine:*

(*i*) *The velocity and shear stress distribution between the plates,*
(*ii*) *The maximum flow velocity, and*
(*iii*) *The shear stress on the upper plate.* **[MU]**

Solution. Viscosity of the fluid, μ = 0.9 Ns/m²

Specific gravity of the fluid = 1.26

Distance between the plates, b = 10 mm = 0.01 m